Long term simulation of binary neutron star merger

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Abstract

General relativistic simulation for the merger of binary neutron stars is performed in a modified method, as an extension of a previous work [1]. We prepare binary neutron stars with a large initial orbital separation and employ the moving-puncture formulation, which enables to follow merger and ringdown phases for a long time, even after black hole formation. Three equal-mass binaries with each mass $1.4M_\odot, 1.45M_\odot, 1.5M_\odot$ and two unequal-mass binaries with mass 1.3–1.6$M_\odot$, 1.35–1.65$M_\odot$ are prepared. We focus primarily on the black hole formation case, and explore mass and spin of the black hole, mass of disks which surround the black hole, and gravitational waves emitted during the black hole formation.

1 Introduction

Coalescence of binary neutron stars is one of the most promising sources for kilometer-size laser interferometric detectors such as the LIGO, GEO,VIRGO, and TAMA. Latest statistical estimate indicates that detection rate of gravitational waves from binary neutron stars will be 1 event per 4–100 years for the first-generation interferometric detectors and 10–500 events per year for the advanced detectors. This suggests that gravitational waves from binary neutron stars will be detected within the next decade.

For theoretically studying the late inspiral, merger, and ringdown phases of the binary neutron stars, numerical relativity is the unique approach. Until quite recently, there has been no general relativistic simulation that quantitatively clarifies the inspiral and merger phases because of limitation of the computational resources or difficulty in simulating a black-hole spacetime, although a number of simulations have been done for a qualitative study. The most crucial drawbacks in the previous works were summarized as follows ; (i) the simulation were not able to be continued for a long time after formation of a black hole and/or (ii) the simulations were short-term for the inspiral phase; the inspiral motion of the binary neutron stars is followed only for $\sim$ 1–2 orbits.

In the present work, we perform an improved simulation overcoming these drawbacks; (i) we adopt the moving-puncture approach [2], which enables to evolve black hole spacetimes for an arbitrarily long time; (ii) we prepare binary neutron stars in quasicircular orbits of a large separation as the initial condition. In the chosen initial data, the binary neutron stars spend in the inspiral phase for 3–4 orbits before the onset of merger, and hence, approximately correct non-zero approaching velocity and nearly zero eccentricity will be results. We focus in particular on quantitatively clarifying formation process of a black hole for the case that it is formed promptly (i.e., in the dynamical time scale $\sim$ 1–2 ms) after the onset of merger. More specifically, the primary purpose of this paper is (1) to determine the final mass and spin of the black hole formed after the merger, (3) to clarify quantitative features of gravitational waves emitted in the merger and ringdown phases.

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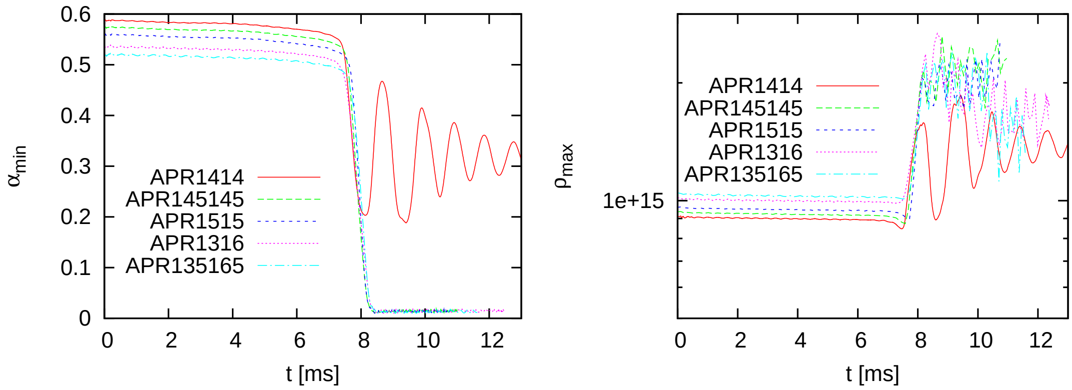

Figure 1: Evolution of the minimum value of the lapse function, $\alpha_{\rm min}$, and the maximum rest-mass density $\rho_{\rm max}$ for models APR1414, APR145145, APR1515, APR1316, and APR135165.

2 Summary of basic equation and numerical issue

For solving the Einstein evolution equation, we use the original version of the Baumgarte-Shapiro-Shibata-Nakamura formulation [3]: The numerical code for solving the Einstein equation and for the hydrodynamics is the same as that in Ref. [4].

3 Result

3.1 General feature for merger process

We have already performed the simulation for binary neutron stars with realistic EOSs [1]. Although the qualitative feature for the merger process found in the present work is the same as in the previous works, we here summarize generic feature of the merger again.

Figure 1 plots the evolution of the minimum lapse function, $\alpha_{\rm min}$, and maximum baryon rest-mass density, $\rho_{\rm max}$, for all the models studied in this paper. For models APR145145, APR1515, APR1316, and APR135165 for which a black hole is formed soon after the onset of the merger, $\alpha_{\rm min}$ ($\rho_{\rm max}$) decreases (increases) monotonically. For these runs, two neutron stars come into the first contact at $t \sim 7.5$ ms. For all the cases, apparent horizon is formed when $\alpha_{\rm min}$ reaches ~ 0.03. For model APR1414, $\alpha_{\rm min}$ ($\rho_{\rm max}$) steeply decreases (increases) after the onset of the merger, but then, they start oscillation and eventually settle down to relaxed values. This indicates that a hypermassive neutron star is the outcome.

In the unequal-mass case, less massive neutron star is tidally deformed ~ 1 orbit before the onset of merger. Then, mass shedding occurs, and as a result, the material in the less massive neutron star acrretes onto the massive companion. During the merger, it is highly tidally deformed, and thus, an efficient angular momentum transport occurs. Due to this, the material in the outer region of the less massive neutron star spreads outward to form a spiral arm. This process helps formation of accretion disks around the formed black hole.

3.2 Black hole mass and spin

As mentioned in the previous subsections, the final outcome after the merger for models APR145145, APR1515, APR1316, and APR135165 is a rotating black hole. We here determine the black-hole mass, $M_{\rm BH,f}$, and spin, $a_{\rm f}$, using the same method as those used in Refs. [1]. All the results for the black-hole mass and spin are summarized in Table 1. We find that $M_{\rm BH,f}$ and $C_e/4\pi$ agree within 0.2% error for all

Table 1: Black-hole mass $M_{\rm BH,f}$ and nondimensional spin parameter $a_{\rm f}$ for models APR145145, APR1515, APR1316, and APR135165. ΔE, ΔJ, $M_{r>r_{\rm AH}}$, $J_{r>{\rm AH}}$, C_p, C_e, and $f_{\rm QNM}$ denote energy and angular momentum carried by gravitational waves, rest mass and angular momentum of the material located outside apparent horizon, and polar and equatorial circumferential radii of the apparent horizon, respectively. $M_{r>r_{\rm AH}}$ and $J_{r>r_{\rm AH}}$, are given in units of M_0 and J_0, respectively.

Model	$\Delta E/M_0$	$\Delta J/J_0$	$M_{r>r_{\rm AH}}$	$J_{r>r_{\rm AH}}$	$M_{\rm BH,f}/M_0$	$C_e/4\pi M_0$	C_p/C_e	$a_{\rm f1}$	$a_{\rm f2}$
APR145145	1.15%	16.7%	–	–	0.988	0.9880	0.8628	0.77	0.77
APR1515	1.19%	16.9%	0.004%	–	0.988	0.9864	0.8576	0.79	0.78
APR1316	1.14%	16.8%	1.13%	2.69%	0.977	0.9768	0.8692	0.75	0.75
APR135165	1.10%	16.4%	0.26%	0.58%	0.986	0.9859	0.8625	0.77	0.77

the models. This indicates that both quantities at least approximately denote the black-hole mass and that we obtain the black-hole mass within $\sim 0.2\%$ error.

The spin parameters $a_{\rm f1}$ and $a_{\rm f2}$, which are determined from the aparent horizon area $\hat{A}_{\rm AH}$ and horizon shape C_p and C_e, respectively, agree within 2% error for all the models. We find that C_p/C_e and $\hat{A}_{\rm AH}$ depend weakly on the grid resolution, and so do $a_{\rm f1}$ and $a_{\rm f2}$. This suggests that the convergent value of the black-hole spin is close to $a_{\rm f1}$ and $a_{\rm f2}$, and for all the models, we estimate the black-hole spin as $a_{\rm f} \approx 0.77 \pm 0.03$.

In the merger of equal-mass, nonspinning binary black holes, $a_{\rm f}$ is ≈ 0.69, which is by about 0.1 smaller than that for the merger of binary neutron stars. This difference arises primarily from the magnitude of ΔJ in the final phase of coalescence. In the merger of binary black holes, a significant fraction of angular momentum is dissipated by gravitational radiation during the last inspiral, merger, and ringdown phases, because a highly nonaxisymmetric state is accompanied with a highly compact state from the last orbit due to the high compactness of black holes. By contrast, compactness of the neutron stars is by a factor of ~ 5 smaller, and as a result, such a highly nonaxisymmetric and compact state is not achieved for the binary neutron stars. Indeed, angular momentum loss rate by gravitational waves during the last phases is much smaller than that of binary black holes.

3.2.1 Merger and ringdown gravitational waves

The waveforms in the ringdown phase are primarily characterized by the fundamental quasinormal mode of the formed black holes for models APR145145, APR1515, APR1316, and APR135165. To clarify this fact, Fig. 2 plots Ψ_4 together with a fitting formula in the form

$$A{\rm e}^{-t/t_d}\sin(2\pi f_{\rm QNM}t+\delta) \tag{1}$$

where A and δ are constants, and the frequency and damping time scale are predicted by a linear perturbation analysis as

$$f_{\rm QNM} \approx 10.7\left(\frac{M_{\rm BH,f}}{3.0M_\odot}\right)^{-1}[1-0.63(1-a_{\rm f})^{0.3}]~{\rm kHz}, \qquad t_d \approx \frac{2(1-a_{\rm f})^{-0.45}}{\pi f_{\rm QNM}}. \tag{2}$$

Figure 2 (a) shows that gravitational waves for model APR1515 are well fitted by the hypothetical waveforms given by Eq. (1) for $t_{\rm ret} \gtrsim 8.4$ ms. [Here, we set $a_{\rm f} = a_{\rm f1}$ for the fitting (cf. Sec. 3.2)]. This is reasonable because the final outcome for model APR1515 is a stationary rotating black hole with negligible disk mass, and hence, the black-hole perturbation theory (i.e., Eq. (1)) should work well. This is also the case for the gravitational waveforms for model APR145145, for which the merger proceeds in essentially the same manner as that for model APR1515.

3.2.2 Fourier spectrum

We define Fourier power spectrum of gravitational waves by

$$h(f) \equiv \frac{D}{M_0}\sqrt{\frac{|h_+(f)|^2+|h_\times(f)|^2}{2}}, \qquad h_+(f)=\int e^{2i\pi ft}h_+(t)dt, \quad h_\times(f)=\int e^{2i\pi ft}h_\times(t)dt. \tag{3}$$

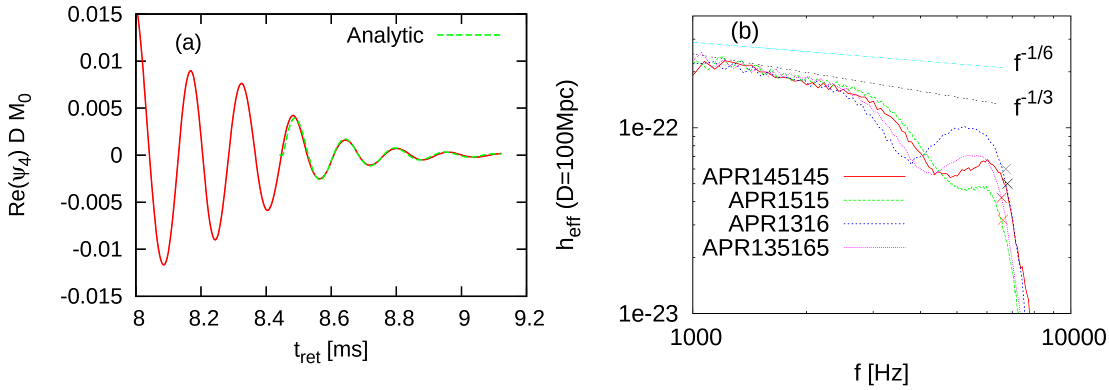

Figure 2: Ringdown waveforms associated primarily with the fundamental quasinormal mode for model APR1515. We plot the real part of Ψ_4. For the figure(a), the dashed curves denote the fitting curves calculated by Eq. (1). (b) Spectrum of gravitational waves for all the black hole formation models.

Here, h_+ and $h_\times$ denote the $+$ and $\times$ modes of gravitational waves of $l = |m| = 2$. From $h(f)$, we define a nondimensional spectrum (or effective amplitude) as $h_{\rm eff}(f) \equiv h(f) f M_0/D$.

In Fig. 2, we show the spectrum ($h_{\rm eff}$) of gravitational waves for models APR1414, APR1515, APR1316, and APR13516. To plot Fig. 2, we assume $D = 100$ Mpc. The Fourier spectrum shows universal features, irrespective of the total mass and mass ratio of the binary neutron stars. We quantitatively summarized as follows: (i) For $f \le f_{\rm cut} \approx 2.5$–3 kHz, the spectrum amplitude gradually decreases according to the relation $\propto f^{-n}$ where n is a slowly varying function of f; $n = 1/6$ for $f \to 0$, and $n \sim 1/3$ for $f \to f_{\rm cut}$. $f_{\rm cut}$ is much larger than the frequency at the ISCO. This is due to the fact that even after the onset of the merger, the merged object has two high-density peaks, and emits gravitational waves for which the waveform is similar to the inspiral one. (ii) For $f \ge f_{\rm cut}$, the spectrum amplitude steeply decreases. This seems to reflect the fact that at such frequency, two density peaks disappear duering the collapse to a black hole. (iii) For $f = f_{\rm peak} \approx 5$–6 kHz, which is slightly smaller than $f_{\rm QNM} \sim 6.7$–6.9 kHz, a broad peak appears. Because the frequency is always smaller than $f_{\rm QNM}$, this peak is not associated with the ringdown gravitational waveform but with the merger waveform. (iv) For $f > f_{\rm fin} \approx f_{\rm QNM}$, the amplitude damps in an exponential manner.

Although the features (i)–(iv) are qualitatively universal, the values of $f_{\rm cut}$, $f_{\rm peak}$, $f_{\rm QNM}$, and $f_{\rm fin}$, and the height and width of the peak at $f = f_{\rm peak}$ depend on the total mass and mass ratio of binary neutron stars, i.e., merger and black hole formation processes. This indicates that if we can detect gravitational waves of high frequency $f = 2$–8 kHz, we will be able to get information about merger and black hole formation precesses.

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