

Running soft parameters in SUSY models with multiple $U(1)$ gauge factors

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Abstract. The two-loop renormalization group equations for the parameters of softly broken supersymmetric gauge theories given in the literature are generalized to the case when the gauge group contains more than a single abelian factor. This contribution to the proceedings of the DISCRETE 2012 symposium is based on reference [1].

1. Introduction

The two-loop renormalization group equations (RGEs) of a generic softly broken supersymmetric (SUSY) model have been known for some time [2, 3]. However, these expressions are not completely general, as they do not handle all possible situations. For instance, if the gauge group contains one $U(1)$ factor, it is possible to form a Fayet-Illiopoulos term κD in the superpotential (the RGEs for the additional parameter κ were given in [4, 5, 6]). Another potential issue is the presence of Dirac gaugino mass terms $m_D^{iA} \psi_i \lambda_A$, if there are superfields in the adjoint

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representation of one of the gauge factor groups (see [7, 8, 9]). Yet another problem occurs when there are multiple $U(1)$ factor groups, since this usually leads to kinetic mixing between the gauge bosons of the different $U(1)$'s [10, 11]. A two-loop renormalization group analysis of non-SUSY theories with this feature is available in [12], while the supersymmetric case, which we shall be discussing, was addressed in [1].

There are many practical applications of these results. For instance, the $U(1)$ -mixing effects in SUSY grand unified theories (GUTs) featuring an extended intermediate $U(1)_R \times U(1)_{B-L}$ stage (see for example [13]) can shift the effective MSSM Bino soft mass by several per cent with respect to the naive estimate where such effects are neglected. In principle, this can have non-negligible effects for the low-energy phenomenology.

2. Gauge theories with multiple $U(1)$ factor groups

Consider a model with a $U(1)^n$ gauge group, containing m supermultiplets Φ_i , $i = 1, \dots, m$.² As usual, n gauge bosons A_μ^a must be introduced, which can be seen as components of a vector \mathbf{A}_μ . Under a gauge transformation with parameters α^a we have the following:

$$\Phi_i \rightarrow \exp(iQ_i^a \alpha^a) \Phi_i \equiv \exp(i\mathbf{Q}_i^T \boldsymbol{\alpha}) \Phi_i, \quad (1)$$

$$\mathbf{A}_\mu \rightarrow \mathbf{A}_\mu + \mathbf{G}^{-1} \partial_\mu \boldsymbol{\alpha}, \quad (2)$$

where \mathbf{Q}_i and $\boldsymbol{\alpha}$ are also vectors in $U(1)$ space, with components Q_i^a and α^a . In the last equation, a \mathbf{G} matrix shows up. In the spirit of making the most general gauge transformation, \mathbf{G} can be any real $n \times n$ matrix. This means in particular that the transformation of A_μ^a may depend on some gauge transformation parameter α^b with $b \neq a$. It is straightforward to see that the Lagrangian will be invariant under this transformation if the covariant derivative of the supermultiplet Φ_i takes the form

$$D_\mu \Phi_i = (\partial_\mu - i\mathbf{Q}_i^T \mathbf{G} \mathbf{A}_\mu) \Phi_i, \quad (3)$$

which supports the idea that \mathbf{G} is a $U(1)$ gauge couplings matrix. Notice that even though it is a square matrix in $U(1)$ space, its left and right indices contract with different vectors: on the left we have the vector with the hypercharges of Φ_i , while on the right there is the $U(1)$ gauge bosons' vector. To complete the picture, there is also to consider a generic gauge kinetic term³

$$-\frac{1}{4} \mathbf{F}_{\mu\nu}^T \boldsymbol{\xi} \mathbf{F}^{\mu\nu} \quad (4)$$

and also, in a softly broken supersymmetric theory, the $U(1)$ gaugino mass term

$$-\frac{1}{2} \boldsymbol{\lambda}^T \mathbf{M} \boldsymbol{\lambda} + \text{h.c.} \quad (5)$$

Here we have introduced additional vectors in $U(1)$ space, $\mathbf{F}_{\mu\nu}$ and $\boldsymbol{\lambda}$, with components $F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$ and λ^a (the gaugino field associated with $U(1)^a$). The new $n \times n$ symmetric matrices $\boldsymbol{\xi}$ and \mathbf{M} are parameters of the theory. The advantage of having $\boldsymbol{\xi} \neq \mathbb{1}$ is that the gauge coupling matrix can be made diagonal with a rotation of the gauge boson and gaugino fields. On the other hand, as far as the renormalization group analysis is concerned, it then becomes necessary to include the effect of $\boldsymbol{\xi}$ on the evolution of the other parameters, as well as to describe the evolution of $\boldsymbol{\xi}$ itself [12]. As such, we follow an alternative approach: $\boldsymbol{\xi} \neq \mathbb{1}$ means that the gauge boson fields (as well as the gaugino fields) are not canonically normalized, so they may

² If m is smaller than n , the $U(1)$'s can be rotated such that the fields are only changed under m of them.

³ The same mixing parameter $\boldsymbol{\xi}$ appears in the gaugino kinetic term, and in the $1/2 D^a D^a$ term as well.

be rotated and rescaled such that in the new basis $\xi = \mathbb{1}$. In this way, the $U(1)$ -mixing in the kinetic term is transferred to the matrix of gauge couplings \mathbf{G} and the gaugino mass matrix \mathbf{M} .

We note that even with $\xi = \mathbb{1}$, there is still some freedom to rotate the vectors and matrices \mathbf{Q}_i , \mathbf{A}_μ , λ , \mathbf{G} and \mathbf{M} living in $U(1)$ space. Indeed, in general we may perform two unphysical rotations, \mathcal{O}_1 and \mathcal{O}_2 , such that

$$\mathbf{Q}_i \rightarrow \mathcal{O}_1 \mathbf{Q}_i, \quad \mathbf{A}_\mu \rightarrow \mathcal{O}_2 \mathbf{A}_\mu, \quad \lambda \rightarrow \mathcal{O}_2 \lambda, \quad \mathbf{G} \rightarrow \mathcal{O}_1 \mathbf{G} \mathcal{O}_2^T, \quad \mathbf{M} \rightarrow \mathcal{O}_2 \mathbf{M} \mathcal{O}_2^T. \quad (6)$$

3. Inclusion of $U(1)$ -mixing effects in the running of the parameters

The RGEs must reflect the symmetries in equation (6), and this can be used to derive the structure of the RGEs, with $U(1)$ -mixing effects fully included. For instance, only those combinations of \mathbf{G} and $\gamma \propto \sum_i \mathbf{Q}_i \mathbf{Q}_i^T$ which transform as $(\dots) \rightarrow \mathcal{O}_1 (\dots) \mathcal{O}_2^T$ are allowed to enter the right-hand side of the renormalization group equation of \mathbf{G} . However, at one-loop level, there is only one such structure that can come up from a matter-field loop in the gauge propagator, namely $\mathbf{G} \mathbf{G}^T \gamma \mathbf{G}$, so one immediately concludes that

$$\beta_{\mathbf{G}}^{(\text{one-loop})} \propto \mathbf{G} \mathbf{G}^T \gamma \mathbf{G}. \quad (7)$$

The proportionality coefficient is trivially obtained by matching this to the single $U(1)$ case.

It might be tempting to think that without a gaugino mass matrix \mathbf{M} in non-SUSY theories, \mathbf{G} can be diagonalized thereby eliminating the $U(1)$ -mixing effects. This however is not the case, as radiative effects will reintroduce off-diagonalities in the gauge couplings matrix. In fact, already at the one-loop level, the anomalous dimension γ which controls the RGE of \mathbf{G} is in general a non-diagonal matrix in $U(1)$ space and, as such, \mathbf{G} cannot be kept diagonal at all energy scales. One exception, where $U(1)$ -mixing effects can be eliminated, is when all the relevant $U(1)$ gauge couplings originate from a common gauge factor and thus, barring threshold effects, all of them happen to be equal at a certain scale. In such a case $\mathbf{G} \propto \mathbb{1}$ at some scale, and both charges and gauge fields can be simultaneously rotated at the one-loop level so that no off-diagonalities appear in γ [14, 2]. This *rotated basis* method will only work in non-SUSY cases where only the gauge sector has to be taken into account. Also, because of the appearance of Yukawa and trilinear soft SUSY breaking couplings in the two-loop RGEs of \mathbf{G} , this approach ceases to be valid beyond the one-loop level.

4. The generalized two-loop β -functions

In [2] one can find the RGEs for models with a simple gauge group, and also a list of replacement rules which extends these results to models with a semi-simple gauge group (with one $U(1)$ at most). Therefore, in order to obtain the RGEs of softly broken SUSY gauge theories with multiple $U(1)$'s, we simply need to generalize these replacement rules by including $U(1)$ -mixing effects. The technique of using the symmetries in equation (6) can be exploited to this end in many situations. However, in a few cases even a detailed inspection of the underlying expressions does not allow an unambiguous identification of their generalized form. When this happens, a careful analysis of the structure of the contributing Feynman diagrams is necessary.

The gauge group is taken to be $G_A \times G_B \times \dots \times U(1)^n$, where the G_X 's are simple groups. We use uppercase indices for simple group-factors and lowercase indices for $U(1)$ groups. As mentioned above, the $U(1)$ sector should be treated as a whole and described in terms of a real $n \times n$ gauge-coupling matrix \mathbf{G} , a $n \times n$ symmetric soft-SUSY breaking gaugino mass matrix \mathbf{M} and column vectors of charges \mathbf{Q}_i for each chiral supermultiplet Φ_i . To eliminate the \mathcal{O}_1 rotation freedom in equation (6), we define $\mathbf{V}_i \equiv \mathbf{G}^T \mathbf{Q}_i$ for each i . Since this is the only combinations of the \mathbf{Q}_i and \mathbf{G} appearing in the Lagrangian, all the RGEs can be written in terms of \mathbf{V} 's and \mathbf{M} only. In this regard, the RGEs of \mathbf{G} are noteworthy because they require an isolated \mathbf{G} (this

would not happen if $d\mathbf{V}_i/dt$ was computed instead of $d\mathbf{G}/dt$). For additional clarifications on the notation used in the following expressions, see [1].

Depending on the group sector (abelian or simple), we get different RGEs for the gauge couplings and the gaugino masses. The parameters are then either the matrices \mathbf{G} , \mathbf{M} or the numbers g_A , M_A . For the abelian sector, it is necessary to perform the following replacements to the simple gauge group expressions in [2]:

$$C(G) \rightarrow 0, \quad (8)$$

$$g^3 S(R) \rightarrow \mathbf{G} \sum_p \mathbf{V}_p \mathbf{V}_p^T, \quad (9)$$

$$g^5 S(R) C(R) \rightarrow \sum_p \mathbf{G} \mathbf{V}_p \mathbf{V}_p^T \left[\sum_B g_B^2 C_B(p) + \mathbf{V}_p \mathbf{V}_p^T \right], \quad (10)$$

$$\frac{g^3 C(k)}{d(G)} \rightarrow \mathbf{G} \mathbf{V}_k \mathbf{V}_k^T, \quad (11)$$

$$2g^2 S(R) M \rightarrow \mathbf{M} \sum_p \mathbf{V}_p \mathbf{V}_p^T + \sum_p \mathbf{V}_p \mathbf{V}_p^T \mathbf{M}, \quad (12)$$

$$g^2 C(k) \rightarrow \mathbf{V}_k \mathbf{V}_k^T, \quad (13)$$

$$2g^2 C(k) M \rightarrow \mathbf{M} \mathbf{V}_k \mathbf{V}_k^T + \mathbf{V}_k \mathbf{V}_k^T \mathbf{M}, \quad (14)$$

$$16g^4 S(R) C(R) M \rightarrow \sum_p \left\{ 4 (\mathbf{M} \mathbf{V}_p \mathbf{V}_p^T + \mathbf{V}_p \mathbf{V}_p^T \mathbf{M}) \left[\sum_B g_B^2 C_B(p) + \mathbf{V}_p \mathbf{V}_p^T \right] + 8 \mathbf{V}_p \mathbf{V}_p^T \left[\sum_B M_B g_B^2 C_B(p) + \mathbf{V}_p^T \mathbf{M} \mathbf{V}_p \right] \right\}. \quad (15)$$

For a simple factor group G_A , the substitution rules of [2] do not need to be changed except for two cases:

$$g^5 S(R) C(R) \rightarrow g_A^3 S_A(R) \left[\sum_B g_B^2 C_B(R) + \mathbf{V}_R^T \mathbf{V}_R \right], \quad (16)$$

$$16g^4 S(R) C(R) M \rightarrow 8g_A^2 M_A S_A(R) \left[\sum_B g_B^2 C_B(R) + \mathbf{V}_R^T \mathbf{V}_R \right] + 8g_A^2 S_A(R) \left[\sum_B M_B g_B^2 C_B(R) + \mathbf{V}_R^T \mathbf{M} \mathbf{V}_R \right]. \quad (17)$$

As for the rest of the parameters in a SUSY model, the relevant substitution rules read:

$$g^2 C(r) \rightarrow \sum_A g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{V}_r, \quad (18)$$

$$M g^2 C(r) \rightarrow \sum_A M_A g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{M} \mathbf{V}_r, \quad (19)$$

$$M^* g^2 C(r) \rightarrow \sum_A M_A^* g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{M}^\dagger \mathbf{V}_r, \quad (20)$$

$$M M^* g^2 C(r) \rightarrow \sum_A M_A M_A^* g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{M} \mathbf{M}^\dagger \mathbf{V}_r, \quad (21)$$

$$g^4 C(r) S(R) \rightarrow \sum_A g_A^4 C_A(r) S_A(R) + \sum_p (\mathbf{V}_r^T \mathbf{V}_p)^2, \quad (22)$$

$$Mg^4C(r)S(R) \rightarrow \sum_A M_A g_A^4 C_A(r) S_A(R) + \sum_p (\mathbf{V}_r^T \mathbf{M} \mathbf{V}_p) (\mathbf{V}_r^T \mathbf{V}_p), \quad (23)$$

$$g^4 C^2(r) \rightarrow \left[\sum_A g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{V}_r \right]^2, \quad (24)$$

$$Mg^4 C^2(r) \rightarrow \left[\sum_A M_A g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{M} \mathbf{V}_r \right] \left[\sum_A g_A^2 C_A(r) + \mathbf{V}_r^T \mathbf{V}_r \right], \quad (25)$$

$$g^4 C(G)C(r) \rightarrow \sum_A g_A^4 C(G_A)C_A(r), \quad (26)$$

$$Mg^4 C(G)C(r) \rightarrow \sum_A M_A g_A^4 C(G_A)C_A(r), \quad (27)$$

$$MM^* g^4 C(G)C(r) \rightarrow \sum_A M_A M_A^* g_A^4 C(G_A)C_A(r), \quad (28)$$

$$g^2 \mathbf{t}_i^{Aj} \text{Tr}(\mathbf{t}^A m^2) \rightarrow \delta_i^j \sum_p \mathbf{V}_i^T \mathbf{V}_p (m^2)_p^p, \quad (29)$$

$$g^2 \mathbf{t}_i^{Aj} (\mathbf{t}^A m^2)_r^l \rightarrow \delta_i^j \mathbf{V}_i^T \mathbf{V}_l (m^2)_r^l, \quad (30)$$

$$g^4 \mathbf{t}_i^{Aj} \text{Tr}[\mathbf{t}^A C(r) m^2] \rightarrow \delta_i^j \sum_p \mathbf{V}_i^T \mathbf{V}_p \left[\sum_B g_B^2 C_B(p) + \mathbf{V}_p^T \mathbf{V}_p \right] (m^2)_p^p, \quad (31)$$

$$g^4 C(i) \text{Tr}[S(r) m^2] \rightarrow \sum_A g_A^4 C_A(i) \text{Tr}[S_A(r) m^2] + \sum_p (\mathbf{V}_i^T \mathbf{V}_p)^2 (m^2)_p^p, \quad (32)$$

$$\begin{aligned} 24g^4 MM^* C(i)S(R) &\rightarrow 24 \sum_A g_A^4 M_A M_A^* C_A(i) S_A(R) \\ &+ 8 \sum_p \left[(\mathbf{V}_i^T \mathbf{M} \mathbf{V}_p) (\mathbf{V}_i^T \mathbf{M}^\dagger \mathbf{V}_p) + (\mathbf{V}_i^T \mathbf{M} \mathbf{M}^\dagger \mathbf{V}_p) (\mathbf{V}_i^T \mathbf{V}_p) \right. \\ &\left. + (\mathbf{V}_i^T \mathbf{M}^\dagger \mathbf{M} \mathbf{V}_p) (\mathbf{V}_i^T \mathbf{V}_p) \right], \end{aligned} \quad (33)$$

$$\begin{aligned} 48g^4 MM^* C(r)^2 &\rightarrow \sum_{A,B} g_A^2 g_B^2 C_A(r) C_B(r) (32M_A M_A^* + 16M_A M_B^*) \\ &+ \sum_A g_A^2 C_A(r) \left(32M_A M_A^* \mathbf{V}_r^T \mathbf{V}_r + 16M_A \mathbf{V}_r^T \mathbf{M}^\dagger \mathbf{V}_r \right. \\ &+ 32\mathbf{V}_r^T \mathbf{M} \mathbf{M}^\dagger \mathbf{V}_r + 16M_A^* \mathbf{V}_r^T \mathbf{M} \mathbf{V}_r \left. \right) \\ &+ 32 \left(\mathbf{V}_r^T \mathbf{M} \mathbf{M}^\dagger \mathbf{V}_r \right) (\mathbf{V}_r^T \mathbf{V}_r) + 16 \left(\mathbf{V}_r^T \mathbf{M} \mathbf{V}_r \right) (\mathbf{V}_r^T \mathbf{M}^\dagger \mathbf{V}_r). \end{aligned} \quad (34)$$

5. Numerical results

In this section, we shall see the importance of the kinetic mixing effects with two examples. Consider first the SUSY $SO(10)$ model of [13] in which the unified gauge symmetry is broken down to the MSSM in three steps, namely, $SO(10) \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \rightarrow \text{MSSM}$; the corresponding breaking scales shall be denoted by m_G , m_R and m_{B-L} , respectively. For our purposes, it is crucial that in this model the ratio m_R/m_{B-L} can be as large as 10^{10} and, hence, the $U(1)$ -mixing effects become important. Figure (1) shows the importance of $U(1)$ -mixing effects in the one-loop running of the gauge coupling constants: with otherwise identical conditions, it shifts the low energy value of α_Y^{-1} by as much as 4%. In order to compensate for this, the intermediate scale m_R would have to

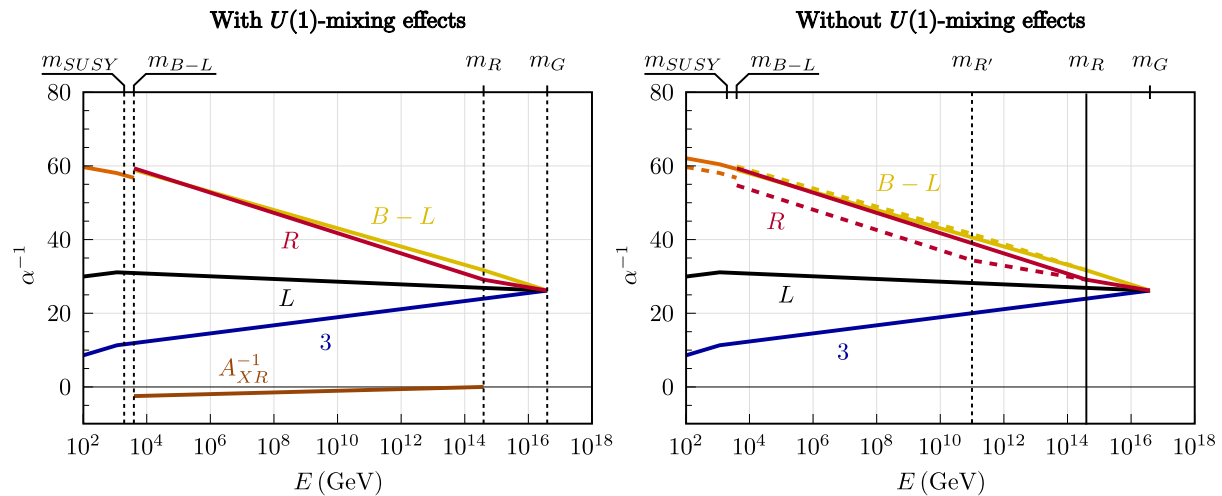


Figure 1. One-loop gauge-coupling evolution in the MRV model [13] with and without $U(1)$ -mixing effects. The position of the GUT scale, the unified gauge coupling and the intermediate symmetry-breaking scale m_R were chosen in such a way as to fit the electroweak data. Close to the $\alpha^{-1} = 0$ axis on the left plot, there is a brown line in the $[m_{B-L}, m_R]$ energy range depicting the evolution of the off-diagonal entries of the matrix $\mathbf{A}^{-1} \equiv 4\pi (\mathbf{G}\mathbf{G}^T)^{-1}$ which, at the one-loop level, scales linearly with $\log E$. Without the kinetic mixing effects taken into account, the low-energy value of α_Y^{-1} (orange solid line on the right) differs from the one obtained in the full calculation by as much as 4%. If one attempts to obtain the right value of $\alpha_Y^{-1}(m_Z)$ by adjusting the $SU(2)_R$ breaking scale, the new m'_R scale must be shifted with respect to the correct m_R by as much as 4 orders of magnitude (vertical solid and dashed lines on the right).

be lowered by as much as 4 orders of magnitude. We also note that the *rotated-basis* method mentioned previously is only partially successful in this model, because the g_R and g_{B-L} gauge couplings do not coincide at the m_R scale. In fact, the value of $\alpha_Y^{-1}(m_Z)$ obtained in this way is about 2% off from the correct one, which nevertheless is still better than assuming no mixing at all.

As for gaugino masses, it can be easily shown that the combination $\mathbf{G}\mathbf{M}^{-1}\mathbf{G}^T$ is a one-loop invariant of the RG flow, which generalizes the traditional one, $\alpha/M = \text{constant}$. This relation can be used to compute the Bino's mass at low energies,

$$M_Y(m_{SUSY}) = \frac{\alpha_Y(m_{SUSY})}{\alpha_G} p_Y^T \mathbf{M}_{1/2} p_Y, \quad (35)$$

where $\mathbf{M}_{1/2}$ is the GUT-scale gaugino soft mass matrix and $p_Y^T = (\sqrt{3/5}, \sqrt{2/5})$ is the vector describing the combination of $U(1)_R \times U(1)_{B-L}$ charges which constitutes the MSSM hypercharge. From equation (35) we see that the ratio $\alpha_Y(m_{SUSY})/M_Y(m_{SUSY})$ depends on whether or not we include the mixing effects, as was already noticed in reference [15]. Note that if $\mathbf{M}_{1/2}$ is not proportional to the unit matrix at the GUT scale, the $p_Y^T \mathbf{M}_{1/2} p_Y$ term will mix all entries of $\mathbf{M}_{1/2}$. Moreover, in the special case that the abelian gauge couplings unify, even the one-loop gaugino sector evolution can be fully accounted for by the *rotated-basis* trick.

Even in cases where the $U(1)$ gauge coupling unify at a certain scale, our two-loop formulas can produce relevant effects. We illustrate this by taking as an example the model presented in reference [16] where an intermediate $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge symmetry

is assumed to originate from a grand-unified framework. We consider two cases: (i) full gauge coupling unification at 2×10^{16} GeV and (ii) a small difference of 5% between the two $U(1)$ couplings caused by possible GUT-scale threshold effects. In the gaugino sector we assume universal boundary conditions in both cases, but the effect gets even stronger if one considers in addition threshold effects in the gaugino sector as well.

The results are contained in table (1). Interestingly, besides the expected equivalence of the *rotated-basis* method and the full-fledged calculation at the one-loop level, the relevant effective hypercharge gauge coupling turns out to be identical to the one obtained even at two-loop level if exact gauge coupling unification is assumed. The reason is that all additional states not present in the MSSM are charged only with respect to $U(1)_{B-L}$ but are neutral under the MSSM gauge group. In the gaugino sector the first deviations show up already in this case, which however are only at the per-mille level. If one also includes threshold corrections at the GUT-scale, then the effects are at the percent level, leading to shifts in the masses potentially measurable already at the LHC. Lastly, it should be kept in mind that the effects would be even larger if the $U(1)_Y$ would result from the breaking of $U(1)_R \times U(1)_{B-L}$ as discussed in the previous example.

Table 1. Low energy values of the entries of the gauge coupling and gaugino mass matrices (g_{ab} and M_{ab} with $a, b = Y, BL$) and the properly fitted MSSM parameters (g_Y, M_Y). All gaugino masses are in GeV. We have set the GUT scale at 2×10^{16} GeV with $g_G = 0.72$, and imposed an mSUGRA boundary condition taking $M_{1/2} = 1 \times 500$ GeV. At the one-loop level, we compare the case with no kinetic mixing effects included, the *rotated basis*, and the full-fledged calculation. At the two-loop level, we include the case where g_Y and g_{BL} are split at the GUT scale due to threshold corrections.

	One-loop results			Two-loop results			
	No kinetic mixing	Rotated basis method	Complete RGEs	No kinetic mixing	Complete RGEs	No kinetic mixing	Complete RGEs
g_{YY}	0.4511	0.4700	0.4700	0.4487	0.4677	0.4487	0.4686
g_{BLBL}	0.4083	0.4243	0.4243	0.4070	0.4231	0.4131	0.4298
g_{BLY}, g_{YBL}	0.0	-0.0723	-0.0723	0.0	-0.0725	0.0	-0.0725
g_Y	0.4511	0.4511	0.4511	0.4487	0.4487	0.4487	0.4500
M_{YY}	196.34	218.13	218.13	185.82	207.96	185.80	208.71
M_{BLBL}	160.83	178.67	178.67	154.88	173.19	144.26	161.97
M_{BLY}, M_{YBL}	0.0	-62.39	-62.39	0.0	-63.10	0.0	-62.15
M_Y	196.34	196.34	196.34	185.82	185.96	185.80	187.04
Exact unification						$g_{BL}^{\text{GUT}} = 1.05 g_Y^{\text{GUT}}$	

6. Conclusions

We have derived the renormalization group equations of softly broken supersymmetric models with more than a single abelian gauge factor group. In such models there are $U(1)$ -mixing effects which must be taken into consideration. Although formally the evolution equations available in the literature do not exhibit any obvious pathologies if such subtleties are not taken into account, the calculations based on these formulas are in general incomplete and, thus, the results are internally inconsistent. In spite of this, the issue of $U(1)$ -mixing in softly broken supersymmetric gauge theories had never been addressed in full generality, even at one loop.

The effect of the derived equations has been illustrated for two cases: at the one-loop level, for a model with different gauge coupling strengths due to a breaking of the original simple group at a high scale; and at the two-loop level, in a model where gauge coupling unification occurs but

only with threshold corrections taken into consideration. In both case we obtain effects in the percent range which none of the previously proposed partial treatments can fully account for.

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