Theory of Bipolar Outflows

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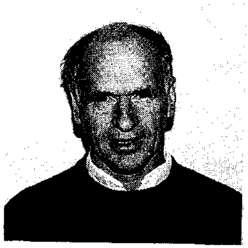

Abstract

I discuss briefly but completely the implications of time dependent accretion for the history of bipolar outflows in section 1. It is shown that the standard Shu estimate is good initially, but that either a 'bang' or 'whimper' mode may develop subsequently. In the case of a whimper mode the current outflow luminosity may not reflect the current infra-red luminosity. Section 2 discusses various outflow driving mechanisms. A general argument is given to explain observed correlations in terms of radiative driving. Recent wind and magneto-disc models are briefly summarized and criticized. Finally in section 3 a new type of outflow model is introduced in its 'toy', analytic form.

1. What Goes Up Must Have Come Down Inflow, Outflow and Luminosity

In this section I wish to review certain subtleties of the accretion phase which may not be sufficiently well known. Such a study may help us to answer questions of the type: Is inflow/outflow occurring simultaneously? Is the bipolar phenomenon part of the actual star building process or is it rather an early secondary phase? Has the initial cloud been used up? If not, then out to what radius has it been disturbed? What are the possible histories of the luminosity due to accretion?

Now there is a commonly used estimate of the spherically symmetric accretion rate in an isothermal cloud due to Shu[1),2)] . But such a single value can only either be some average estimate or can only apply at some special epoch in what is more generally a time dependent flow . Let us first examine under what conditions this is a valid order of magnitude estimate.

If a is the isothermal sound speed in a spherically symmetric self- gravitating medium, and if the mass inside a sphere of interest is not changing rapidly in a dynamical time, then the accretion zone extends to the Bondi-Parker type critical point at the sonic or 'virial' radius r_s, where $r_s = Gm_s/(2a^2)$, and m_s refers to mass enclosed by the sonic radius[3)] . Since at this point the radial velocity is the sound speed (that is the radius moves outward relative to the material with the speed a), we see that an estimate of the accretion rate per unit solid angle *at this stage* is given by $A \approx r_s^2\rho_s a$. Here ρ_s is the density at the sonic point. If in addition we suppose that in order of magnitude $\rho_s \approx 3m_s/(4\pi r_s^3)$ (see below for a justification), then simple manipulation gives $A \approx (3/2\pi)\left(\frac{a^3}{G}\right)$, and $m_s^2 \approx (6/\pi)\left(\frac{a^6}{G^3\rho_s}\right)$. The latter expression gives essentially the "Bonnor-Ebert" mass [31)], which is the maximum mass of a spherical, isothermal, self- gravitating, cloud in hydrostatic equilibrium. Our estimate is in fact slightly larger by a numerical factor ≈ 1.17, and estimates the minimum mass required for transonic, steady inflow. For an observed density it may give the best estimate of the mass in the protostellar condensation in terms of a measured density.

I now turn briefly to an exact solution, which permits one to see when the Shu estimate needs to be generalized. We examine self-gravitating, zero temperature accretion of a cloud that has at $t = 0$ a power law density distribution[4)]. One can only expect this solution to be applicable out to an initial radius $r = r_s$ where we will assume that the collapsing region detaches from its surroundings. We shall therefore use the virial radius as our fiducial scale r_o, which is arbitrary without such physical considerations. In addition, for simplicity, we study here only the case of virialized or zero energy radial shells .

One measures time in units of $(2Gm_s/r_s^3)^{-1/2}$, space in units of r_s, hence velocity in units of $\sqrt{2Gm_s/r_s}$, and density in units of $m_s/(4\pi r_s^3)$. Throughout, m_s is the mass initially inside r_s and is the unit of mass. Then the initial density and mass distributions are $\rho_o(r) = (3-2/D)r^{-(2/D)}$, and $m_o(r) = r^{(3-2/D)}$.

The location of a shell initially at r namely $R(r,t)$ is found from the formulae[5),6)] $R = rS(\xi)$, where $S(\xi) = (1-3\xi/2)^{2/3}$, and $\xi = t/r^{(1/D)}$.

In these formulae D is any real number $\geq 2/3$ or < 0 so that they describe a considerable range of power law initial states of zero temperature. We observe that every shell finds itself at the centre when $\xi = 2/3$, so that the time to enter the singularity is simply $(2/3)r^{(1/D)}$ for the shell initially at r.

It is frequently necessary to invert $R(r,t)$ for $r(R,t)$, which one achieves by writing the equations for R, S and ξ together as

$$x^3 + (3/2)\varsigma x^{(2/D)} - 1 = 0, \tag{1}$$

where $x^2 \equiv S = R/r$, and $\varsigma \equiv t/R^{(1/D)} = \xi/x^{(2/D)}$. Fortunately this is readily solved for interesting choices of D. Note that ξ and ς are two equally good choices of self-similar variables since $\varsigma = \varsigma(\xi)$ by (1) and the definitions.

The radial velocity is given as a function of R, t, by the formula

$$U(R,t) = -\frac{R^{(1-1/D)}}{x^{(3-2/D)}}, \tag{2}$$

and the density by the expression (remembering that $r \equiv R/x^2$)

$$\rho(R,t) = \left(\frac{\rho_o(R/x^2)}{x^3(x^2(1-2/(3D)) + 2/(3D))}\right). \tag{3}$$

The most important result from this treatment then follows as an expression for the accretion rate per unit solid angle $A(R,t) \equiv \dot{m}/4\pi$ in units of $r_s^2(m_s/(4\pi r_s^3)\sqrt{2Gm_s/r_s} \equiv a^3/(\pi G)$ as

$$A(R,t) = \left(\frac{R^{(3-1/D)}\rho_o(R/x^2)}{x^{(6-2/D)}(x^2(1-2/(3D)) + 2/(3D))}\right). \tag{4}$$

The first example is to choose $D = 1$ which corresponds to the 'isothermal' density law[7),8)] and makes $\varsigma = t/R$. Equation (1) is now a cubic equation and the one positive real root is; for $\varsigma^3 \leq 2$ $x = -\varsigma/2 + \varsigma^2/(2\Delta) + \Delta/2$, where $\Delta \equiv (4 - \varsigma^3 + \sqrt{8(2-\varsigma^3)})^{(1/3)}$, and for $\varsigma^3 > 2$ $x = -\varsigma/2 + \varsigma\cos(\phi/3)$, where $\cos\phi \equiv (4-\varsigma^3)/\varsigma^3$. We observe that $x = 1$ at $\varsigma = 0$ and that $x \to 0$ as $\varsigma \to \infty$ as is required to maintain $x^2\varsigma = \xi \leq 2/3$.

The striking behaviour in this case is that (see equation (4)) A varies from 1 when $x = 1$ to 3/2 as $x \to 0$, according to $A(R,t) = (3/2)/(x^2/2+1)$. One sees that the accretion flow is strictly self-similar in this case with the same spatial profile at all times and the same temporal variability at each point in space. On remembering the units of A and taking $r_o \equiv r_s$, this gives the asymptotic accretion rate *at all R* as $(3/2\pi)a^3/G$, precisely as in our estimate of the Shu mode above.

Equation (3) reveals that the density is asymptotically $\propto 1/(t^{1/2}R^{3/2})$, so that this region has the free-fall density profile, as have all cases in fact. The outer density variation continues however to reflect the transition from the initial law to the free-fall profile, for which equation (1) is needed.

However this behaviour is *critically* dependent on the initial density power law D. For example if $D = 3/4$ then equation (1) is no longer analytic, but $x^{8/3}\varsigma \leq 2/3$ plus this equation suffices to show that $x \to (3\varsigma/2)^{-3/8}$ as $\varsigma \to \infty$. Equation (4) yields $A(R,t) = (3x)^2/(x^2+8)R^{-1}$ and so (note that now $\varsigma = t/R^{4/3}$) one finds asymptotically that $A \to (9/8)x^2/R$ or $\propto t^{-3/4}$ for all R. The density is asymptotically $\propto 1/(t^{7/8}R^{3/2})$. I refer to this mode that occurs for $2/3 < D < 1$ as a 'whimper' mode since the accretion inevitably declines in time, although it increases with decreasing R. In this sense it is a true 'inside out' mode and is not strictly self-similar. This applies mostly at small R for physical reasons since the outer boundary falls in at $t = 2/3$ and abruptly ends the accretion. The mode with $D = 1$, appropriately referred to as the 'Shu' or 'stationary' mode, will end also with the falling in of the initial outer boundary, assuming a discontinuous density decrease beyond this surface.

The cases with $D > 1$ furnish us with another mode of considerable interest. We choose $D = 2$ so that we accrete an intially virialized zero temperature cloud whose density is proportional to r^{-1}. Equation (1) is now a cubic in standard form and gives $x = -\varsigma/\Delta' + \Delta'/2$, where $\Delta' \equiv (4+2\sqrt{4+2\varsigma^3})^{1/3}$ for the real root, and we note that $\varsigma = t/R^{(1/2)}$. Equation (4) now shows that $A(R,t) = 3/(x^3(x^2+1/2))\,R^{3/2}$, which is not strictly self- similar. However in the asymptotic limit $x\varsigma \to 2/3$ suffices to show with equations (3) and (4) that $\rho \to 9t/R^{(3/2)}$ and $A \to (81/4)t^3$! Thus this initial distribution yields a 'bang' mode. In fact one sees from the form of $A(R,t)$ that initially ($x \leq 1$) the mass flow decreases towards the center ('outside in' mode in this sense), while at every R it ultimately increases without limit as $t \to \infty$. But this latter phase is probably terminated by the infall of the initial surface r_s.

The apparently zero density special case when $D = 2/3$ corresponds in fact to accretion of nonself- gravitating virialized matter onto a central point mass m_s. The above formulae apply, except that the expression for the initial self- gravitating density must be replaced by an equation for the initial test particle density of the same form, but wherein $(3-2/D)$ is replaced by an arbitrary constant λ, and the power law dependence also becomes arbitrary in the form $r^{-(2/\delta)}$. One then finds that the bang and whimper modes exist respectively for $\delta = 2$ and $\delta = 1$ respectively, although as emphasized below the non self-gravitating phase may generally be considered a whimper mode relative to the self-gravitating phase since λ will be much smaller than one. Thus the different time dependences in this mode are perhaps best named as 'whimper-bang' and 'whimper-whimper' modes.

In essence the arguments of this section manage to extend the discussion of isothermal accretion to more general initial mass distributions, by using this device of neglecting the pressure in the inner collapsing core region. This region has presumably become unstable due to the outward diffusion of magnetic flux, heat and angular momentum[2),9)] . But what have these simple models got to dc with the questions raised at the beginning of this section? Most directly we see that *the protostellar accretion should be strongly time dependent* in general. This means that in the whimper mode for example, an earlier phase of rapid accretion might have launched an energetic bipolar outflow, but that the currently observed low IR luminosity might reflect rather the tail of the accretion phase.

If this latter idea were to be correct it would imply that a bipolar phenomenon is already launched in the protostellar collapse. The characteristic time for this phase is 2/3 or $Gm_s/(6a^3)$ in regular units ($r_o = r_s$) or $1.6 \times 10^5 \times m_s T_{10}^{-(3/2)}$ years, where in this paragraph the core mass is in units of $2m_\odot$ and a mean molecular mass of 2 amu is used. The outer cloud temperature is given in units of $10K$. This time is near the upper end of the range of dynamical life-times of bipolar outflows so that they may indeed be sensitive to variations in the protostellar flow . Note also that in this period the cloud has been disturbed out to a radius of r_s, which is $0.1 \times m_s/T_{10}$ pc. Interestingly enough, for plausible ranges of core mass and cloud temperature, this scale encompasses that of the observed bipolar outflows.

To illustrate the possible luminosity history of a proto-stellar object (PSO) that is first condensing under the influence of its own self-gravity, and subsequently becomes a young stellar object (YSO) that accretes essentially massless surrounding material, let us follow in detail a plausible special case.

There is a very early proto-stellar phase during which the cold cloud is contracting but has not yet formed a substantial opaque core. There will be nevertheless a thermal luminosity due to the efficient radiation of the work done by the force of gravity on the collapsing cloud. Assuming that the gas remains isothermal throughout, it is easy to calculate this luminosity thermodynamically as

$$L_{th} = -a^2 \int \rho \nabla.\mathbf{v}\, dV, \tag{6}$$

or in our standard units for the case $D = 1$ as

$$L_{th} = 9\frac{a^5}{G} \ln \frac{r_s}{R_o}.$$

Here $R_o(t)$ is an inner cutoff radius that coincides with the small but growing opaque core of the cloud. If one supposes that $a \approx 0.2$ $km\ s^{-1}$ as below then this luminosity is only $\approx 2 \times 10^{-3} L_\odot$ if $R_o/R_s = 10^{-8}$, and it decreases slightly as the opaque core grows. It increases rapidly with the ambient temperature however and it will be somewhat larger for the flatter density profiles, as may be calculated from the formulae above.

During the later proto-stellar phase the thermal luminosity due to the isothermal compression is generally dominated by the accretion shock luminosity released at the radius of the opaque core R_o, where $\tau \equiv \kappa\rho(R_o,t)R_o \approx 1$. This may be estimated as

$$L_{sh} = (4\pi f)Gm_o/R_o \times A_o(R_o,t),$$

which becomes, on expressing the various quantities in terms of our standard units,

$$L_{sh} = (4\pi f)(2a^5/\pi G)\frac{m_o(t)}{R_o(t)}A_o(R_o,t). \tag{5}$$

We will take the efficiency factor $4\pi f$ as essentially unity in this illustration to allow for a net accretion efficiency of $\approx 10\%$ due to geometrical effects (see e.g. section 3 and references therein). The dependence of m_o on time follows from $\dot{m}_o = (4\pi f)A_o$ with A_o given by equation (4) above and thus, besides the efficiency, only the time dependence of the opaque core is not calculated consistently in this estimate.

Now if we suppose that the PSO is in the Shu isothermal sphere mode, then $D = 1$ and at small radii $A_o \approx 3/2$ by equation (4) so that $m_o \approx (3/2)t$, whence by (5)

$$L_{sh} \approx \left(\frac{9}{2\pi}\right)\frac{a^5}{G}\frac{t}{R_o}.$$

This becomes on putting $R_o = 10^{-6}$ (essentially several times the eventual stellar dimension), and $a \approx 0.2\ km\ s^{-1}$

$$L_{sh} \approx 6.9 \times 10^{34} t\ ergs\ s^{-1},$$

that is of order 10 solar luminosities when the surface initially at r_s falls in $(t = 2/3)$.

On the other hand if the protostar is still primarily a virialized turbulent cloud[32] so that the initial density profile is with $D = 2$, then the same considerations as above yield

$$L_{sh} \approx 1.02 \times 10^2 \frac{t^4}{R_o}\left(\frac{2a^5}{\pi G}\right) \approx 3.2 \times 10^{30}\frac{t^4}{R_o}\ ergs\ s^{-1}.$$

Thus choosing once again $R_o \approx 10^{-6}$ and $t = 2/3$, one finds $L_{sh} \approx 160 L_\odot$. The onset of the bright period is however rather abrupt compared to the isothermal case above, being a real bang mode near the end of the protostellar phase.

If as above the initial cloud boundary is taken to be rather close to r_s, then it is reasonable to assume that there is a discontinuous decrease in the density beyond this radius and that the mass already accreted in the protostellar phase now dominates that in the surroundings. In this case we may continue to follow the luminous history of what is now a YSO by switching to the appropriate formulae for the accretion of test particle matter with an initial density $\rho_o(r) = (4\pi r_s^2 \rho_{os}/m_s) r^{-2/\delta} \equiv \lambda r^{-2/\delta}$ and $D = 2/3$. Supposing further that $\delta = 1$ gives by equations (4) and (5) with $m_o = 1$

$$L_{sh} = (4\pi f)\lambda R_o^{1/2} \left(1 + \frac{3t}{2R_o^{3/2}}\right)^{1/3} \times \frac{2a^5}{\pi G}.$$

This result shows clearly that in comparison with the protostellar luminosities above this continuing accretion is a 'whimper-bang' mode since the 'whimper factor' $\lambda = 4\pi r_s^2 \rho_{os}/m_s$ should be small when the self-gravity of the surroundings is negligible, and nevertheless the luminosity increases slightly with time. This 'after birth' phase might continue until the supply of surrounding material is exhausted; but since the accreting material originates beyond r_s and hence beyond the direct gravitational domination of the star, the luminosity is rather sensitive to fluctuations of the density, specific angular momentum, and comoving magnetic flux in the surroundings. Such perturbations may provoke the anisotropic instabilities such as discussed in section 3.

All of this discussion ignores in fact the inevitable anisotropy that must be present if inflow and outflow are to coexist. Such anisotropy is likely to come from the increasing effects of angular momentum in the outer core-cloud material, and ultimately perhaps also the magnetic field. The effect of angular momentum added to an initial Bondi flow has been studied numerically[10),11)] . The results show core accretion and the formation of *thick* discs with maximum scales of order r_s, which are maintained dynamically in a sort o f convective circulation of the cloud material. The eventual accretion of such 'storage discs' is likely to be the second, longer lived phase of young stellar activity. Section three discusses the existence of a sort of 'bipolar instability' provoked by the same effects.

2. Outflow Mechanisms

(i) Radiation Driving

The radiation force per unit mass in an opaque spherically symmetric medium driven by a central luminosity L_* is $\kappa L_*/(4\pi r^2 c)$, where κ is the Rosseland mean opacity. The luminosity may be taken to be constant in a steady state. We equate this specific force to the acceleration in a radially accelerating outflow $v(dv/dr)$, and then integrate this equation times the constant mass flux $4\pi r^2 \rho v$ from zero to a radial optical depth τ to obtain the radial component of momentum flux imparted to the layer of optical thickness τ as $F_r = \frac{\tau L_*}{c}$. Taking into account that the photons will not all be moving purely radially in an opaque medium, reduces this estimate by about a factor of 3. This is of course the basic way in which the envelope of a star supports itself.

All of this is fine, but the essential question is rather how quickly are the photons diffused in frequency into gaping spectral holes? If for example the initial photons are absorbed by cool dust grains after very few scatterings or high temperature absorptions and reemitted to escape directly in the IR (the Rosseland mean is dominated by the most transparent spectral regions), then the mechanism is defeated . Thus we infer that such a momentum flux must be established in a region where the 'scattering' plus thermal absoption opacity is greater than the dust opacity by the requisite factor τ . Such a region is likely to be close to the star, depending among other things on the radius of grain formation. *It may in the limit be the source of the high velocity wind that is frequently invoked* to explain the outflows.

The requirements on the optical depth are most severe for the low luminosity sources where factors of 10^4 or so are necessary[12]. This can only be true rather close to the 'stellar' photosphere. It may be rather that we have to invoke a 'whimper' accretion mode wherein the current outflows were driven by an earlier more vigorous luminosity and larger opacity. If so then in figure 1[12] , the low luminosity sources (say $L_* < 10^{2.5} L_\odot$) have to be shifted horizontally by some step (roughly constant?) to meet the true radiation driven line presumably delineated by the high luminosity sources.

But what is this 'radiation driven line'? If we could deduce it a priori, then the shift introduced above might provide a direct measure of the accretion age according to the theory of section 1. It turns out that the slope at least may be simply calculable.

Much can be learned simply by placing a point source of constant luminosity L_* inside an opaque (at least in the inne r regions we assume LTE) self-gravitating medium. The angular dependence of the flow and such questions as the asymptotic velocity will be discussed elsewhere, but it is instructive to carry out a dimensional analysis. From L_*, G, and the radiation constants one can construct more or less uniquely:

A characteristic mass[13]; $m_o \equiv \sqrt{3k^4/(a\mu^4 G^3)}$, which is almost exactly $1 m_\odot$ for a mean molecular mass μ of 2 amu (this might more appropriately be 1 in this phase), and which amusingly is also expressible as $\sqrt{45/\pi^2}\, m^3_{planck}/\mu^2$. It is really a kind of self-gravitating 'Eddington' mass, given L_* and the diffusion limit.

A characteristic scale; $r_o^2 \equiv (3k^4/a\mu^4 G^{9/5}) L_*^{-4/5}$, which is numerically $r_o = 3.7 \times 10^{15} (L_*/L_\odot)^{-2/5}$ cm and is probably an estimate of the scale over which the outflow attains its characteristic velocity.

A characteristic velocity; $V_o \equiv (GL_*)^{(1/5)}$, which is numerically $2(L_*/L_\odot)^{1/5}$ km s^{-1}. This must be distinguished from the maximum velocity in the outflow which must be calculated asymptotically.

A characteristic density follows as $\rho_o \equiv m_o/r_o^3$, or numerically $4 \times 10^{-14} (L_*/L_\odot)^{6/5}$ gm cm^{-3}.

These scales allow us to calculate the characteristic momentum flux in the flow $r_o^2 \rho_o V_o^2$ as $\propto L_*^{4/5}$, the characteristic mass flux $r_o^2 \rho_o V_o$ as $\propto L_*^{3/5}$, and of course the characteristic energy flux $r_o^2 \rho_o V_o^3$ as $\propto L_*$.

These last proportionalities are then a relatively general estimate of the slopes of 'radiation driving' lines in the three planes of interest. The observational measures are a slope of 0.6 in the mass flux- luminosity plane[14], which is as expected (ignoring error!), and more recently[12] 0.69 $\pm$ .05 in the momentum flux - luminosity plane, and 0.8 $\pm$ .06 in the energy flux-luminosity plane. These latter two measures are rather flatter than expected above. However in each case an interpretation of the data wherein the sources with luminosity greater than $\approx 10^{2.5}L_{\odot}$ define the radiation driving lines with slopes close to the predicted values, while the low luminosity sources define a second line with the same slope but shifted horizontally to higher luminosities, is possible. It is particularly compelling in the energy flux luminosity plane[12].

I conclude then that although the case is not proven, since no working detailed models yet exist, there is some support for radiation driving.

(ii) Stellar Winds

Snow plough or thin shell models[15],[16] have led to a clear discussion of the wind-ambient medium interface in terms of energy-driven or momentum-driven outflows. This picture has been tested against the observed 6cm radio emission and the observed correlations between L_*, and the two mechanical fluxes in the CO outflows[12]. It is concluded that all constraints are met by fast ($500 - 1000 km\ s^{-1}$) mostly ionized winds whether the ultimate interaction be energy or momentum driven, and by low velocity ($300 km\ s^{-1}$) mostly neutral, momentum driven winds that are subsequently shock ionized. The energy driven case is the most efficient relative to the observed bolometric luminosity in the IR. However it requires delayed cooling, which seems problematical in a dense expanding wind. A similar problem is posed by the remark[12] that ionizing photons at the Balmer limit lead to predicted 6 cm emission that is close to that observed. This suggests that high temperatures must be maintained in the wind region so as to populate the Balmer ground state and that the wind is optically thick. Thus radiative driving may play a role and in fact the observed[12] correlation between the 6 cm emission and L_{CO} is reminiscent of that found[17] for extra-galactic bipolar outflows, between the mechanical lumino sity of the outflow and the nuclear narrow line luminosity.

Another model[18] treats both the temperature and opacity problems for the high luminosity sources. The radiation from a polar accretion shock heats a 10 au cavity in a thick accretion disc that intermittenly 'backfires' along the polar axis. It predicts outflows to be associated with soft x-ray sources however, at least when one is looking down the polar axis in an outburst phase. Admittedly this phase is only thought to last a few years, and the moving object found recently[19] in Cepheus A bears some ressemblence to the type of ejected object that is predicted .

(iii) Magneto-Disc Driven Outflow

These are wind generation models[20],[21] wherein a centrifugally driven outflow is launched from the open magnetosphere of a Keplerian disc. The equatorial disc material is dense and dominates the magnetic field (super Alfvénic) but, just as is the case for the Sun, there is a gradual transition with height to a magnetically dominated (sub Alfvénic) 'corona' (of relatively low temperature). The interesting idea is that the angular momentum carried off by the spun up material permits in turn the steady inflow of material in the disc. Thus the wind mass flux becomes related to the desired accretion rate onto the central object. There seems to be no clear opinion as to the phase in which this disc phenomenon occurs, but it may of course operate at various times on different scales.

The most detailed ideal MHD model[20] is that of a 2D steady flow. Each flux tube is defined by the cylindrical radius of its foot point in the disc, r_{fp}, and boundary conditions at the disc are fixed simply by means of conservation laws. Although a family of non-self-similar models is found, there is no real matching to the boundary conditions either at infinity (in cylindrical radius) or at the protostellar boundary layer. However the authors do select a model with an axial current flow that is constant with disc radius as the one that avoids both central and surrounding singularities. In their preferred example the asymptotic outflow velocity $v_\infty \propto r_{fp}^{-1}$, where the value is around 41 km s^{-1} at 1 au. Thus this is really a kind of jet model where the high velocities are produced on rather small scales. The mass outflow in the wind is very small compared to the mass flow through the disc, by the square of the lever arm ratio r_{fp}/r_A (r_A is the distance of the Alfvén point on each flux tube from the axis) since this is the ratio of the specific angular momentum carried by a mass element at each end of the 'lever'.

The preceding treatment ignores the necessary disc structure and internal dissipation. Assuming ambipolar diffusion in a weakly ionized disc, one finds[22] a consistent vertical structure that can eventually be matched onto the centrifugal type models, and which is correctly sub-Keplerian in the disc mid-plane. It should be noted however that they believe that this applies on much larger scales than the inner few au. This latter region they expect to be dominated by Ohmic diffusion and is really the beginning of the boundary layer. At 100 au, they suggest a mid-plane density of $10^{10} cm^{-3}$. The thermal structure of the disc as a function of radius has also been calculated[23]. It is found that the ambipolar heating in such a weakly ionized disc can maintain temperatures near $10^4 K$ out to distances approaching 1000 au. This has the virtue of explaining the forbidden line emission in the disc, although it has also been argued[24] that the recollimation of the centrifugal winds will produce these lines in an oblique shock structure. Such large discs are virtually identical in location to the 'storage' discs discussed previously, except that they are Keplerian. They might be the next evolutionary phase after extensive internal dissipation and cooling of the thick discs.

These models are not without their difficulties. The most worrying of these may be the boundary condition at infinity which must serve to hold the disc magnetosphere open. For unlike solar wind type models where the open field is 'combed' out by an organized differential flow, it is rather the open field here that must create the large scale flow and so it must be otherwise created. Such fields are subject to various instabilities, of which the pinching or recollimation mode and the kink and helical instabilities are just a few examples. The non-linear relaxation of dominant magnetic fields to either a force free or an equipartition configuration is another. Moreover the open field is necessarily anchored both at the Keplerian disc and at infinity and so it is subject to strong 'twisting'. It is not clear that the open field structure can be maintained in the presence of such effects, even in the mean.

There is a variant[25),26)] of these models that was also suggested previousl y in the context of extra-galactic sources[27)]. Here a magnetic field is wound up in the disc near an Alfvénic point until a magnetic explosion is driven vertically in both directions along the axis. Angular momentum is extracted during this process so that material falls onto the star simultaneously. Unlike the centrifugal models, the mass flux is equally balanced between the accretion and the outflow. The asymptotic velocity is limited however to about twice the Keplerian velocity at the ejection radius. The model produces a time dependent (bursting) jet model. The scales at which high velocities are produced are comparable to the stellar radius so that this is a near boundary layer phenomenon in contrast to the disc models discussed above. Nevertheless this model avoids having to maintain the globally ordered magnetic field.

Overall I feel that it is not very satisfying that a Keplerian disc should be inserted as an ad hoc component, separate from both the surrounding outflow and the distant accretion. In effect the study of the vertical velocity structure of the disc suggests that we seek to integrate it into the combined inflow/outflow. We turn to such a model in the final section.

3. Bipolar Circulation Models

An analytic example of a new class of model has recently been described[28)]. The essential characteristic of the class is to represent the central point in centrally symmetric accretion flow as a saddle singularity rather than as a node. That is, the spherically symmetric nodal 'Bondi' type accretion meridional stream lines are parametrically unstable to the development of hyperbolic circulation under the addition of rotation and/or a magnetic field. In the steady state, the magnetic field is parallel to the stream lines. Since the flow is super-Alfvénic everywhere the stability of the global structure is maintained by the familiar 'combing' of the magnetic field due to the differential motion. The self-similar structure of the stream -field lines does not strictly match an arbitrary boundary condition at infinity, but this symmetry seems to be a natural small scale limit of a more general self-gravitating circulation or 'convection' flow. I call it a circulation flow since the material is not accreted unless it lies on those relatively few stream lines that physically intersect the stellar disc. It is in this way that these models minimize the luminosity associated with a massive outflow/inflow, in contrast to the low efficiency disc wind models. One expects that they should be mainly relevant in an early post protostellar formation phase as the more distant, high specific angular momentum, material falls in.

In addition to the meridional behaviour, each stream-field line in the bipolar outflow version of these models makes a spiraling approach to the axis roughly parallel to the equatorial plane and then emerges in the form of a conical helix wrapped about the axis of symmetry. Interestingly enough, there is an alternate mode wherein the sense of the motion is reversed.

In order to produce realistic models of bipolar outflows including the asymptotically fast velocities, it seems that the models must be worked out with self-gravity and radiative heating included. However the discovery of sources such as VLA 1623[29] with large relatively uniformly distributed external masses stimulated[28] a reassesment of an analytic solution previously discussed[30]. This 'toy' model is incompressible, viscous and does not produce the requisite velocities at large distances, but it does illustrate nicely the behaviour characteristic of this class of model. Figure 1 shows on the left the meridional stream-field lines for a typical case, while on the right it shows the pressure contours and the pressure in a gray scale representation with the lighter shades representing the higher pressure. One sees that a thick pressure disc forms as a result of the inflow and as a cause of the outflow. In effect the material falls in and rebounds into a bipolar pattern rather than attain the star radially . Most material 'misses' the target as it were. If as it turns the corner some free energy is added, then reasonable velocities at infinity may be attained. The bottom image of Figure 1 shows the path in space of one of the self-similar family of stream-field lines.

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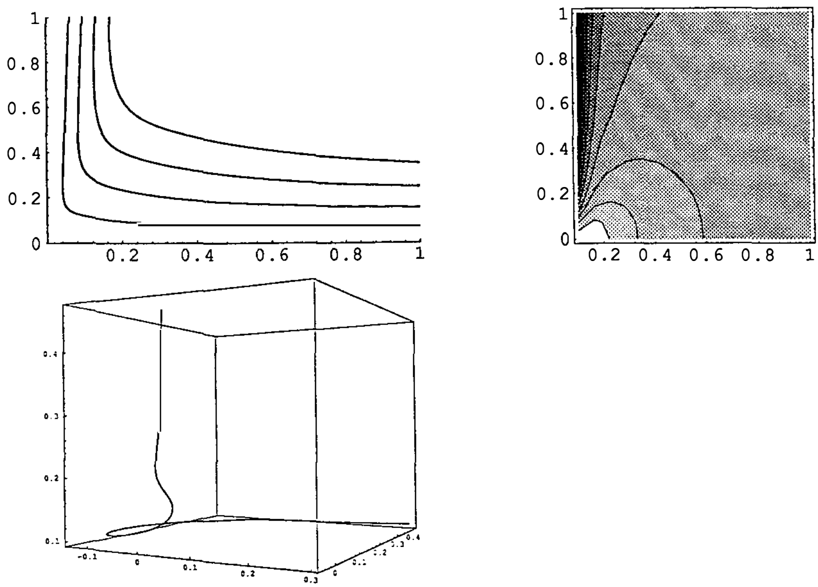
Figure 1



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