

# FOCUSING PROPERTIES OF THE MAGNETIC STRUCTURE OF ISOCHRONOUS CYCLOTRONS WITH LARGE SPIRALITY ANGLE OF POLE TIPS

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## Abstract

Magnetic structures with a large spirality angle of pole tips have been investigated in a number of works and are used in superconducting cyclotrons, H<sup>+</sup> ion cyclotrons, etc. With the design and construction of an 80 MeV isochronous H<sup>+</sup> cyclotron, such studies were continued and extended. In this work, a relatively simple approach for analyzing the spiral structure is proposed.

## INTRODUCTION

The magnetic structure with a large spirality angle of the pole tips is used in cases when vertical focusing from the flutter (field difference in the valley and the sector) is insufficient and it is necessary to add the angle focusing. This situation is typical for superconducting cyclotrons and for cyclotrons that accelerate negative hydrogen ions. Moreover, at JINR in Dubna, such structures have been investigated and a cyclotron and a synchrocyclotron with sectors in the form of an Archimedes spiral with a maximum spirality angle of up to 70 degrees were built. Numerical calculations of the magnetic field for an isochronous superconducting cyclotron with spiral sectors in the approximation of their uniform magnetization were carried out [1].

Two effects were noted: a decrease in the flutter in the central region with the introduction of spirality and a mismatch between the spirality of the sector Irion and the magnetic field. However, calculations made for a specific geometry are not applicable in the case of a different design. With the construction of an isochronous cyclotron for accelerating H-ions up to 40-80 MeV [2, 3], studies of the focusing properties of spiral structures were continued and expanded. Modern 3D software codes simplify the design of the magnetic field of any configuration by using trial- and- error method. However, to speed up the procedure and reduce the number of options for 3D analysis, it is useful to first perform a simplified and visual analysis of the system and estimate the importance of various parameters in the framework of a simpler 2D approximation.

## OPTIMIZATION OF THE MAGNET GAPS

As a first approximation, the hill and the valley gaps have been chosen using the 2D POISCR code calculations with the fill factor method. In this method, a 3D problem is reduced to a 2D one. The iron rings or the so-called shims mounted on the magnet poles and providing an isochronous rise in the field are calculated using a 2D

program with a reduced value of the magnetic permeability  $\mu_{new}(B) = \mu(B) \cdot C$ . The permeability is reduced by a factor  $C$  – so-called filling factor equal to the ratio of the azimuthal length of the sector to the length of the periodicity element at a radius  $r$ . The gap of the magnet obtained in this way corresponds to the gap of the hill, and there is no additional shim in the valley. Thus, two variants of the gaps of the main magnet and the gaps of the hills and the valleys were analyzed. The parameters of these variants are presented in the caption to Fig. 1.

## FLUTTER

The azimuthal variation of the magnetic field [1] is determined by the so-called flutter  $F(r)$ :

$$F(r) = \langle (B - \langle B \rangle)^2 \rangle / \langle B \rangle^2, \quad \langle \dots \rangle = (2\pi)^{-1} \int_0^{2\pi} \dots d\theta$$

Flutter can be represented as a Fourier harmonics expansion of the azimuthal variation of the magnetic field. The fundamental contribution to the expansion is made by the general focusing harmonic associated with the number of sectors and periodicity elements (in our case,  $N = 4$ ). If we denote the value of the fundamental focusing harmonic  $f = B_N / \langle B \rangle$ , then  $F = f^2/2$ .

Analytical calculation of flutter is a complex problem and therefore approximate methods have been used. In particular, in [4], an expression was obtained for the general harmonic of the magnetic field variation in an isochronous cyclotron in the approximation of uniform magnetization of the sectors of a magnet

$$B_N = 8M \sin\left(\frac{2\pi a}{d}\right) \exp\left(-\frac{2\pi g_h}{d}\right),$$

where  $2a$  is the length of the sector along the azimuth for a given radius  $r$ ,  $2g_h$  is the gap in the hill,  $d$  is the period of the structure, equal to the total length of the hill and valley,  $4\pi M = 21\text{kG}$ . It follows from this expression that for an isochronous cyclotron with a period of the magnetic field structure equal to  $d = 2\pi r / N$ , where  $N$  is the number of sectors, the flutter grows with increasing radius according to the law

$$F \sim B_N^2 / 2 \sim \exp(-2/x), \quad x = r / N \cdot g_h \quad (1)$$

Although this approximation is insufficient for obtaining accurate quantitative estimates, it allows, in a unified manner, to get an idea of the relationship between the different parameters of the magnetic structure. Moreover, the introduction of the dimensionless parameter  $x$  enables the comparison of different variants of structures. In particular, flutter rises as the gap in the hill decreases and falls as the number of sectors increases.

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The maximum flutter value corresponds to the case when the azimuthal extent of the sector and the valley are equal. In this case,  $a = 0.25d$  and  $\sin(\pi/2) = 1$ . For the case when the parameter  $x$  becomes less than 0.5, the flutter drops very sharply, i.e., at radii  $r \leq 0.5 \cdot N \cdot g_h$ , azimuthally variation becomes ineffective, and focusing tends to zero. The results of these calculations are shown with curve 3 in Fig. 1.

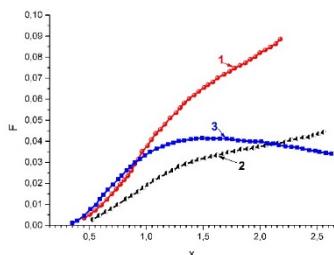


Figure 1: The dependence of flutter on the dimensionless parameter  $x = r / N \cdot g_h$ , where  $N = 4$ , for two options: 1 -  $2g_v = 386$  mm,  $2g_h = 170$  mm,  $h1 = 108$  mm; 2 -  $2g_v = 284$  mm,  $2g_h = 145$  mm,  $h2 = 69$  mm; 3 - a variant of uniform magnetization for the case of the geometry of the first variant.

The dependence of flutter on the radius can be calculated using 3D programs. However, in our case, to speed up and simplify the calculations, we used approximate calculations. Flutter can be estimated using a 2D program if we replace the calculation of the edge effect along the azimuth with the calculation of the edge effect along the radius of the two-dimensional magnet with a gap in the form of teeth and valleys along the radius. In this case, gap variation of 2D-magnet along radius corresponds to the length and gaps of the sector and valley along the azimuth of the investigated magnet.

The simple estimates carried out for the C-80 cyclotron show that the structure with straight sectors does not provide the necessary vertical focusing; therefore, it is necessary to use the next effect.

## SPIRALITY EFFECT

As it is well known, the frequency of vertical oscillations, which determines the vertical focusing, can be expressed using the following approximation:

$$v_z^2 \approx -k + F \cdot S(r, \gamma), \quad S(r, \gamma) = 1 + 2tg^2 \gamma, \quad (2)$$

where  $k$  is the rate index of average magnetic field growth along the radius

$$k = (r / \langle B \rangle) (d \langle B \rangle / dr) \approx 2W / E_0.$$

$W$  is the kinetic energy,  $E_0 = 938$  MeV is the rest mass of the proton,  $\gamma$  is the spirality angle. The frequency of axial oscillations is determined by two oppositely acting terms - the defocusing of the isochronous rise of the mean-field and the focusing action of the azimuthal variation of the field. The task is to select  $F$  and  $\gamma$  for each value of the energy  $W$  so that  $v_z^2$  remains greater than zero during acceleration. At the same time, to limit the dissociation of negative ions, it is necessary to strive for the minimum

possible value of the magnetic field in the hill, i.e. to the minimum flutter value. The spiraling sector provides an increase in focusing force due to the non-perpendicular angle of particle entry into the sector region. Effectively, the flutter  $F$  is multiplied by the coefficient  $S(r, \gamma)$ ; however, in a structure with a large spirality angle, the increase in focusing force is noticeably less [1] than could be expected from the above formula. This is due to a decrease of flutter when the spirality angle is introduced and a mismatch between the iron and the magnetic spirality angles.

There is a simple geometric explanation for the first effect. With a large spirality angle, the difference between the sector length along the azimuth (AB in Fig. 2) and the width of the  $A_1B_1$  sector determined by the perpendicular to its centerline becomes significant.

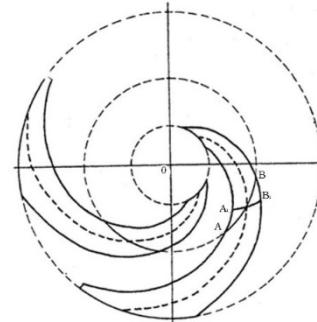


Figure 2: There is a difference between the sector length along the azimuth AB and the "effective" length  $A_1B_1$  at large spiraling angles. The effective width of the sector long the perpendicular to the sector centerline is equal  $A_1B_1 \approx AB \cdot \cos \gamma$ .

In the case of straight sectors, the flutter is determined by the magnetic field difference in the hill and the valley. In this case, the field falls off along the azimuth. When a spiraling angle is introduced, the distance between the sectors along a line perpendicular to the centerline of the sector is much smaller than the distance along the azimuth. This leads to a decrease in the effective length of sector  $A_1B_1$  and, accordingly, to a decrease in the length of the valley. From geometric considerations, we can conclude that the effective length of the  $A_1B_1$  sector is approximately equal to the length of the sector along the azimuth AB multiplied by a factor equal to  $\cos \gamma$ , i.e.  $A_1B_1 \approx AB \cdot \cos \gamma$ . With a decrease in the length of the sector, the effective length of the valley and the period of periodicity decrease. In this case, the dimensionless parameter  $x$ , introduced in expression (1) and related to the length of the periodicity period, will also decrease and become  $x_{\text{eff}} = x \cdot \cos \gamma$ . According to Fig. 1, a decrease in the value of the parameter  $x_{\text{eff}} = x \cdot \cos \gamma$  leads to a shift along the graph towards lower flutter values. At intermediate and large radii, the spirality causes an increase in edge focusing and a decrease in flutter and the total effect leads to an overall increase in focusing. However, at small radii, the introduction of spiraling reduces the effective sector length and this can lead to a very sharp drop in flutter and a total decrease in focusing. The total effect of the introduction of

the spiral sectors can be characterized by a parameter that is the product of two factors: flutter  $F$  and  $S(r, \gamma)$ . However, at small radii, the introduction of spiraling reduces the effective sector length and this can lead to a very sharp drop in flutter and a decrease in focusing. The total effect of the introduction of the spiraling sectors can be characterized by a parameter that is the product of two factors: flutter  $F$  and  $S(r, \gamma)$ . Since the flutter drops sharply at  $x_{\text{eff}} < 0.5$ , the introduction of spiraling in the central region leads to a decrease in focusing. For each radius, it is possible to calculate the limiting value of the spiraling angle, exceeding which spiraling does not give an increase in focusing. This value for each value of the parameter  $x$  can be estimated to be by finding a solution for the following expression:

$$U(x, \gamma) = (F(x \cos \gamma) / F(x)) \cdot (1 + 2 \tan^2 \gamma) - 1 = 0, \quad (3)$$

where  $F(x)$  is a function of the type shown in Fig. 1.

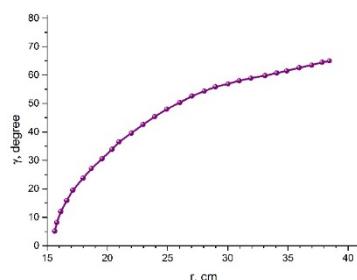


Figure 3: Ultimate spirality angle for C-80 cyclotron in dependence on the radius for  $2g_v = 386$  mm,  $2g_h = 170$  mm,  $N = 4$ .

Figure 3 shows the limiting spiraling angle calculated by the formula (3) depending on the radius for the case of the C-80 cyclotron. According to the Fig. 3, spiraling leads to the decreasing of vertical focusing at radii smaller than 35 cm, and it is advisable to use a structure with a large spirality angle at radii greater than 35 cm. Thus, a structure with a large spiraling angle is effective only at radii larger than the hill gap value.

### 3D COMPUTATION AND EXPERIMENT

When choosing the spiral angle in the cyclotron design, it is necessary to take into account the fact that the magnetic field does not completely repeat the iron sector geometry.

According to the Fig. 4, there is the effect of magnetic spirality "penetration" into the region of straight sectors. And this effect leads to a decrease in vertical focusing at radii smaller than 35 cm. This effect explains the widespread use of direct sectors in the central region. It is also seen that at radii of  $\sim 75 \leq r \leq 88$  cm there is a "lag" between the magnetic field spiraling angle the geometrical one. The maximum lag reaches  $7^\circ$ , which at a spirality angle of  $65^\circ$  leads to a 30% decrease in focusing. After preliminary assessments related to the choice of the parameters of the magnetic structure, the final variants were calculated in detail using the 3D MERMAID program.

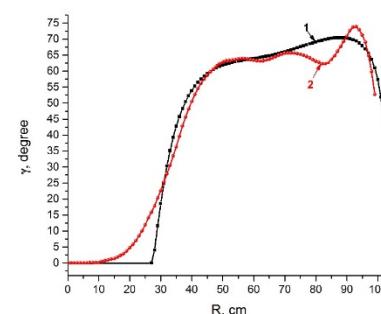


Figure 4: The spirality angle  $-\gamma$  (deg.) in dependence on the radius of the cyclotron: 1 - the data of the sector geometry spirality, 2 – the spirality of the fourth focusing harmonic of the magnetic field, obtained in 3D calculations and measurements of the magnetic field.

In the calculations, to achieve maximum accuracy, the magnetic structure was described using  $\sim 20.5$  million straight prisms. During the design process, two variants of the magnetic structure were considered at a finite radius: flutter  $F = 0.04$ , spirality angle  $\gamma = 55^\circ$  and  $F = 0.025$  with  $\gamma = 65^\circ$ , and field variation amplitudes of 4.14 and 3.28 kG, respectively. Ultimately, the second option was adopted, providing a lower field in the hill, at which the loss of H minus ions due to electrodissociation does not exceed 2.6%.

### CONCLUSION

The analysis of the structure of the magnetic field of a cyclotron with a high spirality angle, presented in this work, makes it possible to investigate the effect of different parameters of the structure and promptly compare various options. Such an approach provides the means to qualitatively analyze the effect of a decrease in the vertical focusing of the spiral structure at the center of the cyclotron. It is expedient to use the structure with a large spiraling angle only at radii larger than the gap in the hill. In the central region, it is advisable to use direct sectors. The paper presents a technique that was used in the design of the magnetic structure of the C-80 cyclotron. The use of the limiting large values of the spirality angle in the C-80 cyclotron made it possible to obtain limiting energy as high as 80 MeV in a magnet with a diameter of 2 meters at an extraction radius of 0.9 m. The magnetic structure allows us to limit the of negative ions electrodissociation to less than 3%.

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