The black hole/qubit correspondence

M. J. Duff

Theoretical Physics, Blackett Laboratory, Imperial College London, London SW7 2AZ, United Kingdom

E-mail: `m.duff@imperial.ac.uk`

Abstract. We review recent progress in the black hole/qubit correspondence.

1. Introduction

The work connects two previously disparate areas of theoretical physics:

 (i) Hawking black hole entropy.

(ii) Qubit entanglement in quantum information theory .

In the 1970s Hawking showed that quantum black holes can radiate energy and hence carry entropy. Although Hawking gave a macroscopic explanation, a microscopic understanding was provided only in 1996 by superstring theory. In 2006 it was noticed that the entropy formula for a particular 8-charge black hole appearing in string theory is given by the 'hyperdeterminant', a quantity introduced by the mathematician Cayley in 1845. Remarkably, this same quantity also appears in the completely different field of quantum information theory (QIT). A qubit is a two-state (up/down) quantum system; so three qubits (Alice, Bob and Charlie) can have 8 different states. These states can get entangled in a way that Einstein described as "Spooky" (Alice, Bob and Charlie could live in different galaxies) and Cayley's hyperdeterminant is the '3-tangle' which measures the degree of entanglement.

This turned out to be the tip of an iceberg and since then a two-way dictionary between black holes and qubits has been established and knowledge of string theory has been used to discover new things about QIT and vice-versa. These include (a) a microscopic interpretation of these 8 states in terms of 8 ways of wrapping D3-branes around the extra dimensions of string theory; (b) the three-way entanglement of seven qubits, suggesting that predictions based on the mathematics of octonions may be testable in the laboratory; (c) the discovery of "Freudenthal duality", a new symmetry of black holes; (d) a new way of classifying multi-qubit entanglement based on Jordan algebras.

Current goals include:

 (i) To pursue the notion of "superqubits" which introduces the concept of bose-fermi symmetry into QIT and finds applications in condensed-matter physics.

(ii) To use intersecting D-branes to solve long-outstanding problems in the currently topical field of multi-qubit entanglement, thus providing falsifiable predictions of string theory.

(iii) To seek a physical origin of these black hole/qubit coincidences, which would have far-reaching implications for a unified theory of fundamental forces.

2. Black holes, string theory and M-theory

If current ideas are correct, a unified theory of all physical phenomena will require some radical ingredients:

2.1. Supersymmetry

For each known *boson* (integer spin 0, 1, 2 and so on), there is a *fermion* (half-integer spin 1/2, 3 /2, 5/2 and so on), and vice versa. The number of these supersymmetries is denoted $\mathcal{N}$ and ranges from 1 to 8 in four spacetime dimensions. The Large Hadron Collider in Geneva will be looking for these superparticles. Moreover, supersymmetry implies gravity: if Einstein had not already discovered general relativity, supersymmetry would force us to invent it. Many theorists favour supersymmetry because it provides a framework within which the weak, electromagnetic and strong forces may be united with gravity.

2.2. Extra dimensions

Curiously, supergravity places an upper limit of eleven on the dimension of spacetime. The familiar universe, of course, has three space dimensions and one time dimension. But in the 1920s Kaluza and Klein suggested that spacetime may have a hidden fifth dimension. This extra dimension would not be infinite, like the others; instead it would close in on itself, forming a circle. The kind of real, four-dimensional world supergravity ultimately predicts depends on how the extra seven dimensions are rolled up, a la Kaluza and Klein.

2.3. Strings

In 1984, however, 11-dimensional supergravity was knocked off its pedestal by superstring theory in 10 dimensions. There were five competing theories: the $E_8 \times E_8$ heterotic, the $SO(32)$ heterotic, the $SO(32)$ Type I, and the Type IIA and Type IIB strings. The $E_8 \times E_8$ seemed, at least in principle, capable of explaining the elementary particles and forces, including their handedness. And strings seemed to provide a theory of gravity consistent with quantum effects.

2.4. Branes and M-theory

Yet the spacetime of eleven dimensions allows for a membrane, which may take the form of a bubble or a two-dimensional sheet. In 1987 it was shown [1] that if one of the 11 dimensions is a circle, we can wrap the sheet around it once, pasting the edges together to form a tube. If the radius becomes sufficiently small, the rolled-up membrane ends up looking like a string in 10 dimensions; it yields precisely the Type IIA superstring. In a landmark talk at the University of Southern California in 1995, Witten [2] drew together all this work on strings, branes and 11 dimensions under the umbrella of M-theory in 11 dimensions.

2.5. Black holes

Such breakthroughs have led to a new interpretation of black holes as intersecting black-branes wrapped around seven curled dimensions. As a result, there are strong hints that M-theory may even clear up the paradoxes of black holes raised by Hawking. In 1974 Hawking showed that black holes are not entirely black but may radiate energy [3]. In that case, they must possess entropy, which measures the disorder by accounting for the number of quantum states available. Yet the microscopic origin of these states stayed a mystery. Using Polchinski's *D-branes*, Strominger and Vafa [4] were able to count the number of quantum states in black-branes. They found an entropy that agrees with Hawking's prediction, placing another feather in the cap of M-theory. Branes now occupy centre stage as the microscopic constituents of M-theory, as the higher-dimensional progenitors of black holes and as entire universes in their own right. Despite all these successes, physicists are glimpsing only small corners of M-theory; the big

picture is still lacking. Over the next few years we hope to discover what M-theory really is. Understanding black holes will be an essential pre-requisite.

3. Entanglement and quantum information theory

In completely separate developments, exciting things were happening in the world of QIT. Quantum entanglement is a quantum mechanical phenomenon in which the states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. In their seminal 1935 paper, Einstein, Podolsky, and Rosen (EPR) demonstrated that quantum mechanics cannot provide a complete description of "local realism" for two spatially separated but quantum mechanically correlated particles. It was not until 1964, however, that CERN theorist John Bell [5] proposed a falsifiable experiment to test EPR and it was not until 1983 that Alain Aspect actually performed it. Quantum mechanics won and local realism lost. As quantum information science developed, the impact of entanglement went far beyond the testing of the conceptual foundations of quantum mechanics. Entanglement is now of central importance in QIT as a physical resource essential to numerous quantum information tasks including: quantum cryptography, superdense coding, and quantum computation.

Utilizing the paradigm of *stochastic local operations and classical communication* (SLOCC) it was established in [6] that three qubits (Alice, Bob and Charlie: $|\Psi\rangle = a_{ABC}|ABC\rangle$ with $A, B, C = 0, 1$) can be entangled in several physically distinct ways: tripartite GHZ (Greenberger-Horne-Zeilinger), tripartite W, biseparable A-BC, separable A-B-C and null:

$$
\begin{array}{ccccc}
 & & \text{GHZ} & & \\
 & \text{W} & & \text{W} & \\
A\text{-}BC & & A\text{-}BC & & A\text{-}BC \\
 & A\text{-}B\text{-}C & & A\text{-}B\text{-}C & \\
 & & \text{Null} & &
\end{array}
\tag{1}
$$

The GHZ state is distinguished by a non-vanishing invariant known as the 3-tangle τ_{ABC}, given by Cayley's hyperdeterminant of a_{ABC}.

However, the crucial problem of classifying arbitrary n-qubit entanglement remains unsolved.

4. Black holes and qubits

Studying the entanglement/entropy correspondence, we have begun to see interesting and non-trivial links between the two fields of string theory and QIT. Further work may lead to the realisation that entanglement and entropy are dual descriptions of the same phenomenon. Successes so far include:

4.1. Black holes in 4D and three qubits

An explicit correspondence between the above 3-tangle and the entropy S of the 8-charge STU black hole of supergravity [8]:

$$S = \tfrac{\pi}{2}\sqrt{\tau_{ABC}}.$$

Moreover, the physically distinct forms of 3-qubit entanglement correspond directly to the physically distinct black hole solutions. Similar results relate $5D$ black holes to qutrits (3-state systems) [9].

4.2. Wrapped branes as qubits

We have demonstrated that the eight states of the 3-qubit system correspond to eight ways of wrapping four D3-branes around three 2-tori of Type II string theory compactified to four dimensions [10]. Similarly, the nine states of the 2-qutrit system correspond to nine ways of wrapping two M2-branes around two 3-tori of M-theory compactified to 5 dimensions.

Table 1. A 4-charge black hole in $4D$ spacetime 0,1,2,3 from four D3-branes wrapping the six extra dimensions 4,5,6,7,8,9 yields a GHZ state. Each row represents a D3 brane and the first row represents a brane wrapping dimensions 4,6 and 8.

0	1	2	3	4	5	6	7	8	9	brane	$\lvert ABC \rangle$
x	o	o	o	x	o	x	o	x	o	D3	$\lvert 000 \rangle$
x	o	o	o	x	o	o	x	o	x	D3	$\lvert 011 \rangle$
x	o	o	o	o	x	x	o	o	x	D3	$\lvert 101 \rangle$
x	o	o	o	o	x	o	x	x	o	D3	$\lvert 110 \rangle$

4.3. Black holes, qubits and octonions

We have extended the $\mathcal{N} = 2$ *STU* model example to the most general case of black holes in $\mathcal{N} = 8$ supergravity where the global symmetry, referred to as U-duality, is $E_{7(7)}$, one of the exceptional Lie groups appearing in Cartan's classification. Noting that $E_{7(7)} \supset [SL(2)]^7$, we have shown that the corresponding system in quantum information theory is that of seven qubits (Alice, Bob, Charlie, Daisy, Emma, Fred and George), $[SL(2)]^7$ being the corresponding SLOCC equivalence group. However, the larger symmetry requires that they undergo at most a tripartite entanglement of a very specific kind. The entanglement measure will be given by the quartic Cartan $E_7(\mathbb{C})$ invariant. The entanglement may be represented by the Fano plane where the vertices A, B, C, D, E, F, G represent the seven qubits and the seven lines $ABD, BCE, CDF, DEG, EFA, FGB$ and GAC represent the tripartite entanglement. See Figure 1. The Fano plane also corresponds to the multiplication table of the imaginary octonions.
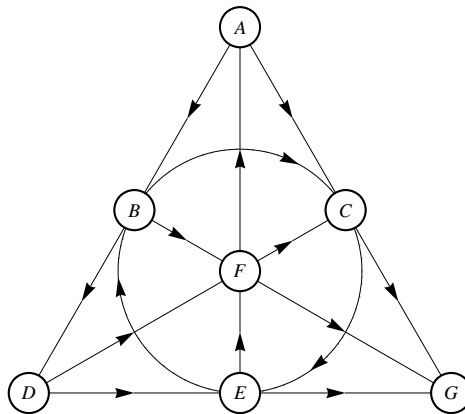

Figure 1. The Fano plane. The vertices A, B, C, D, E, F, G represent the seven qubits and the seven lines $ABD, BCE, CDF, DEG, EFA, FGB, GAC$ represent the tripartite entanglement.

From the point of view of M-theory, this suggests a whole new way of studying its symmetries based on the 7 imaginary octonions (completely different from the Jordan algebra approach that uses all 8 *split* octonions.) From the QIT point of view, seven qubits is just at the limit of what can be reached experimentally, so predictions based on the mathematics of octonions, described by the Fano Plane, may now be testable in the laboratory. Octonions have fascinated mathematicians and physicists for decades but have yet to find any physical application. In recent books both Roger Penrose and Ray Streater have characterised octonions as one of the

great 'lost causes' in physics. Yet we hope that the tripartite entanglement of seven qubits will prove them wrong and provide away of seeing the effects of octonions in the laboratory [11, 12].

4.4. Freudenthal classification of three qubits

New methods to classify three qubits exploiting that elegant branch of mathematics involving Jordan algebras and the related Freudenthal triple system (FTS), familiar from black holes in string theory [13]. The FTS naturally reproduces the conventional 3-qubit entanglement classification (1). Moreover, using the FTS analysis we determined the mathematical *coset* characterisation of both the entanglement and black hole classes, hitherto unknown results.
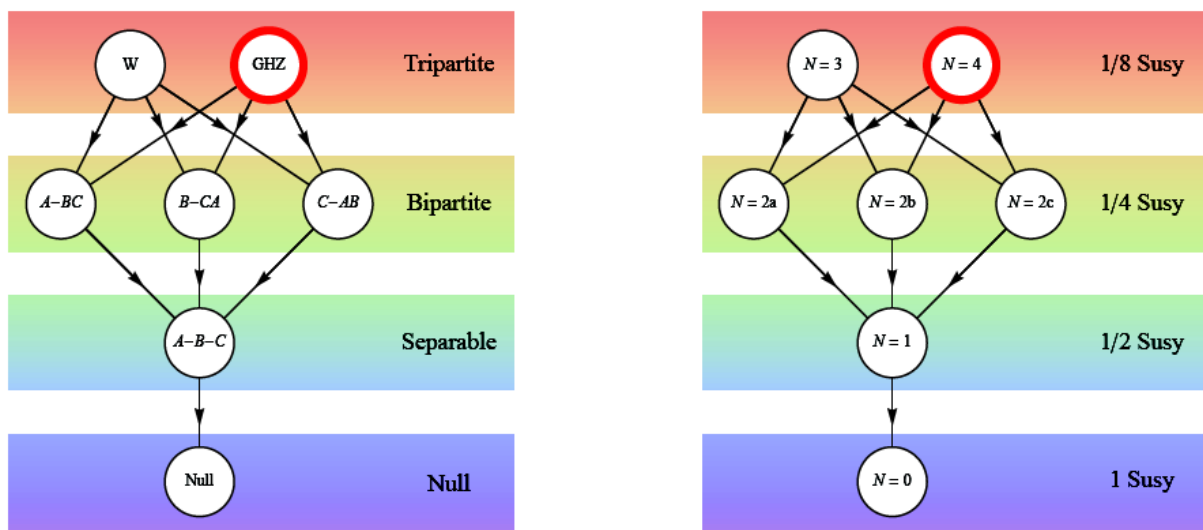

Figure 2: The classification of three qubits (left) exactly matches the classification of black holes from N wrapped branes (right). Only the GHZ state has a nonzero 3-tangle and only the $\mathcal{N} = 4$ black hole has nonzero entropy.

Figure 2.

4.5. Freudenthal duality

When the black hole charges are quantized the U-duality symmetry is broken to an infinite discrete subgroup and the square of the entropy becomes integer valued. Invoking recent important developments in number theory, we have introduced a new symmetry in the case when this integer is a perfect square. This "Freudenthal duality" is best understood in terms of the *integral* FTS and may relate quantized black holes that are not necessarily U-duality equivalent while preserving their lowest order entropy [14]. It would be interesting to understand their role in the physics of stringy black holes. For example, this symmetry rules out small black holes whose masslessness would spoil the conjectured finiteness of $\mathcal{N} = 8$ supergravity.

5. Current research

5.1. Superqubits

Although this topic grew out of our black hole/qubit correspondence, it is logically independent of it. In QIT n-qubit states lie in the fundamental representation of the SLOCC equivalence group $[SL(2, \mathbb{C})]^n$. We propose a supersymmetric generalisation of the qubit, the *superqubit*,

by extending this to the supergroup $[OSp(2|1)]^n$. For $n = 1$ a single superqubit forms a 3-dimensional representation of $OSp(2|1)$ consisting of two commuting "bosonic" components a_A and one anti-commutating "fermionic" component $a_\bullet$.

$$|\Psi\rangle = a_0|0\rangle + a_1|1\rangle + a_\bullet|\bullet\rangle$$

This mathematical construction seems a very natural one. For $n = 2$ and $n = 3$ we introduce
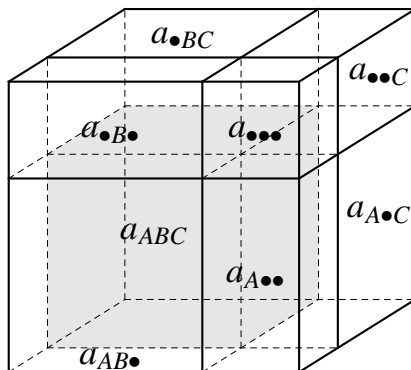

Figure 3. The $3 \times 3 \times 3$ cubic superhypermatrix

the appropriate supersymmetric generalisations of the conventional entanglement measures. In particular, super-Bell and super-GHZ states are characterised respectively by non-vanishing superdeterminant (distinct from the Berezinian) and superhyperdeterminant. We have already uncovered some unexpected consequences. For example, the two superqubit state

$$|\Psi\rangle = i|\bullet\bullet\rangle$$

looks naively separable but is entangled because the product of two Grassmann numbers $a_\bullet b_\bullet$ cannot yield a complex number. In fact it is *maximally* entangled.

From a physical point of view, the supergroup $OSp(2|1)$ is also used in some models of strongly correlated electrons in condensed matter physics. The *t-J* model is a specialisation of the Hubbard model where the effective field theory describes a system in which the charge and spin degrees of freedom separate and are free to move across the lattice. In certain regions of the parameter space the *t-J* model becomes supersymmetric and the chargeon and the spinon transform in the fundamental representation of $OSp(2|1)$ [15]. This means that they form a valid realisation of a superqubit (cf. how the two polarisations of a photon can be a realisation of a qubit). Long chains of these superqubits are described in the lattice and we can use our techniques to quantify any super-entanglement that may be present and, in particular, discover if the supersymmetry is able to control the decoherence times of these systems. The influence that this may have on quantum error correction and quantum computing should not be ignored. Already the role of superqubits in super quantum computing is being explored by Castellani et al [16].

Another example where our superqubits can be used is the *supersymmetric quantum Hall effect* [17], observed in two-dimensional electron systems subjected to low temperatures and strong magnetic fields, whereby conductivity is quantised.

5.2. D-branes and qubits: falsifiable predictions of string theory
The partial nature of our understanding of exactly how the extra dimensions of string/M-theory are curled up has so far prevented any kind of *smoking gun* experimental test. This has led some

critics of string theory to suggest that it is not true science. This is easily refuted[1] by studying the history of scientific discovery; the thirty year time lag between EPR's germ of an idea and Bell's falsifiable prediction provides a nice example. Nevertheless it cannot be denied that such a prediction in string theory would be very welcome. One of our aims is to do just that, namely to use D-brane intersection rules to predict new results in multi-qubit entanglement.

Macroscopically, the entropy is just one quarter the area of the event horizon of the black hole. To give a microscopic derivation [4] we need to invoke ten-dimensional string theory whose associated Dp-branes wrapping around the six compact dimensions provide the string-theoretic interpretation of the black holes. A Dp-brane wrapped around a p-dimensional cycle of the compact directions $(x^4, x^5, x^6, x^7, x^8, x^9)$ looks like a D0-brane from the four-dimensional (x^0, x^1, x^2, x^3) perspective. In the string literature one may find *D-brane intersection rules* which tell us how N branes can intersect over one another and the fraction ν of supersymmetry that they preserve. Up to $N = 4$ the results are given by [18]

$$
\begin{array}{cccc}
 & & N = 4, \nu = 1/8 & \\
 & N = 3, \nu = 1/8 & & N = 3, \nu = 1/8 \\
N = 2, \nu = 1/4 & & N = 2, \nu = 1/4 & & N = 2, \nu = 1/4 \\
 & N = 1, \nu = 1/2 & & N = 1, \nu = 1/2 \\
 & & N = 0, \nu = 1 &
\end{array}
\tag{2}
$$

In our black hole/qubit correspondence, the microscopic description of a GHZ state is that of the $N = 4, \nu = 1/8$ case of D3-branes of Type IIB string theory wrapping the (469), (479), (569), (578) cycles of a six-torus and intersecting over a string. The wrapped circles are denoted by crosses and the unwrapped circles by noughts. $|0\rangle$ corresponds to xo and $|1\rangle$ to ox, as in Table 1. The number of qubits here is three because the number of extra dimensions is six. This also explains where the two-valuedness enters on black hole side. To wrap or not to wrap; that is the qubit.

Repeating the exercise for the $N < 4$ cases and using our dictionary, we see that string theory predicts the three qubit entanglement classification (1), in complete agreement with the standard results of QIT. Allowing for different p-branes wrapping tori of different dimensions, we can also describe qutrits and more generally qudits. Furthermore, for the well-documented cases of 2×2, 2×3, 3×3, $2 \times 2 \times 3$ and $2 \times 2 \times 4$, our D-brane intersection rules are also in complete agreement. However, for higher entanglements, such as $2 \times 2 \times 2 \times 2$, then the QIT results are partial or not known or else contradictory. This is currently a very active area of research in QIT because the experimentalists can now control entanglement with greater numbers of qubits. Our goal is to use the allowed wrapping configurations and D-brane intersection rules to predict new qubit entanglement classifications.

5.3. Physical underpinnings

Despite these mathematical coincidences, however, the underlying physical reason why black hole and qubits should be related is still a mystery. There are intriguing hints that the answer may lie in the above wrapping of D-branes around the extra dimensions of string theory. If so, this qubit interpretation will radically change the way we look at string and M-theory.

Quantum entanglement lies at the heart of quantum information theory, with applications to quantum computing, teleportation, cryptography and communication. So whether or not there is a physical underpinning, the esoteric techniques of string theory might yet have practical applications.

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[1] See or listen to my 2007 debate with Lee Smolin at the Royal Society for the Arts `http://www.thersa.org/events/speakers-archive/d/michael-duff`

Acknowledgments

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