Effects of the modification of gravity on the production of primordial black holes

Sergio Andrés Vallejo Peña

ICRANet, Piazza della Repubblica 10, I–65122 Pescara, Italy
Instituto de Fisica, Universidad de Antioquia, A.A.1226, Medellin, Colombia
E-mail: sergio.vallejo@udea.edu.co

The enhancement of the spectrum of primordial comoving curvature perturbation $\mathcal{R}$ can induce the production of primordial black holes (PBH) which could account for part of present day dark matter. As an example of the effects of the modification of gravity on the production of PBHs, we investigate the effects on the spectrum of $\mathcal{R}$ produced by the modification of gravity in the case of G-inflation, deriving the relation between the unitary gauge curvature perturbation ζ and the comoving curvature perturbation $\mathcal{R}$, and identifying a background dependent enhancement function $\mathcal{E}$ which can induce large differences between the two gauge invariant variables.

When ζ is not constant in time it is different from $\mathcal{R}$, for example on sub-horizon scales, or in models exhibiting an anomalous super-horizon growth of ζ, but since this growth cannot last indefinitely, eventually they will coincide. We derive the general condition for super-horizon growth of ζ, showing that slow-roll violation is not necessary. Since the abundance of PBHs depends on the statistics of the peaks of the comoving density contrast, which is related to the spectrum of $\mathcal{R}$, it is important to take into account these effects on the PBHs abundance in modified gravity theories.

Keywords: Modified gravity, primordial curvature perturbations, primordial black holes.

1. Introduction

The study of primordial perturbations is fundamental in any cosmological model, since it allows to make predictions of the conditions which provided the seeds for the anisotropies of the cosmic microwave background (CMB) radiation or for the process of structure formation. Among the different theoretical scenarios proposed to explain the accelerated expansion of the Universe, Horndeski's theory[1] has received a lot of attention, both in the context of inflation and dark energy.

The calculation of the equation for cosmological perturbations for these theories have been so far performed in the so called unitary gauge, also known as uniform field gauge. While the unitary gauge has some computational convenience in general relativity when only a scalar field is present, in general it is not directly related to observations, which depend on the comoving curvature perturbations $\mathcal{R}$. The production of PBHs[2–6] is an example of phenomenon depending on $\mathcal{R}$[7] and not on the unitary gauge curvature perturbations ζ. Another example are the numerical codes developed for the solution of the Boltzman's equations in a perturbed Friedman-Lemaître-Robertson-Walker (FLRW) Universe, which are using equations in the synchronous gauge,[8] which for adiabatic perturbations coincides

approximately with the comoving gauge,[9] justifying the use of the comoving slices gauge for early Universe calculations.

The comoving gauge can differ from the unitary gauge in modified gravity theories because the effective energy momentum tensor arising from the modification of gravity can produce some effective entropy terms, which are absent in $K(X)$ theories, but are present in any more complicated Hordenski's theory. In this letter we compute the general relation between $\mathcal{R}$ and ζ in G-Inflation.[10, 11] As an application we use this relation to investigate the effects of the modification of gravity on the power spectrum of $\mathcal{R}$, and its implications on the production of PBHs.

2. G-inflation

In G-inflation the scalar field Φ is minimally coupled to gravity according to the action[10, 12]

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{Pl}^2}{2} R + L(\Phi, X) \right) ,$$

where $X = -g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi / 2$, R is the Ricci scalar and we use a system of units in which $c = \hbar = 1$. The Lagrangian density of the scalar field corresponds to

$$L(\Phi, X) = K(\Phi, X) + G(\Phi, X)\Box\Phi , \tag{1}$$

where K and G are arbitrary functions. The corresponding effective stress-energy-momentum tensor (EST) is given by

$$T_{\mu\nu} = L_{,X}\nabla_\mu \Phi \nabla_\nu \Phi + P_\Phi g_{\mu\nu} + \nabla_\mu \Phi \nabla_\nu G + \nabla_\nu \Phi \nabla_\mu G , \tag{2}$$

where

$$L_{,X} = \partial_X L = K_X(\Phi, X) + G_X(\Phi, X)\Box\Phi , \tag{3}$$

$$P_\Phi = L - \nabla_\mu \left(G\nabla^\mu \Phi \right) = K - g^{\mu\nu}\nabla_\mu \Phi \nabla_\nu G . \tag{4}$$

3. The perturbed effective energy-stress-momentum tensor

The most general scalar perturbations with respect to a flat FLRW background can be written as

$$ds^2 = a^2 \Big\{ -(1 + 2A)d\tau^2 + 2\partial_i B dx^i d\tau + [\delta_{ij}(1 - 2C) + 2\partial_i\partial_j E] dx^i dx^j \Big\} . \tag{5}$$

For the decomposition of the scalar field and the EST into their background and perturbation parts we use the notation

$$\Phi(x^\mu) = \phi(\tau) + \delta\phi(x^\mu) , \tag{6}$$

$$T^\mu{}_\nu = \overline{T}^\mu{}_\nu + \delta T^\mu{}_\nu . \tag{7}$$

The background components of the EST are

$$\overline{T}^0{}_0 = -\overline{\rho} = K(\phi,\chi) + \frac{3\mathcal{H}\phi'^3}{a^4}G_\chi(\phi,\chi) - \frac{\phi'^2}{a^2}\left[K_\chi(\phi,\chi) + G_\phi(\phi,\chi)\right], \qquad (8)$$

$$\overline{T}^0{}_i = \overline{T}^i{}_0 = 0, \qquad (9)$$

$$\overline{T}^i{}_j = \delta^i{}_j\overline{P},$$

$$\overline{P} = K(\phi,\chi) - \frac{\mathcal{H}\phi'^3}{a^4}G_\chi(\phi,\chi) + \frac{\phi'^2}{a^2}\left[G_\phi(\phi,\chi) + \frac{\phi''}{a^2}G_\chi(\phi,\chi)\right], \qquad (10)$$

where the primes stand for derivatives with respect to τ, χ is given by $\chi = \frac{\phi'^2}{2a^2}$, and the subscripts ϕ and χ denote partial derivatives with respect to these quantities, i.e. $G_\phi(\phi,\chi) = \partial_\phi G(\phi,\chi)$ and $G_\chi(\phi,\chi) = \partial_\chi G(\phi,\chi)$. In order to define the comoving slices gauge we need this component of the perturbed EST

$$\delta T^0{}_i = -\left(K_\chi + 2G_\phi - \frac{3\mathcal{H}\phi'}{a^2}G_\chi\right)\frac{\phi'^2}{a^2}\partial_i\delta\phi - \frac{\phi'^2}{a^4}G_\chi\partial_i\left(\delta\phi' - \phi'A\right), \qquad (11)$$

where $\mathcal{H} = a'/a$. The remaining components of the perturbed EST are not relevant to the computations done in this letter, and we will give them in a future work. Under a gauge transformation of the form $(\tau, x^i) \to (\tau + \delta\tau, x^i + \delta x,^i)$ the perturbations $\delta\phi$, A, B, C, and E transform according to[13]

$$\delta\phi \to \delta\phi - \phi'\delta\tau, \qquad (12)$$

$$A \to A - \mathcal{H}\delta\tau - \delta\tau', \qquad (13)$$

$$B \to B + \delta\tau - \delta x', \qquad (14)$$

$$C \to C + \mathcal{H}\delta\tau, \qquad (15)$$

$$E \to E - \delta x. \qquad (16)$$

4. Evolution of curvature perturbations in the unitary gauge

In single scalar field models the unitary gauge is defined by the condition $\delta\phi_u = 0$. From the gauge transformation in eq.(12) we can see that the time translation $\delta\tau_u$ necessary to go to the unitary gauge is given by

$$\delta\tau_u = \frac{\delta\phi}{\phi'}. \qquad (17)$$

Using eq.(15) we can compute the curvature perturbation in the unitary gauge ζ

$$\zeta \equiv -C_u = -C - \mathcal{H}\delta\tau_u = -C - \mathcal{H}\frac{\delta\phi}{\phi'}. \qquad (18)$$

which is by construction gauge invariant. We can also define other gauge invariant quantities such as the unitary gauge lapse function

$$A_u \equiv A - \mathcal{H}\delta\tau_u - \delta\tau_u' = A - \mathcal{H}\frac{\delta\phi}{\phi'} - \left(\frac{\delta\phi}{\phi'}\right)'. \qquad (19)$$

The second order action for ζ in Horndeski's theories was computed in[14]

$$S_\zeta^{(2)} = \int dt d^3 x a^3 \left[\mathcal{G}_S \dot\zeta^2 - \frac{\mathcal{F}_S}{a^2} (\partial_i \zeta)^2 \right] , \tag{20}$$

where $\mathcal{G}_S$ and $\mathcal{F}_S$ are functions of $K(\phi, \chi)$ and $G(\phi, \chi)$ and their derivatives. The Lagrange equations for this action give the equation of motion of ζ

$$\zeta'' + \left(2\mathcal{H} + \frac{\mathcal{G}_S'}{\mathcal{G}_S} \right) \zeta' - c_s^2 \overset{(3)}{\Delta} \zeta = 0 , \tag{21}$$

where $c_s^2(\tau) = \mathcal{F}_S / \mathcal{G}_S$.

For the Fourier transform of the above equation we use the notation

$$\zeta_k'' + \left(2\mathcal{H} + \frac{\mathcal{G}_S'}{\mathcal{G}_S} \right) \zeta_k' + c_s^2 k^2 \zeta_k = 0 . \tag{22}$$

5. Comoving slices gauge in G-inflation

The comoving slices gauge is defined by the condition $\delta T^0{}_i = 0$. In G-inflation, combing eqs.(12-13) with eq.(11) we have that under an infinitesimal time translation

$$\delta T^0{}_i \to \delta T^0{}_i + \partial_i \left(\frac{\phi'^2}{a^4} D \delta \tau \right) , \tag{23}$$

where

$$D = a^2 (2G_\phi + K_\chi) + G_\chi (-4\mathcal{H}\phi' + \phi'') , \tag{24}$$

from which we get the time translation $\delta\tau_c$ required to go to the comoving slices gauge

$$\delta\tau_c = \frac{1}{\phi' D} \left[-\phi' G_\chi (3\mathcal{H}\delta\phi + \phi' A - \delta\phi') + a^2 (2G_\phi + K_\chi)\delta\phi \right] . \tag{25}$$

Note that in the particular case in which G does not depend explicitly on χ, i.e. $G(\phi, \chi) = G(\phi)$ the above transformation reduces to

$$\delta\tau_c = \frac{\delta\phi}{\phi'} , \tag{26}$$

and the comoving gauge coincides with the unitary gauge, since in this case the system is equivalent to a $K(X)$ theory.[15, 16]

The comoving curvature perturbation $\mathcal{R}$ is then defined as

$$\mathcal{R} \equiv -C_c = -C - \mathcal{H}\delta\tau_c . \tag{27}$$

Our goal is to derive the relation between ζ and $\mathcal{R}$, and we can achieve this by performing the gauge transformation between the unitary and comoving slices gauge.

Using the general gauge transformation defined in eq.(25), when $\delta\phi = 0$ and $A = A_u$, we get

$$\delta\tau_{uc} = -\frac{\phi' G_\chi}{D} A_u , \tag{28}$$

from which we obtain

$$\mathcal{R} = \zeta + \mathcal{H}\frac{\phi' G_\chi}{D}A_u \,. \tag{29}$$

The gauge invariant variable A_u can be expressed in terms of ζ using the perturbed Einstein's equation $\delta G^0{}_i = \delta T^0{}_i/M_{Pl}^2$ in the unitary gauge, which using eq.(11) gives

$$-\zeta' + \mathcal{H}A_u = -\frac{\phi'^3 G_\chi}{2M_{Pl}^2 a^2}A_u \,. \tag{30}$$

We can then combine eq.(29) and eq.(30) to obtain the relation between $\mathcal{R}$ and ζ only

$$\mathcal{R} = \zeta + \mathcal{H}\frac{\phi' G_\chi}{D}\left(\frac{\phi'^3 G_\chi}{2M_{Pl}^2 a^2} + \mathcal{H}\right)^{-1}\zeta' = \zeta + \mathcal{E}(\tau)\zeta' \,, \tag{31}$$

where we have defined the enhancement factor $\mathcal{E}(\tau)$, a quantity depending only on the background, which can induce a significant difference between the curvature perturbations on comoving and uniform field slices. The relation between the power spectrum of ζ and $\mathcal{R}$ is then given by

$$P_\mathcal{R} = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = P_\zeta + \frac{k^3}{2\pi^2}\Delta \,, \tag{32}$$

where

$$\Delta = \left[\mathcal{E}\zeta^*\zeta' + \mathcal{E}^*\zeta'^*(\zeta + \mathcal{E}\zeta')\right] \,. \tag{33}$$

Note that the above relations are valid on any scale, since they are just based on gauge transformations, without assuming any sub or super horizon limit. This implies that the spectra of $\mathcal{R}$ and ζ could be different due to a change in the evolution of both sub-horizon and super-horizon modes during the time interval when $\mathcal{E}(\eta)$ is large. On sub-horizon scales the effect is always present, since ζ is oscillating and $\zeta' \neq 0$, while for super-horizon scales the effect could be suppressed if $\zeta \approx 0$, but even for models conserving ζ there could be an effect, since the freezing does not happen immediately after horizon crossing. We will discuss later the implication on the production of PBHs.

6. Conservation of $\mathcal{R}$ and ζ

From eq.(31) we can reach the important conclusion that

$$\zeta = const \Rightarrow \zeta = \mathcal{R} = const \,; \tag{34}$$

however the opposite is not true, i.e.

$$\mathcal{R} = const \nRightarrow \zeta = const \,, \tag{35}$$

which can have important implications for conservation laws of $\mathcal{R}$ and non-Gaussianity consistency conditions.[17] As explained previously, $\mathcal{R}$ is the quantity

related to observations, so it would be inconsistent to infer constraints on ζ from CMB observations for example, since the latter depend on $\mathcal{R}$. From a theoretical point of view the models approximately conserving ζ on super-horizon scales may be incompatible with observations for large enhancement functions $\mathcal{E}(\tau)$, because $\mathcal{R}$ could be not conserved, implying for example a violation of the non-Gaussianity consistency condition or a miss-estimation of PBHs abundance.

Nevertheless it should be noted that the super-horizon growth of perturbations cannot last indefinitely, or the entire perturbative treatment of the problem would breakdown, leading to inhomogeneities much larger than those imprinted in the CMB for example. For this reason it is expected that for any model compatible with observations the super-horizon growth of ζ should be only temporary, and according to eq.(34), at some time after horizon crossing $\zeta \approx \mathcal{R}$. This simplifies the calculation of $\mathcal{R}$, whose evolution can be then traced during and after reheating, too, contrary to ζ.

The only exception to this argument could be very small scales ζ modes which leave the horizon very late, and whose super-horizon growth could continue until horizon re-enter, without affecting the validity of the perturbative treatment of the problem. For these small scale modes the difference between $\mathcal{R}$ and ζ could be important, but it would still be computationally convenient to solve the equation for ζ and then obtain $\mathcal{R}$ using the gauge transformation given in eq.(31).

7. Enhancement of curvature perturbations

As already observed for comoving curvature perturbation $\mathcal{R}$ in general relativity for standard kinetic term single field models,[18] a temporary violation of slow-roll conditions can lead to the anomalous growth of what would normally be a decaying mode. A similar mechanism can induce the growth of ζ, as we will show in this section. We can re-write eq.(22) in the form

$$\frac{\mathrm{d}}{\mathrm{d}a}\left(a^3 \mathcal{H} \mathcal{G}_S \frac{\mathrm{d}\zeta_k}{\mathrm{d}a}\right) + a\mathcal{F}_S \frac{k^2}{\mathcal{H}}\zeta_k = 0 \,, \tag{36}$$

from which it is possible to find a super-horizon scale solution of the form

$$\zeta_k = A + B \int \frac{\mathrm{d}a}{a} f \,, \quad f = \frac{1}{a^2 \mathcal{H} \mathcal{G}_S} \,, \tag{37}$$

where A and B are constants. For standard slow-roll models the function f decreases as the scale factor increases, implying that ζ tend to a constant value, i.e. the second term in eq.(37) is a decaying mode. If the function f is a growing function of a then the second term in eq.(37) becomes a growing mode, and there can be a super-horizon growth. It follows that the general condition for super-horizon growth of ζ_k is then

$$\frac{\mathrm{d}f}{\mathrm{d}a} \geq 0 \,, \tag{38}$$

or equivalently

$$\frac{\mathrm{d}f}{\mathrm{d}a} = \frac{1}{a'}\frac{\mathrm{d}f}{\mathrm{d}\tau} = \frac{1}{a\mathcal{H}}f' \geq 0 \,. \tag{39}$$

During inflation $a\mathcal{H} > 0$ and this condition reduces to

$$f' \geq 0 \,. \tag{40}$$

In the case of a minimally coupled single scalar field the unitary gauge and the comoving gauge coincide, and the general condition given above takes the form[18]

$$3 - \epsilon + \eta \leq 0 \,, \tag{41}$$

where the slow-roll parameters are defined according to

$$\epsilon \equiv -\frac{a}{\mathcal{H}^2}\left(\frac{\mathcal{H}}{a}\right)' = \frac{a^2(\overline{\rho}+\overline{P})}{2M_{Pl}^2\mathcal{H}^2} \quad , \quad \eta \equiv \frac{\epsilon'}{\epsilon\mathcal{H}} \,. \tag{42}$$

In G-Inflation the condition given in eq.(40) implies that

$$f' = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{1}{a^2\mathcal{H}\mathcal{G}_S}\right) = \frac{3 - \epsilon + \mathcal{G}_S'/\mathcal{H}\mathcal{G}_S}{a^2\mathcal{G}_S} = \frac{\gamma}{\delta} \leq 0 \,, \tag{43}$$

which gives the general condition for super-horizon growth in an expanding Universe. For a contracting Universe the inequality would be inverted.

As can be seen from the above equation the super-horizon growth can be achieved in different cases, corresponding to γ and δ having opposite signs, contrary to what happened for the standard kinetic term single field scenario, in which δ sign is fixed. Note also that contrary to standard kinetic term single field models, the super-horizon growth does not depend only on the slow-roll parameters, implying that it can occur also during slow-roll.

The anomalous super-horizon growth of ζ, and consequently of $\mathcal{R}$, can increase the abundance of PBHs, since it affects the statistics of the density perturbations peaks which can seed the PBHs.

8. Production of primordial black holes

The super-horizon growth of $\mathcal{R}_k$ could produce primordial black holes which could possibly account for part of dark matter[2,4,7,19–26] and produce gravitational waves (GW) detectable with future GW detectors such as LISA.[5,7] In this session we will show how to obtain some approximate estimation of the effetcs of the modification of gravity on the PBH production, without considering any specific model, leaving this to a future work.

The mass M of PBHs produced by the mode $\mathcal{R}_k$ re-entering the horizon during the radiation domination can be approoximated as[7]

$$M = \gamma M_H\Big|_F \,, \tag{44}$$

where $\gamma \approx 0.2$ is a correction factor, and $M_H\big|_F$ is the horizon mass $M_H \equiv (4\pi/3)\overline{\rho}(a\mathcal{H})^{-3}$ at the time of PBH formation, corresponding to the horizon crossing time

$$k = (a^2\mathcal{H})\big|_F . \tag{45}$$

Note the above is just a rough estimation, and a more accurate treatment would involve the use of a scaling relation.[27, 28]

The present time fraction f_{PBH} of PBHs of mass M against the total dark matter component can then be approximated as[7]

$$f = 2.7 \times 10^8 \left(\frac{\gamma}{0.2}\right)^{1/2} \left(\frac{g_{*F}}{106.75}\right)^{-1/4} \left(\frac{M}{M_\odot}\right)^{-1/2} \beta ,$$

where g_{*F} is the number of relativistic degrees of freedom at formation, the quantity β is the energy density fraction of PBHs at formation time

$$\beta \equiv \frac{\overline{\rho}_{PBH}}{\overline{\rho}}\bigg|_F , \tag{46}$$

which can be written in terms of the probability of the density contrast $P(\delta)$ as[3, 29]

$$\beta(M) = \gamma \int_{\delta_t}^{1} P(\delta)\mathrm{d}\delta , \tag{47}$$

where δ_t is the threshold for PBH formation. Assuming the density perturbations follow a Gaussian distribution β is given by

$$\beta(M) \approx \frac{\gamma}{\sqrt{2\pi}\nu(M)} \exp\left[-\frac{\nu(M)^2}{2}\right] , \tag{48}$$

where $\nu(M) \equiv \delta_t/\sigma(M)$, and $\sigma(M)$ is an estimation of the standard deviation of the density contrast on scale R from the variance

$$\sigma^2(M) = \int \mathrm{d}\ln k W^2(kR)\mathcal{P}_\delta(k) = \int \mathrm{d}\ln k W^2(kR)\left(\frac{16}{81}\right)(kR)^4\mathcal{P}_\mathcal{R}(k) , \tag{49}$$

where $W(kR)$ is a window function smoothing over the comoving scale $R(M) = (a^2\mathcal{H})^{-1}\big|_F = 2GM/a_F\gamma^{-1}$, and the relation between δ and $\mathcal{R}$ has been used in the second equality. It should be mentioned that eq.(48) can be used as a guideline, but more accurate calculations would involve the use of the results of numerical simulations.[30, 31] The choice of the window function could also affect[32, 33] the results of the calculation.

Our aim here is not make an accurate estimation of the PBHs abundance for a specific model, but to show why in general it can be impacted by the modification of gravity, and the approximations adopted so far are enough to serve this general purpose. According to the equations above, the PBH fraction β is affected by the

power spectrum of $\mathcal{R}$ since this can increase the standard deviation of the density field $\sigma(M)$. Note that the above approximations to estimate the PBHs abundance can receive important corrections depending on the shape of power spectrum, on non- gaussianity, and non-linear statistics.[34–36] Due to the importance of all these different effects it is difficult to find a general model independent analytical formula to estimate the PBHs abundance for a generic G-inflation theory, but any enhancement of the power spectrum is expected, according to eq.(32), to affect the probability of production of PBHs. Beside this, numerical relativity simulations of the PBHs formation are based on general relativity, so the effects of the modification of gravity on the process of gravitational collapse are at the moment not fully understood and would require investigations beyond the scope of this paper.[37]

At the end of its anomalous super-horizon growth, ζ will coincide with $\mathcal{R}$, and the consequent enhancement of the spectrum will lead to an increased PBH abundance. Contrary to what happens for standard kinetic term single field models in general relativity,[18] in the case of G-inflation this power spectrum enhancement can be achieved also during slow-roll, as long as the condition in eq.(43) is satisfied, which can be attained by an appropriate choice of the function $\mathcal{G}_S$. We expect a similar behavior for more complex modified gravity theories as well.

9. Conclusions

We have computed the effective energy-stress-tensor for G-inflation theories in the comoving slices gauge and have used it to derive a general relation between the unitary gauge curvature ζ and the comoving curvature perturbation $\mathcal{R}$, involving an enhancement function which depends on the evolution of the background, and which can cause a large difference between the two gauge invariant quantities.

When ζ is not constant in time it differs from $\mathcal{R}$, for example on sub-horizon scales, or in models exhibiting an anomalous super-horizon growth of ζ, but since this growth cannot last indefinitely, eventually they will coincide. We have derived the general condition for super-horizon growth of ζ, showing that slow-roll violation is not necessary, and discussed how the the enhancement of the spectrum of $\mathcal{R}$ can affect the PBH abundance.

We expect similar results to hold for other modified gravity theories such as other Horndeski's theories,[1] since also for these theories there can be effective entropy or anisotropy terms which can modify the evolution of curvature perturbations. In the future it will be interesting to extend this study to other modified gravity theories or to multi-fields systems, and to use observations to constraints the different types of theories. It would also be important to perform numerical simulations of the PBHs formation taking into account the non perturbative effects of the modification of gravity on the process of black hole formation.

References

1. G. W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, *Int. J. Theor. Phys.* **10**, 363 (1974).
2. K. M. Belotsky, A. D. Dmitriev, E. A. Esipova, V. A. Gani, A. V. Grobov, M. Yu. Khlopov, A. A. Kirillov, S. G. Rubin and I. V. Svadkovsky, Signatures of primordial black hole dark matter, *Mod. Phys. Lett.* **A29**, p. 1440005 (2014).
3. B. J. Carr, The primordial black hole mass spectrum., *Astrophysical Journal* **201**, 1 (Oct 1975).
4. M. Y. Khlopov, Primordial Black Holes, *Res. Astron. Astrophys.* **10**, 495 (2010).
5. M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914, *Phys. Rev. Lett.* **117**, p. 061101 (2016), [erratum: Phys. Rev. Lett.121,no.5,059901(2018)].
6. A. S. Josan, A. M. Green and K. A. Malik, Generalised constraints on the curvature perturbation from primordial black holes, *Phys. Rev.* **D79**, p. 103520 (2009).
7. M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, Primordial black holes—perspectives in gravitational wave astronomy, *Class. Quant. Grav.* **35**, p. 063001 (2018).
8. M. Bucher, K. Moodley and N. Turok, The General primordial cosmic perturbation, *Phys. Rev.* **D62**, p. 083508 (2000).
9. A. E. Romano, S. Mooij and M. Sasaki, Adiabaticity and gravity theory independent conservation laws for cosmological perturbations, *Phys. Lett.* **B755**, 464 (2016).
10. T. Kobayashi, M. Yamaguchi and J. Yokoyama, G-inflation: Inflation driven by the Galileon field, *Phys. Rev. Lett.* **105**, p. 231302 (2010).
11. S. Hirano, T. Kobayashi and S. Yokoyama, Ultra slow-roll G-inflation, *Phys. Rev. D* **94**, p. 103515 (2016).
12. C. Deffayet, O. Pujolas, I. Sawicki and A. Vikman, Imperfect Dark Energy from Kinetic Gravity Braiding, *JCAP* **1010**, p. 026 (2010).
13. H. Kodama and M. Sasaki, Cosmological Perturbation Theory, *Prog. Theor. Phys. Suppl.* **78**, 1 (1984).
14. T. Kobayashi, M. Yamaguchi and J. Yokoyama, Generalized G-inflation: Inflation with the most general second-order field equations, *Prog. Theor. Phys.* **126**, 511 (2011).
15. A. E. Romano, General background conditions for K-bounce and adiabaticity, *Eur. Phys. J.* **C77**, p. 147 (2017).
16. J. Garriga and V. F. Mukhanov, Perturbations in k-inflation, *Phys. Lett.* **B458**, 219 (1999).
17. A. E. Romano, S. Mooij and M. Sasaki, Global adiabaticity and non-Gaussianity consistency condition, *Phys. Lett.* **B761**, 119 (2016).
18. R. Saito, J. Yokoyama and R. Nagata, Single-field inflation, anomalous enhancement of superhorizon fluctuations, and non-Gaussianity in primordial black hole formation, *JCAP* **06**, p. 024 (2008).
19. B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, New cosmological constraints on primordial black holes, *Phys. Rev. D* **81**, p. 104019 (2010).
20. B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Constraints on Primordial Black Holes (2 2020).
21. J. Yokoyama, Formation of MACHO primordial black holes in inflationary cosmology, *Astron. Astrophys.* **318**, p. 673 (1997).
22. J. Garcia-Bellido and E. Ruiz Morales, Primordial black holes from single field models of inflation, *Phys. Dark Univ.* **18**, 47 (2017).
23. J. Garcia-Bellido, A. D. Linde and D. Wands, Density perturbations and black hole formation in hybrid inflation, *Phys. Rev.* **D54**, 6040 (1996).

24. P. Ivanov, P. Naselsky and I. Novikov, Inflation and primordial black holes as dark matter, *Phys. Rev.* **D50**, 7173 (1994).
25. G. F. Chapline, Cosmological effects of primordial black holes, *Nature* **253**, 251 (Jan 1975).
26. K. M. Belotsky, V. I. Dokuchaev, Y. N. Eroshenko, E. A. Esipova, M. Yu. Khlopov, L. A. Khromykh, A. A. Kirillov, V. V. Nikulin, S. G. Rubin and I. V. Svadkovsky, Clusters of primordial black holes, *Eur. Phys. J.* **C79**, p. 246 (2019).
27. J. Yokoyama, Cosmological constraints on primordial black holes produced in the near critical gravitational collapse, *Phys. Rev. D* **58**, p. 107502 (1998).
28. J. C. Niemeyer and K. Jedamzik, Near-critical gravitational collapse and the initial mass function of primordial black holes, *Phys. Rev. Lett.* **80**, 5481 (1998).
29. A. M. Green, A. R. Liddle, K. A. Malik and M. Sasaki, A New calculation of the mass fraction of primordial black holes, *Phys. Rev.* **D70**, p. 041502 (2004).
30. T. Nakama, T. Harada, A. G. Polnarev and J. Yokoyama, Identifying the most crucial parameters of the initial curvature profile for primordial black hole formation, *JCAP* **01**, p. 037 (2014).
31. M. Shibata and M. Sasaki, Black hole formation in the Friedmann universe: Formulation and computation in numerical relativity, *Phys. Rev. D* **60**, p. 084002 (1999).
32. K. Ando, K. Inomata and M. Kawasaki, Primordial black holes and uncertainties in the choice of the window function, *Phys. Rev. D* **97**, p. 103528 (2018).
33. K. Tokeshi, K. Inomata and J. Yokoyama, Window function dependence of the novel mass function of primordial black holes, *JCAP* **12**, p. 038 (2020).
34. C. Germani and I. Musco, Abundance of Primordial Black Holes Depends on the Shape of the Inflationary Power Spectrum, *Phys. Rev. Lett.* **122**, p. 141302 (2019).
35. V. Atal and C. Germani, The role of non-gaussianities in Primordial Black Hole formation, *Phys. Dark Univ.* **24**, p. 100275 (2019).
36. C. Germani and R. K. Sheth, Nonlinear statistics of primordial black holes from Gaussian curvature perturbations, *Phys. Rev. D* **101**, p. 063520 (2020).
37. C.-Y. Chen, Threshold of primordial black hole formation in modified theories of gravity (2019).