THE INTERACTIONS IN $K_{\mu 3}$ AND K_{e3} DECAY

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INTRODUCTION

An attractive feature common to all present theories of weak interactions is the assumption that they result from the coupling (either direct or via intermediate bosons) of vector-axial vector currents. The V-A coupling of leptonic currents and baryonic currents conserving strangeness successfully explains the characteristic properties of β-decay, π-decay and μ-decay. On the other hand, the leptonic decay modes of K-mesons provide the best source of information on the nature of the currents carrying strangeness. From $K_{\mu 2}$ one learns that there must exist an axial-vector current carrying strangeness; further, if the leptonic weak interaction is universal, one can explain the absence of the K_{e2} mode by excluding a pseudo-scalar interaction. From $K_{\mu 3}$ and K_{e3} decay one can obtain information on scalar, vector and tensor coupling of strangeness non-conserving currents with leptons. In recent experiments on K_{e3} decay [1, 2] a good fitting of the pion spectrum was obtained with a constant vector form factor. Although regarding this result as strong evidence of pure vector coupling, one might still argue that other possibilities, such as a mixture of vector and tensor couplings with energy-dependent form factors, are not excluded.

I shall discuss here, in the first place, some properties of the transition amplitudes which are independent of the structure of form factors. They provide a test of universality and a criterion for unambiguous determination of the nature of the weak interaction involved in these processes. Secondly, I shall discuss the structure of form factors, by means of dispersion relations and introducing explicitly the effect of the K^*-resonance in the $K\pi$-interaction.

1. NATURE OF INTERACTION

The basic assumption involved in this discussion is that the lepton pair is locally produced through an interaction containing no derivative coupling (Fig. 1).

1.1. Kinematics

The kinematical configuration of the decay products depends on two scalar variables. I choose them as the total energy W of the lepton pair in its centre-of-mass system and the angle α between the direction of the neutrino and pion in that system. The advantage of this choice of variables will soon be-

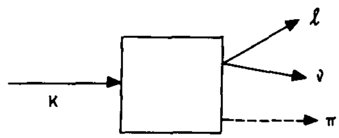

Fig. 1

$K \to \pi + \ell + \nu$

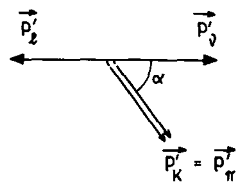

Fig. 2

K-decay kinematics in the centre-of-mass system of the lepton pair

come apparent (Fig. 2). These variables are related to the energies E_π, E_ℓ, E_ν for decay at rest by

$$W^2 = m_K^2 + m_\pi^2 - 2 m_K E_\pi , \tag{1}$$

$$\cos\alpha = [(E_\ell - E_\nu) W^2 - m_\ell^2 (m_K - E_\pi)] / (W^2 - m_\ell^2)\, p_\pi . \tag{2}$$

The range of values of W^2 is independent of $\cos\alpha$: $[m_\ell^2,\ (m_K - m_\pi)^2]$.

1.2. Dynamics

The general form of the matrix element is

$$M^I = (4\, p_{K0} p_{\pi 0})^{\frac{1}{2}} \langle \pi \,|\, J_\alpha^I(0)\, | K \rangle \bar{u}_\ell O^\alpha u_\nu = \bar{u}_\ell \mathfrak{M}^I u_\nu , \tag{3}$$

$$\mathfrak{M} = \frac{1}{m_K} g_s + \frac{1}{m_K^2} \gamma_\alpha (g_\nu p_K^\alpha + g'_\nu p_\pi^\alpha) + \frac{i}{m_K^3} g_T \sigma_{\alpha\beta} p_K^\alpha p_\pi^\beta . \tag{4}$$

The index I in (3) stands for the isospin of the current. The g's are form factors that, following our basic assumption, depend only on the pion energy E_π. They are real if time reversal invariance holds.

1.3. Transition rate [3]

$$dT = \frac{1}{(4\pi m_K)^3} \rho(s, \cos\alpha) \frac{p_\pi}{m_K}\left(1 - \frac{m_\ell^2}{s}\right) ds \, d\cos\alpha, \tag{5}$$

$$\rho = \frac{1}{4}\left(1 - \frac{m_\ell^2}{s}\right)\left\{ s\left|(g_s + \frac{m_\ell}{m_K} f_0) + (g_T - \frac{m_\ell m_K}{s} f_1)\frac{p_\pi}{m_K}\cos\alpha\right|^2 + \left|f_1 - \frac{m_\ell}{m_K} g_T\right|^2 p_\pi^2 \sin^2\alpha \right\}, \tag{6}$$

where $s = W^2$ and

$$f_0 = (1/2)(g_V - g_{V'}) + (1/2)(g_V + g_V')(m_K^2 - m_\pi^2)/s$$
$$f_1 = g_V + g_V'. \tag{7}$$

One can write

$$\frac{dT}{ds \, d\cos\alpha} = \frac{1}{(4\pi m_K)^3}(a_0(s) + \sqrt{2}\, a_1(s)\cos\alpha + a_2(s)\cos 2\alpha). \tag{8}$$

1.4. Universality

I shall first draw attention to the coefficient

$$a_2(s) = \frac{p_\pi^3}{8 m_K}\left(1 - \frac{m_\ell^2}{s}\right)^3 \left(\frac{s}{m_K^2}|g_T|^2 - |f_1|^2\right)$$

which depends on the lepton mass only through the factor $(1 - \frac{m_\ell^2}{s})^3$. Hence if the couplings with muons and electrons are identical, then [3]

$$(s - m_\mu^2)^{-3} a_{2\mu}(s) \equiv (s - m_e^2)^{-3} a_{2e}(s). \tag{9}$$

The verification of this identity provides a test of the universality of the weak interaction, independent of the structure and nature of form factors.

1.5. Angular correlation

The simple structure of the angular correlation (8) stems from the fact that in its centre of mass system the lepton pair is produced in the singlet state for scalar coupling g_s and f_0 and in a triplet for vector f_1 and tensor g_T. In K_{e3}, if the electron mass is neglected, the spins would be parallel and anti-parallel, respectively, for vector and tensor coupling ($s_z = 1$ and $s_z = 0$). The advantage of our choice of variables is that the simple angular correlation is preserved after integration over the pion energy or any part of the pion spectrum. This angular correlation is particularly suitable for the identification of the type of coupling. We give in Table I a summary of values of $\lambda_{1,2} = a_{1,2}/a_0$ for K_{e3} decay when the electron mass is neglected [3] in the angular correlation

$$(dT/d\cos\alpha) \sim 1 + \sqrt{2}\,\lambda_1 \cos\alpha + \lambda_2 \cos 2\alpha. \tag{10}$$

TABLE I

VALUE OF λ IN THE ANGULAR CORRELATION

$(dT/d\cos\alpha) \sim 1 + \sqrt{2}\,\lambda_1 \cos\alpha + \lambda_2 \cos 2\alpha$ for K_{e3} decay

	V	S	T	(VS)	(VT)	(ST)
λ_1	0	0	0	0	0	$\neq 0$
λ_2	-1	0	1	<0	$\neq 0$	>0

The absolute value of λ is always less than or equal to one. By measuring this angular correlation one obtains clear-cut discrimination of the type of coupling. In particular for pure vector coupling $(dT/d\cos\alpha) \sim \sin^2\alpha$.

1.6. K_{e3} decay

In K_{e3} decay the contribution to the transition rate of terms that depend on the electron mass is practically negligible. The expressions for the pion and electron spectrum are greatly simplified by neglecting the electron mass.

(a) Pion spectrum

$$\frac{dT}{dE_\pi} = \frac{p_\pi}{(4\pi m_K)^3}\left(s|g_s|^2 + \frac{1}{3\,m_K^2}\, s p_\pi^2 |g_T|^2 + \tfrac{2}{3}\, p_\pi^2 |f_1|^2\right). \tag{11}$$

Since at the lower end of the pion spectrum $p_\pi^2 = 0$ and at the upper end $s = 0$, these factors are expected to dominate the pion spectrum, giving specific features for each type of coupling. However, I would like to point out that a mixture of tensor and vector coupling cannot be distinguished from pure vector when the energy dependence of the form factors is taken into account.

A determination of the π^0 spectrum in K_{e3} decay was carried out by BROWN et al. [2]. They measured the angular correlation of the two γ-rays from the decay of π^0 produced in K_{e3}^+ decay at rest. The frequency dis-

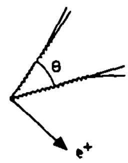

Fig. 3

K^+_{e3} decay at rest

tribution of events as a function of $u = m_\pi \operatorname{cosec}(\theta/2)$ is related to the pion spectrum by

$$F(u) = \int_u^{W_\pi} \frac{u}{\sqrt{E_\pi'^2 - u^2}} \frac{1}{p_\pi'} \frac{dT}{dE_\pi'} \, dE_\pi' \tag{12}$$

where $W_\pi = \frac{1}{2m_K}(m_K^2 + m_\pi^2 - m_l^2)$ is the maximum pion energy. Using for (dT/dE_π) the expression given by (11) with constant form factors, they obtained a remarkably good fitting taking $g_s = g_T = 0$. One can also determine directly (dT/dE_π) in terms of $F(u)$. Multiplying (12) by $1/\sqrt{u^2 - E_\pi^2}$ and integrating between E_π and W_π, one obtains, after interchanging the order of integrations on the right,

$$\int_{E_\pi}^{W_\pi} \frac{F(u)du}{\sqrt{u^2 - E_\pi^2}} = \frac{\pi}{2} \int_{E_\pi}^{W_\pi} \frac{1}{p_\pi'} \frac{dT}{dE_\pi'} \, dE_\pi' .$$

Hence,

$$\frac{1}{p_\pi} \frac{dT}{dE_\pi} = -\frac{2}{\pi} \frac{d}{dE_\pi} \int_{E_\pi}^{W_\pi} \frac{F(u)\,du}{\sqrt{u^2 - E_\pi^2}} = -\frac{2}{\pi} \int_{E_\pi}^{W_\pi} \frac{E_\pi}{\sqrt{u^2 - E_\pi^2}} \frac{d}{du}\left(\frac{F(u)}{u}\right) du. \tag{13}$$

This expression may be useful in analysing a similar experiment for $K_{\mu 3}$. The pion spectrum for pure vector coupling in K_{e3} decay is as shown in Fig. 4. The shape of the spectrum determines the vector form factor $f_1(s)$

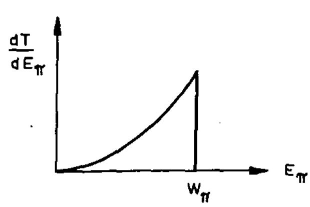

Fig. 4

Pion spectrum for pure vector coupling in K_{e3} decay which was found to be constant within experimental errors.

(b) Electron spectrum

Some definite predictions can be made about the electron spectrum if one assumes pure vector coupling [3, 4].

Let us introduce the variables

$$\eta = E\,(1 - E/m_K),$$
$$w = 2\,E\,(W_e - E)/(m_K - 2\,E), \tag{14}$$

where $W_e = (1/2\,m_K)\,(m_K^2 - m_\pi^2)$ is the maximum value of the electron energy E; w is positive in the physical region, vanishes at both ends of the spectrum ($E = 0$, $E = W_e$) and has a maximum at $E_0 = \frac{1}{2}\,(m_K - m_\pi) = 175$ MeV. The electron energy is a double-valued function of w:

$$E = \tfrac{1}{2}\,(W_e + w \pm [(W_\pi - w)^2 - m_\pi^2]^{\frac{1}{2}}).$$

Throughout this discussion the electron mass is being neglected. For pure vector coupling the spectrum $dT/d\eta$ of η is given by

$$\frac{dT}{d\eta} = \frac{1}{(4\,\pi\,m_K)^3} \int_0^{2m_K w} (w - \frac{s}{2\,m_K})\,|f_1(s)|^2\,ds \tag{15}$$

which is an increasing function of w. Hence the maximum of the spectrum is at the energy E_0, i.e. $\eta = W_e/2$, independently of the structure of the form factor (Fig. 5). For any combination (VS) or (VT) one can show that the maximum would be at a higher energy. If $dT/d\eta$ is plotted against w, the two branches, corresponding to energies $E < E_0$ and $E > E_0$, will coincide for pure vector coupling (Fig. 6); for any combination (VS) or (VT) the first branch always remains below the second one.

The common shape of the two branches in Fig. 6 depends on the structure of the vector form factor $f_1(s)$. One can show that

$$|f_1(s)|^2 = \frac{(4\pi\,m_K)^3}{2\,m_K}\;\frac{d^2}{d\,w^2}\left(\frac{dT}{d\eta}\right)_{w\,=\,s/2m_K}. \tag{16}$$

Since the form factor depends only on the second derivative of the electron spectrum, this spectrum does not provide a good determination of $f_1(s)$. Conversely, one can conclude that the electron spectrum is not very sensitive to the structure of the vector form factor.

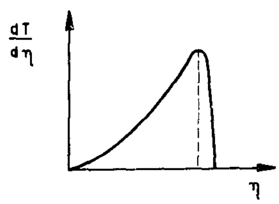

Fig. 5

η spectrum for pure V-coupling

Maximum of $\frac{dT}{d\eta}$ at η = 112 MeV

Maximum value of η = 122 MeV

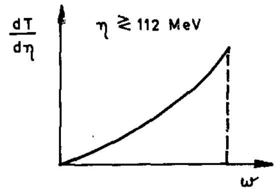

Fig. 6

K_{e3} decay plot for V-coupling

1.7. Polarization

We have seen that from K_{e3} decay one can obtain the vector form factor $f_1(s)$, but $f_0(s)$ has to be determined from $K_{\mu 3}$ decay. Here besides the energy

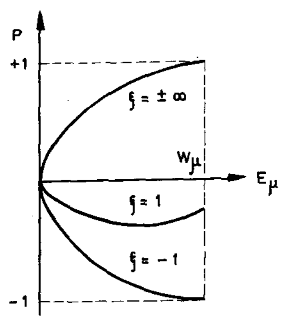

Fig. 7

Energy dependence of longitudinal polarization of muons

spectra and angular correlation one has another source of information, namely, the longitudinal polarization of muons. In Fig. 7 this polarization is given for pure vector coupling and constant form factors, for different values of the ratio $\xi = (g_V - g_V')/(g_V + g_V')$.

2. FORM FACTORS

I turn now to our second topic, an investigation of the structure of form factors. I restrict myself to vector form factors.

One starts with the assumption that

$$f_+ = g_V + g_V' = f_1 \quad \text{and} \quad f_- = g_V - g_V'$$

are analytic functions of s with a cut from $(m_K + m_\pi)^2$ to $+\infty$. The discontinuity across this cut is given by the absorptive part of the matrix element $(4\, p_{\pi 0}\, p_{K0})^{\frac{1}{2}} \langle 0 | J_\alpha | \pi k \rangle$. One can then write dispersion relations for f_+ and f_- which might require subtractions, depending on the behaviour at infinity. I shall assume that the dispersion relation for f_+ needs one subtraction and that for f_- no subtraction [5]. According to this assumption, f_- is entirely induced by strong interactions. One can show that the contribution to the absorptive amplitude of nucleon-anti-hyperon intermediate states is compatible with this assumption [5]. Another possible justification is to invoke some higher symmetry in strong interactions which would ensure the conservation of the current. Suppose that there is a higher symmetry in the strong interactions connected with the conservation of a current carrying strangeness. In the level where this possible underlying symmetry holds, the masses of non-strange and strange particles within the same multiplet would be equal, in particular the kaon and pion masses. In order to get the mass splitting some interactions have to be introduced that break symmetry. However, the existence of such higher symmetry would still lead to some definite consequences. When the interactions that break the symmetry are turned on and the mass differences are thereby introduced, the current carrying strangeness is no longer conserved. However, "in the limit where the mass differences within a multiplet are neglected, the current is conserved. In this limit the divergence of the current is zero". One speaks then of a partially conserved current. Now the matrix element of the divergence of the current between the states of a kaon and a pion is given by

$$(4\, p_{\pi 0}\, p_{K0})^{\frac{1}{2}}\, \partial^\alpha \langle \pi | J_\alpha(x) | K \rangle = \frac{1}{2\, m_K^2} [f_+ (m_K^2 - m_\pi^2) + s f_-]\, e^{-i(p_K - p_\pi)\cdot x}, \quad (17)$$

and this must be zero when $(m_K^2 - m_\pi^2) \to 0$, which implies that

$$\lim_{(m_K - m_\pi) \to 0} f_-(s) = 0. \quad (18)$$

But at sufficiently high energies the mass differences can be neglected; therefore condition (18) leads to

$$\lim_{s \to \infty} f_-(s) = 0, \quad (19)$$

which is the assumption we have made. Therefore this assumption holds in a theory where there exists an underlying symmetry of strong interactions that ensures the conservation of a current carrying strangeness. However, one should point out that from (19) the existence of a partially conserved current does not follow necessarily. A slightly different version of a partially conserved current hypothesis was proposed by BERNSTEIN and WEINBERG [5a]. They require the vanishing of (17) as $s \to \infty$. This requirement is stronger than (18).

Now we have the dispersion relations:

$$f_+(s) = f_+(0) + \frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} \frac{s}{s'}\,\frac{ds'}{s'-s}\,\mathrm{Im}\;f_+(s'),$$
$$f_-(s) = \frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'-s}\,\mathrm{Im}\;f_-(s'). \qquad (20)$$

The form factor f_+ corresponds to the state of the $K\pi$ system with $J = 1$ and $f_0 = \frac{1}{2}(f_- + \frac{m_K^2 - m_\pi^2}{s} f_+)$ to the state with $J = 0$. Thus f_- is coupled to f_+ by unitarity. Now I shall take into account in Im f only the contribution from the lowest massive intermediate state, which is a $K\pi$-state, and neglect the contributions from states of higher masses. In this way the relations (20), together with unitarity, become linear integral equations, which can be solved in terms of S and P wave $K\pi$-phase shifts [5]; f_+ depends only on δ_1 and f_0 on δ_0. One knows about the $K\pi$-interaction that it has a resonance K^* at $W = 888$ MeV with a width $\Gamma \lesssim 50$ MeV in the state $I = \frac{1}{2}$, $J = 0$ or $J = 1$. The effect of the width is negligible in the region of energies we are interested in $(0 < s < (m_K - m_\pi)^2)$, and one can replace the resonant phase shift by a step function $\theta(s - m_{K^*}^2)\pi$; the other phase shift will be neglected. The results of our calculations are equivalent to the Feynman diagrams shown in Fig. 8.

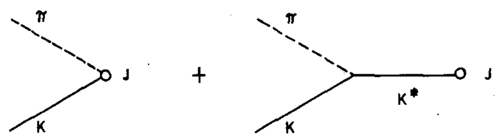

Fig. 8

One obtains [6]:

(a) $J_{K^*} = 0$

$$f_+(s) = f_+(0),$$
$$f_-(s) = \frac{m_K^2 - m_\pi^2}{m_{K^*}^2 - s}\,\chi\, f_+(0), \qquad (21)$$

(b) $J_{K^*} = 1$

$$f_+(s) = \left(1 + \frac{\chi s}{m_{K^*}^2 - s}\right) f_+(0),$$
$$f_-(s) = \frac{m_K^2 - m_\pi^2}{m_{K^*}^2 - s}\,\chi\, f_+(0). \qquad (22)$$

The parameter χ measures the influence of K* in the decay. The effect of an intermediate vector boson would be the same as of a vector K* with $\chi = 1$. The above expressions should then be modified in the following way:

$$f_+(s) \to f_+(s)\frac{M^2}{M^2 - s} \; ; \; f_-(s) \to f_-(s) - f_+(s)\frac{m_K^2 - m_\pi^2}{M^2 - s} \qquad (23)$$

where M is the mass of the vector boson which has to be greater than the kaon mass $M > m_K$.

3. EXPERIMENTAL RESULTS

The experimental value for the branching ratio $(K_{\mu 3}/K_{e3})$ is [7] R = 0.96 ± 0.15. Assuming universal vector interaction with constant form factors, the branching ratio is given in terms of $\xi = f_-/f_+$ by [8]

$$R = 0.651 + 0.126\,\xi + 0.019\,\xi^2 \qquad (24)$$

which gives two possible values for ξ:

$$\xi = 1.85 \pm 0.8 \text{ and } \xi = -8.15 \pm 0.85.$$

In order to decide between the two solutions the muon spectrum was measured by two different groups, BROWN et al. [7] and DOBBS et al. [9]. Unfortunately their data are incompatible. The first group obtains the best fit with $\xi = 1.46$ while the second one found $\xi = -9$. The pion spectrum in $K_{\mu 3}$ might provide a better determination of ξ.

In addition to the branching ratio one knows from the K_{e3} experiment [2, 7] that $f_+(s)$ has very little energy dependence. It was found that

$$m_K^2\, f_+'(0)/f_+(0) = 0.45 \pm 0.56. \qquad (25)$$

Let us compare these results with the theoretical predictions. If the spin of K* is one then one, obtains

$$m_K^2\, f_+'(0)/f_-'(0) = -m_K^2/(m_K^2 - m_\pi^2) = -1.1 \qquad (26)$$

even if there exists an intermediate vector boson. Then

$$m_K^2\, f_+'(0)/f_+(0) = -1.1\,(f_-(0)/f_+(0)). \qquad (27)$$

Taking into account (25) one can see that the value $\xi = -9$ is inconsistent with this result. The other value $\xi = 1.46$ is barely compatible.

For scalar K* all the energy dependence of $f_+(s)$ would come from the intermediate vector boson and is consistent with (25).

Another theoretical prediction valid in the absence of an intermediate vector boson is

$$m_K^2 f_-'(0)/f_-(0) = (m_K/m_{K^*})^2 = 0.3 \quad (28)$$

for both values of the K* spin.

It should be pointed out that these predictions can be made only in so far as neglecting the contributions to the dispersion integrals of higher mass intermediate states is actually justified.

A last point I want to make very briefly is concerned with the isospin of the current. So far we have been dealing with a I = 1/2 current. There is now evidence for a mixture of I = 1/2 and I = 3/2 currents. Then the form factors for K^+, K_1 and K_2 decay will not be the same. However, if the assumptions we have made hold for both currents and there exists no sizeable $K\pi$-interaction in the I = 3/2, S and P states, then the structure of the form factors remain unchanged; in particular, one still obtains the relations (26) and (28) for all three decays.

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