

Dark matter and radiative neutrino masses

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Abstract. The $R\nu$ MDM model relates radiative seesaw and minimal dark matter mass scales without imposing any beyond the Standard Model (SM) gauge symmetry. The model contains, besides the SM particles, a Majorana fermion multiplet N_R and a scalar multiplet χ which transform under the SM gauge group as $(1, 5, 0)$ and $(1, 6, -1/2)$. We study the dark matter physics of this model and the relevant lepton flavor violating processes.

1. Introduction

The existence of dark matter has been firmly established for decades by various astrophysical and cosmological experiments [1]. The nature of dark matter, however, has remained elusive since then, except for the fact that dark matter make up 23.3% of the universe [1]. Physicists have proposed many theoretical models accommodating this ingredient, a large majority of which introduce certain discrete symmetry to stabilize the dark matter particle. It is also possible to stabilize the dark matter particle without imposing any new symmetry through the minimalistic approach, by adding specific SM gauge multiplets such that any dark matter decay channel is automatically forbidden by the SM gauge symmetry itself [2]. The extra SM gauge multiplet is chosen according to three conditions: (a) there is an electrically neutral component in the multiplet; (b) the neutral component has the lightest mass; (c) this component is automatically stable on cosmological time-scales. The relic abundance of the dark matter candidate is then completely determined by the SM gauge interactions, which in turn fixes the mass of the dark matter to be several TeVs, the only new parameter in these Minimal Dark Matter (MDM) models.

Besides the dark matter problem, it is also necessary to extend the SM to decipher the neutrino oscillation myths [3]. Such extensions include seesaw models [4, 5, 6] and Zee models [7], all of which share a common feature, large neutrino masses above the electroweak scale, which is not determined *a priori*. These neutrino mass scales can also be linked to the dark matter mass scales by generating neutrino masses radiatively [8]. Most of these radiative seesaw models (RSSM), however, impose discrete symmetries to stabilize the dark matter particle. Incorporating the MDM and the RSSM in the same model, the so-called $R\nu$ MDM provides the possibility to solve the dark matter problem and the neutrino oscillation problem in the same

frame work with no beyond the SM gauge and discrete symmetries, which also has testable phenomenological predictions.

2. The model

The extra matter content of R ν MDM includes a Majorana right-handed fermion N_R and a scalar χ which transform under the SM gauge group as

$$N_R : (1, 5, 0) \quad \chi : (1, 6, -1/2), \quad (1)$$

which are selected according to the following considerations. In order to radiatively generate neutrino masses, a Majorana mass term of N_R is necessary, which means N_R should have zero hypercharge. In the mean time, $\bar{L}HN_R$ type of Yukawa coupling should not be allowed to forbid N_R from mixing with the SM leptons and thus decaying to the SM particles. These two requirements exclude the possibility of N_R as $SU(2)_L$ singlet and triplet. The next possibilities are (2, 0) and (4, 0) with $SU(3)_C$ quantum number omitted. Together with the existence of $\bar{L}N_R\chi$ necessary for the RSSM ingredient, these two possibilities demand χ to be (1, -1/2), (3, -1/2) or (5, -1/2), which makes all the component fields in both N_R and χ fractionally charged and leaves us with no dark matter candidate. Thus the minimal choice for N_R should start with (5, 0), which corresponds to a choice of χ to be (4, -1/2) or (6, -1/2). A $H^\dagger H H \chi$ term is allowed if χ is (4, -1/2), which will induce vacuum expectation value (VEV) for χ and break the accidental Z_2 symmetry that stabilize the dark matter candidate.

With this choice of matter content, the most general Lagrangian containing N_R and χ is

$$\begin{aligned} L_{new} &= \bar{N}_R i \gamma^\mu D_\mu N_R + (D^\mu \chi)^\dagger D_\mu \chi - (\bar{L}_L Y \chi N_R + \frac{1}{2} \bar{N}_R^c M N_R + h.c.) - V_{new} \\ V_{new} &= \mu_\chi^2 \chi^\dagger \chi + \lambda_\chi^\alpha (\chi \chi^\dagger \chi \chi^\dagger)_\alpha + \lambda_{H\chi}^\alpha (H^\dagger H \chi^\dagger \chi)_\alpha + [\tilde{\lambda}_{H\chi} (H\chi)^2 + h.c.] , \end{aligned} \quad (2)$$

which is invariant under an accidental Z_2 symmetry even after electroweak symmetry breaking, under which only N_R and χ are odd. This discrete symmetry makes the lightest component field of N_R and χ stable.

3. Dark matter and neutrino masses

3.1. Dark matter

The dark matter candidate should be the lightest neutral component of either N_R or χ . In the minimal model, χ^0 , however, is ruled out since its direct detection cross section is 2 or 3 orders of magnitude bigger than the experimental limits due to its nonzero hypercharge. N_R^0 , the neutral component of N_R , has the lightest mass at one-loop level according to the formula of mass splitting

$$\begin{aligned} m_N^Q - m_N^{Q'} &= \frac{g^2 m_N}{16\pi^2} (Q^2 - Q'^2) \left[\sin^2 \theta_W f(m_Z/m_N) + f(m_W/m_N) - f(m_Z/m_N) \right] , \\ f(x) &= \frac{x}{2} \left\{ 2x^3 \ln x - 2x + (x^2 - 4)^{1/2} (x^2 + 2) \ln[(x^2 - 2 - x\sqrt{x^2 - 4})/2] \right\} , \end{aligned} \quad (3)$$

where Q and Q' are the electric charges of different component. So N_R^0 serves as a good dark matter candidate, whose mass is determined by the relic abundance constraint to be 9.6 TeV [2]. The direct DM detection cross section is predicted to be around 10^{-44} cm² which is safely below the current upper limits from CDMSII and Xenon100, but can be tested in the future by superCDMS and xenon-1ton experiments [9].

3.2. Neutrino masses and mixing

The neutrino masses are generated radiatively. The mass matrix of the neutrinos at one-loop is

$$(M_\nu)_{ll'} = \frac{1}{16\pi^2} \frac{14}{5} \frac{Y_{li} Y_{l'i}}{m_{N_i}} \tilde{\lambda}_{H\chi} v^2 I(m_k, m_m, m_{N_i}),$$

$$I(m_{\chi_k}, m_{\chi_m}, m_{N_i}) = \frac{m_{N_i}^2}{m_{\chi_k}^2 - m_{\chi_m}^2} \left(\frac{m_{\chi_k}^2 \ln(m_{N_i}^2/m_{\chi_k}^2)}{m_{N_i}^2 - m_{\chi_k}^2} - \frac{m_{\chi_m}^2 \ln(m_{N_i}^2/m_{\chi_m}^2)}{m_{N_i}^2 - m_{\chi_m}^2} \right), \quad (4)$$

where $m_{\chi_{k,m}}$ and m_{N_i} are the masses of the scalars, and fermions in the loop. The mass matrix will be diagonalized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix V_{PMNS} . Thus the Yukawa coupling matrix can be solved

$$Y = \left(\frac{v^2}{16\pi^2} \frac{14}{5} \tilde{\lambda}_{H\chi} \right)^{-1/2} V_{PMNS} \hat{M}_\nu^{1/2} O \hat{M}_N^{1/2} \hat{I}^{-1/2}, \quad (5)$$

where \hat{M}_n and \hat{I} are diagonal matrices with $\hat{M}_N = \text{diag}(m_{N_1}, m_{N_2}, \dots)$ and $\hat{I} = \text{diag}(\tilde{I}(z_1), \tilde{I}(z_2), \dots)$. O satisfies $OO^T = I$ and can be complex in general. The Yukawa coupling matrix varies with $\tilde{\lambda}_{H\chi}$, the choice of O and different neutrino mass patterns. It can be seen later that the entries of the Yukawa coupling matrix take sizable values when constraints from flavor physics are considered.

4. Lepton flavor violation

Since the mass of the dark matter candidate is set to be around 10 TeV, it is impossible to test the theory at the LHC. Flavor physics, however, can be an effective probe. For $\ell_i \rightarrow \ell_j \gamma$, the decay amplitudes for $\ell_i \rightarrow \ell_j \gamma$ can in general be parameterized as

$$\mathcal{M} = \epsilon^{*\mu} \bar{\ell}_j i \sigma_{\mu\nu} q^\nu (\tilde{A}_R P_R + \tilde{A}_L P_L) \ell_i. \quad (6)$$

Neglecting small mass splitting of component fields within a given N_R multiplet, we obtain

$$\tilde{A}_R = \frac{m_i e}{32\pi^2 m_\chi^2} \sum_k Y_{ik}^* Y_{jk} F(z_k),$$

$$F(z) = P_\chi \frac{2z^3 + 3z^2 - 6z + 1 - 6z^2 \ln z}{6(1-z)^4} - P_N \frac{z^3 - 6z^2 + 3z + 2 + 6z \ln z}{6(1-z)^4}, \quad (7)$$

where $z_k = m_{N_k}^2/m_\chi^2$. Summarizing all one loop contributions, we have $P_\chi = -5$ and $P_N = 2$. \tilde{A}_L can be obtained by replacing m_i by m_j in the above. In order to have a large branching ratio for $\mu \rightarrow e \gamma$ close to the upper bound in both normal and inverted hierarchies with m_{ν_1} larger than 0.08 eV, the parameter $\tilde{\lambda}_{H\chi}$ is smaller than 10^{-11} , which is plotted in Fig. 1. The smallness of $\tilde{\lambda}_{H\chi}$ is technically natural since the model has a global $U(1)$ lepton number symmetry if this coupling is set to zero. In this regime, the elements of the Yukawa matrix Y are of order a few, which may be close to the non-perturbative region wherein the results may become less accurate compared with smaller m_{ν_1} . But for m_{ν_1} less than 0.08 eV, the perturbative predictions should be reasonably reliable.

For $\mu - e$ conversion, the effective Lagrangian from Ref. [10] is

$$L_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} [m_\mu \bar{e} \sigma^{\mu\nu} (A_R P_R + A_L P_L) \mu F_{\mu\nu} + h.c.]$$

$$- \frac{G_F}{\sqrt{2}} \left[\bar{e} (g_{LS(q)} P_R + g_{RS(q)} P_L) \mu \bar{q} q + \bar{e} (g_{LP(q)} P_R + g_{RP(q)} P_L) \mu \bar{q} \gamma_5 q + h.c. \right]$$

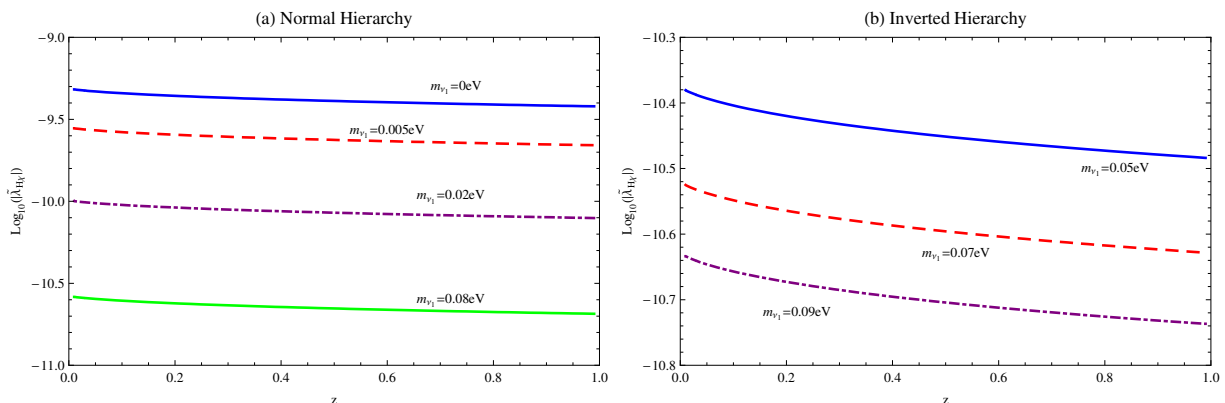


Figure 1. $\log_{10} |\tilde{\lambda}_{H\chi}|$ vs. $z = m_N^2/m_\chi^2$ for (a) normal hierarchy, and (b) inverted hierarchy for different choices of m_{ν_1} . The curves are obtained using the recent MEG upper limit $\text{Br}(\mu \rightarrow e\gamma) = 2.4 \times 10^{-12}$.

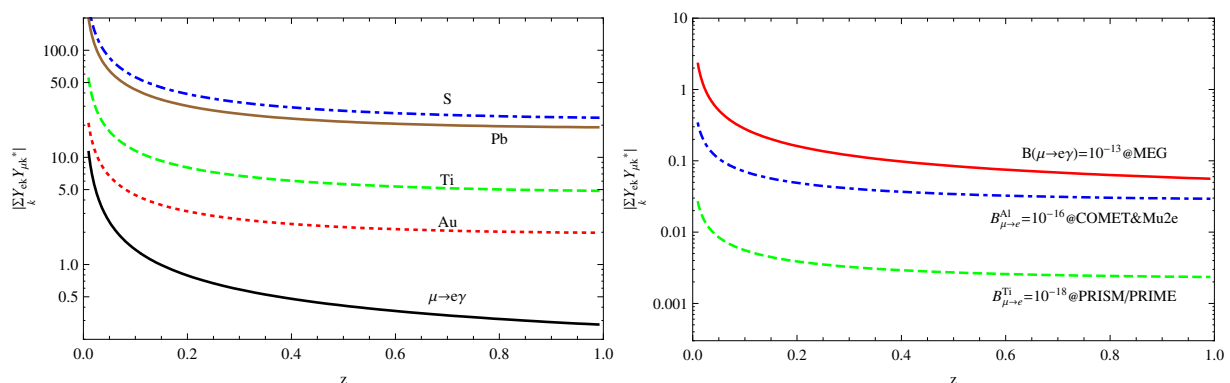


Figure 2. Comparison of present (left panel) and prospective (right panel) constraints from $\mu - e$ conversion on various nuclei and $\mu \rightarrow e\gamma$.

$$\begin{aligned}
& - \frac{G_F}{\sqrt{2}} \left[\bar{e}(g_{LV(q)}\gamma^\mu P_L + g_{RV(q)}\gamma^\mu P_R)\mu \bar{q}\gamma_\mu q + \bar{e}(g_{LA(q)}\gamma^\mu P_L + g_{RA(q)}\gamma^\mu P_R)\mu \bar{q}\gamma_\mu \gamma_5 q + h.c. \right] \\
& - \frac{G_F}{\sqrt{2}} \left[\frac{1}{2}\bar{e}(g_{LT(q)}\sigma^{\mu\nu} P_R + g_{RT(q)}\sigma^{\mu\nu} P_L)\mu \bar{q}\sigma_{\mu\nu} q + h.c. \right]. \quad (8)
\end{aligned}$$

where at the one-loop level, $g_{(L,R)(S,P,A,T)(q)} = 0$, $g_{RV(q)} = 0$, and

$$A_R = \frac{\sqrt{2}}{8} \frac{\tilde{A}_R}{G_F m_\mu}, \quad A_L = \frac{m_e}{m_\mu} A_R, \quad (9)$$

$$g_{LV(q)} = -\frac{s_W^2}{2\pi^2} \sum_k \frac{m_W^2}{m_{N_k}^2} Q_q Y_{ek} Y_{\mu k}^* z_k [P_\chi G_\chi(z_k) + P_N G_N(z_k)]. \quad (10)$$

The contribution from A_L can be neglected compared with that from A_R . The constraints on $\sum_k Y_{ek} Y_{\mu k}^*$ from $\mu - e$ conversion on various nuclei and $\mu \rightarrow e\gamma$ are shown in Fig.2 on the left panel assuming that N_k are degenerate. We see that among the current available experimental limits,

$\mu \rightarrow e\gamma$ gives the strongest constraints. The current bounds on $\mu - e$ conversion do not provide significant constraints on the parameter since the Yukawa couplings can still be much larger than of order 1. On the other hand, the sensitivities of prospective future $\mu - e$ conversion searches Mu2E[11]/COMET[12] and PRISM[13] exceed that of the MEG experiment. Constraints from $\tau \rightarrow \mu(e)\gamma$ are also studied and are less stringent than those from $\mu \rightarrow e\gamma$ but may be probed by future experiment at the super B factory[14] as shown Fig. 2. Magnetic dipole moments of μ and neutrinos are also studied and the new contributions are well under current experimental limits.

5. Conclusion

In the work we have constructed a minimal model, $R\nu$ MDM, incorporating the MDM and the RSSM without beyond the SM gauge symmetry. This model provides a fermionic dark matter candidate with a mass around 10 TeV that is not testable at the LHC. It predicts a direct DM detection cross section of order 10^{-44} cm², which can be tested in future experiments, such as superCDMS, and Xenon-1ton. Present constraints obtained from $\mu - e$ conversion searches are weaker than those from $\mu \rightarrow e\gamma$, which will be the other way around in the future experiments. This model also can imply a sizable $\tau \rightarrow \mu\gamma$ decay rate that may be observable at super B factories.

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