

INDEPENDENT COMPONENT ANALYSIS OF TEVATRON TURN-BY-TURN BPM MEASUREMENTS

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Abstract

Transverse dipole coherent beam oscillations in the Tevatron were analyzed with different independent component analysis algorithms. This allowed to obtain the model-independent values of coupled beta-functions as well as betatron phase advances and dispersion function along the ring from a single kick measurement. Using a 1-turn shift of turn-by-turn BPM readings for virtual doubling of the number of BPMs it is also possible to measure the fractional part of betatron tunes with high accuracy. Good agreement with the linear optical model of the Tevatron has been observed.

INTRODUCTION

The Tevatron collider has 118 horizontal and 118 vertical beam position monitors (BPMs) distributed uniformly along the ring. These BPMs can simultaneously record beam oscillation histories over many thousand turns. This produces a lot of precisely measured data which can be analysed with statistical methods, namely independent component analysis (ICA) decomposition. The basic idea of ICA is to represent all the recorded signals $\mathbf{x}_i = [x_i(t_1), x_i(t_2), \dots, x_i(t_{N \text{ TURNS}})]^T$ as a linear combination of a small number of unknown independent sources, so called temporal modes of ICA $\mathbf{s}_j = [s_j(t_1), s_j(t_2), \dots, s_j(t_{N \text{ TURNS}})]^T$:

$$\mathbf{X} = \mathbf{A}\mathbf{S},$$

where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]^T$ is the matrix of n measured signals, $\mathbf{A}$ is the unknown mixing matrix and $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_m]^T$ is the matrix of m source signals to be estimated.

The specific criterion of mutual source signal independence varies from one particular ICA algorithm to another [1] but most of them employ diagonalization of the covariance matrix $\mathbf{R}_x = \mathbf{X}\mathbf{X}^T = \mathbf{A}\mathbf{R}_s\mathbf{A}^T$ via singular value decomposition (SVD) as a first step to estimate source signals:

$$\mathbf{R}_x = [\mathbf{u}_1, \dots, \mathbf{u}_n] \text{diag}(\lambda_1^2, \dots, \lambda_n^2) [\mathbf{u}_1, \dots, \mathbf{u}_n]^T$$

The number of sources m is determined as a number of significant singular values and the transformation of $\mathbf{X}$ producing the covariance matrix $\mathbf{R}_s = \mathbf{I}$ is performed as $\mathbf{S}_0 = \mathbf{Q}\mathbf{X}$, where

$$\mathbf{Q} = \text{diag}\left(\frac{1}{\lambda_1}, \dots, \frac{1}{\lambda_m}\right) [\mathbf{u}_1, \dots, \mathbf{u}_m]^T$$

The mixing matrix $\mathbf{A}$ is pseudoinverse of the $\mathbf{Q}$ matrix.

The described $\mathbf{X}$ matrix decomposition is called principal component analysis (PCA) and it serves as a first step in many ICA algorithms. It is followed by some additional rotation in the m -dimensional space to find the basis of source signals that best fit a given mutual independence criterion.

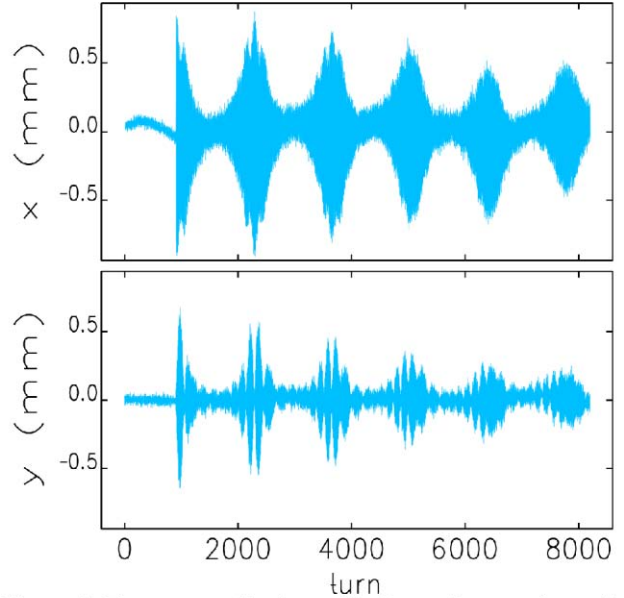


Figure 1: Transverse dipole oscillations of proton beam in the Tevatron recorded by horizontal and vertical BPMs. At about 900th turn beam was kicked in horizontal plane.

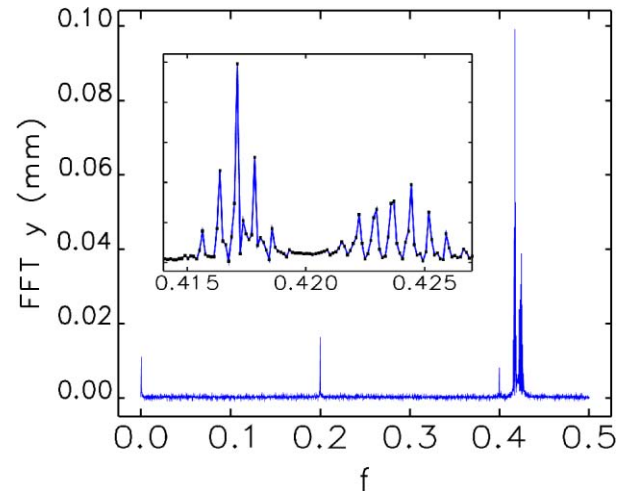


Figure 2: FFT-spectrum of signal from vertical BPM (see Fig. 1).

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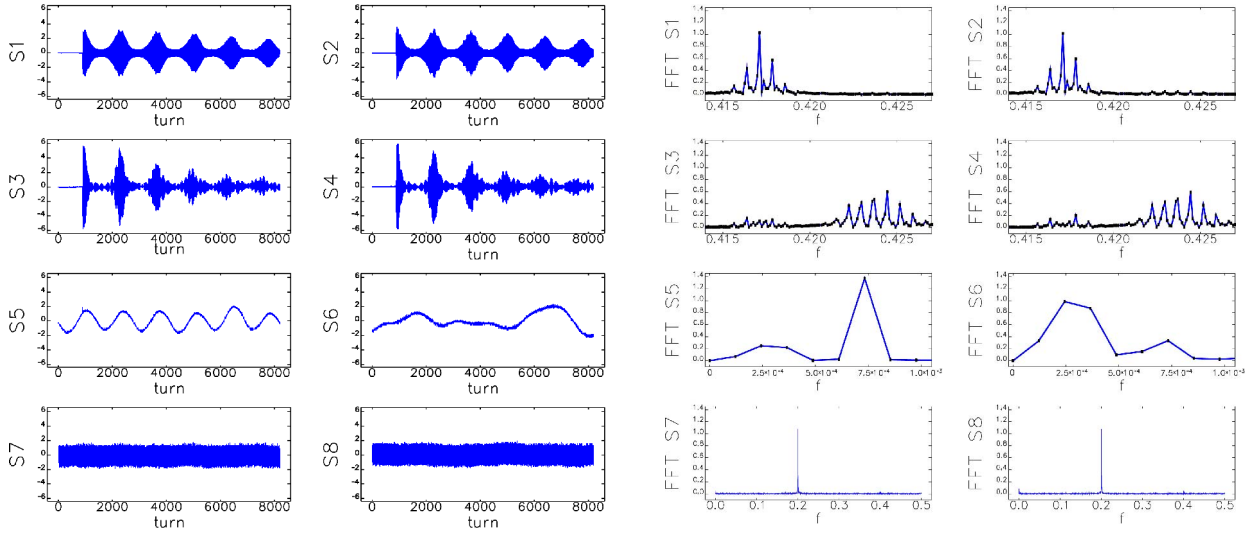


Figure 3: Temporal modes of ICA (AMUSE) decomposition and their FFT-spectra.

ICA DECOMPOSITION RESULTS

Several ICA implementations for Matlab/Octave (Octave [6] is an open-source alternative to Matlab) were tested, namely the AMUSE, EVD and JADE algorithms from the ICALAB package [4] and the FAST ICA algorithm [5]. Data preprocessing and post processing like fast discrete Fourier transformation (FFT) and filtering was performed with the help of SDDS-toolkit [3]. A graphical user interface was written in tcl/tk scripting language. The resulting turn-by-turn data analysis program is available for downloading [7].

The temporal modes of ICA decomposition of the Tevatron turn-by-turn measurements are shown in Fig. 3. The Algorithm for Multiple Unknown Signals Extraction [8] (AMUSE) was used in this case. Both horizontal and vertical BPM signals were included into X matrix. Modes in Fig. 3 are ordered according to their contribution to the X matrix. The contribution of j -th independent component to the signal from i -th BPM is given by the A_{ij} . We can define the significance of the j -th component as a norm of j -th column of matrix A (the column is considered as a vector and it's called the spatial ICA mode). So **S1** is the most significant component, **S8** is the least significant one.

First four modes **S1-S4** correspond to the betatron oscillations and have the largest spatial modes. **S5** is the low-frequency synchrotron-motion mode though it has some mixing with the **S6** mode. **S6** mode is caused by mechanical vibrations of Tevatron final focus quadrupoles. **S7** and **S8** modes are excited by timing errors of BPM electronics with periodicity of five turns resulting coherent lines at tunes of 0.2 and 0.4. If we retain more singular values during the PCA decomposition other less significant noise signals will appear as independent components. For example the noise with the tune of 0.4 is below the PCA threshold in Fig. 3.

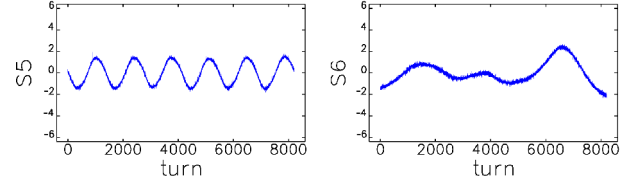


Figure 4: Low frequency ICA-modes after rotation transformation was used to obtain the pure synchrotron motion mode.

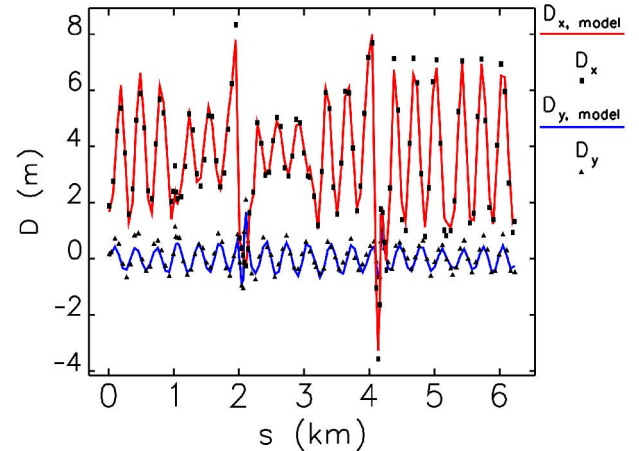


Figure 5: Dispersion function at BPM locations obtained from spatial synchrotron motion ICA mode and compared to ORM-calibrated linear model of the accelerator.

Different ICA algorithms produce the similar picture but the mixing among resulting low-frequency modes and among betatron modes may be different. AMUSE is good at separating the betatron oscillations, JADE and FAST ICA usually produce unmixed low-frequency modes but they fail to separate betatron modes properly when the tunes are very close to each other as they are in the presented case.

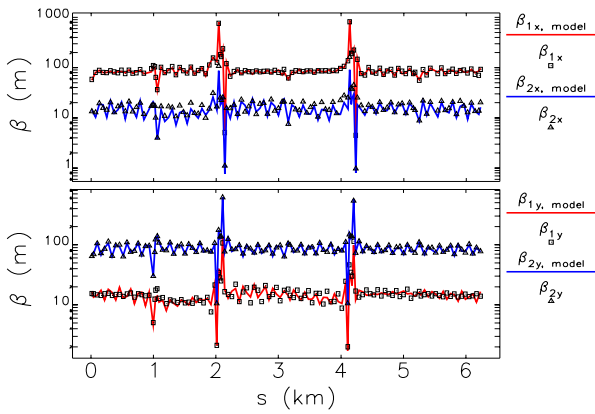


Figure 6: Coupled beta-functions at BPM locations obtained from spatial modes of S1-S4. Solid lines represent model values. FFT-filter from 0.413 to 0.429 was applied to signals before ICA for better accuracy.

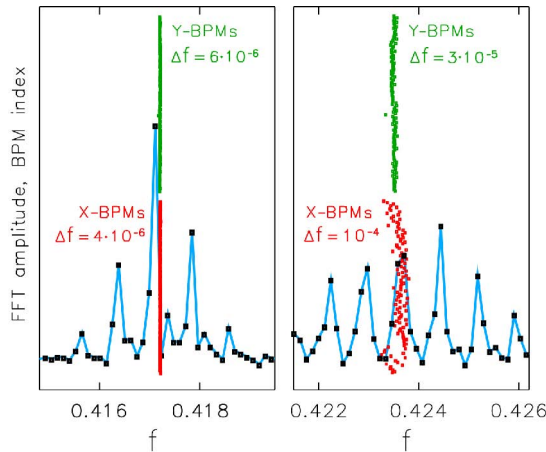


Figure 7: Betatron tune calculated as phase advance between BPM and its virtual twin shifted by 1 turn. RMS-errors of tune determination are shown.

To untangle the mixed modes a rotation transformation O has to be applied [2]:

$$X = AO^T OS = AS,$$

$$S_{new} = OS, A_{new} = AO^T.$$

One still needs a criterion of mode separation to perform this rotation. In the case of mixing between S5 and S6 modes on the Fig. 3 such criterion may be the monochromaticity of synchrotron motion mode S5 i. e. there should not be any lower-frequency components in the FFT-spectrum of the S5 mode. Fig. 4 shows the result of such rotation transformation. The spatial mode of pure synchrotron motion mode is proportional to dispersion function as can be seen from the Fig. 5. For the presented measurement we also have a good linear model of the Tevatron so we can compare the measurement to the model optics. The model was obtained with the orbit response matrix (ORM) fitting technique [9].

The Tevatron has a strong coupling between horizontal and vertical motion. If a beam is kicked horizontally vertical oscillations are also excited in less than 100 turns (see Fig. 1). The fractional parts of betatron tunes are close to each other and normally synchro-betatron

sidebands overlap. That makes it difficult for ICA to separate the betatron motion modes since the conditions of statistical independence may be fulfilled poorly for two finite-length sinusoidal signals with slightly different frequencies. AMUSE algorithm performs well but often it still leaves some residual mixing. To achieve the rotational untangling of betatron modes beside the mode separation criterion based on the frequency spectra analysis [2] it is also possible to use the different approach. We can double the number of analysed BPM signals in the X matrix if we consider each of the BPMs as two monitors separated exactly by one turn. The signal from the second BPM is the signal from the first one shifted by 1 turn. The betatron phase advance between these two monitors should be exactly equal to the betatron tune. The fractional part of betatron phase advance between any two BPMs in the ring can be calculated from the spatial modes of betatron motion [10]. Then one needs to perform the ICA with the doubled number of input signals and then to find a rotation transformation O that gives the smallest spread of tune values calculated at different BPMs. This technique was successfully applied for the Tevatron turn-by-turn measurements and simulation and it proved to be efficient in untangling the mixed betatron modes even with overlapping synchrotron sidebands. Also it gives us the precise values of betatron tunes (see Fig. 7).

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REFERENCES

- [1] S. Choi et al. "Blind Source Separation and Independent Component Analysis: A Review". Neural Information Processing - Letters and Reviews Vol. 6, No. 1, Jan. 2005.
- [2] Chun-xi Wang. "Untangling mixed modes in model-independent analysis of beam dynamics in circular accelerators". Phys. Rev. ST Accel. Beams 7, 114001 (2004).
- [3] M. Borland and L. Emery, "The Self-Describing Data Sets File Protocol and Program Toolkit". Proc. ICALEPCS 1995, Chicago, Illinois, pp. 653-662 (1996). <http://www.aps.anl.gov/asd/oag/software.shtml>
- [4] A. Cichocki et al., ICALAB Toolboxes, <http://www.bsp.brain.riken.jp/ICALAB>
- [5] <http://www.cis.hut.fi/projects/ica/fastica>
- [6] <http://www.octave.org>
- [7] <http://www-bdnew.fnal.gov/tevatron/lifetrac/tbt>
- [8] Lang Tong et al. "Indeterminacy and Identifiability of Blind Identification". IEEE transactions on circuits and systems, vol. 38, N. 5, May 1991.
- [9] A. Valishev et al. "Progress with Collision Optics of the Fermilab Tevatron Collider". Proceedings of EPAC 2006, Edinburgh, Scotland.
- [10] Chun-xi Wang, et al. "Phase advance and beta function measurements using Model-Independent Analysis". Phys. Rev. ST Accel. Beams 6, 104001 (2003).