

Transport properties of the hot quark-gluon plasma

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Abstract. A phase where quark and gluon are the relevant degrees of freedom is expected for nuclear matter at energy density $\epsilon \geq 1 \text{ GeV}/\text{fm}^3$ and a temperature $T > 160 \text{ MeV}$. A transient state of such a matter can be created by mean of ultra-relativistic heavy-ion collisions. We briefly overview some main results on the properties of the quark-gluon plasma emphasizing the necessity to develop a transport theory for quarks and gluons able to incorporate the main developments in lattice QCD and perturbative QCD. First results show that Boltzmann-Vlasov transport theory correctly predict the elliptic flow observed at both RHIC and LHC energies.

1. Introduction

The study of the fundamental theory of strong interactions, Quantum Chromo Dynamics (QCD), under extreme conditions of temperature and density has been one of the most challenging problems in physics during the last 20 years, capturing increasing experimental and theoretical attention. There are several reasons underlying such a vivid interest. QCD is a quantum field theory with an extremely rich dynamical content (asymptotic freedom, confinement, chiral symmetry, nontrivial vacuum ...). Besides Heavy-Ion Collisions at ultrarelativistic energies ($\sqrt{s} > 10 \text{ AGeV}$) provide the unique possibility to create a transient state of matter at energy densities and temperatures similar to those of the Universe in the first $10^{-6} - 10^{-5} \text{ s}$ after the Big Bang. Furthermore the most dramatic event during the first second was the quark-to-hadron phase transition associated to a reduction in the number of degrees of freedom by about a factor three [1]. Finally, in recent years the discovery of a duality between gauge and string theory has led to the development of a new field of intense research [2].

In 1965 Hagedorn conjectured the existence of a limiting temperature $T_c \sim 160 \text{ MeV}$ for the hadronic system due to an envisaged exponential increase of the density of hadronic states. The existence of a matter made by quark in a deconfined state was suggested for the first time in 1975 by Cabibbo and Parisi [3] soon after the Nobel Prize paper on the asymptotic freedom of non-abelian gauge theory [4]. They pointed out that the so-called Hagedorn limiting temperature was associated to a divergency in the hadronic gas partition function and hence a sign of a phase transition to quark matter. However first evidences of the possibility to realize by mean of heavy-ion collision a transient state of such a matter came only in the 90's thanks to the SPS facility able to realize heavy-ion collisions up to $\sqrt{s} = 17 \text{ AGeV}$. It was however only with the RHIC project conducted at Brookhaven National Laboratory (BNL) that it was possible to create a quark-gluon plasma (QGP) phase (expected at energy density $\epsilon_c > 1 \text{ GeV}/\text{fm}^3$) lasting for about 4-5 fm/c at a maximum initial temperature $T \sim 2T_c$. A temperature and a

duration time sufficiently long for strong interactions that have made possible to achieve several discoveries about the properties of the QGP and its hadronization [5, 6].

The present knowledge about the properties of the QGP is mainly based on three complementary sources: QCD calculations on the lattice (lQCD) and in the perturbative regime (pQCD), theoretical and phenomenological models, empirical information from heavy-ion experiments. The lQCD computations have clearly shown that a phase transition at energy density $\epsilon_c \sim 1 \text{ GeV}/\text{fm}^3$ and a temperature $T_c \sim 160 \text{ MeV}$ occurs being most likely a cross over in the case of realistic quark masses. Furthermore the energy density and entropy density reach about 80% of their ideal gas limit value already relatively close to the critical temperature T_c , but then the full value is reached only for asymptotically large temperatures. Even more interestingly a large deviation from a non-interacting gas behavior is found in the large trace anomaly $T_\mu^\mu = \epsilon - 3P$ up to $T \sim 2T_c$ indicating a system far from a mere gas of quarks and gluons and hinting to a strongly interacting one. The development of pQCD calculations at high temperature has shown a slow convergency with both temperature and mass of the quarks. It is necessary to go to temperature $T > 3 - 4T_c$ [7] or at masses larger than $m_c \sim 1.3 \text{ GeV}$ [8] to have a higher order pQCD scheme applicable.

Our focus in these Proceedings is on the phenomenological models and in particular on the development of a transport theory for quarks and gluons able to embed the information coming from lQCD and pQCD from one hand and to provide a tool for a direct comparison with the experimental observables on the other hand.

2. Main results at RHIC

The theoretical and experimental efforts around RHIC in the last decade has allowed a first important breakthrough in the knowledge of the properties of the QGP at least up to $T \sim 2T_c$. It became soon quite clear that a new state of matter has been created and there are several novel discoveries and results. We will discuss some of them focusing on two main type of observables that has allowed a first survey of the QGP. A first observable is called nuclear modification factor and provides a measure of the modification of the hadrons momentum spectra in ion-ion (AA) collisions respect to the pp collisions through the ratio of the respective spectra rescaled by the number of collisions N_{coll} according to a geometrical Glauber model:

$$R_{AA}(p_T) = \frac{d^2 N^{AA}/d^2 p_T}{N_{coll} d^2 N^{NN}/d^2 p_T} \quad (1)$$

It is clear that $R_{AA} = 1$ means that AA collisions are merely a superposition of nucleon-nucleon collisions. First observations at RHIC and more recently at LHC have shown $R_{AA}(p_T) \sim 0.2$ for most central collisions (see figure 2 for the case of heavy quarks) corresponding to a strong interaction of the partons initially created as one could expect if a QGP plasma fireball has been really created.

The other main observables that allow to characterize several properties of the QGP medium is the elliptic flow. Its origin is the initial space eccentricity of the QGP coming from the non-central overlapping of the colliding ions, quantified by $\epsilon = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$. Due to the pressure gradients of the QGP such a space eccentricity is converted into an anisotropic momentum distribution respect to the azimuthal angle ϕ_p that can be expressed by mean of a Fourier expansion:

$$f_q(p_T, \phi_p) = f_q(p_T) \left(1 + 2 \sum_n v_n \cos(n \phi_p) \right) \quad (2)$$

where the first coefficient (v_1) is vanishing on the average to the space symmetry of the system while the second term v_2 namely the elliptic flow is the dominant one. Both at RHIC and LHC values of v_2 up to about 0.25 has been observed, see figure 1 which means that in the $\phi_p = 0$

direction the abundancy of hadrons can be about 3 times larger than at $\phi_p = \pi/2$. It is clear that it is a strong effect carrying information on both the EoS and the shear viscosity of the QGP.

We now briefly focus on three main results of relevant to the study of the transport properties of QGP and its hadronization:

- *The QGP is a nearly perfect fluid with very low viscosity* - The plasma created at temperatures $T \sim 200 - 300$ MeV and small baryon chemical potential exhibits a nearly perfect fluid behavior in the bulk of the system, opposite to the asymptotic freedom expectations. Such a statement is mainly corroborated by the observation of large anisotropic flows that develop due to the initial anisotropy of the created fireball. Similarly to what has been observed in the same years for ultra-cold trapped atoms [9] the elliptic flow has values close to the ideal hydrodynamical predictions. A first estimate would suggest a $\eta/s \leq 0.4$ very close to the conjectured lower bound for supersymmetric gauge theories in the infinite coupling limit [2] and to that suggested by quite general Quantum Mechanics considerations [10]. It remains to be determined is the value of η/s of what could be the most ideal fluid ever observed and in particular its microscopic origin. As for the microscopic origin of the ideal fluid behavior, a possible explanation is the presence of quark-antiquark resonances that are reminiscent of hadronic-like or gluonic states [8] or could be a more subtle competition between electrically charged quasiparticles (quarks and gluons) and magnetically charged ones (magnetic monopoles) [11]. Lattice results trying to identify and isolate these objects and their contribution to thermodynamics are also becoming available [12]. In the next Section we will describe more in detail the issue of the QGP shear viscosity.
- *Hadronization is modified respect to the vacuum one* - The statement is justified by the fact that the ratio of baryons to mesons is up to a factor of 4 larger than the one in pp collision in an intermediate range of $p_T \sim 2 - 6$ GeV. The most convincing explanation of this phenomena is that most of the hadrons come from a coalescence of the quarks in the plasma [13, 14]. The basic idea is that instead of pop-up quarks from the vacuum as in the standard fragmentation picture one can hadronize combining the quark of the medium. In such a picture calling $f_q(p)$ the (anti-)quark distribution function, the spectra of the mesons is given by two different mechanisms:

$$\frac{d^2 N_H}{d^2 p_T} = \int_{\Sigma(1)} f_q(p) \otimes D_{q \rightarrow H}(z) + \int_{\Sigma(n_q)} f_q^{n_q}(p/n_q) \otimes \Phi_H(p_{rel}) \quad (3)$$

The first term represent the standard fragmentation contribution while the second is the coalescence one. The $\Sigma(n)$ is the n -particle phase space, $D(z)$ is the fragmentation function giving the probability that a parton q will give an hadron H of momentum $p_H = z p_q$ and Φ_H is the hadron wave function. It has been shown in several works that the fragmentation dominates parametrically at high p_T [15, 16, 13], but with the expected density and temperature of the quark plasma there is a dominance quark coalescence up to $p_T \sim 5 - 6$ GeV. It is easy to understand that in a coalescence process the baryon can be produced more abundantly respect to fragmentation, in fact the quarks are already present and in particular because the distribution function at low p_T are exponentials $f_q \sim e^{-p/T}$ therefore the integrand in Eq.(3) is the same for baryons and mesons:

$$f_q^{n_q} \sim \left[e^{-p/n_q T} \right]^{n_q} = e^{-p/T}$$

which means that baryons and mesons can be produced with the same probability apart from wave function and finite quark mass effect and differences in the degeneracy of

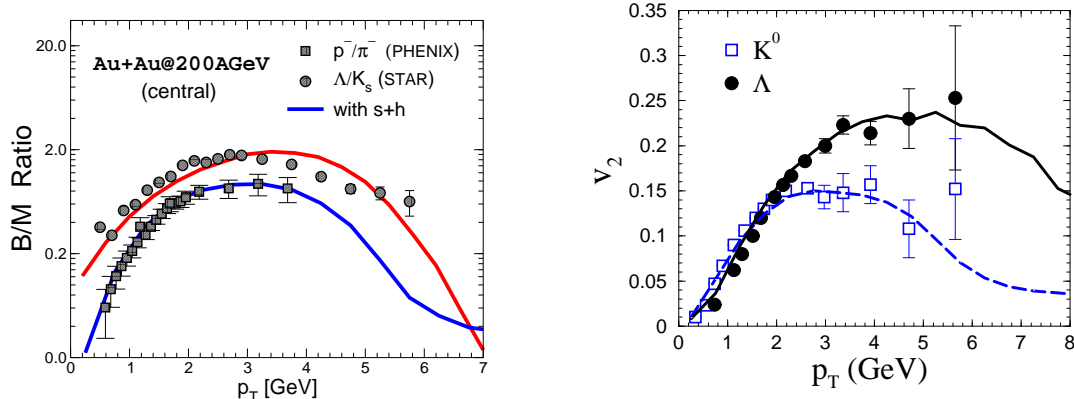


Figure 1. (Color online) Left Panel: The ratio of \bar{p}/π^- (circles) and Λ/K_s (squares) in $Au + Au$ at 200 AGeV is shown as a function of p_T and compared to the prediction of a coalescence model (solid lines) [15]. Right Panel: elliptic flow in $Au + Au$ at 200 AGeV for K and Λ compared to a coalescence model prediction.

hadronic states. Therefore can account for the observed enhancement of the baryon over meson ratio observed in the data and shown in figure 1 for proton/pions (squares) and lambda/kaon (circles). The solid lines are the correspondent prediction of a coalescence plus fragmentation model [14, 15, 17].

A coalescence mechanism brings with it another and even more peculiar feature that is the scaling of the elliptic flow with the number of constituent quarks, a property firmly observed at RHIC and more recently also at the LHC energy. The understanding of QNS is straitforward in the simplified version of collinear quarks coalescing with momenta p_q/n_q . Each quark distribution can be written as $f_q(p_T, \phi) = f_q(p_T) (1 + 2v_{2q}(p_T) \cos(2\phi))$. Under this approximation substituting in the second term of Eq.3 it can be easily shown that

$$v_{2M}(p_T) = 2v_2(p_T/2) \quad v_{2B}(p_T) = 3v_2(p_T/3) \rightarrow v_{2H}(p_T/n_q) \quad \text{universal flow} \quad (4)$$

In figure 1 (right) it is shown how the coalescing scaling is able to predict for example the Kaon and the Lambda $v_2(p_T)$ once the quark $v_{2q}(p_T)$ has been fixed fitting the pion v_2 .

There several other observable that appear to be consistent with a quark coalescence mechanism like triggered angular correlation, charge fluctuations, R_{AA} and v_2 for heavy meson with charm and bottom quarks.

Of course what it has been briefly describe is a quite simplified version of a realistic coalescence models that has to include also the r -space coalescence, the possibility of quarks with different momenta, the radial flow, the feeddown from resonance decays, the contribution from higher Fock states and so on. A review of this aspect of the QGP physics can be found in Ref. [13, 14] microscopic scale is important and the specific mechanism of hadronization can modify the observables from the partonic to the hadronic case.

- *Heavy Quarks strongly interact with the medium* - The trivial expectation was that due the large mass respect to the plasma temperature, $m_Q \gg T$ and a presumable pQCD behavior due to the $m_Q \gg \Lambda_{QCD}$ their in medium interaction relatively weak and the relaxation time of heavy quarks was much larger that the light quark one. Furthermore the main mechanism responsible for the in-medium energy loss, the gluon brehmsstrahlung, should be suppressed by a dead cone effect in the gluon radiation. Despite such pQCD expectation the experimental data [18] revealed a a strong suppression of the spectra, small R_{AA} , and a quite large elliptic flow both nearly comparable with the light quark ones, see figure 2.

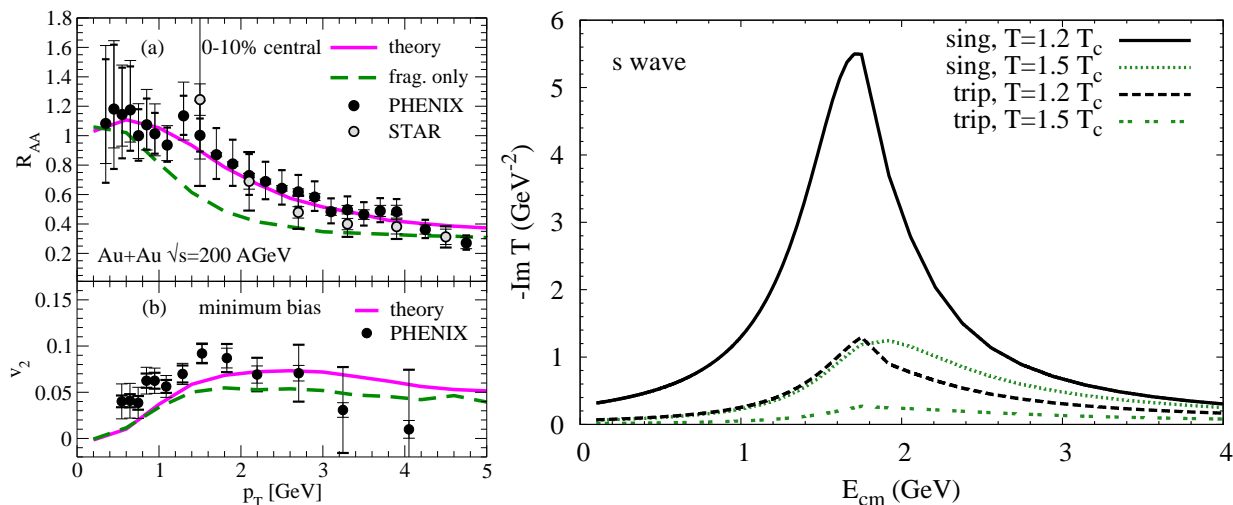


Figure 2. (Color online) Left Panel: the nuclear modification factor (upper panel) and elliptic flow (lower panel) of single electrons coming from D and B decay with/without (solid/dashed lines) quark coalescence in Au-Au collisions compared to RHIC data [23]. Right Panel: Imaginary part of the in-medium T -matrix for S -wave $c\bar{q}$ and cq scattering in color-singlet and antitriplet channels, respectively, at two different temperatures.

Models based on jet quenching or upscaled pQCD correction failed to explain the observed $R_{AA}(p_T)$ and the $v_2(p_T)$ measured. Only a non-perturbative approach to the heavy quark dynamics based on the solution of the T-matrix scattering under a potential derived from lQCD has been capable of account for the data [19, 20]. The main ingredients are the presence of a resonant scattering that leads to a peak in the imaginary part of the T-matrix especially in the color singlet state and again the presence of a coalescence mechanism for hadronization. In figure 2 it is shown by solid line the prediction of the T-matrix approach while the dashed line show the result if one discard the coalescence mechanism and assume an hadronization only by parton fragmentation. Predictions for at the LHC appears to be also quite succesfull showing a fairly good agreement with eralier results from the ALICE Collaboration [21, 22].

We have briefly discussed three main surprising and relevant discoveries at RHIC, of course there are other several aspects that could be discussed as the evidence of a strong jet quenching at high momenta, first possible signs of a Color Glass Condensate matter, the confirmation of the enhancement of strangeness. We note that the main findings about the properties of the Quark-Gluon Plasma both in the light and heavy quark sector ask for the development of a transport theory of quarks and gluons, as we discuss more in detail in the next section.

3. Transport Theory for the Quark-Gluon Plasma

A first comparison with preliminary data has shown an agreement of both the p_T spectra and the elliptic flow of different hadrons with the prediction of ideal hydrodynamics [24]. This has lead to the announcement of the creation of an almost ideal fluid. Nonetheless thanks to a more accurate comparison it has been found that dissipative effects cannot be neglected and even a small shear viscosity to entropy ratio η/s produce sizeable effect increasing with the transverse momentum p_T of the particles [25]. This has triggered a lot of activity in developing a relativistic theory of viscous hydrodynamics. The basic idea is to add a dissipative part in the energy momentum tensor by mean of a first order expansion in the space-momentum gradients:

$$T^{\mu\nu} = T_{ideal}^{\mu\nu} + \delta T_{NS}^{\mu\nu} = \epsilon u^\mu u^\nu - P \Delta^{\mu\nu} + \eta_s (\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \partial^\alpha u_\alpha) \quad (5)$$

where $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$, $\Delta^\mu \equiv \Delta^{\mu\nu} \partial_\nu$. However viscous corrections to ideal hydrodynamics are indeed large and a simple relativistic extension at first order, the so-called Navier-Stokes, is affected by causality and stability pathologies [26]. It is therefore necessary to go to second order gradient expansion, and in particular the Israel-Stewart theory has been implemented to simulate the RHIC collisions providing an upper bound for $\eta/s \leq 0.4$ [27].

Such an approach, apart from the present limitation to 2+1D simulations, has the more fundamental problem of a limited range of validity in η/s and in transverse momentum for $p_T > 1.5$ GeV. In this p_T region viscous hydrodynamics breaks its validity because the relative deviation of the equilibrium distribution function $\delta f/f_{eq}$ increases probably with p_T^2 becoming large already at $p_T \geq 3T \sim 1$ GeV. In fact viscous terms have two main effects one is the dissipative correction to the flow velocity $u^\mu(x)$ and to the density and temperature evolution, the other is the non-equilibrium corrections to the distribution function $f \rightarrow f_{eq} + \delta f$. It has to be realize that however there is no biunivocal correspondence between the non-equilibrium distribution and the non-equilibrium energy momentum tensor. Therefore the theory of hydrodynamics cannot determine δf and an ansatz has to be chosen typically it used the Grad's ansatz:

$$\frac{\delta f}{f_{eq}} = \frac{\delta T^{\mu\nu}}{\epsilon + P} \frac{p_\mu p_\nu}{T^2}.$$

In this contest it appears important the development of a more complete transport theory for quarks and gluons that would have a more wide range of validity recovering the limiting case of hydrodynamics. This appears important not only for the issued of the viscosity of the QGP but more in general also because a transport theory has also microscopic scale that could result essential to treat consistently the hadronization mechanism. Furthermore heavy quark significantly deviates from a full thermalization and cannot be described by viscous hydrodynamics while they can be self-constitently included in the transport theory and treated on equal footing as the light ones.

We are therefore developing a relativistic Boltzmann-Vlasov transport theory for on-shell particles [28, 29, 30]. Such a transport approach has the advantage to be a 3+1D approach not based on a gradient expansion in viscosity that is valid also for large η/s and for out of equilibrium momentum distribution allowing a reliable description also of the intermediate p_T range where the important property of quark number scaling (QNS) of $v_2(p_T)$ has been observed [13].

Furthermore Boltzmann-Vlasov transport theory distinguishes between the short range interaction associated to collisions and long range interaction associated to the field interaction, responsible for the change of the Equation of State (EoS) respect to that of a free gas. This last feature allows to unify two main ingredients that are relevant for the formation of collective flow. In ideal hydrodynamics the $v_2(p_T)$ depends only on the EoS namely on the sound velocity $c_s^2 = dP/d\epsilon$, while the mean free path λ is assumed to be vanishing. In the parton cascade approach the EoS is fixed to be the one of a free gas $c_s^2 = 1/3 = P/\epsilon$, while the mean free path $\lambda = 1/\rho\sigma$ is finite. In the first stage of RHIC the two different approaches were able to account for the large v_2 observed; in particular the parton cascade with large scattering cross section predicted the saturation of v_2 vs p_T [31]. Anyway once viscosity is finite both a finite λ and the EoS are important for the generation of the momentum anisotropies and this is naturally present in the Boltzmann-Vlasov transport approach.

The basic equation of transport for the (anti-) quark phase-space distribution function f^\pm for the case of mean field interaction that generate massive quasi particles can be written as:

$$p^\mu \partial_\mu f^\pm(x, p) + M(x) \partial_\mu M(x) \partial_p^\mu f^\pm(x, p) = \mathcal{C}(x, p) \quad (6)$$

where the first term is related to the free streaming, the second term represents the effect of a scalar field modifying the $\epsilon = 3P$ relation (giving a finite interaction measure) and the last term is the effect of the collisions directly associated to a finite λ and therefore to a finite η/s . The collisions term if only two body collisions are considered can be written as:

$$\mathcal{C}(x, p) = \iint_{21'2'} (f_{1'} f_{2'} - f_1 f_2) |\mathcal{M}_{1'2' \rightarrow 12}|^2 \delta^4(p_1 + p_2 - p'_1 - p'_2) \quad (7)$$

where $f_j = \int_j d^3 p_j / [(2\pi)^3 2E_j]$, \mathcal{M} denotes the transition matrix for the elastic processes and f_j are the particle distribution functions.

The relevance of the transport equation for quasiparticles with a space-time dependent mass resides in the success of quasi particles in describing correctly the behavior of energy density and pressure of the QGP as computed in the IQCD approach.

3.1. Quasiparticle model

A successful way to account for non-perturbative dynamics is a quasi-particle approach, in which the interaction is encoded in the quasi-particle masses.

The model is usually completed by introducing a finite bag pressure that can account for further non-perturbative effects and could be directly linked to the gluon condensate at least in the pure gauge case [32]. It is already well known that, in order to be able to describe the main features of lattice QCD thermodynamics, a temperature-dependent mass has to be considered. This also implies that the bag constant has to be temperature-dependent, in order to ensure thermodynamic consistency.

The temperature-dependent effective mass for quarks and gluons can be evaluated in a perturbative approach that suggests the following relations [33]:

$$m_g^2 = \frac{1}{6} g^2 \left(N_c + \frac{1}{2} n_f \right) T^2 \quad m_{u,d}^2 = \frac{N_c^2 - 1}{8N_c} g^2 T^2 \quad (8)$$

where n_f is the number of flavors considered, N_c is the number of colors, $m_{u,d}$ is the mass of the light quarks.

The coupling g is generally temperature-dependent. However, as mentioned in the introduction, the calculation of such a T -dependence by means of perturbation theory does not allow to have a good description of lattice QCD thermodynamics. Therefore, usually $g(T)$ is left as a function to be determined through the fit to lattice QCD data.

The pressure of the system can then be written as the sum of independent contributions coming from the different constituents, which have a T -dependent effective mass, plus a bag constant:

$$P_{qp}(m_u, m_d, \dots, T) = \sum_{i=u,d,g} d_i \int \frac{d^3 p}{(2\pi)^3} \frac{\vec{p}^2}{3E_i(p)} f_i(p) - B(T), \quad (9)$$

where $f_i(p) = [1 \mp \exp(\beta E_i(p))]^{-1}$ are the Bose and Fermi distribution functions, with $E_i(p) = \sqrt{\vec{p}^2 + m_i^2}$; $d_i = 2 \times 2 \times N_c$ for quarks and $d_i = 2 \times (N_c^2 - 1)$ for gluons.

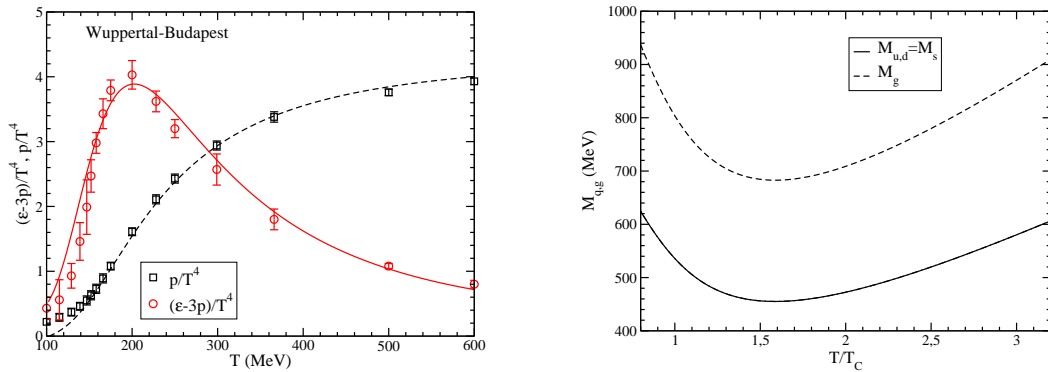


Figure 3. (Color online) Left panel : Lattice data [34] for the pressure and trace anomaly as functions of the temperature, together with the quasi-particle model curves. Right panel: quark and gluon quasi-particle masses as functions of T/T_c [34, 35].

In order to have thermodynamic consistency, the following relationship has to be satisfied:

$$\left(\frac{\partial P_{qp}}{\partial m_i}\right)_{T,\mu} = 0, \quad i = u, d, \dots, \quad (10)$$

which gives rise to a set of equations of the form

$$\frac{\partial B}{\partial m_i} + d_i \int \frac{d^3p}{(2\pi)^3} \frac{m_i}{E_i} f_i(E_i) = 0. \quad (11)$$

Only one of the above equations is independent, since the masses of the constituents all depend on the coupling g through relationships of the form: $m_i(T, \mu = 0) = \alpha_i g(T) T$, where α_i are constants depending on N_c and N_f according to Eqs. (8). The energy density of the system is then obtained from the pressure through the thermodynamic relationship $\epsilon(T) = T dP(T)/dT - P(T)$ and will have the form

$$\epsilon_{qp}(T) = \sum_i d_i \int \frac{d^3p}{(2\pi)^3} E_i f_i(E_i) + B(m_i(T)) = \sum_i \epsilon_{kin}^i(m_i, T) + B(m_i(T)). \quad (12)$$

In the model there are therefore two unknown functions, $g(T)$ and $B(T)$, but they are not independent, they are related through the thermodynamic consistency relationship (10). Therefore, only one function needs to be determined, which we do by imposing the condition:

$$\epsilon_{qp}(T) = \epsilon_{lattice}(T). \quad (13)$$

We performed our fit to the lattice data for the energy density [34]; in figure 3 we show the good agreement between our curves and lattice results for other quantities like pressure and trace anomaly.

We notice that, at sufficiently high temperature, $m \sim T$, as we can expect because T remains the only scale of the problem. When we are approaching the phase transition, there is instead a tendency to increase the correlation length of the interaction: the quasiparticle model tells us that this can be described as a plasma of particles with larger masses. This essentially determines the fall and rise behavior of $m(T)$ seen in all quasiparticle model fits to lattice data in $SU(N)$ gauge theories including the $SU(3)$ case for QCD. We notice a quite smoother behavior of the masses when the Wuppertal-Budapest IQCD are considered respect to older data of the HotQCD

collaboration [35], indicating that the strength of such correlation is significantly reduced when lattice simulations are performed at the physical quark masses and the continuum limit is taken.

The main point for our purposes here is the direct application of such a simple quasiparticle model in the transport theory supplies the possibility to include the correct equation of state of the QGP as computed in lattice QCD.

3.2. Transport at fixed shear viscosity

Our aim is to exploit the transport approach fixing the value of the η/s in order to make possible a direct comparison to viscous hydrodynamic approach and more generally to have a tool to directly estimate the viscosity of the plasma. To this end we do not calculate the cross section from a microscopic model (which could very well be an impossible task) but determine the local cross section σ in order to have the wanted local viscosity. Here we illustrate the procedure for the simplest case of a massless gas for simplicity, the extension to the finite mass case is easily achieved [36], In kinetic theory under ultra-relativistic conditions the shear viscosity can be expressed as [10]: $\eta = \frac{4}{15}\rho\langle p\rangle\lambda$, with ρ the parton density, λ the mean free path and $\langle p\rangle$ the average momentum. Therefore considering that the entropy density for a massless gas is $s = \rho(4 - \mu/T)$, μ being the chemical potential or fugacity, we get:

$$\eta/s = \frac{4}{15} \frac{\langle p\rangle}{\sigma_{tr}\rho(4 - \mu/T)} \quad (14)$$

where σ_{tr} is the transport cross section, i.e. the $\sin^2\theta$ weighted cross section.

From Eq. (14) we see that assuming locally the thermal equilibrium this can be obtained evaluating in each α cell the cross section according to:

$$\sigma_{tr,\alpha} = \frac{4}{15} \frac{\langle p\rangle_\alpha}{\rho_\alpha(4 - \mu_\alpha/T)} \frac{1}{\eta/s} \quad (15)$$

with $4\pi\eta/s$ set in the range 1–4. This approach is equivalent to have a total cross section of the form $\sigma_{Tot} = K(\rho, T)\sigma_{pQCD} > \sigma_{pQCD}$ where K takes into account the non perturbative effects responsible for that value of viscosity. This approach have been shown to recover the viscous hydrodynamics evolution of the bulk system [26].

We notice that a guideline on the temperature and time dependence of the cross section can be obtained considering the simple case of a free massless gas for which $s = g\frac{2\pi^2}{45}T^3$, and therefore neglecting μ in Eq. (15) one gets $\sigma_{tr} \sim T^{-2}$ for $4\pi\eta/s = 1$. Furthermore a simple Bjorken expansion which means $T \sim \tau^{-1/3}$ gives $\sigma_{tr} \propto \tau^{2/3}$ which is the approximate prescription adopted in [21]. In figure 4 it is shown $\sigma_{tr}(\tau)$ evaluated locally in space in a cylinder of radius 3 fm as a function of time for $Au + Au$ at $\sqrt{s} = 200$ AGeV, we see on the left the approximate $\tau^{2/3}$ and on the right the agreement with the estimated T^{-2} behavior.

4. First preliminary results

In our calculation the initial condition are longitudinal boost invariant with the initial parton density $dN/d\eta(b=0) = 1250$ at RHIC and $dN/d\eta(b=0) = 2250$ at LHC. The partons are initially distributed in coordinate space according to the Glauber model while in the momentum space at RHIC (LHC) the partons with $p_T \leq p_0 = 2$ GeV ($p_T \leq p_0 = 4$ GeV) are distributed according to a thermalized spectrum with a maximum temperature in the center of the fireball of $2T_c$ ($3.5T_c$), while for $p_T > p_0$ we take the spectrum of non-quenched minijets according to standard NLO-pQCD calculations. We also start our simulation at the time $t_0 = 0.6fm/c$ at RHIC and $t_0 = 0.3fm/c$ at LHC.

In order to study the effect of the kinetic freezeout on the generation of the elliptic flow we have performed two calculations one with a constant $4\pi\eta/s = 1$ during all the evolution

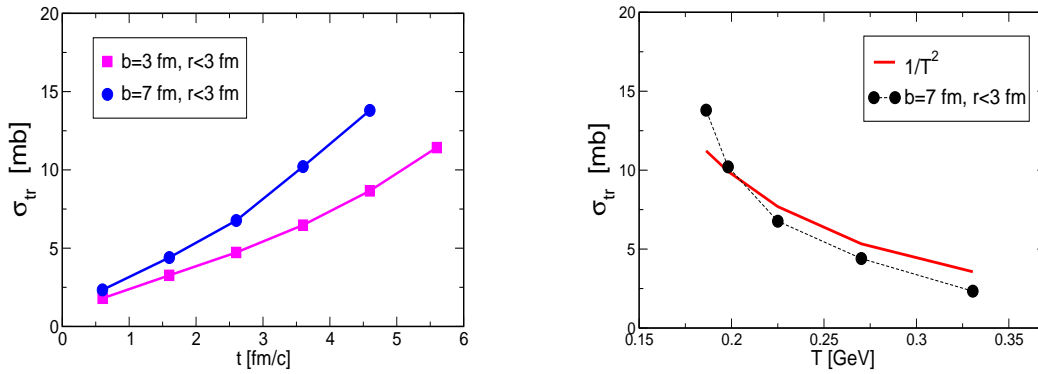


Figure 4. (Color online) Left: Time dependence of the cross section in a central region of rapidity ($|y| < 0.2$) and transverse radius $r < 3$ fm. Right: Temperature dependence of the cross section in the central region compared with the T^{-2} dependence.

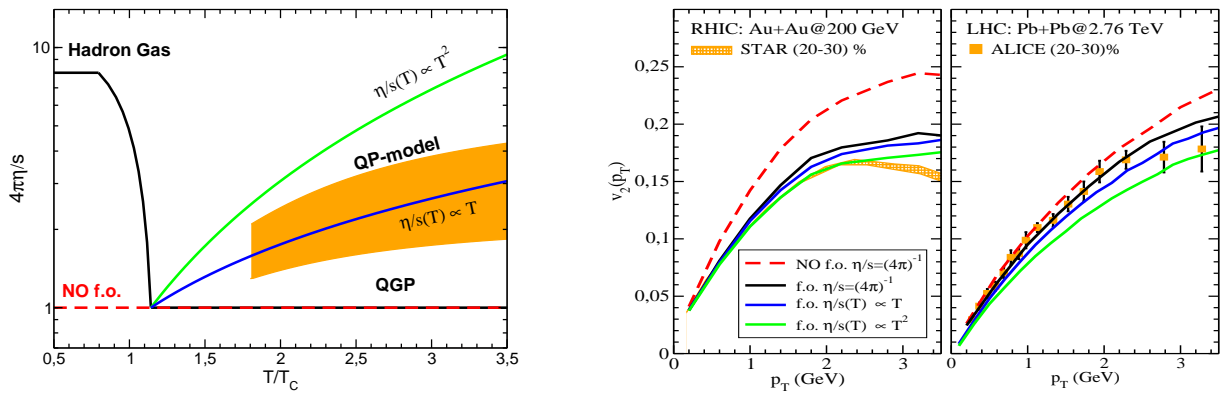


Figure 5. (Color online) Different temperature dependent parametrizations for η/s . The orange area take into account the quasi-particle model predictions for η/s [35]. Right: Differential elliptic flow $v_2(p_T)$ at mid rapidity for 20% – 30% collision centrality. The red dashed line is the calculation with $4\pi\eta/s = 1$ during all the evolution of the fireball and without the freeze out condition, while the black blue and green lines are calculations with the inclusion of the kinetic freeze out and with $4\pi\eta/s = 1$, $4\pi\eta/s \propto T$ and $4\pi\eta/s \propto T^2$ in the QGP phase respectively as shown in the left panel.

of the system (red dashed line of figure 5) the other (shown by black solid line in figure 5) with $4\pi\eta/s = 1$ in the QGP phase and an increasing η/s in the cross over region towards the estimated value for hadronic matter $4\pi\eta/s \sim 8$. Such an increase allows for a smooth realistic realization of the kinetic freeze-out. In figure 5 (right) it is shown the elliptic flow $v_2(p_T)$ at mid rapidity for 20% – 30% centrality for both RHIC Au+Au at $\sqrt{s} = 200$ GeV and LHC Pb+Pb at $\sqrt{s} = 2.76$ TeV. As we can see at RHIC energies, left panel of figure 5, the v_2 is sensitive to the hadronic phase and the effect of the freeze out is to reduce the v_2 of about of 20%, from red dashed line to black solid line in left panel of figure 5 (see also figure 6. For the p_T range shown we get a good agreement with the experimental data for a minimal viscosity $\eta/s \approx 1/(4\pi)$ once the f.o. condition is included. At LHC energies, right panel of figure 5, the scenario is different, we have that the v_2 is less sensitive to the increase of η/s at low temperature in the hadronic phase. The effect of large η/s in the hadronic phase is to reduce the v_2 by less than 5% as shown by the solid line for LHC in figure 6 while the RHIC case is shown by red dashed line and compared to the case at $4\pi\eta/s = 2$ but without the f.o. dynamics. This different behaviour of

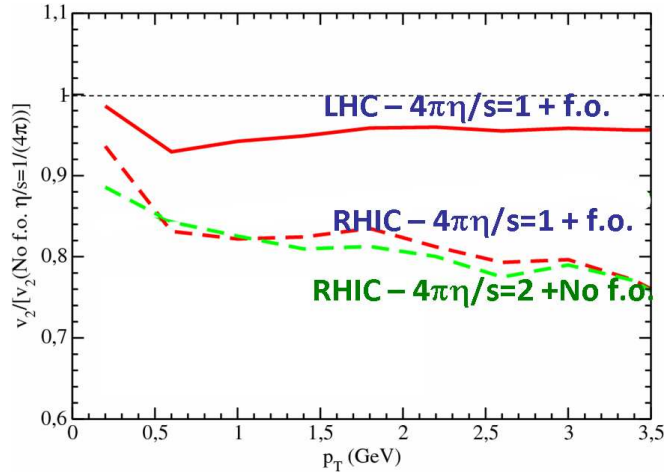


Figure 6. (Color online) The ratio between the elliptic flow $v_2(p_T)$ developed when the increase of η/s in the hadronic phase is included realizing a kinetic freeze-out (f.o.), see figure 5 and the maximal $v_2(p_T)$ obtained discarding the freeze-out. Results are shown for $Au + Au$ at $\sqrt{s} = 200$ AGeV (RHIC) and $Pb + Pb$ at $\sqrt{s} = 2.75$ ATeV (LHC) in minimum bias centrality.

v_2 between RHIC and LHC energies can be explained looking at the life time of the fireball. In fact at RHIC energies the life time of the fireball is smaller than that at LHC energies, $5fm/c$ at RHIC against the about $10fm/c$ at LHC. Therefore at RHIC the elliptic flow has not enough time to fully develop in the QGP phase. While at LHC we have that the v_2 can develop almost completely in the QGP phase.

Due to this large life time of the fireball at LHC and the larger initial temperature is interesting to study the effect of a temperature dependence in η/s . In the QGP phase η/s is expected to have a minimum of $\eta/s \approx (4\pi)^{-1}$ close to T_C as suggested by lQCD calculation. While at high temperature quasi-particle models (see previous Section) seems to suggest a temperature dependence of the form $\eta/s \sim T^\alpha$ with $\alpha \approx 1 - 1.5$ [35]. To analyze these possible scenarios for η/s in the QGP phase we have considered two different situation one with a linear dependence $4\pi\eta/s = T/T_0 = (\epsilon/\epsilon_0)^{1/4}$ (blue line) and the other one with a quadratic dependence $4\pi\eta/s = (T/T_0)^2 = (\epsilon/\epsilon_0)^{1/2}$ (green line) where $\epsilon_0 = 1.7$ GeV/fm³ is the energy density at the beginning of the cross over regions where the η/s has its minimum, see figure 5.

At RHIC energies the v_2 is essentially not sensitive to the dependence of η/s on temperature in the QGP phase, see the blu and green lines in the left panel of figure 5. However the effect on average is to decrease the value of v_2 but at low $p_T < 1.5$ GeV the $v_2(p_T)$ appears to be insensitive to $\eta/s(T)$ while a quite mild dependence appears at higher p_T where however the transport approach tends always to overpredicted the elliptic flow observed experimentally. In any case still a strong temperature dependence in η/s has a small effect on the generation of v_2 we found that with a constant or at most linearly dependent $\eta/s(T)$ the transport approach can describe the data at both RHIC and LHC at least up to $p_T \sim 2$ GeV. It is quite likely that a more detailed analisis of all the anisotropic harmonics measurable up to $v_5 = \langle \cos(5\phi_p) \rangle$ will allow to better constraint the $\eta/s(T)$ as anticipated for the v_4/v_2^2 in Ref. [37].

5. Perspectives and conclusions

We have reviewed some of the main results of the QGP at high temperature created in ultrarelativisti heavy-ion collisions (HIC) at both RHIC and LHC energies. We are developing a transport approach to study the properties of the QGP and have the possibility of interpreting the rich fenomenology coming from ultrarelativistic HIC. We have shown that a Boltzmann-

Vlasov transport approach has the potential to properly include the dynamics associated to an EoS as evaluated in the lattice QCD and the viscosity dissipation mechanisms. First results indicate that without any parameter tuning the approach is able to correctly predict the behavior of the elliptic flow going from the $Au + Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV to the $Pb + Pb$ at $\sqrt{s_{NN}} = 2.75$ TeV showing its validity. A first important result is that at LHC a key observables like the elliptic flow is much less contaminated by the hadronic phase allowing a better study of the QGP properties.

In the next future the capability to naturally extend transport theory to self-consistently include the developments in the quark-gluon quasiparticle models, the possibility extend the study of the collective flows to higher harmonics up to $v_5 = \langle \cos(5\phi) \rangle$ and the extension of the transport approach to heavy quarks dynamics and the long-standing issue of the J/Ψ suppression-regeneration will potentially allow to obtain a deeper insight into the QGP properties and their microscopic origin.

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References

- [1] Schwarz D J 2003 *Annalen Phys.* **12** 220 astro-ph/0303574)
- [2] Kovtun P, Son D T and Starinets A O 2005 *Phys. Rev. Lett.* **94** 111601
- [3] Cabibbo N and Parisi G 1975 *Phys. Lett. B* **59** 67
- [4] Gross D and Wilczek F 1973 *Phys. Rev. Lett.* **30** 1343
- [5] Adams J *et al.* (STAR Collaboration) 2005 *Nucl. Phys. A* **757** 102
- [6] Adcox K *et al.* (PHENIX Collaboration) 2005 *Nucl. Phys. A* **757** 184
- [7] Blaizot J P, Iancu E and Rebhan A 2003 (*Preprint hep-ph/0303185*)
- [8] Caron-Huot S and Moore G D 2008 *Phys. Rev. Lett.* **100** 052301
- [9] O'Hara K M, Hemmer S L, Gehm M E, Granade S R and Thomas J E 2002 *Science* **298** 2179
- [10] Danielewicz P and Gyulassy M 1985 *Phys. Rev. D* **31** 53
- [11] Ratti C and Shuryak E 2009 *Phys. Rev. D* **80** 034004
- [12] Chernodub M, D'Alessandro A, D'Elia M and Zakharov V 2009 (*Preprint 0909.5441*)
- [13] Fries R J, Greco V and Sorensen P 2008 *Ann. Rev. Nucl. Part. Sci.* **58** 177
- [14] Greco V 2008 *Eur. Phys. J. ST* **155** 45
- [15] Greco V, Ko C and Levai P 2003 *Phys. Rev. C* **68** 034904
- [16] Fries R, Muller B, Nonaka C and Bass S 2003 *Phys. Rev. C* **68** 044902
- [17] Greco V, Ko C and Levai P 2003 *Phys. Rev. Lett.* **90** 202302
- [18] Adler S S *et al.* (PHENIX Collaboration) 2006 *Phys. Rev. Lett.* **96** 032301
- [19] van Hees H, Mannarelli M, Greco V and Rapp R 2008 *Phys. Rev. Lett.* **100** 192301
- [20] van Hees H, Greco V and Rapp R 2006 *Phys. Rev. C* **73** 034913
- [21] Armesto N e, Borghini N e, Jeon S e, Wiedemann UA e, Abreu S *et al.* 2008 *J. Phys. G* **35** 054001
- [22] Dainese A (ALICE Collaboration) 2011 (*Preprint 1106.1341*)
- [23] Adare A *et al.* (PHENIX Collaboration) 2007 *Phys. Rev. Lett.* **98** 172301
- [24] Heinz U W 2004 *J. Phys. G* **30** S251
- [25] Romatschke P and Romatschke U 2007 *Phys. Rev. Lett.* **99** 172301
- [26] Huovinen P and Molnar D 2009 *Phys. Rev. C* **79** 014906
- [27] Song H and Heinz U W 2009 *J. Phys. G* **36** 064033
- [28] Ferini G, Colonna M, Di Toro M and Greco V 2009 *Phys. Lett. B* **670** 325
- [29] Greco V, Colonna M, Di Toro M and Ferini G 2008 *Prog. Part. Nucl. Phys.* **62** 562
- [30] Xu Z and Greiner C 2005 *Phys. Rev. C* **71** 064901
- [31] Molnar D and Gyulassy M 2002 *Nucl. Phys. A* **697** 495
- [32] Castorina P, Miller D E and Satz H 2011 *Eur. Phys. J. C* **71** 1673
- [33] Levai P and Heinz U W 1998 *Phys. Rev. C* **57** 1879
- [34] Borsanyi S, Endrodi G, Fodor Z, Jakovac A, Katz S D *et al.* 2010 *JHEP* **1011** 077
- [35] Plumari S, Alberico W M, Greco V and Ratti C 2011 *Phys. Rev. D* **84** 094004
- [36] Plumari S, Baran V, Di Toro M, Ferini G and Greco V 2010 *Phys. Lett. B* **689** 18
- [37] Plumari S, Baran V, Di Toro M and Greco V 2011 *J. Phys. Conf. Ser.* **270** 012061