Hadron-hadron scattering by energy-dependent and independent interactions- a supersymmetric approach

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Introduction:

Utilizing the theoretical underpinnings, such as Schrödinger's equation, Supersymmetric Quantum Mechanics (SQM), and the Phase Function Method (PFM) both charged and uncharged systems are investigated, with a focus on a diverse array of local and nonlocal potentials, including the intricate landscape of screened Coulomb potentials. The conversion of non-local nuclear interactions into simplified local forms addresses challenges inherent in solving the Schrödinger's equation for complex potentials.

The impact of short-range interactions between nuclei, primarily due to multi-pion exchange processes and nucleon recoil necessitates consideration of non-local nuclear interactions, represented by two variables 'r' and 'r''. Mathematical manipulation enables the conversion of multivariable non-local potentials to simplified local forms. While exact solutions exist for specific potentials, such as Coulomb, Square well, etc., others like Hulthén and Yamaguchi are solvable only for the S (l=0) state. Various mathematical approaches, including supersymmetry factorization and Phase function methods, are employed to address this limitation, focusing on charged and uncharged systems. The α−α system is extensively studied with different potentials to explain scattering phase shifts. The Hulthén potential is identified as an effective model for deuteron and α-nucleon interactions.

The relationship between regular and irregular solutions of non-local and local potentials has been explored, emphasizing the energy dependence of the Yamaguchi potential and the need to assess if an energy-dependent potential out-performs an energy-independent one.

The Jost Function plays a central role in developing a relativistic S-matrix theory, crucial for understanding particle interactions during scattering processes. the Jost function is instrumental in analyzing bound and resonant states, as well as low-energy scattering data. Its holomorphic nature in the upper complex k-plane (Im K>0) is key, as the zeros of the Jost function in this plane are directly linked to bound state energies. The On-Shell Jost Function is easily calculated from its integral representation, correlating phase values with scattering phase shifts with a pi phase difference. For accurate modeling of few-nucleon systems (e.g., α-nucleon, p-p, n-3He), a customized centrifugal barrier term is incorporated, aligning with nuclear interaction requirements. Literature has explored diverse approaches to approximate higher partial waves.

The supersymmetry transformation operators (T_1, T_2, T_3, T_4), integral to particle physics are exploited to create phase-equivalent potentials. The SUSY operators transition between fermionic and bosonic states, unveiling super partners in quantum field theory. They modify the initial Hamiltonian uniquely: T_1 eliminates the ground state, T_2 adds an extra bound state, and T_3 and T_4 alter the Jost Function and potential singularity. Applying these operators to a Manning-Rosen potential yields phase-equivalent potentials, and comparisons with experimental data validate the approach.

We make use of deformed Hulthén potential, delving into the derivation of regular and Jost solutions. The incorporation of energy-dependent correction terms enhances the accuracy of predictions, particularly for deuteron and triton nuclei, contributing to a nuanced understanding of nuclear forces.

Results and Discussions:

Figures 1 to 4 depicts scattering phase shifts and cross sections data of some of the system studied

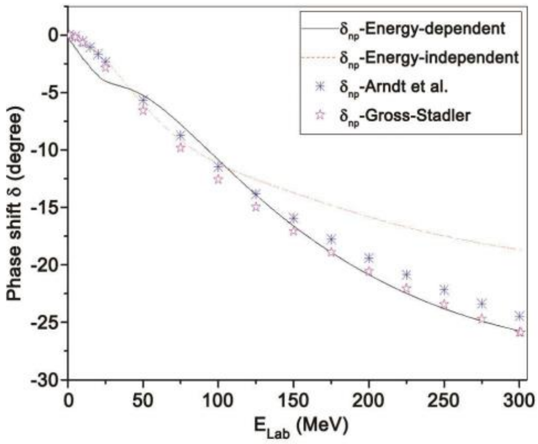

Fig.1: $^3D_1$ energy dependent and independent phase shifts as a function of E_{Lab} with ref. [1, 2].

In Fig.1 we see phase shifts due to energy dependent potential out performs energy independent one.

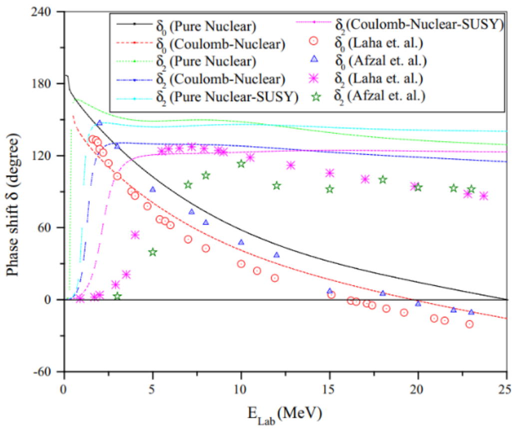

Fig. 2: $\alpha - \alpha$ scattering phase shifts for S and D waves with standard data.

The $\alpha - \alpha$ phase shifts for S and D waves have better match with standard data for Coulomb-Nuclear-SUSY interactions, as shown in Fig. 2. Fig. 3 shows that differential scattering cross section, for α-p system match well with Brockman *et. al.* data. Phase-shifts generated from the phase equivalent potentials, using four SUSY transformations match well with experimental data.

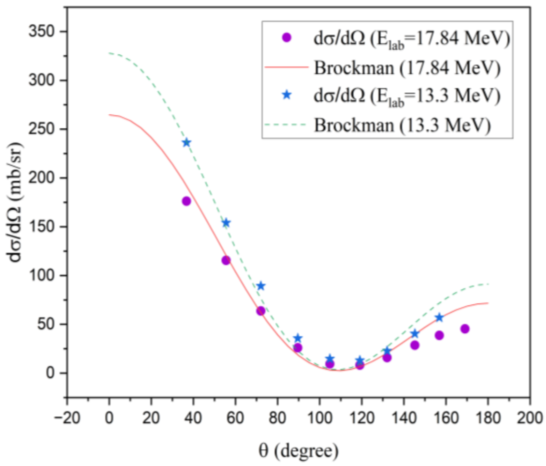

Fig. 3: Differential scattering cross section for α-p system at two different lab energies with standard data [3].

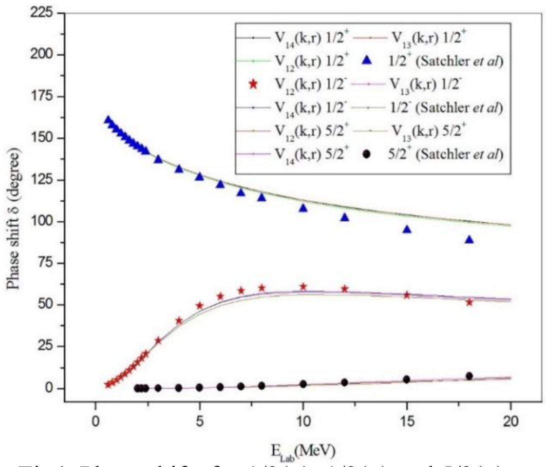

Fig4: Phase shifts for 1/2(+); 1/2(−) and 5/2(+) α-n states for Phase equivalent potentials with reference data [4].

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