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# Benchmark quantum computer with analytical single spin-flip dynamics

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## Abstract

The physical world is inherently out of equilibrium, and understanding the non-equilibrium behavior of quantum many-body systems remains a key open challenge in condensed matter physics. Recent advances in quantum computing platforms, such as Rydberg atoms and superconducting qubits, have opened promising avenues for studying non-equilibrium dynamics of many-particle systems using quantum simulators. In this work, we propose a benchmark scheme for validating the dynamical evolution outcomes of future large-scale qubit systems, which far exceeds the capability of classical numerical benchmark. Based on the  $J_1 - J_2$  Heisenberg model, we provide tunable analytical results including quantum walk dynamics, the out-of-time-ordered correlator (OTOC), and the butterfly velocity. Furthermore, taking IBM's programmable quantum platform as an example, we design a scheme for simulating quantum walk dynamics and experimentally demonstrate the feasibility of benchmarking with our proposed dynamical observables.

**Keywords:** Quantum computation, Benchmark, Quantum walk, Out-of-time-ordered correlator

## 1 Introduction

The study of non-equilibrium phenomena in quantum many-body systems remains a central frontier of condensed matter physics [1–3]. Recent programmable quantum computer utilizing Rydberg-atom, trapped ions and superconducting circuit platforms incorporating hundreds of qubits, opens a direct path to simulating quantum many-body dynamics on an unprecedented scale [4–8]. As these devices continue to grow in size and complexity, the pressing need arises for corresponding theoretical protocols and benchmarking methodologies to validate the outcomes of such dynamical simulations.

As one of the few tunable dynamical processes in such systems, quantum walks (QW) offer promising prospects for technological applications in quantum computing and related areas [9–16]. The pioneering work by Childs *et al.* in 2009 established that continuous-time QW can serve as a universal computational model for programmable quantum computers [11, 12], sparking extensive research in subsequent years. To date, QW has been experimentally demonstrated across a wide range of simulation platforms, including optical systems [17–30], trapped ions [31–35], ultracold atomic gases [36–42], and superconducting circuits [43–46], among others.

On another front, the transport properties of quantum many-body systems are intimately connected to the speed at which quantum information propagates. This has motivated growing attention to out-of-time-ordered correlators (OTOCs) as key indicators of thermalization and information scrambling [47–52]. Serving as a quantum generalization of the classical Poisson bracket, OTOCs capture the growth of non-commutativity between operators over time. They have been widely employed to characterize information spreading in diverse settings, including

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quantum many-body dynamics [53–61], many-body localized phases [62–71], quantum chaotic systems [72–77], and even holographic black hole models [78–81]. Notably, Maldacena, Shenker, and Stanford provided the first theoretical proof of a fundamental bound, the exponential growth limit, on operator growth in black hole physics and the SYK model [79], thereby reinforcing the AdS/CFT correspondence [80, 81]. Experimentally, OTOCs have been measured in several table-top quantum simulators [59, 60, 62, 82–89], establishing them as essential observables for studying information dynamics.

In this work, we present an analytical scheme based on the  $J_1 - J_2$  Heisenberg model and linear spin-wave theory. We demonstrate that the dynamics of a single-spin-flip QW, the OTOCs, and the butterfly velocity can be accurately and efficiently characterized within this scheme. Owing to its computational efficiency and tunability, our scheme provides a practical approach for benchmarking non-equilibrium dynamics on future large-scale quantum processors. The dynamical results of our scheme are verified by multiple numerical approaches at various scales. Furthermore, we implement a proof-of-principle experiment on an IBM programmable quantum computer, confirming that the non-equilibrium observables derived from our theory are directly measurable on current quantum devices.

## 2 Visualized QW

### 2.1 Spin wave theory

The model adopted in this letter is a square-lattice ferromagnetic  $J_1$ - $J_2$  Heisenberg model, and the related Hamiltonian can be written as

$$\begin{aligned} \hat{H} = & J_1 \sum_{m,n} (\hat{S}_{m,n} \hat{S}_{m\pm 1,n} + \hat{S}_{m,n} \hat{S}_{m,n\pm 1}) \\ & + J_2 \sum_{m,n} \hat{S}_{m,n} \hat{S}_{m\pm 1,n\pm 1}, \end{aligned} \quad (1)$$

where  $J_1$  and  $J_2$  correspond to the nearest-neighbor (nn) and the next-nearest-neighbor (nnn) spin exchange coupling, respectively. For simplicity, we consider only the ferromagnetic case, i.e.,  $J_1 < 0$  and  $J_2 < 0$ .

Based on the linear spin wave (SW) theory, one can exactly diagonalize the Hamiltonian of the system (see supplementary materials [90] for details). By performing Holstein-Primakoff transformation and Fourier transformation for Eq. (1), Hamiltonian of the magnon can be obtained as

$$\begin{aligned} \hat{H} = & \sum_{\mathbf{k}} \omega_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}} + J_1 \sum_{i,j} (\hat{n}_{i,j} \hat{n}_{i\pm 1,j} + \hat{n}_{i,j} \hat{n}_{i,j\pm 1}) \\ & + J_2 \sum_{i,j} (\hat{n}_{i,j} \hat{n}_{i\pm 1,j\pm 1}), \end{aligned} \quad (2)$$

where  $\hat{n}_{i,j}$  is the particle number operator and the second(third) term represents two-body interaction between nn(nnn) sites.  $\omega_{\mathbf{k}}$  stands for the energy of magnons, which has the following expressions,

$$\omega_{\mathbf{k}} = 2[J_1(1 - \gamma_{1\mathbf{k}}) + J_2(1 - \gamma_{2\mathbf{k}})], \quad (3)$$

$$\gamma_{1\mathbf{k}} = \frac{1}{2}(\cos k_x + \cos k_y), \quad (4)$$

$$\gamma_{2\mathbf{k}} = \frac{1}{2}[\cos(k_x + k_y) + \cos(k_x - k_y)]. \quad (5)$$

### 2.2 Spin-flip dynamics and magnon's QW

Then, one can readily study the propagation behavior of magnons with the diagonalized Hamiltonian, and figure out how the single spin flip information diffuses in the background of the ferromagnetic ground state. Let's start with calculating the density distribution versus time, which is a key observable easy to measure experimentally in studying QW, with its expression as [37–39],

$$\xi(\mathbf{r}, t) = \langle \phi | \hat{S}_{\mathbf{r}_0}^+ \hat{S}_{\mathbf{r}_{m,n}}^z(t) \hat{S}_{\mathbf{r}_0}^- | \phi \rangle, \quad (6)$$

with

$$\hat{S}_{\mathbf{r}_{m,n}}^z(t) = e^{iHt/\hbar} \hat{S}_{\mathbf{r}_{m,n}}^z e^{-iHt/\hbar}, \quad (7)$$

where  $|\phi\rangle$  is the initial state, i.e., the ferromagnetic ground state and  $\mathbf{r} = \mathbf{r}_{m,n} - \mathbf{r}_0$ . In the expression,  $\hat{S}_{\mathbf{r}_{m,n}}^z$  represents the spin operator at  $\mathbf{r}_{m,n}$  in  $z$ -direction, while  $\hat{S}_{\mathbf{r}_0}^+$  and  $\hat{S}_{\mathbf{r}_0}^-$  denote the spin flip up and down operators at  $\mathbf{r}_0$ , respectively. The diffusion behavior of the magnon can be revealed by measuring the evolution of density distribution with time.

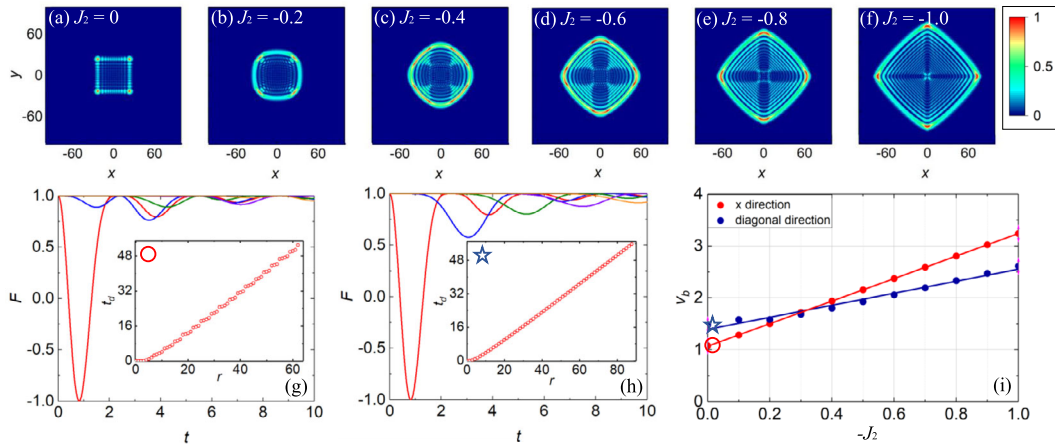
Since this is a ferromagnetic (FM) system, the single spin-flip on top of the fully polarized FM ground state can be exactly mapped to a single hard-core boson in the vacuum state. As is known to all, in FM case, the Hamiltonian ignoring the interacting terms is still exact for single-magnon sectors. Hence, we can substitute the effective Hamiltonian

$$\hat{H}_{eff} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}} \quad (8)$$

into Eq. (6). Considering the operation rules of creation and annihilation operators, one can obtain the analytical expression of QW (see supplementary material [90] for details), i.e.,

$$\xi(\mathbf{r}, t) = \left| \sum_{\mathbf{k}} \frac{e^{-i(\mathbf{k}\cdot\mathbf{r} + \omega_{\mathbf{k}}t/\hbar)}}{N^2} \right|^2. \quad (9)$$

Thus, we can obtain the exact solution of the time-evolution that involves a great number of qubits on two-dimensional computing platform.



**Figure 1** (Color online). Visualized magnon QW and spread properties of ferromagnetic  $J_1$ - $J_2$  Heisenberg model with nn and nnn interaction. (a-f) The density distributions of magnon with different nnn interaction strengths  $J_2$  at  $t = 25$ . (g) The OTOC along  $x$  direction for different distance  $|\mathbf{r}| = \mathbf{r}_{m,0} - \mathbf{r}_0 = 0$  (red), 2 (blue), 4 (green), 6 (purple), 8 (orange). (h) The OTOC along diagonal direction for different  $|\mathbf{r}| = \mathbf{r}_{m,n=m} - \mathbf{r}_0 = 0$  (red),  $2\sqrt{2}$  (blue),  $4\sqrt{2}$  (green),  $6\sqrt{2}$  (purple),  $8\sqrt{2}$  (orange). Insets: The  $t_d$  lines (line of first decline moments) for  $x$ -axis and diagonal. (i) Butterfly velocities for different nnn interaction. Throughout,  $J_1 = -1$ . The unit of  $x$  and  $y$  is the lattice constant  $a = 1$ , whereas the time unit here is  $\hbar/|J_1| = 1$  with  $\hbar = 1$

For convenience, we refer to the above technique as analytical single spin-flip dynamics (ASSD). The process of magnon’s dynamic evolution under different parameters  $J_2$  is visualized in Figs. 1(a-f). As shown in the figure, when  $J_2 = 0$ , the system degenerates to the standard Heisenberg model. The density distribution then features ballistic propagation, typical for QW. When  $J_2 \neq 0$ , the nnn coupling comes into play. As shown in Fig. 1, the influence of the nnn coupling on dynamics of the system can be distilled into three points:

I.  $J_2$  causes the magnon to travel faster in all directions. As we know, the computation speed of quantum computers is closely related to the speed of information propagation between qubits. The observed acceleration of information propagation is connected to the computational speedups in quantum algorithms such as spatial search based on quantum walks [91].

II. The acceleration is anisotropic, maximum in the  $x$ - and  $y$ -axis directions and minimum in the diagonal direction. Therefore, new quantum computers can be designed on the basis of the anisotropic propagation, which will play a pivotal role in conducting anisotropic computation and quantum simulation effectively.

III. The competition between nn and nnn coupling cannot destroy the ballistic propagation behavior. However, the geometric structure of ballistic propagation can be changed by manipulating parameters  $J_1$  and  $J_2$ . As shown in Figs. 1(a-f), the system transforms from a square structure to a rhombic QW.

### 3 OTOCs and butterfly velocities

Furthermore, we calculate OTOCs of the system and the corresponding butterfly velocity to reveal the propagation

characteristics [53–60, 82–85]. Note that butterfly velocity and the Lieb-Robinson bounds (the upper limit of the information propagation speed in quantum systems) are highly interdependent, which reflects information propagation behavior in many-body systems [47, 92–98]. Detailed derivation of OTOCs and butterfly velocity in  $J_1$ - $J_2$  model is included in Supplementary Material [90].

By simple operator gymnastics, one can obtain the analytic expression of the system’s OTOCs, which reads

$$\hat{F}(t) = \langle \phi | \hat{S}_{\mathbf{r}_0}^+ \hat{S}_{\mathbf{r}_{m,n}}^z(t) \hat{S}_{\mathbf{r}_0}^z(0) \hat{S}_{\mathbf{r}_{m,n}}^z(t) \hat{S}_{\mathbf{r}_0}^z(0) \hat{S}_{\mathbf{r}_0}^- | \phi \rangle. \quad (10)$$

Then, we get OTOCs’ analytical expression as

$$\hat{F}(t) = 1 - 8/N^2 \Omega_1 \Omega_2 + 8/N^4 \Omega_1 \Omega_2 \Omega_1 \Omega_2, \quad (11)$$

where,

$$\begin{aligned} \Omega_1 &= \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot (\mathbf{r}_0 - \mathbf{r}_{m,n})} e^{i\omega_{\mathbf{k}} t / \hbar}, \\ \Omega_2 &= \sum_{\mathbf{k}} e^{i\mathbf{k} \cdot (\mathbf{r}_0 - \mathbf{r}_{m,n})} e^{-i\omega_{\mathbf{k}} t / \hbar}. \end{aligned} \quad (12)$$

Based on the ASSD technique, the propagation properties of magnetic information (a spin-flip signal) in the large-scale  $J_1$ - $J_2$  Heisenberg model can be revealed at a lower computational cost.

The results of OTOCs and the butterfly velocity are plotted in Figs. 1(g-i). We define the time when OTOC begins to decline as the critical time  $t_d$ . The illustration reflects the relationship between  $t_d$  and  $\mathbf{r}$ . As shown in Fig. 1(g) [(h)], the OTOCs along the  $x$ -,  $y$ -axis [diagonal] direction

oscillate versus time, where the amplitude of OTOCs decreases and  $t_d$  increases with the increasing  $r$ . Since the magnon wave packet itself is conservative in the process of evolution, the probability of magnon distribution at a distant point is relatively small, hence the relatively small amplitude [47–49]. Besides, the propagation mode of the system is determined by the growth of the local Heisenberg operator, therefore, the longer the propagation time of the distant lattice magnon, the larger the corresponding  $t_d$ . The above two points well reflect that the magnon information diffuses in a way of QW. Note that the slope of  $t_d$  in the illustration is the butterfly velocity [72, 73]. Further, we calculate how the butterfly velocity of the system changes with  $J_2$  [see Fig. 1(i)]. As shown in the figure, when  $J_2$  increases, the butterfly velocity in the diagonal direction will slowly surpass that in the  $x$ -direction, which indicates that the butterfly velocity can very well reflect the competitive relationship between nn coupling and nnn coupling in the system.

In short, based on OTOCs and butterfly velocity, one can understand the features of magnon QW in  $J_1$ - $J_2$  model from the following three perspectives:

I. The superposition of another QW on top of the previous one will lead to the acceleration of information propagation;

II. The competition between the two results in the change from diagonal dominance to  $x$ - and  $y$ -axis dominance of QW, and correspondingly, the geometric structure changes from square to rhombus;

III. The introduction of nnn coupling will not destroy the ballistic propagation mode, therefore, the system still displays QW.

It is noticeable that the QW and OTOCs results of a single spin-flip are totally accurate, which is supported by mathematical proofs in the previous sections and double-checked by Exact Diagonalization and TDVP in the Supplementary Material [90].

#### 4 Digital quantum simulation

In principle, SW theory can very effectively describe the diffusion process of information in large-scale or even infinite lattice ferromagnetic systems, thus making it an ideal way to test the computational power of quantum computers. We believe that the coming decades will see a great increase in the number of qubits, as well as the realization of error-correcting qubits. With the unstoppable progress of technological infrastructure, more phenomena in many-body dynamics, such as QW in large-scale systems and controllable magnetic transport, will be verified by more programmable quantum processors in the future.

As a principle testing, here we conduct experiments on a 5-qubit quantum processor and compare the experimental results with the theoretical ones. In order to conduct an experiment of scrambling dynamics on the IBM quantum processor, one must translate the time evolution into

the language of quantum circuits, i.e., a series of standard quantum gates. First, the Heisenberg model described in boson language can be written as

$$H = -\frac{1}{2}N + 2 \sum_l (\hat{a}_l^\dagger \hat{a}_l) - \sum_l (\hat{a}_l \hat{a}_{l+1}^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}). \quad (13)$$

Here we take  $J_1 = -1$  and  $S = 1/2$ . By ignoring the constant term, which only brings in a global phase factor, the time evolution operator can be written as

$$\hat{U}(\delta t) = e^{-iH\delta t} = e^{-i[2 \sum_l (\hat{a}_l^\dagger \hat{a}_l) - \sum_l (\hat{a}_l \hat{a}_{l+1}^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1})]\delta t}. \quad (14)$$

For tiny  $\delta t$ , the Trotter decomposition can be used to break the evolutionary operator into the following steps,

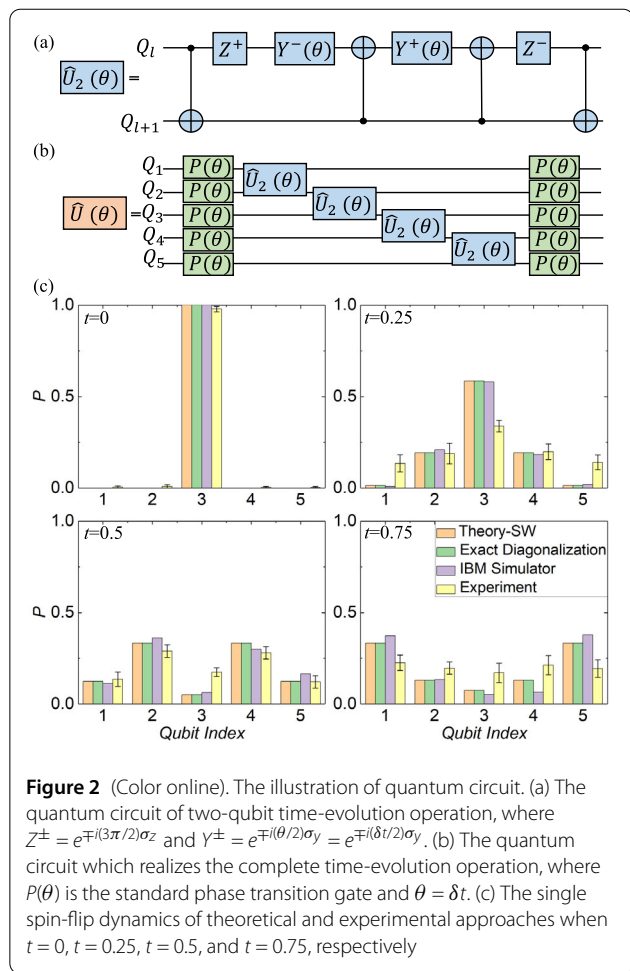
$$\begin{aligned} \hat{U}(\delta t) &\simeq e^{-i \sum_l (\hat{a}_l^\dagger \hat{a}_l) \delta t} e^{i \sum_l (\hat{a}_l \hat{a}_{l+1}^\dagger + \hat{a}_l^\dagger \hat{a}_{l+1}) \delta t} e^{-i \sum_l (\hat{a}_l^\dagger \hat{a}_l) \delta t} \\ &= \hat{U}_1(\delta t) \hat{U}_2(\delta t) \hat{U}_1(\delta t). \end{aligned} \quad (15)$$

where  $\hat{U}_1$  and  $\hat{U}_2$  represent the contribution of the on-site term and nn term. Next, we need to implement the  $\hat{U}_1$  and  $\hat{U}_2$  operators with a series of single and double qubit logic gates.  $\hat{U}_1$  can be readily implemented via a single qubit logic gate, while  $\hat{U}_2$  can be formed by the combination of  $C$ -NOT gate, Y gate and Z gate, as shown in Fig. 2(a). The input parameter  $\theta$  is a function of  $\delta t$  and  $J_1$ . In the experiment,  $\delta t = 0.1$  and  $J_1 = -1$ . Thus, a complete life cycle of time-evolving operation can be written as the quantum circuit in Fig. 2(b).

Figure 2(c) shows the magnon dynamic evolution behavior obtained via four different methods (analytical solution of SW theory, exact diagonalization method, IBM simulator and real IBM quantum processor). The results obtained by all these methods are consistent, i.e., the system exhibits the feature of QW. It is worth noting that the analytical solution calculated by SW stands out as the ideal choice, for it can predict the dynamic evolution behavior of large-scale magnetic models at a lower computational cost of classical computers.

Since the analytical solution given here has universality, it can be used as a “standard” to measure the performance and fidelity of large-scale quantum chips in the future. Currently, since quantum computers are limited in fidelity and number of qubits, only small-scale models can be simulated. Recent experiments with quantum circuits, however, have shown more and more controllable qubits [45, 46]. In the future, with the increase of the number of qubits and the advent of error-correcting qubits, more phenomena of many-body dynamics can be studied using the method presented in this paper.

It is important to discuss the restriction of our ASSD scheme to systems with spatial translational symmetry.



This focus, however, is a deliberate strength for a benchmark. Systems without such symmetry are inherently challenging for fidelity benchmarking, as the necessary average over disorder realizations obscures the intrinsic error profile of the quantum hardware. The central aim of our work is to identify a sweet spot that balances benchmarking accuracy with the simulation of large-scale quantum dynamics. By utilizing translationally symmetric systems, we achieve a clear, unambiguous measure of device performance.

### 5 Conclusion

As the accuracy and the number of entangled qubits continuously increase, numerical methods will start to lose their reliability due to the ascending calculation cost and error when solving non-equilibrium problems. Thus, effective techniques that can verify the capability of quantum computers that involve a large number of qubits are needed. In this connection, we propose a technique from the perspective of dynamics. By conducting a single spin-flip on top of the ground state of a two-dimensional ferromagnetic  $J_1$ - $J_2$  Heisenberg model and simulating the time-

evolution of QW and OTOCs with an analytical procedure, we can provide large-scale quantum computer with a decent benchmark. Moreover, the accuracy and superiority of the analytical procedure overtake those of the numerical methods of TDVP and ED. Finally, we prove an experiment on IBMQ to justify the selection of dynamical observable, and look forward to seeing future realization of this scheme on larger-scale quantum computation platforms. Our work will provide quantum computation researchers with a useful tool and shed light on quantum information, quantum chaos and traditional condensed matter physics.

### Author contributions

Z-XG and X-DH did the theoretical and numerical work with the input from ZL and X-JY. All authors participated in the analysis of the results. ZL and X-JY provided the original idea. Z-XG wrote the manuscript with the input from all authors. Q-QS wrote the quantum computing part of the supplementary material. All authors have read and approved the manuscript.

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### Data availability

All data underlying the results are available from the authors upon reasonable request.

### Declarations

#### Competing interests

The authors declare no competing interests.

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