



Alternative $[SU(3)]^4$ model of leptonic color and dark matter

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Received 9 January 2018; received in revised form 26 January 2018; accepted 29 January 2018

Available online 3 February 2018

Editor: Hong-Jian He

Abstract

The alternative $[SU(3)]^4$ model of leptonic color and dark matter is discussed. It unifies at $M_U \sim 10^{14}$ GeV and has the low-energy subgroup $SU(3)_q \times SU(2)_l \times SU(2)_L \times SU(2)_R \times U(1)_X$ with $(u, h)_R$ instead of $(u, d)_R$ as doublets under $SU(2)_R$. It has the built-in global $U(1)$ dark symmetry which is generalized $B-L$. In analogy to $SU(3)_q$ quark triplets, it has $SU(2)_l$ hemion doublets which have half-integral charges and are confined by $SU(2)_l$ gauge bosons (stickons). In analogy to quarkonia, their vector bound states (hemionia) are uniquely suited for exploration at a future e^-e^+ collider.

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1. Introduction

To venture beyond the Standard Model (SM) of quarks and leptons, there have been many trailblazing ideas. One is the notion of grand unification, i.e. the embedding of the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ in a single larger symmetry such as $SU(5) \sim E_4$, $SO(10) \sim E_5$, or E_6 . There are indeed very many papers devoted to this topic. Less visited are the symmetries $[SU(3)]^N$, where $N = 3, 4, 6$ have been considered [1–8]. Another idea is that the $SU(2)_R$ quark doublet may not be $(u, d)_R$ but rather $(u, h)_R$ where h is an exotic quark

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of charge $-1/3$. This was originally motivated by superstring-inspired E_6 models [9,10] and later generalized to nonsupersymmetric models [11–14], but is easily implemented in $[SU(3)]^N$ models. A third idea is quark–lepton interchange symmetry [15,16] which assumes $SU(3)_l$ for leptons in parallel to $SU(3)_q$ for quarks, but with $SU(3)_l$ broken to $SU(2)_l \times U(1)_{Y_l}$. This is naturally embedded in $[SU(3)]^4$ [4] and implies that only one component of the color lepton triplet is free, i.e. the observed lepton, whereas the other two color components (with half-integral charges) are confined in analogy to the three color components of a quark triplet. Finally a fourth idea has been put forward recently [6,17] that a dark symmetry may exist within $[SU(3)]^N$ itself or perhaps $[SU(3)]^N \times U(1)$. This new insight points to the possible intrinsic unity of matter with dark matter [18–20].

In this paper, all four of the above ideas are incorporated into a single consistent framework based on the symmetry $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$. The three families of quarks and leptons are contained in the bifundamental chain $(3, 3^*, 1, 1) + (1, 3, 3^*, 1) + (1, 1, 3, 3^*) + (3^*, 1, 1, 3)$ which also include other fermions beyond the SM. This unifying symmetry is broken by two bifundamental scalars at M_U to $SU(3)_q \times SU(2)_l \times SU(2)_L \times SU(2)_R \times U(1)_X$ in such a way that a residual global $U(1)_D$ symmetry remains. This important property guarantees that a dark sector exists for a set of fermions, scalars, and vector gauge bosons. Because of the necessary particle content of $[SU(3)]^4$, this $U(1)_D$ may be identified as generalized $B-L$ [21], under which quarks have charge $1/3$ and leptons have charge -1 , but the other particles have different values.

At M_R of order a TeV, $SU(2)_R \times U(1)_X$ is broken to $U(1)_Y$ of the SM, with particle content of the SM plus possible light particles transforming under the leptonic color $SU(2)_l$ symmetry. We will discuss their impact on cosmology as well as their possible revelation at a future e^-e^+ collider, following closely our previous work [5] on the subject. We will also consider the phenomenology associated with the $SU(2)_R$ gauge symmetry and the possible dark-matter candidates of this model.

2. Fermion content and dark symmetry

All fermions belong to bitriplet representations $(3, 3^*)$ under $SU(3)_A \times SU(3)_B$, where $SU(3)_A$ acts vertically from up to down with $I_{3A} = (1/2, -1/2, 0)$ and $Y_A = (1, 1, -2)/(2\sqrt{3})$, and $SU(3)_B$ horizontally from left to right with $I_{3B} = (-1/2, 1/2, 0)$ and $Y_B = (-1, -1, 2)/(2\sqrt{3})$. The dark symmetry we will consider is

$$D = \sqrt{3}(-2Y_L + \sqrt{3}I_{3R} + Y_R - 2Y_l). \tag{1}$$

Under $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$, the fermion content of our model is then given by

$$q \sim (3, 3^*, 1, 1) \sim \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \quad D_q \sim \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 1 & 1 & -2 \end{pmatrix}, \tag{2}$$

$$l \sim (1, 3, 3^*, 1) \sim \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & n \end{pmatrix}, \quad D_l \sim \begin{pmatrix} 0 & 0 & -3 \\ 0 & 0 & -3 \\ 3 & 3 & 0 \end{pmatrix}, \tag{3}$$

$$l^c \sim (1, 1, 3, 3^*) \sim \begin{pmatrix} z_1^c & y_1^c & x_1^c \\ z_2^c & y_2^c & x_2^c \\ n^c & e^c & \nu^c \end{pmatrix}, \quad D_{l^c} \sim \begin{pmatrix} -3 & 0 & 0 \\ -3 & 0 & 0 \\ 0 & 3 & 3 \end{pmatrix}, \tag{4}$$

$$q^c \sim (3^*, 1, 1, 3) \sim \begin{pmatrix} h^c & h^c & h^c \\ u^c & u^c & u^c \\ d^c & d^c & d^c \end{pmatrix}, \quad D_{q^c} \sim \begin{pmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}, \quad (5)$$

where u has charge $2/3$, d, h have charge $-1/3$, x, z have charge $1/2$, y has charge $-1/2$, v, n have charge 0 , and e has charge -1 . Using

$$R_D = (-1)^{D+2j}, \quad (6)$$

we see that $u, u^c, d, d^c, v, v^c, e, e^c, z, z^c$ are even, and $h, h^c, x, x^c, y, y^c, n, n^c$ are odd. Further, the gauge bosons which take h to u, d in $SU(3)_L$ and h^c to u^c, d^c in $SU(3)_R$ are odd, as well as the corresponding ones in $SU(3)_I$, and the others even, including all those of the SM. Hence R_D would remain a good symmetry for dark matter provided that the scalar sector responsible for the symmetry breaking obeys it as well.

The scalar bitriplets responsible for the masses of the fermions in Eqs. (2) to (5) come from three chains, each of the form $(3, 1, 3^*, 1) + (1, 3, 1, 3^*) + (3^*, 1, 3, 1) + (1, 3^*, 1, 3)$. Specifically,

$$\phi^{(1,3,5)} \sim (1, 3, 1, 3^*) \sim \begin{pmatrix} \eta^0 & \phi_2^+ & \phi_1^0 \\ \eta^- & \phi_2^0 & \phi_1^- \\ \chi^0 & \chi^+ & \lambda^0 \end{pmatrix}, \quad D_\phi \sim \begin{pmatrix} -3 & 0 & 0 \\ -3 & 0 & 0 \\ 0 & 3 & 3 \end{pmatrix}, \quad (7)$$

$$\bar{\phi}^{(2,4,6)} \sim (1, 3^*, 1, 3) \sim \begin{pmatrix} \bar{\eta}^0 & \eta^+ & \bar{\chi}^0 \\ \phi_2^- & \bar{\phi}_2^0 & \chi^- \\ \bar{\phi}_1^0 & \phi_1^+ & \bar{\lambda}^0 \end{pmatrix}, \quad D_{\bar{\phi}} \sim \begin{pmatrix} 3 & 3 & 0 \\ 0 & 0 & -3 \\ 0 & 0 & -3 \end{pmatrix}. \quad (8)$$

From the $q^c q \phi$ terms, we obtain masses of hh^c from $\langle \chi^0 \rangle^{(1)}$, dd^c from $\langle \phi_1^0 \rangle^{(3)}$, uu^c from $\langle \phi_2^0 \rangle^{(5)}$. From the $ll^c \bar{\phi}$ terms, we obtain masses of nn^c, zz^c from $\langle \bar{\chi}^0 \rangle^{(2)}$, vv^c, xx^c from $\langle \bar{\phi}_1^0 \rangle^{(4)}$, ee^c, yy^c from $\langle \bar{\phi}_2^0 \rangle^{(6)}$. It is clear that D and thus R_D remain unbroken by the above vacuum expectation values.

3. Symmetry breaking pattern

We consider the breaking of $[SU(3)]^4$ at M_U by two scalar bitriplets, one transforming as $\phi^{L+} \sim (1, 3, 3^*, 1) \sim l$, belonging to a chain in parallel to the fermions, the other transforming as $\phi^{R-} \sim (1, 1, 3, 3^*) \sim l^c$, belonging to a chain with an additional overall imposed assignment of odd R_D , i.e. an additional Z_2 factor [6]. This preserves the relative R_D among its components, but prevents it from coupling to the fermions. Using $\langle \phi_{33}^{L+} \rangle$ with even R_D to break $SU(3)_L \times SU(3)_I$ to $SU(2)_L \times SU(2)_I \times U(1)_{(Y_L+Y_I)/\sqrt{2}}$ and $\langle \phi_{33}^{R-} \rangle$ which also has even R_D to break $SU(3)_I \times SU(3)_R$ to $SU(2)_I \times SU(2)_R \times U(1)_{(Y_I+Y_R)/\sqrt{2}}$, the resulting theory preserves R_D . Assuming also that all the particles of the chain associated with ϕ^{R-} are superheavy, the low-energy theory with the residual gauge symmetry $SU(3)_q \times SU(2)_I \times SU(2)_L \times SU(2)_R \times U(1)_X$, where $X = (Y_L + Y_R + Y_I)/\sqrt{3}$, also preserves D .

Since there are three fermion chains, and five scalar chains, the b coefficients for the renormalization-group running of each $SU(3)$ gauge coupling are all given by

$$b = -11 + \frac{2}{3} \left(\frac{1}{2} \right) (2)(3)(3) + \frac{1}{3} \left(\frac{1}{2} \right) (2)(3)(5) = 0. \quad (9)$$

This shows that we have a possible finite field theory [3] above M_U .

At M_R , the $SU(2)_R \times U(1)_X$ gauge symmetry is broken to $U(1)_Y$ of the SM, where $Y = I_{3R} - X$, by an $SU(2)_R$ doublet whose neutral component is a linear combination of

χ^0 from $\phi^{(1)}$, the conjugate of $\bar{\chi}^0$ from $\bar{\phi}^{(2)}$, and ϕ_{31}^{R+} from the (1, 1, 3, 3*) component of the chain containing ϕ^{L+} discussed previously. From the allowed antisymmetric trilinear term $l^c l^c \phi^{R+}$, the mass term $x_1^c y_2^c - x_2^c y_1^c$ is then obtained. Note that the corresponding mass term $x_1 y_2 - x_2 y_1$ is superheavy because it comes from $\langle \phi_{33}^{L+} \rangle$. Note also that the corresponding term $l^c l^c \phi^{R-}$ is forbidden because of the overall assignment of odd R_D for ϕ^{R-} . Finally the symmetry $SU(2)_L \times U(1)_Y$ is broken by two $SU(2)_L$ doublets to $U(1)_{em}$ with $Q = I_{3L} + Y$.

4. Renormalization-group running of gauge couplings

The renormalization-group evolution of the gauge couplings is dictated at leading order by

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu')} + \frac{b_i}{2\pi} \ln\left(\frac{\mu'}{\mu}\right), \tag{10}$$

where b_i are the one-loop beta-function coefficients. From M_U to M_R , we assume that all fermions are light except the three families of (x, y) hemions. As for the scalars, we assume that only the following multiplets are light under $SU(2)_L \times SU(2)_R \times U(1)_X$: 1 copy of (1, 2, -1/2), 6 copies of (2, 2, 0), 3 copies of (2, 1, -1/2), and 4 copies of (2, 1, 1/2). This choice requires fine tuning in the scalar sector as in other models of grand unification such as $SU(5)$ and $SO(10)$. As a result, the five b coefficients are given by

$$b_q = -11 + \frac{2}{3} \left(\frac{1}{2}\right) (6)(3) = -5, \tag{11}$$

$$b_l = -\frac{22}{3} + \frac{2}{3} \left(\frac{1}{2}\right) (4)(3) = -\frac{10}{3}, \tag{12}$$

$$b_L = -\frac{22}{3} + \frac{2}{3} \left(\frac{1}{2}\right) (3+1)(3) + \frac{1}{3} \left(\frac{1}{2}\right) [7+6(2)] = -\frac{1}{6}, \tag{13}$$

$$b_R = -\frac{22}{3} + \frac{2}{3} \left(\frac{1}{2}\right) (3+2+1)(3) + \frac{1}{3} \left(\frac{1}{2}\right) [1+6(2)] = \frac{5}{6}, \tag{14}$$

$$b_X = \frac{2}{3} \left[\frac{1}{6}(3) + \frac{1}{6}(3) + \frac{1}{4}(4) + \frac{1}{4}(4) \right] (3) + \frac{1}{3} \left(\frac{1}{4}\right) [2+7(2)] = \frac{22}{3}. \tag{15}$$

From M_R to M_Z , we assume the SM quark and lepton content together with 1 copy of (x^c, y^c) hemions and two $SU(2)_L$ Higgs scalar doublets. The massless $SU(2)_l$ stickons are of course included but they affect only α_l . The four b coefficients are then

$$b_q = -11 + \frac{2}{3} \left(\frac{1}{2}\right) (4)(3) = -7, \tag{16}$$

$$b_l = -\frac{22}{3} + \frac{2}{3} \left(\frac{1}{2}\right) (2) = -\frac{20}{3}, \tag{17}$$

$$b_L = -\frac{22}{3} + \frac{2}{3} \left(\frac{1}{2}\right) (3+1)(3) + \frac{1}{3} \left(\frac{1}{2}\right) (2) = -3, \tag{18}$$

$$b_Y = \frac{1}{2} \left[\frac{2}{3} \left\{ \frac{10}{3}(3) + \frac{1}{4}(4) \right\} + \frac{1}{3} \left(\frac{1}{4}\right) (4) \right] = \frac{23}{6}, \tag{19}$$

where a factor of 1/2 has been inserted to normalize b_Y . The boundary condition at M_R for $SU(2)_R \times U(1)_X$ to become $U(1)_Y$ is

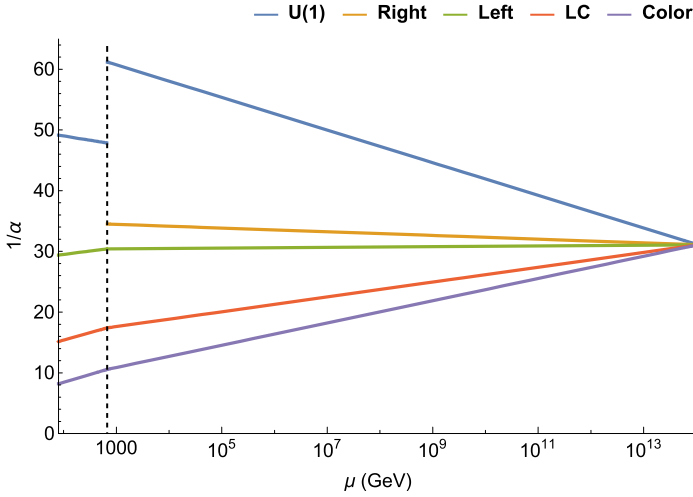


Fig. 1. Evolution of α_i^{-1} as a function of energy scale. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

$$\frac{2}{\alpha_Y(M_R)} = \frac{1}{\alpha_R(M_R)} + \frac{1}{\alpha_X(M_R)}. \tag{20}$$

We then obtain

$$\frac{1}{\alpha_q(M_Z)} = \frac{1}{\alpha_U} - \frac{7}{2\pi} \ln \frac{M_R}{M_Z} - \frac{5}{2\pi} \ln \frac{M_U}{M_R}, \tag{21}$$

$$\frac{1}{\alpha_L(M_Z)} = \frac{1}{\alpha_U} - \frac{3}{2\pi} \ln \frac{M_R}{M_Z} - \frac{1}{6(2\pi)} \ln \frac{M_U}{M_R}, \tag{22}$$

$$\frac{1}{\alpha_Y(M_Z)} = \frac{1}{\alpha_U} + \frac{23}{6(2\pi)} \ln \frac{M_R}{M_Z} + \frac{49}{12(2\pi)} \ln \frac{M_U}{M_R}. \tag{23}$$

Using the experimental inputs

$$\alpha_q(M_Z) = 0.1185, \tag{24}$$

$$\alpha_L(M_Z) = (\sqrt{2}/\pi) G_F M_W^2 = 0.0339, \tag{25}$$

$$\alpha_Y(M_Z) = 2\alpha_L(M_Z) \tan^2 \theta_W = 0.0204, \tag{26}$$

where a factor of 2 has been used to normalize α_Y , we find

$$\frac{1}{0.0339} - \frac{1}{0.1185} = 21.06 = \frac{4}{2\pi} \ln \frac{M_R}{M_Z} + \frac{29}{6(2\pi)} \ln \frac{M_U}{M_R}, \tag{27}$$

$$\frac{1}{0.0204} - \frac{1}{0.0339} = 19.52 = \frac{41}{6(2\pi)} \ln \frac{M_R}{M_Z} + \frac{17}{4(2\pi)} \ln \frac{M_U}{M_R}. \tag{28}$$

This implies $M_R \simeq 600$ GeV and $M_U \simeq 10^{14}$ GeV, as shown in Fig. 1. The 5 lines emanating from a common point at 10^{14} GeV represent $U(1)_X$, $SU(2)_R$, $SU(2)_L$, $SU(2)_I$, and $SU(3)_q$ from top to bottom. The line between M_R and M_Z represents normalized $U(1)_Y$. Since there are uncertainties (both theoretical and experimental) in the above estimate, the value of M_R should not be taken too literally, but rather an indication that particles transforming under $SU(2)_R$ have masses of an order of magnitude greater than those of the SM. As a result, $\alpha_U = 0.0322$. Using

Table 1
Particle content of proposed model.

Particles	$SU(3)_q$	$SU(2)_l$	$SU(2)_L$	$SU(2)_R$	$U(1)_X$	D	S	$I_{3R} + S$
$(u, d)_L$	3	1	2	1	-1/6	(1, 1)	1/3	1/3
$(u, h)_R$	3	1	1	2	-1/6	(1, -2)	-1/6	(1/3, -2/3)
d_R	3	1	1	1	1/3	1	1/3	1/3
h_L	3	1	1	1	1/3	-2	-2/3	-2/3
$(\nu, l)_L$	1	1	2	1	1/2	(-3, -3)	-1	-1
$(n, l)_R$	1	1	1	2	1/2	(0, -3)	-1/2	(0, -1)
ν_R	1	1	1	1	0	-3	-1	-1
n_L	1	1	1	1	0	0	0	0
$(z, y)_R$	1	2	1	2	0	(3, 0)	1/2	(1, 0)
x_R	1	2	1	1	-1/2	0	0	0
z_L	1	2	1	1	-1/2	3	1	1
(ϕ_1^0, ϕ_1^-)	1	1	2	1	1/2	0	0	0
(χ^+, χ^0)	1	1	1	2	-1/2	(3, 0)	1/2	(1, 0)
(η, Φ_2)	1	1	2	2	0	(-3, 0)	-1/2	(-1, 0)
λ^0	1	1	1	1	0	3	1	1

$$\frac{1}{\alpha_R(M_R)} = \frac{1}{\alpha_U} + \frac{5}{6(2\pi)} \ln \frac{M_U}{M_R}, \tag{29}$$

we obtain $\alpha_R(M_R) = 0.0290$. Using

$$\frac{1}{\alpha_l(M_Z)} - \frac{1}{\alpha_q(M_Z)} = \frac{1}{3(2\pi)} \ln \frac{M_R}{M_Z} + \frac{5}{3(2\pi)} \ln \frac{M_U}{M_R}, \tag{30}$$

we obtain $\alpha_l = 0.0650$, implying a confining scale of about 0.4 MeV from leptonic color. This is significantly different from the result of the $[SU(3)]^4$ model with $M_R = M_U$, where it is a few keV [4,5].

5. Low-energy particle content

The particles of this model at or below a few TeV are listed in Table 1 under $SU(3)_q \times SU(2)_l \times SU(2)_L \times SU(2)_R \times U(1)_X \times D$, where $X = (Y_L + Y_R + Y_l)/\sqrt{3}$ (each Y normalized according to $\sum Y^2 = 1/2$), $D = \sqrt{3}(-2Y_L + \sqrt{3}I_{3R} + Y_R - 2Y_l)$, and $Q = I_{3L} + I_{3R} - X$. The $SU(2)_L \times SU(2)_R$ scalar bidoublet contains the $SU(2)_L$ doublets $\eta = (\eta^0, \eta^-)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$, with η heavy at the M_R scale. Because of the assumed symmetry breaking pattern, our model actually possesses a conserved global symmetry

$$S = \frac{1}{\sqrt{3}}(Y_R - 2Y_L - 2Y_l) \tag{31}$$

before $SU(2)_R$ breaking, even though the corresponding gauge symmetry has been broken. Whereas both S and I_{3R} are broken by $\langle \chi^0 \rangle$, the combination

$$I_{3R} + S = \frac{D}{3} \tag{32}$$

is unbroken. Although this idea was used previously [11,12], the important observation here is that $I_{3R} + S$ coincides with the usual definition of $B-L$ for the known quarks and leptons, but

takes on different values for the other particles. Hence $D/3$ may be defined as generalized $B-L$ and functions as a global dark $U(1)$ symmetry. Now

$$R_D = (-1)^{3B-3L+2j} \tag{33}$$

so that it is identical to the usual definition of R parity in supersymmetry for the SM particles. Here the odd R_D particles are the h, n, x, y fermions, $(\eta^0, \eta^-), \lambda^0$ scalars, and W_R^\pm vector bosons. Note that leptonic color $SU(2)_l$ confines the x, y hemions to bosons which must then have even R_D .

To verify that generalized $B-L$ is indeed a global dark $U(1)$ symmetry of our model, consider the $SU(2)_R$ gauge bosons (W_R^+, W_R^0, W_R^-) which has $S = 0$. Hence they have $I_{3R} + S$ values $(1, 0, -1)$. This is expected because W_R^+ takes h_R to u_R and l_R to n_R . Consider next the Yukawa terms allowed by the gauge symmetry and S , i.e.

$$\bar{d}_R(u_L\phi_1^- - d_L\phi_1^0), \bar{u}_R(u_L\phi_2^0 - d_L\phi_2^+) + \bar{h}_R(-u_L\eta^- + d_L\eta^0), (\chi^+\bar{u}_R - \chi^0\bar{h}_R)h_L, \tag{34}$$

$$(\phi_1^0\bar{\nu}_L + \phi_1^-\bar{l}_L)\nu_R, \bar{\nu}_L(n_R\eta^0 + l_R\phi_2^+) + \bar{l}_L(n_R\eta^- + l_R\phi_2^0), \bar{n}_L(n_R\chi^0 - l_R\chi^+), \tag{35}$$

$$\bar{z}_L(z_R\chi^0 - y_R\chi^+), \bar{x}_R(\bar{z}_R\chi^+ + \bar{y}_R\chi^0), \bar{d}_R h_L \lambda^0, \bar{n}_L \nu_R \lambda^0, \bar{z}_L x_R \lambda^0, z_R y_R \bar{\lambda}^0, \tag{36}$$

and the scalar trilinear terms

$$\phi_1^-(\eta^0\chi^+ + \phi_2^+\chi^0) - \phi_1^0(\eta^-\chi^+ + \phi_2^0\chi^0), \lambda^0(\eta^0\phi_2^0 - \eta^-\phi_2^+). \tag{37}$$

It is easily confirmed from the above that $I_{3R} + S$ is not broken by $\langle\phi_{1,2}^0\rangle$ and $\langle\chi^0\rangle$. Note that in the familiar case of $SU(5)$ grand unification, neither B nor L is part of $SU(5)$ but both exist as low-energy conserved quantities. Here, B and L are again not part of $[SU(3)]^4$ separately, but a generalized $B-L$ emerges, and remains unbroken to be naturally interpreted as a global dark symmetry.

6. Gauge sector

Let

$$\langle\phi_1^0\rangle = v_1, \quad \langle\phi_2^0\rangle = v_2, \quad \langle\chi^0\rangle = v_R, \tag{38}$$

then the $SU(3)_q \times SU(2)_l \times SU(2)_L \times SU(2)_R \times U(1)_X$ gauge symmetry is broken to $SU(3)_q \times SU(2)_l \times U(1)_{em}$ with a residual global $I_{3R} + S$ as the dark symmetry, as explained previously.

Consider now the masses of the gauge bosons. The charged ones, W_L^\pm and W_R^\pm , do not mix because the latter have dark charge ± 1 . Their masses are given by

$$M_{W_L}^2 = \frac{1}{2}g_L^2(v_1^2 + v_2^2), \quad M_{W_R}^2 = \frac{1}{2}g_R^2(v_R^2 + v_2^2). \tag{39}$$

Since $Q = I_{3L} + I_{3R} - X$, the photon is given by

$$A = \frac{e}{g_L}W_{3L} + \frac{e}{g_R}W_{3R} + \frac{e}{g_X}Z_X, \tag{40}$$

where $e^{-2} = g_L^{-2} + g_R^{-2} + g_X^{-2}$. Let

$$Z = (g_L^2 + g_Y^2)^{-1/2} \left(g_L W_{3L} - \frac{g_Y^2}{g_R} W_{3R} - \frac{g_Y^2}{g_X} Z_X \right), \tag{41}$$

$$Z' = (g_R^2 + g_X^2)^{-1/2} (g_R W_{3R} - g_X Z_X), \tag{42}$$

where $g_Y^{-2} = g_R^{-2} + g_X^{-2}$, then the 2×2 mass-squared matrix spanning (Z, Z') is given by

$$\frac{1}{2} \begin{pmatrix} (g_L^2 + g_Y^2)(v_1^2 + v_2^2) & (\sqrt{g_L^2 + g_Y^2}/\sqrt{g_R^2 + g_X^2})(g_X^2 v_1^2 - g_R^2 v_2^2) \\ (\sqrt{g_L^2 + g_Y^2}/\sqrt{g_R^2 + g_X^2})(g_X^2 v_1^2 - g_R^2 v_2^2) & (g_R^2 + g_X^2)v_R^2 + (g_X^4 v_1^2 + g_R^4 v_2^2)/(g_R^2 + g_X^2) \end{pmatrix}. \tag{43}$$

Their neutral-current interactions are given by

$$\mathcal{L}_{NC} = e A_\mu J_Q^\mu + g_Z Z_\mu (j_{3L}^\mu - \sin^2 \theta_W j_{em}^\mu) + (g_R^2 + g_X^2)^{-1/2} Z'_\mu (g_R^2 j_{3R}^\mu + g_X^2 j_X^\mu), \tag{44}$$

where $g_Z^2 = g_L^2 + g_Y^2$ and $\sin^2 \theta_W = g_Y^2/g_Z^2$. Since Z – Z' mixing is constrained by experiment to be less than 10^{-4} or so, we assume $(g_X^2 v_1^2 - g_R^2 v_2^2)/v_R^2$ to be negligible.

The new gauge boson Z' may be produced at the Large Hadron Collider (LHC) through their couplings to u and d quarks, and decay to charged leptons (e^-e^+ and $\mu^-\mu^+$). Hence current search limits for a Z' boson are applicable. Using $\alpha_R(M_R) = 0.0290$ and $\alpha_X(M_R) = 0.0163$, the $c_{u,d}$ coefficients [22,23] used in the data analysis for our model are

$$c_u = (g_{uL}^2 + g_{uR}^2)B = 0.04 B, \quad c_d = (g_{dL}^2 + g_{dR}^2)B = 0.01 B, \tag{45}$$

where B is the branching fraction of Z' to e^-e^+ and $\mu^-\mu^+$. Assuming that Z' decays to all the particles listed in Table 1, except for the scalars which become the longitudinal components of the various gauge bosons, we find $B = 0.044$. Based on the 2016 LHC 13 TeV data set [24], this translates to a bound of about 3 to 4 TeV on the Z' mass.

7. Scalar sector

Consider the most general scalar potential consisting of $\Phi_L = (\phi_1^0, \phi_1^-)$, $\chi_R = (\chi^+, \chi^0)$, λ^0 , and

$$\eta = \begin{pmatrix} \eta^0 & \phi_2^+ \\ \eta^- & \phi_2^0 \end{pmatrix}, \quad \tilde{\eta} = \sigma_2 \eta^* \sigma_2 = \begin{pmatrix} \bar{\phi}_2^0 & -\eta^+ \\ -\phi_2^- & \bar{\eta}^0 \end{pmatrix}, \tag{46}$$

then

$$\begin{aligned} V = & -\mu_L^2 \Phi_L^\dagger \Phi_L - \mu_R^2 \chi_R^\dagger \chi_R - \mu_\eta^2 Tr(\eta^\dagger \eta) - \mu_\lambda^2 \bar{\lambda} \lambda + [\mu_1 \Phi_L^\dagger \eta \chi_R + \mu_2 \lambda det(\eta) + H.c.] \\ & + \frac{1}{2} f_L (\Phi_L^\dagger \Phi_L)^2 + \frac{1}{2} f_R (\chi_R^\dagger \chi_R)^2 + \frac{1}{2} f_\lambda (\bar{\lambda} \lambda)^2 + \frac{1}{2} f_\eta [Tr(\eta^\dagger \eta)]^2 + \frac{1}{2} f'_\eta Tr(\eta^\dagger \eta \eta^\dagger \eta) \\ & + f_{LR} (\Phi_L^\dagger \Phi_L) (\chi_R^\dagger \chi_R) + f_{L\lambda} (\Phi_L^\dagger \Phi_L) (\bar{\lambda} \lambda) + f_{R\lambda} (\chi_R^\dagger \chi_R) (\bar{\lambda} \lambda) + f_{\lambda\eta} (\bar{\lambda} \lambda) Tr(\eta^\dagger \eta) \\ & + f_{L\eta} \Phi_L^\dagger \eta \eta^\dagger \Phi_L + f'_{L\eta} \Phi_L^\dagger \tilde{\eta} \tilde{\eta}^\dagger \Phi_L + f_{R\eta} \chi_R^\dagger \eta^\dagger \eta \chi_R + f'_{R\eta} \chi_R^\dagger \tilde{\eta}^\dagger \tilde{\eta} \chi_R. \end{aligned} \tag{47}$$

Note that

$$2|det(\eta)|^2 = [Tr(\eta^\dagger \eta)]^2 - Tr(\eta^\dagger \eta \eta^\dagger \eta), \tag{48}$$

$$(\Phi_L^\dagger \Phi_L) Tr(\eta^\dagger \eta) = \Phi_L^\dagger \eta \eta^\dagger \Phi_L + \Phi_L^\dagger \tilde{\eta} \tilde{\eta}^\dagger \Phi_L, \tag{49}$$

$$(\chi_R^\dagger \chi_R) Tr(\eta^\dagger \eta) = \chi_R^\dagger \eta^\dagger \eta \chi_R + \chi_R^\dagger \tilde{\eta}^\dagger \tilde{\eta} \chi_R. \tag{50}$$

The minimum of V satisfies the conditions

$$\mu_L^2 = f_L v_1^2 + f_{L\eta} v_2^2 + f_{LR} v_R^2 + \mu_1 v_2 v_R / v_1, \tag{51}$$

$$\mu_\eta^2 = (f_\eta + f'_\eta) v_2^2 + f_{L\eta} v_1^2 + f_{R\eta} v_R^2 + \mu_1 v_1 v_R / v_2, \tag{52}$$

$$\mu_R^2 = f_R v_R^2 + f_{LR} v_1^2 + f_{R\eta} v_2^2 + \mu_1 v_1 v_2 / v_R. \tag{53}$$

The 3×3 mass-squared matrix spanning $\sqrt{2}Im(\phi_1^0, \phi_2^0, \chi^0)$ is then given by

$$\mathcal{M}_I^2 = \mu_1 \begin{pmatrix} -v_2 v_R / v_1 & v_R & v_2 \\ v_R & -v_1 v_R / v_2 & -v_1 \\ v_2 & -v_1 & -v_1 v_2 / v_R \end{pmatrix}, \quad (54)$$

and that spanning $\sqrt{2}Re(\phi_1^0, \phi_2^0, \chi^0)$ is

$$\begin{aligned} \mathcal{M}_R^2 = \mu_1 & \begin{pmatrix} -v_2 v_R / v_1 & v_R & v_2 \\ v_R & -v_1 v_R / v_2 & v_1 \\ v_2 & v_1 & -v_1 v_2 / v_R \end{pmatrix} \\ & + 2 \begin{pmatrix} f_L v_1^2 & f_{L\eta} v_1 v_2 & f_{LR} v_1 v_R \\ f_{L\eta} v_1 v_2 & (f_\eta + f'_\eta) v_2^2 & f_{R\eta} v_2 v_R \\ f_{LR} v_1 v_R & f_{R\eta} v_2 v_R & f_R v_R^2 \end{pmatrix}. \end{aligned} \quad (55)$$

Hence there are two zero eigenvalues in \mathcal{M}_I^2 with one nonzero eigenvalue $-\mu_1[v_1 v_2 / v_R + v_R(v_1^2 + v_2^2) / v_1 v_2]$ corresponding to the eigenstate $(-v_1^{-1}, v_2^{-1}, v_R^{-1}) / \sqrt{v_1^{-2} + v_2^{-2} + v_R^{-2}}$. In \mathcal{M}_R^2 , the linear combination $H = (v_1, v_2, 0) / \sqrt{v_1^2 + v_2^2}$, is the standard-model Higgs boson, with

$$m_H^2 = 2[f_L v_1^4 + (f_\eta + f'_\eta) v_2^4 + 2f_{L\eta} v_1^2 v_2^2] / (v_1^2 + v_2^2). \quad (56)$$

The other two scalar bosons are much heavier, with suppressed mixing to H , which may all be assumed to be small enough to avoid the constraints from dark-matter direct-search experiments.

The dark scalars are λ^0 , χ^\pm , and (η^0, η^-) . Whereas χ^\pm become the longitudinal components of W_R^\pm , the other scalars have the interaction

$$\mu_2 \lambda^0 (\eta^0 \phi_2^0 - \eta^- \phi_2^+) + H.c. \quad (57)$$

The 2×2 mass-squared matrix linking $(\lambda, \bar{\eta})$ to $(\bar{\lambda}, \eta)$ is given by

$$\mathcal{M}_{\lambda-\eta}^2 = \begin{pmatrix} -\mu_\lambda^2 + f_{L\lambda} v_1^2 + f_{R\lambda} v_R^2 + f_{\lambda\eta} v_2^2 & \mu_2 v_2 \\ \mu_2 v_2 & -\mu_\eta^2 + f_\eta v_2^2 + f'_{L\eta} v_1^2 + f'_{R\eta} v_R^2 \end{pmatrix}. \quad (58)$$

We assume μ_2 to be very small so that there is negligible mixing, with λ^0 as the lighter particle which is our dark-matter candidate. Note of course that η^0 is not a suitable candidate because it has Z^0 interactions.

8. Dark matter interactions

Consider the scalar singlet λ^0 as our dark-matter candidate. Let its coupling with the SM Higgs boson be $f_{\lambda H} \sqrt{2} v_H$, then it has been shown [14] that for $m_\lambda = 150$ GeV, $f_{\lambda H} < 4.4 \times 10^{-4}$ from the most recent direct-search result [25]. With such a small coupling, the λ^0 annihilation cross section in the early Universe through the SM Higgs boson is much too small for λ^0 to have the correct observed relic abundance. Hence a different process is required.

Consider then the Yukawa sector. As noted in Eq. (36), the interactions $f_x \lambda^0 \bar{z}_L x_R$ and $f_y \bar{\lambda}^0 z_R y_R$ exist. Now x_R / y_R forms a Dirac hemion and has been assumed to be light in the previous analysis on the renormalization-group running of gauge couplings. For convenience, the outgoing y_R may be redefined as incoming x_L . Let $m_\lambda > m_x$, then $\lambda^0 \bar{\lambda}^0 \rightarrow x \bar{x}$ through z exchange is possible as shown in Fig. 2. Let $f_y = f_x^*$ so that the $\lambda^0 \bar{z} x$ interaction is purely scalar. The cross section \times relative velocity is then given by

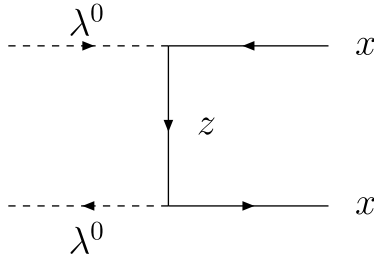


Fig. 2. Dark scalar annihilation to hemions.

$$\sigma v_{rel} = \frac{f_x^4}{4\pi} \left(1 - \frac{m_x^2}{m_\lambda^2}\right)^{3/2} \frac{(m_z + m_x)^2}{(m_z^2 + m_\lambda^2 - m_x^2)^2}. \tag{59}$$

As an example, let $m_\lambda = 150$ GeV, $m_x = 100$ GeV, and $m_z = 600$ GeV, then $\sigma v_{rel} = 1$ pb is obtained for $f_x = 0.385$. The $x\bar{x}$ final states remain in thermal equilibrium through the photon, with their confined bound states (which are bosons with even R_D) decaying to SM particles as described in a following section.

9. Leptonic color in the early Universe

As discussed in our earlier paper [5], the $SU(2)_I$ massless stickons (ζ) play a role in the early Universe. The important difference is that $\alpha_I(M_Z)$ is bigger here than in the Babu–Ma–Willenbrock (BMW) model [4], i.e. 0.065 versus 0.047. Hence the leptonic color confinement scale is about 0.4 MeV instead of 4 keV. At temperatures above the electroweak symmetry scale, the hemions are active and the stickons are in thermal equilibrium with the standard-model particles. Below the hemion mass scale, the stickon interacts with photons through $\zeta\zeta \rightarrow \gamma\gamma$ scattering with a cross section

$$\sigma \sim \frac{\alpha^2 \alpha_I^2 T^6}{64m^8}, \tag{60}$$

where m is the mass of the one light $x_R y_R$ hemion of this model. The decoupling temperature of ζ is then obtained by matching the Hubble expansion rate

$$H = \sqrt{(8\pi/3)G_N(\pi^2/30)g_*T^4} \tag{61}$$

to $[6\zeta(3)/\pi^2]T^3(\sigma v)$. Hence

$$T^{14} \sim \frac{2^{12}}{3^4} \left(\frac{\pi^7}{5[\zeta(3)]^2}\right) \frac{G_N g_* m^{16}}{\alpha^4 \alpha_I^4}. \tag{62}$$

For $m = 100$ GeV and $g_* = 92.25$ which includes all particles with masses up to a few GeV, $T \sim 9$ GeV. Hence the contribution of stickons to the effective number of neutrinos at the time of big bang nucleosynthesis (BBN) is given by [26]

$$\Delta N_\nu = \frac{8}{7}(3) \left(\frac{10.75}{92.25}\right)^{4/3} = 0.195, \tag{63}$$

compared to the value 0.50 ± 0.23 from a recent analysis [27].

As the Universe further cools below a few MeV, leptonic color goes through a phase transition and stickballs are formed. However, they are not stable because they are allowed to mix with a scalar bound state of two hemions which would decay to two photons. For a stickball ω of mass m_ω , we assume this mixing to be $f_\omega m_\omega/m$, so that its decay rate is given by

$$\Gamma(\omega \rightarrow \gamma\gamma) = \frac{\alpha^2 f_\omega^2 m_\omega^5}{256\pi^3 m^4}. \quad (64)$$

Using $m_\omega = 1$ MeV as an example with $m = 100$ GeV as before, its lifetime is estimated to be 1.0×10^7 s for $f_\omega = 1$. This means that it disappears long before the time of photon decoupling, so the $SU(2)_I$ sector contributes no additional relativistic degrees of freedom. Hence N_{eff} remains the same as in the SM, i.e. 3.046, coming only from neutrinos. This agrees with the PLANCK measurement [28] of the cosmic microwave background (CMB), i.e.

$$N_{eff} = 3.15 \pm 0.23. \quad (65)$$

10. Leptonic color at future e^-e^+ colliders

Unlike quarks, all hemions are heavy. Hence the lightest bound state is likely to be at least 200 GeV. Its cross section through electroweak production at the LHC is probably too small for it to be discovered. On the other hand, in analogy to the observations of J/ψ and Υ at e^-e^+ colliders of the last century, the resonance production of the corresponding neutral vector bound states (hemionia) of these hemions is expected at a future e^-e^+ collider (ILC, CEPC, FCC-ee) with sufficient reach in total center-of-mass energy. Their decays will be distinguishable from heavy quarkonia (such as toponia) experimentally.

As discussed in Ref. [5], the formation of hemion bound states is analogous to that of QCD. Instead of one-gluon exchange, the Coulomb potential binding a hemion–antihemion pair comes from one-stickon exchange. The difference is just the change in an $SU(3)$ color factor of $4/3$ to an $SU(2)$ color factor of $3/4$. The Bohr radius is then $a_0 = [(3/8)\bar{\alpha}_I m]^{-1}$, and the effective $\bar{\alpha}_I$ is defined by

$$\bar{\alpha}_I = \alpha_I(a_0^{-1}). \quad (66)$$

Using $\alpha_I(M_Z) = 0.065$ with $m = 100$ GeV, we obtain $\bar{\alpha}_I = 0.087$ and $a_0^{-1} = 3.26$ GeV. Consider the lowest-energy vector bound state Ω of the lightest hemion of mass $m = 100$ GeV. In analogy to the hydrogen atom, its binding energy is given by

$$E_b = \frac{1}{4} \left(\frac{3}{4} \right)^2 \bar{\alpha}_I^2 m = 106 \text{ MeV}, \quad (67)$$

and its wavefunction at the origin is

$$|\psi(0)|^2 = \frac{1}{\pi a_0^3} = 11.03 \text{ GeV}^3. \quad (68)$$

Since Ω will appear as a narrow resonance at a future e^-e^+ collider, its observation depends on the integrated cross section over the energy range \sqrt{s} around m_Ω :

$$\int d\sqrt{s} \sigma(e^-e^+ \rightarrow \Omega \rightarrow X) = \frac{6\pi^2}{m_\Omega^2} \frac{\Gamma_{ee}\Gamma_X}{\Gamma_{tot}}, \quad (69)$$

where Γ_{tot} is the total decay width of Ω , and Γ_{ee} , Γ_X are the respective partial widths.

Since Ω is a vector meson, it couples to both the photon and Z boson through its constituent hemions. Hence it will decay to W^-W^+ , $q\bar{q}$, l^-l^+ , and $\nu\bar{\nu}$. Using

$$\langle 0|\bar{x}\gamma^\mu x|\Omega\rangle = \epsilon_\Omega^\mu\sqrt{8m_\Omega}|\psi(0)|, \tag{70}$$

the $\Omega \rightarrow e^-e^+$ decay rate is given by

$$\Gamma(\Omega \rightarrow \gamma, Z \rightarrow e^-e^+) = \frac{2m_\Omega^2}{3\pi}(|C_V|^2 + |C_A|^2)|\psi(0)|^2, \tag{71}$$

where

$$C_V = \frac{e^2(1/2)(-1)}{m_\Omega^2} + \frac{g_Z^2(-\sin^2\theta_W/4)[(-1 + 4\sin^2\theta_W)/4]}{m_\Omega^2 - M_Z^2}, \tag{72}$$

$$C_A = \frac{g_Z^2(-\sin^2\theta_W/4)(1/4)}{m_\Omega^2 - M_Z^2}. \tag{73}$$

In the above, Ω is composed of the singlet hemions x_R and y_R with invariant mass term $x_{1R}y_{2R} - x_{2R}y_{1R}$. The (x_L, y_L) option, considered in the BMW model, is not available here because they are superheavy from the breaking of $SU(3)_L$ at M_U . Here $\Gamma_{ee} = 139$ eV. Similar expressions hold for the other fermions of the SM.

For $\Omega \rightarrow W^-W^+$, the triple γW^-W^+ and ZW^-W^+ vertices have the same structure. The decay rate is calculated to be

$$\Gamma(\Omega \rightarrow \gamma, Z \rightarrow W^-W^+) = \frac{m_\Omega^2(1-r)^{3/2}}{6\pi r^2} (4 + 20r + 3r^2) C_W^2 |\psi(0)|^2, \tag{74}$$

where $r = 4M_W^2/m_\Omega^2$ and

$$C_W = \frac{e^2(1/2)}{m_\Omega^2} + \frac{g_Z^2(-\sin^2\theta_W/4)}{m_\Omega^2 - M_Z^2}. \tag{75}$$

Because of the accidental cancellation of the two terms in the above, C_W turns out to be very small. Hence $\Gamma_{WW} = 10$ eV. For $\Omega \rightarrow ZZ$, there is only the t -channel contribution, i.e.

$$\Gamma(\Omega \rightarrow ZZ) = \frac{m_\Omega^2(1-r_Z)^{5/2}}{3\pi r_Z} D_Z^2 |\psi(0)|^2, \tag{76}$$

where $r_Z = 4M_Z^2/m_\Omega^2$ and $D_Z = g_Z^2 \sin^4\theta_W/4(m_\Omega^2 - 2m_Z^2)$. Hence Γ_{ZZ} is negligible. The Ω decay to two stickons is forbidden by charge conjugation. Its decay to three stickons is analogous to that of quarkonium to three gluons. Whereas the latter forms a singlet which is symmetric in $SU(3)_C$, the former forms a singlet which is antisymmetric in $SU(2)_I$. However, the two amplitudes are identical because the latter is symmetrized with respect to the exchange of the three gluons and the former is antisymmetrized with respect to the exchange of the three stickons. Taking into account the different color factors of $SU(2)_I$ versus $SU(3)_C$, the decay rate of Ω to three stickons and to two stickons plus a photon are

$$\Gamma(\Omega \rightarrow \zeta\zeta\zeta) = \frac{16}{27}(\pi^2 - 9) \frac{\alpha_l^3}{m_\Omega^2} |\psi(0)|^2, \tag{77}$$

$$\Gamma(\Omega \rightarrow \gamma\zeta\zeta) = \frac{8}{9}(\pi^2 - 9) \frac{\alpha\alpha_l^2}{m_\Omega^2} |\psi(0)|^2. \tag{78}$$

Table 2
Partial decay widths of the hemionium Ω .

Channel	Width
$\sum \nu\bar{\nu}$	36 eV
$e^-e^+, \mu^-\mu^+, \tau^-\tau^+$	0.4 keV
$u\bar{u}, c\bar{c}$	0.3 keV
$d\bar{d}, s\bar{s}, b\bar{b}$	0.1 keV
W^-W^+	10 eV
ZZ	< 0.1 eV
$\zeta\zeta\zeta$	39 eV
$\zeta\zeta\gamma$	7 eV
sum	0.9 keV

Hence $\Gamma_{\zeta\zeta\zeta} = 39$ eV and $\Gamma_{\gamma\zeta\zeta} = 7$ eV. The integrated cross section for $X = \mu^-\mu^+$ is then 1.2×10^{-32} cm²-keV. For comparison, this number is 7.9×10^{-30} cm²-keV for the $\Upsilon(1S)$. At a high-luminosity e^-e^+ collider, it should be feasible to make this observation. Table 2 summarizes all the partial decay widths.

There are important differences between QCD and QHD (quantum hemiodynamics). In the former, because of the existence of light u and d quarks, it is easy to pop up $u\bar{u}$ and $d\bar{d}$ pairs from the QCD vacuum. Hence the production of open charm in an e^-e^+ collider is described well by the fundamental process $e^-e^+ \rightarrow c\bar{c}$. In the latter, there are no light hemions. Instead it is easy to pop up the light stickballs from the QHD vacuum. As a result, just above the threshold of making the Ω resonance, the many-body production of Ω + stickballs becomes possible. This cross section is presumably also well described by the fundamental process $e^-e^+ \rightarrow x\bar{x}$, i.e.

$$\sigma(e^-e^+ \rightarrow x\bar{x}) = \frac{2\pi\alpha^2}{3} \sqrt{1 - \frac{4m^2}{s}} \left[\frac{(s + 2m^2)}{s^2} + \frac{x_W^2}{2(1 - x_W)^2} \frac{(s - m^2)}{(s - m_Z^2)^2} + \frac{x_W}{(1 - x_W)} \frac{(s - m^2)}{s(s - m_Z^2)} - \frac{(1 - 4x_W)}{4(1 - x_W)} \frac{m^2}{s(s - m_Z^2)} \right], \quad (79)$$

where $x_W = \sin^2\theta_W$ and $s = 4E^2$ is the square of the center-of-mass energy. Using $m = 100$ GeV and $s = (250 \text{ GeV})^2$ as an example, we find this cross section to be 0.79 pb.

In QCD, there are $q\bar{q}$ bound states which are bosons, and qqq bound states which are fermions. In QHD, there are only bound-state bosons, because the confining symmetry is $SU(2)_l$. Also, unlike baryon (or quark) number in QCD, there is no such thing as hemion number in QHD, because y is effectively \bar{x} . This explains why there is no stable analog fermion in QHD such as the proton in QCD.

11. Concluding remarks

Candidates for dark matter are often introduced in an *ad hoc* manner, because it is so easy to do. There are thus numerous claimants to the title. Is there a guiding principle? One such is supersymmetry, where the superpartners of the SM particles naturally belong to a dark sector. Another possible guiding principle proposed recently is to look for a dark symmetry embedded as a gauge symmetry in a unifying extension of the SM, such as $[SU(3)]^N$. In this paper, the

alternative $[SU(3)]^4$ gauge model of leptonic color and dark matter is discussed in some detail. The dark global $U(1)$ symmetry is identified as generalized $B-L$ and the dark parity is $R_D = (-1)^{3B-3L+2j}$. The dark sector contains fermions (h, x, y, n) , scalars $[(\eta^0, \eta^-), \lambda^0]$, and vector gauge bosons W_R^\pm , where h is a dark quark of charge $-1/3$, x, y are hemions of charge $\pm 1/2$, and n is a dark neutral fermion. The dark matter of the Universe is presumably a neutral scalar dominated by the singlet λ^0 .

In $[SU(3)]^N$ models, the fermion and scalar content is assumed to consist of moose chains, i.e. bitriplets under $SU(3)_i \times SU(3)_j$. As such $[SU(3)]^3$ is naturally embedded in E_6 , and not $SU(9)$. As for $[SU(3)]^4$, it is a subgroup of $SU(12)$ as well as E_8 . Again the former is not suitable, but the latter also cannot accommodate a moose chain of 4 links, although it does contain E_6 which has a moose chain of 3 links. In short, there is no obvious single unifying symmetry for the $[SU(3)]^4$ model discussed in this paper.

The absence of observations of new physics at the LHC is a possible indication that fundamental new physics may not be accessible using the strong interaction, i.e. quarks and gluons. It is then natural to think about future e^-e^+ colliders. But is there some fundamental issue of theoretical physics which may only reveal itself there? and not at hadron colliders? The notion of leptonic color is such a possible answer. Our alternative $[SU(3)]^4$ model allows for the existence of new half-charged fermions (hemions) under a confining $SU(2)_l$ leptonic color symmetry, with masses below the TeV scale. It also predicts the $SU(2)_l$ confining scale to be 0.4 MeV, so that stickball bound states of the vector gauge stickons are formed. These new particles have no QCD interactions, but hemions have electroweak couplings, so they are accessible in a future e^-e^+ collider, as described in this paper.

Acknowledgment

This work was supported in part by the U.S. Department of Energy Grant No. DE-SC0008541.

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