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THE PETROV CLASSIFICATION AND VACUUM DARK FLUID

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Abstract

The Petrov classification of stress-energy tensors makes it possible to introduce a unified description of dark energy and dark matter as a vacuum dark fluid based on the space-time symmetry. In this approach a vacuum dark energy is described by a variable cosmological term whose symmetry is reduced as compared with the Einstein cosmological term which allows a vacuum energy to be evolving and clustering. The relevant class of solutions to the Einstein equations implies also the existence of compact vacuum objects generically related to a dark energy: regular black holes, their remnants and self-gravitating vacuum solitons with de Sitter vacuum interior – which can be responsible for observational effects typically related to a dark matter. The mass of objects with de Sitter interior is generically related to vacuum dark energy and to breaking of space-time symmetry.

Key words: dark energy, dark matter, regular black holes and solitons with de Sitter core.

Introduction

Quantum field theory in curved space-time does not contain a unique specification for the quantum state of a system, and the symmetry of a vacuum expectation value of a stress-energy tensor does not always coincide with the symmetry of a background space-time [1]. In the case of the de Sitter space the renormalized expectation value of $\langle T_{\mu\nu}\rangle$ for a scalar field with an arbitrary mass m and curvature coupling ξ is proved to have a fixed point attractor behavior at late times ([1] and references therein) approaching, dependently on m and ξ, or the Bunch – Davies de Sitter-invariant vacuum either, for the massless minimally coupled case ($m = \xi = 0$) the de Sitter invariant Allen – Folacci vacuum. The last case is peculiar since the de Sitter invariant two-point function is infrared divergent, and the vacuum states, free of this divergence, are O(4)-invariant Fock vacua; the vacuum energy density in the O(4)-invariant case is not the same (larger) than in de Sitter-invariant case [2].

The Petrov classification of stress-energy tensors provides opportunity to consider vacuum in a model-independent way, as a medium specified by the algebraic structure of its stress-energy tensor [3–5]. The Einstein cosmological term $\Lambda g_{\mu\nu}$ corresponds to the de Sitter vacuum presented by the stress-energy tensor of maximal symmetry, with all three spacelike eigenvalues equal to the timelike eigenvalue. As a result it has an infinite set of co-moving reference frames, so that an observer cannot in principle measure his velocity with respect to it [3]. The maximal symmetry of a vacuum stress-energy tensor can be reduced to the case when less than three spacelike eigenvalues are equal to the timelike eigenvalue [4, 5]. This leads inevitably (by the Bianchi identities) to dynamical vacuum energy represented by anisotropic vacuum dark fluid which can both be distributed and form compact objects [6]. It generates regular space-time with the de Sitter interior whose existence follows from requirements of regularity and certain energy conditions on a source term in the Einstein equations [7].

Vacuum dark fluid provides a unified description of dark energy and dark matter. The key point is that astronomical data testify in favor of a cosmological vacuum dark energy described by the Einstein cosmological term (see [8] and references therein). The problem is that density of de Sitter vacuum must be constant by the contracted Bianchi identities, while the inflationary paradigm requires its much bigger value for the earliest stage of the Universe evolution. Vacuum dark fluid represents a cosmological vacuum by a variable spherically symmetric cosmological term which connects smoothly two de Sitter vacua at $r \to 0$ and $r \to \infty$. Its symmetry is reduced as compared with the Einstein cosmological term which allows a vacuum energy to be evolving and clustering. Time-evolving and space-inhomogeneous cosmological term [5] describes regular cosmological models dominated by vacuum dark energy [9].

The relevant class of solutions to the Einstein equations implies the existence of compact vacuum objects generically related to a dark energy through their de Sitter vacuum interiors: regular black holes [4, 10], their remnants [11, 12] and self-gravitating vacuum solitons [7, 11, 13], which can be responsible for observational effects typically related to a dark matter [6].

The question of the origin of dark matter still remains open [14]. The most popular hypothesis is that dark matter consists of neutral weakly interacting particles created in the hot early Universe. However, recently gathered results lead to the conclusions that known elementary particles can not account for a dark matter, at least in the frame of the Standard Model [15]. Dark energy particles as quanta of the cosmological constant λ (considered as the fundamental constant) were proposed in [16] for a wide range of masses up to 10^{55} g including thus also observable Universe. In models of a unified dark fluid with scalar fields, a dark energy is treated as a remnant density of a complex scalar field and dark matter as particles of this field [17], although the form of the scalar field potential can not be directly derived from high energy theories.

Vacuum dark fluid provides a model-independent dark energy-dark matter unification based on the space-time symmetry. Vacuum gravitational solitons called G-lumps [7] (they are bounded by their own gravity balanced at the surface where the strong energy condition is violated) can be responsible for local effects related to a dark matter in a way similar to λ-particles of [16] and complex scalar field particles of [17].

Black holes (especially primordial) are recognized as good dark matter candidates [18]. Black hole remnants (final products of Hawking evaporation) have been considered as a source of dark matter for more than two decades [19] (for a review see [14]). The open question discussed in the literature concerns the existence of remnants: In the case of a singular black hole it would be a Planck size black hole; however, no evident symmetry or quantum number exists which would prevent complete evaporation. Character and scale of uncertainty concerning an endpoint of the Hawking evaporation of a singular black hole are clearly evident in the case of a multihorizon space-time [20]. The fate of a regular black hole is unambiguous: it leaves thermodynamically stable double-horizon remnant with the positive specific heat [11, 12].

Mass of objects is related to interior de Sitter vacuum and breaking of space-time symmetry from the de Sitter group at the origin [7]. This has been tested by evaluating the gravito-electroweak unification scale from the measured mass-squared differences for solar and atmospheric neutrinos [21]. Nonlinear electrodynamics coupled to gravity provides a non-trivial example of a matter object with dark energy interior [22, 23] which we discuss in Section 2. In Section 1 we present the vacuum dark fluid in general setting, and in Section 2 we show how it can provide a unified description of dark energy and dark matter.

1. Vacuum dark fluid

The Einstein cosmological term $\Lambda g_{\mu\nu}$ with $\Lambda = \text{const}$, corresponds to a vacuum stress-energy tensor of the maximal symmetry

$$\Lambda\delta^{\mu\nu} = 8\pi G T^{\mu\nu}_{\text{vac}}. \tag{1}$$

In the Petrov classification, stress-energy tensors are classified on the basis of their algebraic structure. When eigenvalues of $T_{\mu\nu}$ are real, the eigenvectors of $T_{\mu\nu}$ are non-isotropic and form a comoving reference frame with a timelike eigenvector representing a velocity.

In this classification an anisotropic fluid is specified by [IIII] and [II(II)], and an isotropic fluid by [I(III)]. The first symbol denotes the eigenvalue related to the timelike eigenvector. Parentheses combine degenerate eigenvalues. A comoving reference frame is defined uniquely if and only if none of spacelike eigenvalues $\lambda_k (k = 1, 2, 3)$ coincides with a timelike eigenvalue λ_0. Otherwise there exists an infinite set of comoving reference frames.

The maximally symmetric de Sitter vacuum (1), specified by [(IIII)] in the Petrov classification scheme (all eigenvalues equal, all reference frames comoving), represents the isotropic vacuum fluid. The high symmetry of a vacuum stress-energy tensor (1) can be reduced to the case when one (or two) of the spacelike eigenvalues of $T_{\mu\nu}$ coincides with its timelike eigenvalue

$$p_k = -\rho. \tag{2}$$

A vacuum stress-energy tensor with a reduced symmetry is invariant under Lorentz boosts in the k-direction. This makes impossible to single out a preferred comoving reference frame and thus fix the velocity with respect to a vacuum fluid which is intrinsic property of a vacuum [24].

A vacuum defined by the symmetry of its stress-energy tensor must be evidently anisotropic (except the maximally symmetric de Sitter vacuum (1)). The Petrov classification scheme suggests three types of anisotropic vacuum fluid: [(II)(II)], [(II)II], [(III)I] [6].

A spherically symmetric vacuum fluid corresponds to [(II)(II)] and is specified by [4]

$$T^t_t = T^r_r. \tag{3}$$

It satisfies the equation of state (following from $T^{\mu}_{\nu;\mu} = 0$) for anisotropic perfect fluid

$$p_r = -\rho; \quad p_\perp = -\rho - \frac{r}{2}\frac{d\rho}{dr} \tag{4}$$

and generates space-time with the de Sitter center whose existence follows from requirements of regularity and the weak energy condition on a source term in the Einstein equations [7].

The Einstein equations with a source term specified by (3) admit the class of regular solutions asymptotically de Sitter as $r \to 0$ and $r \to \infty$ [5, 7]

$$(8\pi G)^{-1}\Lambda\delta^{\mu}_{\nu} \Longleftarrow T^{\mu}_{\nu} \Longrightarrow (8\pi G)^{-1}\lambda\delta^{\mu}_{\nu} \tag{5}$$

with $\lambda < \Lambda$. The metric of a space-time is given by

$$ds^2 = g(r)\,dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2 \tag{6}$$

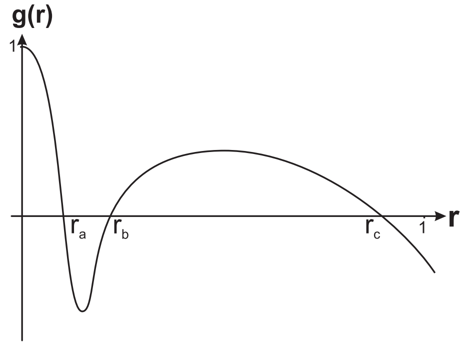

Fig. 1. Metric function in the case of three horizons

with the metric function [10]

$$g(r) = 1 - \frac{2G\mathcal{M}(r)}{r} - \frac{\lambda}{3}r^2; \quad \mathcal{M}(r) = 4\pi \int_0^r \rho(x)x^2\,dx, \tag{7}$$

which evolves from the de Sitter metric function $g(r) = 1 - (\Lambda + \lambda)r^2/3$ as $r \to 0$, to the Kottler – Trefftz metric function $g(r) = 1 - r_g/r - \lambda r^2/3, r_g = 2GM$, for $r \ll r_*$ where $r_* = (r_0^2 r_g)^{1/3}$ with $r_0^2 = 3/\Lambda$, is the characteristic length scale in geometry with de Sitter center ([4] and references therein). The mass parameter (gravitational mass)

$$M = \int_0^\infty \rho(r)r^2 dr \tag{8}$$

is related to interior de Sitter vacuum and breaking of space-time symmetry from the de Sitter group at the origin [7]. Space-time can have not more than three horizons [9], the cosmological horizon r_c, the black bole horizon $r_b < r_c$, and the internal horizon $r_a < r_b$ (see Fig. 1).

The internal horizon $r = r_a$ is the cosmological horizon for a static observer in the R-region $0 \leq r < r_a$. A static observer in the R-region $r_b < r < r_c$ observes T_- region $r_a < r < r_b$ as a regular cosmological black hole. Its mass is limited within $M_{cr1} \leq M \leq M_{cr2}$. The value $M = M_{cr1}$ corresponds to a double-horizon ($r_a = r_b$) state which appears as an end-point of the Hawking evaporation. For $M < M_{cr1}$ the metric (6) describes a G-lump in asymptotically de Sitter space (the upper curve in Fig. 2). Second critical mass M_{cr2} corresponds to the double horizon $r_b = r_c$ and represents a regular modification of the Nariai solution.

This behavior is generic for the class of regular solutions specified by (3) and satisfying the weak energy condition [7, 9]. The pictures are plotted with the density profile [4]

$$\rho(r) = \rho_0 \exp(-r^3/r_0^2 r_g); \;\; r_0 = \sqrt{3/8\pi G\rho_0}; \;\; \rho_0 = \rho(r \to 0) = (8\pi G)^{-1}\Lambda; \;\; r_g = 2GM \tag{9}$$

which describes vacuum polarization effects leading to de Sitter interior in the simple semi-classical model for vacuum polarization in the gravitational field [11].

2. Regular cosmologies with vacuum dark energy

In the coordinates of comoving observers, the metric (6) describes regular vacuum dominated cosmologies (vacuum density evolves smoothly from a big initial value to

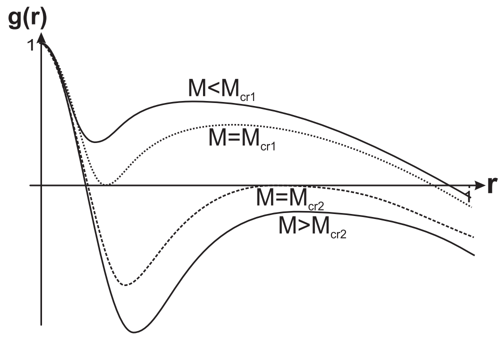

Fig. 2. Metric function for double-horizon and one-horizon configurations

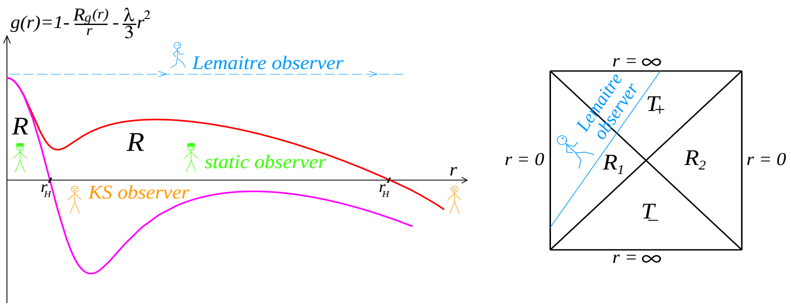

Fig. 3. Spherically symmetric vacuum space-time with one horizon

a small value) of the Lemaitré class and Kantowski – Sachs type whose dynamics depends on the number of horizons.

In the vacuum cosmologies of the Lemaitré class, evolution starts from a nonsingular non-simultaneous de Sitter bang followed by an anisotropic stage at which most of the mass is produced [25]. For cosmologies of Kantowski – Sachs type, evolution starts with a null bang from a horizon, but information about pre-bang history is available for KS observes [9].

Two simplest cases of one-horizon configurations are shown in Fig. 3; the global structure of space-time is the same as for de Sitter geometry but with dynamical vacuum dark energy.

In the Lemaitré coordinates this configuration represents vacuum anisotropic models of the Lemaitré class, in which evolution starts with a nonsingular non-simultaneous de Sitter bang from the regular time-like surface $r(R, \tau) = 0$ for the model with zero and negative spatial curvature, and from $r = r_i$ for the models with the positive spatial curvature [9].

In the Kantowski – Sachs region it corresponds to the class of regular homogeneous T-models with vacuum dark energy [26]. Typical features of homogeneous regular T-models are: the existence of a Killing horizon; beginning of the cosmological evolution from a null bang at the horizon; the existence of a regular static pre-bang region visible to cosmological observers; creation of matter from anisotropic vacuum, accompanied by very rapid isotropization. Detailed calculations of the spherically symmetric regular T-model based on the general exact solution for a mixture of the vacuum fluid and dust-like matter, have shown the ability of cosmological T-models to satisfy the observational constraints [26].

In quantum cosmology it is possible, in frame of the minisuperspace model, to adapt cosmological constant Λ for description of a vacuum dark energy density jumping from the big initial value to the small value suggested by observations [27]. The gauge-non-invariance of quantum cosmology leads to a connection between a choice of the gauge and quantum spectrum for a certain physical quantity which can be specified in the framework of the minisuperspace model. There exists a particular gauge in which the cosmological constant Λ is quantized [27], so that making a measurement of Λ today one can find its small value with the biggest probability, while at the beginning of the evolution, the biggest probability corresponds to its biggest value. Transitions between quantum levels of dark energy Λ in the course of the Universe evolution can be related to several scales of symmetry breaking [27].

3. Dark matter candidates

3.1. Regular black hole remnants. The quantum temperature of a horizon r_h determined by its surface gravity κ_h is given by the Gibbons–Hawking formula:

$$kT_h = \frac{\hbar}{2\pi c}\kappa_h = \frac{\hbar}{4\pi c}|g'(r_h)|. \tag{10}$$

In space-time with three horizons, an observer in the R-region $r_b < r < r_c$ can detect the Hawking radiation from a black hole horizon r_b and from a cosmological horizon r_c, and an observer in the R-region $0 \le r < r_a$ can detect radiation from the cosmological horizon r_a.

Thermodynamics is studied by applying the Padmanabhan approach relevant for a multihorizon space with non-zero pressure and based on a canonical ensemble of metrics (6) at the constant temperature of the horizon determined by the periodicity of the Euclidean time in the Euclidean continuation of the Einstein action [28]. With this approach we find temperature T_h, thermodynamical energy E_h, entropy S_h, free energy F_h, and specific heat written below in the units $c = G = \hbar = 1$ [12]:

on black hol horizon

$$kT_b = \frac{1}{4\pi}\left(\frac{1}{r_b} - \frac{\lambda}{3}r_b - 8\pi\rho(r_b)r_b\right); \quad E_b = \frac{1}{2}r_b; \tag{11}$$

on internal and cosmological horizons

$$kT_h = \frac{1}{4\pi}\left(8\pi\rho(r_h)r_h + \frac{\lambda}{3}r_h - \frac{1}{r_h}\right); \quad E_h = -\frac{1}{2}r_h; \tag{12}$$

on any horizon

$$S_h = 4\pi r_h^2; \quad F_h = E_h - T_h S_h; \tag{13}$$

$$C_h = dE_h/dT_h; \quad C_h^{-1} = -\frac{1}{2\pi}\left[8\pi\rho'(r_h)r_h + 8\pi\rho(r_h) + \lambda + \frac{1}{r_h^2}\right]. \tag{14}$$

Dependence of temperature on the black hole horizon radius is shown in Fig. 4.

Fig. 4 is plotted with the density profile (9), but this curve is generic. Independent of a particular form of the density profile $\rho(r)$, $T_b \to 0$ as $r_a \to r_b$, and as $r_c \to r_b$, since surface gravity vanishes in the extrema of the metric function $g(r)$. Hence the temperature curve should have a maximum, $T_b(r_m) = T_{b\,\max}$. It follows that specific heat on the black hole horizon C_b is negative for $r > r_m$ and positive for $r < r_m$. At the maximum $C_b^{-1} = 0$, hence a specific heat is broken and changes its sign in the course of quantum evaporation [11, 12]. For the case of the density profile (9), maximal

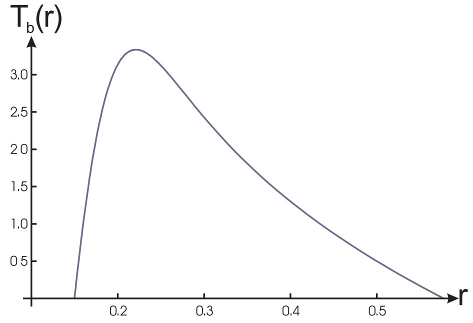

Fig. 4. Temperature of a regular black hole in de Sitter space

temperature corresponding to the phase transition is $T_{b\,\max} = T_{tr} \simeq 0.2 T_{Pl}\sqrt{\rho_0/\rho_{Pl}}$. For $\rho_0 = \rho_{GUT}$ and $M_{GUT} \simeq 10^{15}$ GeV it gives $T_{tr} \simeq 0.2 \cdot 10^{11}$GeV.

The answer to the question what is an endpoint of evaporation, depends on where move horizons. For a metric function (7) with de Sitter asymptotics at the center and Kottler – Trefftz asymptotics for $r \gg r_* = (r_0^2 r_g)^{1/3}$, a density profile involves scaling r/r^*, and for zeros of a metric function (7) we obtain $dr_b/dM > 0$, $dr_a/dM < 0$, $dr_c/dM < 0$. In the region $0 \le r \le r_a$, which is the whole manifold for a static observer, $dr_a \ge 0$ by the second law of thermodynamics for horizons. The horizon r_a moves outwards and $dr_a/dM < 0$, hence M decreases; since $dr_b/dM > 0$, a black hole horizon r_b shrinks. Specific heat C_a is positive near the double horizon, $dT_a/dE_a > 0$ and $dT_a/dr_a < 0$, hence T_a decreases with increasing r_a. With $dT_a/dM > 0$ and $dT_a/dr_a < 0$ this leads to monotonic decreasing M and T_a until T_a vanishes on the double horizon $r_a = r_b = r_d$ where $C_d > 0$ [12].

The specific heat C_h^{-1} can be written as

$$C_h^{-1} = \frac{1}{2\pi}\left(\frac{g'(r_h)}{r_h} + g''(r_h)\right). \tag{15}$$

This formula tells unambiguously that an extreme state with a double horizon $(g' = 0)$ is thermodynamically stable when it appears in a minimum of the metric function $g(r)$, and thermodynamically unstable when it appears in its maximum [12]. We conclude that a regular black hole leaves behind a thermodynamically stable double-horizon remnant. For the case of the density profile (9), its mass is $M_{\rm remnant} \simeq 0.3 M_{Pl}\sqrt{\rho_{Pl}/\rho_0}$.

3.2. Vacuum gravitational solitons – *G*-lumps. This name is owing to Coleman's lumps which are non-singular non-dissipative solutions of finite energy holding themselves together by their own self-interaction [29]. The idea of lumps can be traced back to the Einstein idea to describe an elementary particle by a regular solution of nonlinear field equations as a "bunched field" located in the confined region where field and energy are particularly high [30]. Vacuum soliton G-lump was proposed in 1996 in a model-independent way as a regular solution to the Einstein equations with the de Sitter interior without horizons [11]. In terms of the proposed in 2001 gravastar model with de Sitter core [31], a G-lump corresponds to a model-independent gravastar with continuous density and pressures.

The criterion of stability of G-lumps to external polar perturbations given by [6]

$$r(p_\perp + \rho)' \le \rho + (p_\perp + \rho) \tag{16}$$

is satisfied for a wide class of density profiles.

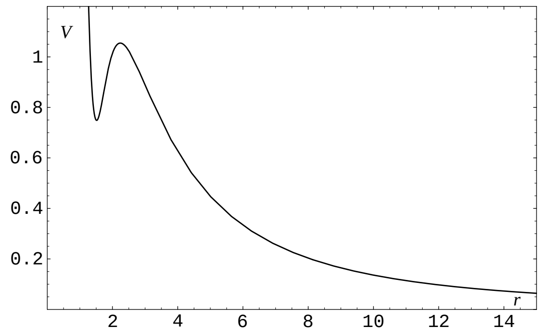

Fig. 5. Potential $V_\gamma(r)$ for G-lump with $r_g/r_0 = 1.5$ $(L = 4)$

In the field of G-lump and of the double-horizon remnant, there exist nontrivial geodesic orbits [32] which can be used in search for their observational signatures as dark matter candidates. Geodesics are described by

$$\left(\frac{dr}{d\sigma}\right)^2 + V_{(p,\gamma)}(r) = E^2; \quad V_p(r) = g(r)\left(1 + \frac{L^2}{r^2}\right); \quad V_\gamma(r) = \frac{L^2}{r^2}g(r), \qquad (17)$$

where σ is the affine parameter along geodesic, V_p is the potential for time-like geodesics, and V_γ for the null geodesics. For a G-lump and extreme black hole the potential curves differ essentially from that for a black hole and evidently depend on the mass M. Potentials V_p have, in a certain range of masses, three extrema and, hence, two branches of stable circular orbits separated by a gap. Potential V_γ shown in Fig. 5 reveals the most striking feature of geodesics in the field of G-lump: the existence of stable bound photon orbits including circular orbits!

3.3. Electromagnetic soliton. Nonlinear electrodynamics coupled to gravity is described by the action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}(R - L(F)); \; F = F_{ik}F^{ik} \qquad (18)$$

with an arbitrary gauge invariant lagrangian $L(F)$ with the Maxwellian asymptotics in the weak field regime. A stress-energy tensor of a spherically symmetric electromagnetic field has the symmetry (3). For a field satisfying the weak energy condition a spherically symmetric electrically charged electrovacuum structure has obligatory de Sitter center in which the electric field vanishes while the energy density of electromagnetic vacuum achieves its maximal value [22]. By the Gürses-Gürsey algorithm based on the Trautman – Newman technique [33], spherically symmetric electrovacuum solution is transformed into a spinning electrovacuum solution asymptotically Kerr – Newman for a distant observer. De Sitter center becomes de Sitter equatorial disk which has both perfect conductor and ideal diamagnetic properties and displays superconducting behavior within a single spinning soliton. This behavior is generic for the class of regular spinning solutions describing electrovacuum black holes and solitons [23]. De Sitter vacuum supplies a particle with the finite positive electromagnetic mass related to breaking of space-time symmetry. These results apply to the cases when the energy scale is less than the Planck scale. Recently they found a certain confirmation in the existence of minimal length scale ("closest approach" of particles) in the annihilation reaction $e^+e^- \to \gamma\gamma(\gamma)$, which can be explained by the existence of the characteristic surface at which electromagnetic attraction is balanced by the gravitational repulsion due to de Sitter interior [34].

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Резюме

И. Дымникова. Классификация Петрова и темная вакуумная жидкость.

Классификация Петрова тензоров энергии-импульса позволяет ввести объединённое описание тёмной энергии и тёмной материи как вакуумной тёмной жидкости на основе симметрии пространства-времени. При таком подходе вакуумная тёмная энергия описывается переменным космологическим членом, симметрия которого нарушена по сравнению с космологическим членом Эйнштейна. В случае сферической симметрии инфляционное уравнение состояния выполняется только для радиального давления, в результате плотность энергии и оба давления становятся зависящими от времени и пространственных координат. Уравнения Эйнштейна с правой частью, представленной тензором энергии-импульса такого типа, допускает также класс решений, описывающих компактные объекты с центром де Ситтера: регулярные чёрные дыры, продукты их испарения и вакуумные гравитационные солитоны, которые могут ответственными за наблюдательные эффекты, свидетельствующие о существовании тёмной материи. Масса объектов с де Ситтеровским ядром связана с тёмной энергией и нарушением симметрии пространства-времени от группы де Ситтера в центре до группы Пуанкаре на бесконечности для асимптотически плоских пространств или до группы де Ситтера с меньшим значением космологической постоянной для асимптотически де Ситтеровских на бесконечности пространств.

Ключевые слова: тёмная энергия, тёмная материя, регулярные объекты с де Ситтеровским ядром.

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