Some Recent Issues in Relativity and Cosmology

Thesis Submitted for a Degree of Doctor of Philosophy in Science (Physics) of the University of North Bengal

Submitted by

Saroj Nepal

(Department of Physics)

North Bengal University

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To

My Parents

DEPARTMENT OF PHYSICS
UNIVERSITY OF NORTH BENGAL
P.O. NORTH BENGAL UNIVERSITY
SILIGURI
DIST. DARJEELING (WB)
PIN: 734013 INDIA

Railway Station: New Jalpaiguri (NFR)
Airport: Bagdogra
Phone: 0353-2776338 Fax: 91-353-2699001
E-mail: sk_ghosal@yahoo.com

Date: December 15, 2008

TO WHOM IT MAY CONCERN

This to certify that the investigations reported in this thesis entitled "**Some Recent Issues in Relativity and Cosmology**" by **Mr. Saroj Nepal** has been carried out here by the candidate himself under my supervision and guidance. He has fulfilled all the requirements for submission of the thesis for Ph.D. degree of the University of North Bengal. Although the research work presented in this dissertation has been performed in collaboration with others, much of the work has been performed by him and his contribution is quite substantial. In character and disposition **Mr. Saroj Nepal** is fit to submit the thesis for Ph. D. degree.

Prof. S. K. Ghosal
Professor of Physics
(Supervisor)

Preface

The present thesis is a compilation of findings of the present author following the investigations on the conceptual issues like twin paradox and its ramifications in relativity theory and issues in the realm of high energy physics (UHECR paradox) and an area touching to VSL cosmology. The thesis is built up by me during the last several years in the Department of Physics, North Bengal University (NBU) as a research scholar under UGC (Nepal) fellowship. It is a great pleasure to offer my sincere and deep gratitudes to all those, whose active interests in my work have been continuous impetus for me for the preparation of this thesis.

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Saroj Nepal

Contents

Notations and Abbreviations

ALT	Approximate Lorentz Transformation
AS	Acoustic Signal
AZT	Approximate Zahar Transformation
BE	Boughn Effect
CMBR	Cosmic Microwave Background Radiation
CS-thesis	Conventionality of Simultaneity thesis
CVL	Constancy of Velocity of Light
DSR	Doubly Special Relativity
DT	Dolphin Transformation
EP	Equivalence Principle
EW	Einstein World
GPS	Global Positioning System
GR	General Relativity
GSDC	Gravitational Slowing Down of Clocks
GW	Galilean World
LCF	Length Contraction Formula
LT	Lorentz Transformation
QSU	Quasi Static Model

MSRP	Modified Second Relativity Postulate
NDSR	Neo Doubly Special Relativity
OWS	One Way Speed
RTD	Relativistic Time Dilation
SR	Special Relativity
TDF	Time Dilation Formula
TE	Transformation Equation(s)
TT	Tangherlini Transformation
TWS	Two Way Speed
VSL	Variable Speed of Light
ZT	Zahar Transformation

Chapter I

General Introduction

1.1 Introduction

The present dissertation deals with some conceptual issues in relativity theory. Some of these issues concern the pedagogical aspect of the theory while some relate a few observational puzzles in the field of cosmic ray physics and relativistic cosmology. It is interesting to note that on the one hand, the fundamental postulates of relativity is being challenged today for example, in the context of ultra high energy cosmic ray (UHECR) paradox (on the theoretical side, in the context of quantum gravity and Lorentz invariance violation (LIV) theories and the likes) on the other hand it seems that even after about hundred years since the advent of relativity theory the scientific community at large is yet to fully reconcile with the hugely counter-intuitive outcomes of the theory. The concepts and implications of the kinematics of relativity, as one may observe, have not yet fully been settled in our minds; paradoxes, resolutions and consequent debates concerning the theory still continue. For a concrete example, one may point out that nearly three hundred articles have already been written on the single issue like the twin paradox or the clock paradox and still more continues to pour in. The authors of these articles should not be accused of splitting hairs sitting on the ivory towers. They have been serving the scientific community by providing a lot of insight into various crucial questions in relativity theory.

Concerning the observational puzzles mentioned in the beginning, one often considers modification of standard relativity theories. In my opinion, to think of any possible violation of a well established theory (for one reason or the other) it is essential first to refine our understanding of the theory. I found that my humble efforts to understand the twin paradox in order also to capture newer insights, has provided me with a vantage point from where I have been able to look beyond. In particular a newer approach to understanding the twin paradox based on the conventionality of simultaneity (CS) thesis has fascinated me. Our CS-thesis approach has also thrown some new light on the famously counter-intuitive issue. My learning of the CS-thesis in connection with the investigations on twin paradox, enabled me to obtain a novel understanding of one of the most puzzling paradoxes of physics, viz UHECR paradox. Again if our approach to deal with the UHECR problem turns out to be correct, one can show that the underlying assumptions of the novel approach may directly lead to a variable speed of light (VSL) cosmology. Indeed a simple minded approach to this effect has been a small part of the programme of my investigations. The present thesis is a compilation of the findings of the present author following investigations on the conceptual issues like the twin paradox and its ramifications, an issue in the realm of high energy physics (UHECR paradox) and an area touching VSL cosmology.

The assortment of topics of investigations has one unifying thread: They all relate to studies of space-time behavior in different inertial frames. In the treatment of twin paradox ordinarily standard relativistic transformations including ones permitted by CS-thesis have been used. In some cases however, non-relativistic transformations have been discussed for purely pedagogical purposes. In other two areas prompted by the search for new physics, a case for a deformed Lorentz transformations (LT) has seriously been considered. However predictions of a deformed LT is expected to be different from those of relativity theory, hence care has been taken so that the difference in predictions be undetectable in the domain where special relativity (SR) has been tested beyond doubt. The main text concerning the present study comprises of chapters III-VII which reports observations and results obtained by me (along with my collaborators) in the last few years. Some of these observations have been published and some have been reported in the national and international meetings and others are under publications.

The whole volume, apart from the introductory chapters (I-II) is divided into two parts. Part-I deals with the conceptual and pedagogical issues concerning aspects of the special relativity theory in particular that of the twin paradox and its ramifications while Part-II reports investigations on UHECR paradox and VSL cosmology based on a novel LIV approach that we have developed. As has been mentioned, the CS-thesis of SR has been found to be an essential tool for the investigations. I therefore devote an entire chapter (chapter II) in order to discuss it as an essential prelude. Some novel space-time transformations (to be used later) are also discussed therein in some detail.

All the chapters of the main text are self contained. However for readers not thoroughly acquainted with the issues discussed, a brief review of the previous works has been presented in the next two sections of this introductory chapter as a background (*Background-I* and *Background-II*). Background-I pertains to topics of Part-I of the thesis which deals with the conceptual and pedagogical issues, while Background-II provides reviews of previous works concerning possible deformation of standard relativity theory in the context of UHECR paradox and VSL cosmologies. These sections will also provide the scope and objective of the present study. Finally in the last section of this chapter, I will give a topic wise summary of the main investigations. This section will provide a gateway for the main contents and the readers will find glimpse in advance of what lies ahead.

1.2 Background-I

1.2.1 Outline of the Problem

In 1905's paper on special relativity Einstein[1] predicts that of the two clocks, the one which is moved away and subsequently brought back to its original position will lag behind the stationary one. He termed the effect as " a peculiar consequence"[2]. In 1911 Einstein restates the result in a more dramatic way in the following form: "If we placed a living organism in a box ... one could arrange that the organism, after any arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had already long since given way to new generations. For the moving organism the lengthy time of the journey was mere instant, provided the motion took place with approximately the speed of light"[3]. Historically the word "twins" first finds its appearance in the discussion when Langevin[4] has posed a thought experiment in the problematic form in which a twin leaves the earth, for a distant-star at a speed closed to the speed of light and returns in the same speed to meet the stay-at-home twin on earth to discover that at the end of the trip he is younger than the earth-bound sibling. This counter-intuitive result has been given the name "twin paradox". Note that although counter-intuitive, both Einstein and Langevin did not suggest that there was any paradox in it. Indeed Einstein and Langevin recognized early that the situations for the earth-bound and the stay-at-home twins were not symmetrical and any expectation or claim of symmetrical outcome regarding their ages itself was erroneous. Hence there was no paradox (see later for more discussions). Perhaps the term "twin paradox" was coined much later. Many authors including the present one use the term "clock paradox" for the problem. We shall use in this thesis the phrases "clock paradox" and "twin paradox" interchangeably. However a clarification is in order. LT predicts reciprocal time dilation of moving clocks. Historically this counter-intuitive feature of SR often used to be called the "clock paradox" in the literature[5]. Whereas non-reciprocal time dilation is predicted for the round-trip journey of a clock and the "twin paradox" focuses on this asymmetrical time dilation. However the term "clock paradox" is also often substituted for "twin paradox" in the attempt to avoid the biological issue of whether a traveller's aging is in accord with the standard clocks that he carries[5, 6]. As mentioned we will use the term "clock paradox" in the later sense.

Before we proceed let us first examine in what sense the differential aging of the twin parable due to Langevin is a paradox. There are three facets of the problem. Scott[7] has noted that "It is paradox, in the dictionary meaning of the word from the view points of (i) absolute time (ii) the special theory of relativity, and (iii) the general theory of relativity."

(i) A common man with an intuitive notion of absolute time is puzzled by any difference between the two clocks. (Even some authors of relativity have been found to express the view that time dilation and length contraction of SR are only apparent and it arises because of distant clock synchronization alone[8]; they cannot accept true time dilation of a moving clock when it is brought back to its original position). In this sense this aspect of paradox is mainly concerned with the counter-intuitive feature of the problem.

(ii) A person aware of time dilation of SR knows that the effect depends on relative velocity and is perfectly reciprocal with respect to observers who are in relative motion hence the person gets puzzled by the contradictory claims by the twins regarding the direction of asymmetry in their ages since although there is acceleration, the effect as has been said depends on relative velocity alone. Therefore this aspect, instead of focusing on the counter-intuitive aspect (as discussed above) concerns the logical contradiction of the issue.

(iii) There are others who take such statement of relativity theory as "the laws of physics are same in all frames of reference" too näively, get puzzled by the asymmetric outcome (differential aging)[1]. In this context they further note that there are accelerations and since SR (as if) cannot deal with accelerated frames of reference, the problem can only be treated by general theory.

The last aspect of paradox is the most trivial one as the principle of relativity states the *equivalence of inertial frames of reference* only. (Indeed often the principle of general covariance is construed as equivalence of *all* reference frames—surely this is incorrect). The aspect of the twin paradox which is found most perplexing is the one which arises within SR. The present investigation therefore mainly concentrates on this facet of the problem. Although the chapter-III discusses the issue in the context of GR through the principle of equivalence, it attempts to focus on resolving the logical contradiction aspect rather than the counter-intuitive aspect of the issue. Coming back to Langevin's account of the paradox[2], to be specific, consider two twins A and B initially on the earth. The sibling B takes a space trip to a distant-star at P, a distance L from A, eventually turns around and returns to earth. If it is assumed that the periods of acceleration are negligible compared to the periods of constant velocity, the time for the round trip of B as measured by A may be

[1]For example Dingle[9] in 1957 stated that Einstein made a "regrettable error" and he argued that "According to the postulates of SR, if two identical clocks separate and reunite, there is no observable phenomenon that will show in absolute sense that one rather than other moved. If the postulates of relativity is true, the clocks must be retarded equally or not at all, their readings will agree on reunion if they agreed at separation..."

[2]One may note here that although the standard account of the paradox considers that the traveller has a twin who stays on earth, Langevin's original parable did not include the term "twin". However that is a matter of history and hence besides the point.

calculated as $2L/v$. Because of the relativistic time dilation B's clock will record $2\gamma^{-1}L/v$ time for the round trip. Since $\gamma^{-1} = (1 - v^2/c^2)^1/2 < 1$, where c is speed of light, at the end of the trip B should be younger than A.

The beginning students of relativity immediately after getting introduced by their teachers to the paradox (which is a consequence of above result) often hears that in spite of the asymmetrical outcome, there is no paradox as such as the situations are not symmetrical for the twins: only one of the siblings experiences acceleration and hence the result is not surprising at all. As mentioned earlier Langevin and Einstein also did not see any paradox in the "peculiar consequence" of SR although in the later years Einstein had to fight his opponents and tried to defend his theory by giving answer to the "paradox" in terms of his General theory.

Authors of repute are often found to dismiss the paradox by pointing out that "The differential aging suggested by Langevin comes directly from the fact that proper time is a path dependent quantity in special relativity"[5]. The statement may be clarified as follows. Consider the Fig.(1.1) below. One calculates the proper time τ along the two trajectories of the Minkowski space-time. It is assumed that the to and fro coordinate-speeds are the same. These trajectories are labeled as path (1) from the origin O of the earth's frame to time $t = 2T$ (say) along y-axis and path (2) from the same origin along an oblique line to the turning point P and back again to O'. These paths corresponds to the earth-bound and travelling twin respectively. The proper times are obtained by integrating along each trajectory,

$$d\tau = \gamma^{-1}dt. \tag{1.1}$$

The non-integrability of $d\tau$ follows from the fact that the results are different for the above two paths. This also follows from the elementary notion of geometry: The sum of two sides is always different from other side of triangle. However since in SR the space-time is pseudo euclidean, $OP + PO' < OO'$. This implies that the traveller will age less. From all this it appears that as if the aspect of non-integrability of proper time is the answer to the paradox and people are often found to express surprise at the suggestion of the paradox itself.

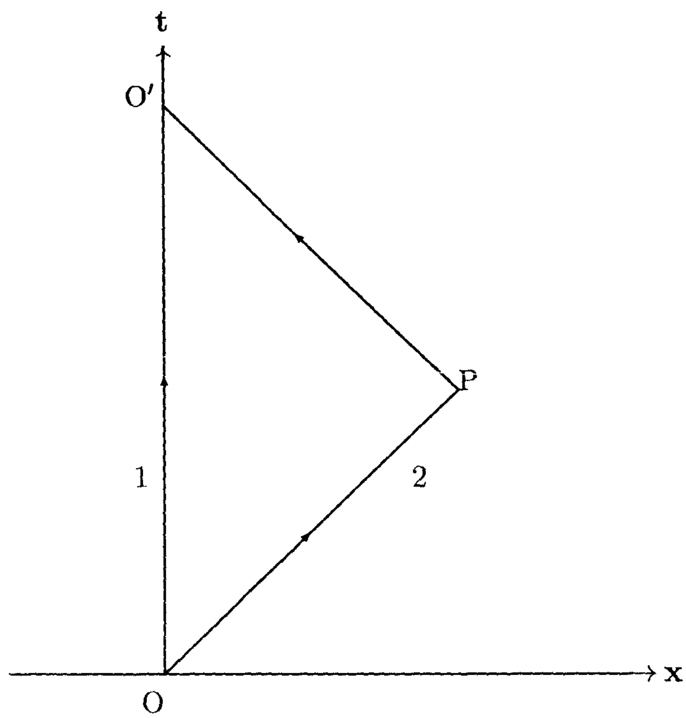

Figure 1.1: **Minkowski diagram**

As another example Bondi[10] compares the aging process of a human being with the mileage of a car and the journey of the traveller twin with that of a vehicle along a curvy road. Referring to the aging of the traveller Bondi has been found to remark "It will *not* therefore come as a surprise to him on his return to earth to find out that he has aged less than the people there, just as the traveller who took the curvy road cannot have been surprised that he covered a longer mileage than the traveller who followed the straight line". "Hence there is no clock "paradox", since it is not paradoxical for two persons with different experiences to find that the consequences of their experiences differ". These trivializing often cavalier statements[3] on the issue are made when one is concerned with the third aspect of the paradox only.

1.2.2 Resolutions

The literature is replete with discussions of twin paradox, its variations and their resolutions. That there exists a plenty of resolutions only suggest the richness of the issue. After about fifty years since Paul Langevin[4] had posed the thought experiment concerning the clock paradox, David Scott[7] identified (of the many solutions suggested at the time) three alternative approaches which to him were particularly instructive for the twin problem. We will now consider them below one by one of which the last one we have briefly mentioned in the last section.

(i)Length Contraction:

The length contraction of moving length can be used to explain the time difference. This has been discussed by Fremlin[11]. The idea is as follows: B is in a uniform motion at speed v, he can measure the distance AP (where points A and P denote the positions of stay-at-home twin and that of the distant star respectively) and will find it is $\gamma^{-1}L$ but not L because of relativistic length contraction effect. Hence B will calculate his time for round trip as $2\gamma^{-1}L/v$. In this approach, the asymmetry can be thought of as related to the fact that the distance point P is fixed relative to A and not to B.

The trouble with this explanation is that although B calculates the travel time of A's round-trip on B's clock correctly but the so-called resolution remains silent about the reading of A's clock from the perspective of the traveller. Will it not be dilated?

[3]Often these statements are found as passing remarks by some authors who in their scholarly discourse also deal with the deeper aspects of the paradox.

(ii)Doppler Shifts:

In the literature Darwin[12] describes each of the two observers can keep a record of the other's time during the course of the trip by sending out regular time signals (see also [5]). It can be speculated that light or radio signals are sent regularly in the sender's time. In the above scenario of twins record, using the relativistic Doppler factor the frequency of a receding source is $[(c-v)/(c+v)]^{1/2}$ and for an approaching source is $[(c+v)/(c-v)]^{1/2}$. While B is on the outward trip, A will receive B's signals at $[(c+v)/(c-v)]^{1/2}$ intervals corresponding to a red Doppler shift. Whereas when B is approaching, A will receive B's signals at $[(c-v)/(c+v)]^{1/2}$ intervals corresponding to a violet Doppler shift.

First consider A's record of B's signal. Observer A will receive slow or $[(c+v)/(c-v)]^{1/2}$ signals form B for the duration of the outward trip or for the time light takes to travel from P to A. For remainder of the total time A will receive fast or $[(c-v)/(c+v)]^{1/2}$ signals from B. Hence A will record time $[(c+v)/(c-v)]^{1/2}L/(c+v) + [(c-v)/(c+v)]^{1/2}L/(c-v)$ or $2\gamma^{-1}L/v$ worth of B's signal.

Now consider B's record of A's signal. B will receive slow signals from A until he reaches P and the upon reversing his motion, he will recest signals. If t be the total time of the trip as measured by B then for time $t/2$, B receives slow signals from A and for time $t/2$ years fast signals. Since A sends out $2L/v$ worth of signals during the trip,

$$[(c-v)/(c+v)]^{1/2}t/2 + [(c+v)/(c-v)]^{1/2}t/2 = 2L/v, \qquad (1.2)$$

hence solving one obtains $t = 2\gamma^{-1}L/v$ as described before.

This treatment in terms of Doppler shifts makes it clear that the asymmetry exists because B reverses his motion and hence *immediately* observes the change in the rate of the signals. For A, he must wait for the time taken by light to traverse the distance P to A before noting the change in the signals. However this is again merely the description of what is taking place but it does not explain the fault of the standard analysis. Besides this Doppler-shift analysis does not give the reason why there should be an abrupt change of the earth-bound clock because of B' turn around.

(iii) Word Lines:

We have mentioned this approach in the earlier section and already criticized the often made cavalier remark associated with the approach that "there is no paradox as such and the differential aging follows from the fact that the proper time is not integrable". Although it is true that there is no paradox (indeed there cannot be a paradox in a time tested physical theory) one cannot but ignore the intricacies of the paradoxical issue. However the world line approach serves one to pose the problem in geometrical terms and provides a smart and uniform basis for discussions of the paradox as well as different variations of the clock paradox.

The time τ for the clock B, according to SR is related to the A-clock time t and displacement (x, y, z) of B as measured by A in the following way:

$$d\tau^2 = dt^2 - (dx^2 + dy^2 + dz^2)/c^2. \tag{1.3}$$

Clearly $d\tau$ is always less than dt as B departs from A. Hence for a return journey along any path (which requires integration of the above expression), the time interval on clock B will be less than that of A. One may note however that the explanation, requires that the frame attached to A is inertial. The path of observers or particles in space-time is known as world lines. Various authors describe and analyze the problem by drawing the world line of the twins (or the clocks) and sometimes for the photons they send. We have already drawn such world lines for the twins in the earlier section. Below we draw[4] world line diagrams for some variations of the twin paradox problem including the standard one.

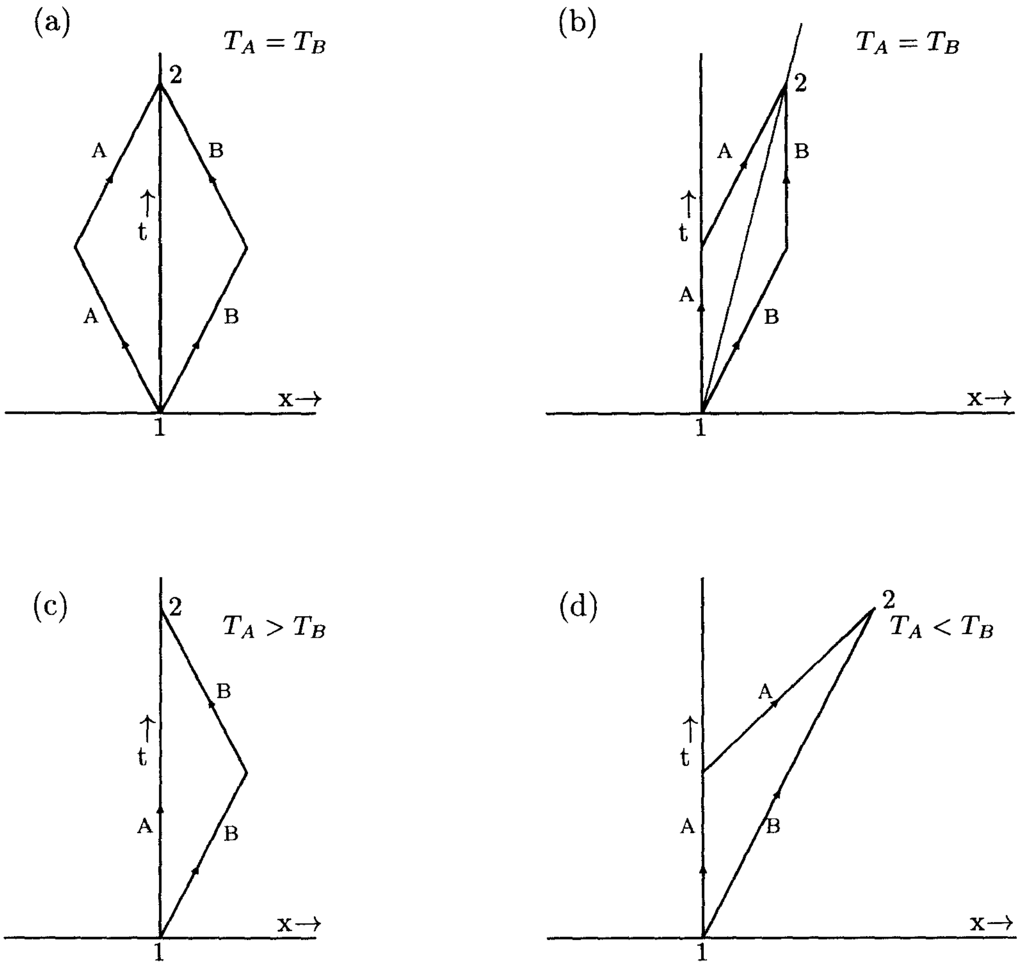

Figure 1.2: **World line diagram for variations on the clock paradox problem.**

In the first two diagrams there is no differential aging ($T_A = T_B$) when the twins meet at point 2, whereas the last two diagrams depict the cases where the twins' ages differ. The reader can easily reconstruct the parables for the twin paradoxes that the figures represent. This shows the power of these diagrams although the approach is unable to address the subtleties of the problem. Debs and Redhead in their most cited paper[5] have extensively used the pedagogical power of these diagrams in connection with their conventionality of simultaneity approach to the

[4]We owe to G. D. Scott's paper[7] for these diagrams.

problem.

1.2.3 The Genesis of the Problem at a Glance

Although the current section intends to review earlier works and ideas concerning the problem, let us briefly discuss the root of the paradox which will be elaborated later in the main chapters. Indeed the reader will then be better equipped to grasp the unsatisfactory features of some of the earlier authors' approach to the problem from the beginning.

The relativistic time dilation effect of SR relates time of two different nature. One concerns the rate of ticking of a moving clock at its positions and the corresponding time is referred to proper time of the clock. The other refers to readings of spatially separated coordinate clocks, as the concerned clock moves past these coordinate clocks. Time recorded by the coordinate clocks are known as coordinate time. Since coordinate clocks are spatially separated, the coordinate time for a given pair of events depends on the synchronization convention or the standard of simultaneity adopted to synchronize these coordinate clocks. In SR we adopt Einstein synchrony or relativity of simultaneity according to which the one-way speed (OWS) of light is stipulated to be equal to its round trip speed[13]. The proper time of a clock however is independent of any synchronization conventions. In twin paradox, the proper time of one twin (clock) is calculated from the knowledge of the coordinate time elapsed in other twin's frame of reference. We shall later see in detail that genesis of the twin paradox lies in the failure to calculate the so called proper time in the frame of reference attached to the traveller twin.

Asymmetry in Twin Parable:

We have already pointed out earlier that the paradox arises if one näively treats the perspectives of the twins symmetrically. Some authors seem to remain satisfied by pointing out the asymmetrical situations for the twins. Terrell[14] elaborated this aspect by pointing out the fact that the observational data of two observers (clocks) will not be at all similar. The data of the accelerated observer obtained by means of "single" Doppler-shift and visual observation method would be peculiar and inconsistent with the data obtained by radar and "double" Doppler-shift methods. It was then remarked that the accelerated observer would be "under no temptation to consider himself in a situation equivalent with that of the unaccelerated observer ...". In other words as Terrell pointed out, the traveller twin would then "not be surprised to discover upon returning that he had aged less than the other observer".

However mere pointing out the asymmetry of the perspectives of the twins can hardly be called a resolution, but I consider it worthwhile here to reproduce the novel approach of the author to highlight the accelerated and unaccelerated scenarios. This will at least clear any doubt that may exist in our mind as to the non-equivalence of the situation.

(i) Unaccelerated Scenario:

Consider Lord Halsbury's "three brothers thought experiment"[5] in which three brothers (clocks) A, B and C are moving uniformly with relative velocities along the straight line. We assume that each observer is equipped with radar, radio and optical equipment for measuring distances, relative velocity and synchronizing clocks. It is however hardly necessary to explain the detailed procedure for such measurements for the present purpose.

For definiteness one may consider that A observes twin B passes him with the velocity v at the time $t_A = 0 = t_B$. He synchronizes his clock in passing. The twin B travels a distance L at a time $t_A = L/v$ and simultaneously passes the third observer C, which moves in the travelling in the opposite direction with the same velocity v relative to A. The sibling C synchronizes his clock to that of B at the position of passing. Finally C crosses the position A at time $t_A = 2L/v$ and A and C compare their clocks.

Now one may very well see that this is no ambiguity if one accepts the time dilation formula of SR. The observer A notes the clock of both B and C run slower by the Lorentz factor γ^{-1} as we have already discussed. Thus A observes the reading of C-clock as $2\gamma^{-1}L/v$ at the position of passing when his clock reads $t_A = 2L/v$. Since A, B and C are all assumed to be in *unaccelerated frames of reference*, according to SR then there is no abstract reason to prefer A than B or C. From the view-point of B, the clock A runs slow by the same Lorentz factor γ^{-1} as A observes for B-clock. Similarly B observes C-clock to be even slower than that of A since B measures the velocity of C to be $2v/(1+v^2/c^2)$ which is greater than v. Hence B predicts that C-clock reads less than that of A when C passes A. Similarly according to C's perspective, A-clock runs slower by the same Lorentz factor γ^{-1} but C observes B-clock is even slower. This clearly accounts for the fact that C-clock (which has been synchronized to that B) reads less time than A-clock when the latter passes C[5]. However, in the present scenario, A, B and C none of has any special status; they are "equally good as observers". Their differing observations is in accordance with SR. So far as they continue their uniform motion there is no basis for arguing that anyone's clock is really indicating the passage of less time than another's clock. If one says so it would imply giving preference to one of the three co-ordinate systems. This would however contradict the basic tenet of SR. Such a possibility may however become a reality if the observers are not equivalent situations.

[5]In essence, the resolution points out that during the transfer of clock information from frame of B to the frame C, the line of simultaneity has changed, with a consequent discrepancy and advance of time at A.

(ii) Accelerated Scenario:

If now one change is made in the above scenario–substituting observer B for observer C at the time of their meeting, the unaccelerated situation becomes the usual twin paradox. We then deal only with two observers i.e A and B (say). As observed by A, B synchronizes his clock in passing, travels with velocity v to distance L, then reverses direction in a time negligible with respect to L/v and returns to the position of A. The observations of A are essentially the same as in the unaccelerated case, so that B's clock will read less than A's upon the second meeting. The observations of B, who does not remain in a single inertial system, can be shown to be confusing and apparently internally inconsistent[14]. One may then be tempted to conclude that the acceleration which B undergoes makes a real difference in the status of two observers. At this point we feel it worthwhile to study the role of acceleration in the twin problem. Below we provide a discussion on the issue by mentioning two important works exemplifying the problem.

1.2.4 Role of Acceleration

The direct role of acceleration of travelling twin in causing the differential aging has been criticized in the literature although it is quite clear that in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of twins have to compare at the same space-time events) one cannot normally avoid acceleration[6]. In an interesting article Gruber and Price[15] dispel the idea of any direct connection between acceleration and asymmetric aging by presenting a variation of the paradox where although one twin is subjected to undergone an arbitrary large acceleration, no differential aging occurs. In their version, the rocket-

[6]Vide section (1.2.7) for a discussion of some novel version of the paradox, where the traveller although "unaccelerated" eventually meet the stay-at-home one.

bound twin undergoes a periodic motion as shown in Fig.(1.3) so that

$$x = (V_{max}/\omega) sin\omega t, \tag{1.4}$$

where x and t are the coordinates of a fixed frame on earth and V_{max} is the maximum speed achieved by the rocket-bound twin relative to the earth. The acceleration of the rocket (the rocket's 4-acceleration) has a maximum magnitude of $V_{max}\omega$, which occurs at times $\omega t = \pm\pi/2, \pm 3\pi/2....$ These relativistic results agree with Newtonian (non-relativistic) answers i.e, the maximum velocity occurs when the particle has zero velocity relative to the fixed frame on earth.

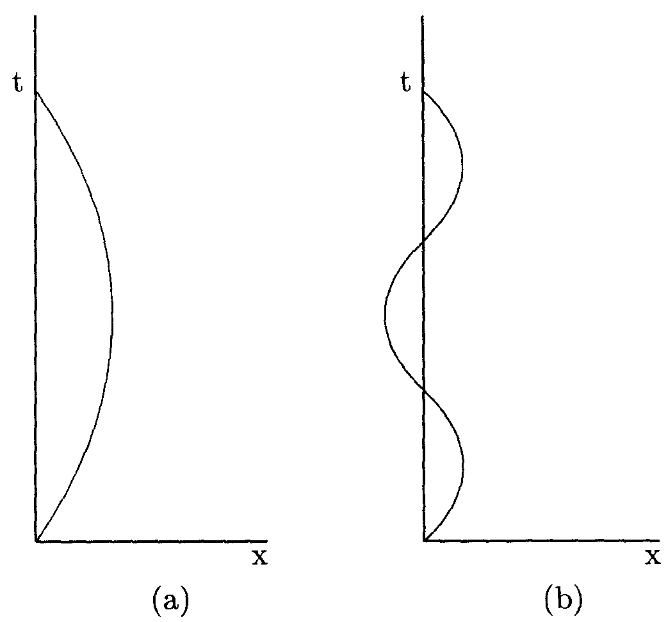

Figure 1.3: **Rocket-bound twin World lines. The world line in (b) has maximum acceleration three times that for the world line in (a).**

The proper time τ of rocket-bound twin relates to the coordinate time t by

$$d\tau = dt[1 - (V_{max}/c)^2 cos^2 \omega t]^{1/2}. \tag{1.5}$$

A rocket trip starting and ending at the earth will take an integer number i.e, staring at $t = 0$ and lasting until $\Delta t = n\pi/\omega$. For such a trip the proper time will be

$$\Delta\tau = n\int_0^{\pi/\omega}[1-(V_{max}/c)^2cos^2\omega t]^{1/2}dt, \tag{1.6}$$

so that the ratio of elapsed rocket time to elapsed earth time is given by

$$\Delta\tau/\Delta t = (2/\pi)\int_0^{\pi/2}[1-(V_{max}/c)^2cos^2\theta]^{1/2}d\theta. \tag{1.7}$$

This integral can be evaluated numerically. The important fact on this equation is that it is independent on ω so that time dilation effect is independent on the maximum acceleration $V_{max}\omega$. These considerations eventually prove that acceleration *per se* cannot be the root of differential aging. Indeed one can show that one can have arbitrary large acceleration without any significant differential aging!

There is also a converse situation as discussed by Boughn[16] in connection with an interesting variation on the twin paradox. It is shown therein that spatially separated twins can age differently although their history of acceleration remains the same.

We shall discuss this scenario in detail in Sec.(1.2.7) where we discuss some important variations of the paradox. That the acceleration *per se* cannot play a role is evident also from the usual calculation of the age difference from the perspective of inertial frame of the stay-at-home twin if one notes that the duration of turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be understood in terms of usual time dilation of the travelling twin during the unaccelerated segment of her journey. In such a calculation the time dilation is also calculated during the acceleration phase (assuming the clock hypothesis to be true[5]) and is shown to contribute arbitrarily small value in the age offset if the duration of the acceleration phase is assumed to tend to zero. Dray[17] and Barrow[18] have stated that the role of the acceleration can be eliminated by posing the problem in a closed universe setting.

Having understood all this it is important to note that although acceleration has a secondary role from the point of view of the earth-bound clock, from the stand point of the traveller the effect of acceleration is far from being trivial. The quantitative role of acceleration (from the point of view of the accelerated observer) in the asymmetry has been studied by Nekolic[19] who has estimated that the influence of acceleration on the differential aging not only depends on the value of the acceleration itself but also on the relative distance between the accelerated observer and the initial clock. The procedure predicts that acceleration has no influence if two clocks (initial and non-inertial) are at the same position. However if travelling twin moves with constant velocity and suddenly reverses the direction of motion, at this time, it will appear to him that the time of the inertial clock instantaneously jumps forward but there is no such jump of the accelerated clock from the point of view of inertial twin.

1.2.5 The Resolution in General Theory of Relativity

In spite of the fact that the clock paradox can be resolved in the context of SR, many authors feel that the introduction of the general theory of relativity (GR) and a gravitational field at the point of acceleration of the travelling twin is the right way to explain the asymmetrical aging. Although the opinion is still divided regarding the usefulness of the GR resolutions, most of the current expositions concern the intricate issues and different facets of the special relativistic resolutions. Historically among the many, Einstein[20], Tolman[21], Bohm[22] and Frisch[23] advocate the use of GR. For example, Bohm in this connection notes that two clocks running at places of different gravitational potential will have different rates. This obvious reference of the phenomenon of gravitational red-shift, a prediction of GR was made by Bohm (and Frisch as well) around the time when the effect had just been tested for the first time by Pound and Rebka[24]. On the other hand Einstein offered his GR argument as early as in 1918[20], which indeed was his second argument. The first one was however the usual special relativistic one. An interesting history as to why Einstein had to invoke the GR argument can be seen in an interesting article by Peter Pesic[2].

In the standard version of the twin paradox the differential aging from the perspective of the stay-at-home (inertial) observer A can easily be calculated assuming that for the most parts of the journey of travelling twin B, the motion remains uniform except that there is a turn-around acceleration of the rocket so that finally the sibling are able to meet and compare their ages. The duration of the acceleration phase can be considered to be arbitrarily small compared to the time it takes during its forward and return journeys and hence the age difference occurs due to usual relativistic time dilation of a clock for its uniform motion.

$$Age\ diff = 2t_A(1-\gamma^{-1}) \approx 2t_A v^2/c^2, \tag{1.8}$$

where $2t_A$ is the time the rocket takes for its entire journey (up and down) in uniform speed v and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor.

The paradox is resolved if one can show that B predicts the same difference in spite of the fact that the time dilation effect is reciprocal. Clearly some new considerations must offset this reciprocal time dilation and this must provide some extra aging to A from the point of view of B so that the age difference remains independent of twins perspectives. One of these considerations, as has already been discussed, is the one of the synchronization gap that B discovers due to her change of inertial frame during the round-trip. This has been clearly demonstrated by Bondi[10] in the context of Lord Hulsubery's three brother approach to understanding the twin paradox. The other way of understanding the same thing, as remarked by Harpaz[25] and others[21, 23] is the consideration of pseudo-gravity of general relativity (GR) experienced by B because of turn-around. "Although there is no need to invoke GR in explaining the twin paradox, the student may wonder that outcome of the analysis would be if we know how to deal with accelerated reference frames. We could use space ship as our reference frame.... We would find that we must have a gravitational field in this frame to account for the accelerations... if as required in general relativity, we then compute the frequency shifts of light in this gravitational field we come to same calculation in special relativity" [26]. Harpaz[25] has argued that although special relativistic approach can correctly account for the age difference between the twins, "it does not manifest the 'physical agent' responsible for creation of a such a difference." It is held that equivalence principle (EP) provides such an agent. In order to demonstrate how EP plays the role in the analysis, Harpaz has used the gravitational red-shift formula, which can be obtained heuristically as

$$\Delta\nu = \nu_0(1 + gh/c^2), \tag{1.9}$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed from a distance h from the source where the frequency of the same light is seen to be ν_0. Interpreting this red-shift effect in terms of gravitational slowing down of clocks (GSDC), the formula can be written as

$$t_1 = t_2(1 + \Delta\phi/c^2), \tag{1.10}$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\phi = gh$, is the potential difference between these points. It has been shown that with respect to B the acceleration plays a role by providing an extra time difference between B and A, because of GSDC during the (arbitrary) short duration of B's acceleration. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount. Indeed Einstein suggested that "according to GR, a clock works faster the higher the gravitational potential at the place where it is situated"[27], and since there are homogeneous gravitational field equivalent to the acceleration experienced by B, one should add this contribution to the calculation. Einstein asserted that "calculation shows that the consequent advancement amounts to exactly twice as much as retardation during stages of inertial motion. This completely clean up the paradox..."[27].

In an interesting paper Perin[28] has set up a specific round-trip situation from the point of view of each twin by using the gravitational field equation as its starting point. The geodesic equations of motion are solved in the travelling twin's reference frame in order to determine the time elapsed on the earth twin during the periods of acceleration. He has pointed that "The equality of the result obtained by each twin is explicitly exhibited". Perrin's method is a generalization of Moller's[29] approach. Moller has solved the problem by transforming Lorentz metric in an arbitrary accelerated reference frame by requiring that its spatial part is to be Euclidean. Very recently Gron[30] has assumed that the travelling twin may be considered at rest in a uniformly reference frame. In such a frame the line element (in the case of time like interval) representing the proper time interval, as measured by a clock following a world line connecting two events has the form

$$-c^2 d\tau^2 = (1 + gx/c^2)c^2 dt^2. \tag{1.11}$$

For a clock at the position ($x = h$) gives

$$\Delta\tau = (1 + gh/c^2)\Delta t, \tag{1.12}$$

so that the travelling twin experiences a gravitational field each time. He has shown that at the starting and arrival points the proper time vanishes, but travelling twin can predicts the aging of earth-bound twin when he experiences a gravitational field at the turning point. The earth bound twin falls freely upwards in gravitational field she experiences the gravitational field for a time $\Delta t = 2v/g$. Hence the extra time in the period of acceleration becomes $2hv/c^2$ as calculated by synchronization gap by virtue of special relativistic treatment.

1.2.6 Flaw of the GR Analysis

The essence of any general relativistic solution of the twin problem thus lies in introducing an equivalent pseudo gravitational potential to be experienced by the traveller twin at the time of her direction reversing acceleration. A consequent gravitational time offset effect then provides the extra aging of the stay-at-home twin A required to make the correct prediction by the traveller twin B. Thus acceleration is absolutely essential for the GR analysis and hence the equivalent homogeneous gravitational field as the physical cause behind the asymmetrical aging of the twins. It can be shown that the application of the equivalence principle of general theory of relativity as in the treatment by Einstein, Harpaz and others [21, 22, 25, 31, 32] is essentially trivial. We clarify this rather a strong (against the GR analysis) statement with reasons as follows:

(i) As Builder[32] has noted " The equivalence principle states that the description of events in terms of the coordinates of an accelerated reference system is indistinguishable from the description of identical events in terms of coordinates of reference system at rest in an equivalent gravitational field". The principle thus allows the course of events in a gravitational field to be predicted by SR, the course of events as describes in terms of the coordinates of the equivalent accelerated reference system.

(ii) In effect the authors advocating GR resolution, have tried to answer by denying the applicability of SR, and then using instead calculations that have been obtained from SR by means of the equivalence principle. Builder[32] writes " This tortuous procedure succeeded in evading the paradox rather than resolving it". The procedure will be quite invalid if SR itself are indeed not properly applicable to the problem.

(iii) The clock paradox can be posed without acceleration by invoking a third inertial observer, thus a flaw of the GR analysis is that this cannot resolve the variations of the paradox which do not involve any acceleration. In addition, as pointed out by Debs and Redhead[5] and also others[33], that since in the twin problem one deals with the flat space-time (Riemann tensor $R^{\alpha}_{\beta\mu\nu} = 0$), any reference to GR in this context is quite confusing. In this context Unnikrishnan[27] has noted that "all standard resolutions of the twin paradox invoking acceleration or an equivalent pseudo-gravity as a physical effect responsible for asymmetric time dilation are flawed...".

(iv) In an effort to find a "physical agent" responsible for the extra aging, the author of Ref.[25] and many others rely on some approximate formulas including that of the gravitational red-shift because of $v^2/c^2 << 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make such small velocity approximation for the problem.

(v) The explanation based on SR relies on the fact that during the direction reversing acceleration, the travelling twin changes from one reference frame to another and the lack of simultaneity of one reference frame with respect to other provides the "missing time" which constitutes the reason for the differential aging. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula, one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $z = gh/c^2$ is only approximate and is derived by making use of the *classical* Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation. In chapter III we will elaborate on this issue.

1.2.7 Some Variations of the Twin Paradox

In the pages of the scientific literature, there are several variations of the clock paradox which are indeed useful additions to the pedagogy of SR in general and twin paradox in particular. In Sec.(1.2.2) we have already referred some of them indirectly through the space-time diagrams (Fig.(1.2)). In one of these diagrams one represented a scenario where neither of the twins are stay at home, instead two of them perform identical journey from a common point on earth first in opposite directions and then turn around and finally meet at the same position. Thus the asymmetry of the usual twin problem is removed in this parable but here any one of the sibling is entitled to think that the other is doing all the moving and hence must suffer time dilation indicating the contradiction! The situation is best exemplified in the so-called "circular twin paradox". Although the paradox first appeared in

Lightman et.al[34] in 1975, a more elaborate discussion on the problem has been given by Cranor et.al[35] in the beginning of this century. The authors presented a variation of the twin paradox where "each twin leaves on one ring of a counter rotating pair of infinitesimally separated rings so that the twins travel on the same circular path but in opposite directions....". The twin of one ring should claim the clock of the other twin be slowed down by time dilation caused by the latter's relative velocity and other contradicting the claim. The resolution of the paradox correctly focuses attention in the connection of time dilation to clock synchronization of coordinate clocks. According to Cranor et.al's parable the two rings have been assumed to rotate clockwise and counter clockwise with equal angular speed ω with respect to the the laboratory frame. The paper assumes that one twin Lisa with a team of observers live stationary at every point on its ring of radius R whereas the other sibling Bart lives on the other identical ring. The authors assume that Bart moves at the speed $v = \omega R$ in the counterclockwise direction through the laboratory while Lisa's ring rotate with the same speed in the other direction. The twins will pass each other during their rotation so that they can easily compare their clocks. They are assumed to start from the same place and they notice that their clocks both read $t = 0$. Using the velocity addition formula of SR one obtains Bart's speed with respect to the observers on Lisa's ring as

$$v_{rel} = 2v(1 + v^2/c^2)^{-1}. \tag{1.13}$$

Hence Lisa's team observes Bart's clock ticking more slowly than the proper time of their clocks by a relative Lorentz factor

$$\gamma_{rel} = (1 - v_{rel}^2/c^2)^{-1/2} = (1 + v^2/c^2)(1 - v^2/c^2)^{-1}. \tag{1.14}$$

This means that his clock will lag behind the clock of next of Lisa's team of observers that he passes. Bart's clock should be seen to lag more and more as he passes members of Lisa's team one by one. Finally after the half a rotation he ultimately passes his counterpart again and should find that their clock would disagree– Bart's clock lags behind Lisa's clock. Believing in the reciprocity of time dilation of SR, Bart and his team can similarly argue that Lisa's clock should lag behind his clock at future meetings indicating a contradiction.

The solution of the problem requires bringing attention to the fact that the time dilation formula of SR is applicable provided the coordinate clocks of the inertial frames are synchronized according to Einstein's method of synchronization. In this connection Cranor et.al correctly point out the difficulty in synchronizing coordinate clocks on the rotating frame in Einstein's way. In our opinion the authors are very close to the truth however some of the remarks made in the article are rather unfortunate. Let us clarify this below.

Observe that Einstein described his synchronization only for inertial frames of reference. As a rotating ring represents a set of non-inertial frames, the authors describe three other schemes of synchronization. This may briefly be reproduced here: Method 1. The ring is initially non rotating and any two infinitesimally separated coordinate clocks of the ring are synchronized according to Einstein's method (standard synchrony convention) by using the light signal following the standard convention by stipulating that light's one-way speed (OWS) is the same as its two-way speed (TWS). The ring can then be set into rotation uniformly such that all points of the ring are "treated identically". Method 2. It uses a light flash from a big laboratory clock stationed at the center of ring. The ring is again at rest to start with and the observers of the ring upon receiving the light flash can all set their clocks to read a same time, say $t = 0$. As before the rings are again uniformly put into the motion after the completion of synchronization process. Method 3.This is almost the same as method 2 in every respect except for the fact that the coordinate clocks on the ring are synchronized when the ring is already in motion.

It is clear that the method 1, 2 and 3 synchronization schemes all suggest absolute synchrony, according to which two spatially separated events that are simultaneous with respect to the rotating ring are also simultaneous with the laboratory. In this connection the authors make the following remarks "If methods 1, 2 or 3 are used for synchronization of ring clocks, then events that are simultaneous to Lisa and Milhouse will also be simultaneous to the Lab observers. It follows that Lisa and Milhouse, and more generally the entire set of observers on Lisa's ring, are not correctly synchronized to constitute special relativity reference frames. This explains what we already know must be true: There will be no lagging of Bart's clock observed as it passes each of Lisa's observers. For the relativistically inappropriately synchronized clocks of Lisa's observers, there is no time dilation of Bart's clock". (In the quotation Milhouse is the closest neighbor of Lisa on the ring in the counter clockwise direction.)

From the above remark one may think that time dilation of SR is the result of the relativity of simultaneity alone. If the latter goes so does the former. This is obviously not true. It can be shown that time dilation can also be observed even if the co-ordinate clocks are inappropriately synchronized. Indeed for absolute synchrony time dilation and "time contraction" (for the inverse transformation) are both observed in the relativistic world.

The reference of "other synchronization" schemes in the context of the circular twin paradox however supports the conventionality of simultaneity thesis (conventionalist's claim, normally denounced by relativists) in an indirect way. Much of what has been just said is a subject matter of another paper by my other colleagues and myself, however the details of which will not be included in the present thesis.

Boughn's Paradox:

In an earlier section we have discussed the relationship of time dilation and the acceleration of the traveller twin to dispel a common (students') misconception that differential aging is caused by acceleration. A case of zero time dilation in an accelerating rocket[15] has been discussed at length. It has also been briefly mentioned therein that a converse situation exists where twins experience differential aging in spite of their accelerations being the same. We now consider this variation of the twin paradox due to S. P. Boughn[16] somewhat in detail. According to Boughn's story two twins A and B on board two identical rockets (with equal amount of fuel) initially at rest a distance x_0 apart in an inertial frame S, get identical accelerations for some time in the direction AB (x-direction say), and when all their fuel has been expended they finally come to rest with respect to another inertial frame S' moving with velocity v along the positive x-direction with respect to S. By applying LT Boughn then obtains a very astonishing result that after the acceleration phases are over, B becomes older than A.

With respect to the first frame S (i.e according to Mom and Dad of the twins) the ages of the siblings (reading of the clocks) do not alter throughout their journey. Since the twins undergo identical accelerations (same velocities) the distance between twins' ships x_0 with respect to S remains same. By applying LT, the age difference and their separation can be determined from LT as,

$$\begin{aligned} x' &= \gamma(x - vt), \\ t' &= \gamma(t - vx/c^2), \end{aligned} \tag{1.15}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, (x, t) and (x', t') are space-time coordinates of Mom and Dad's frame (S) and twins' final frame of reference (S') respectively.

Consider now two events marked by the birthdays of the siblings and denote the times of these events as t_A (t'_A) and t_B (t'_B) in the frame of reference S (S'). Using the time transformation of LT, these birthday times are related as

$$\begin{aligned} t'_A &= \gamma(t_A - v x_A/c^2), \\ t'_B &= \gamma(t_B - v x_B/c^2), \end{aligned} \tag{1.16}$$

where x_A and x_B denote the spatial positions of the twins A and B respectively as measured from S. Hence

$$t'_B - t'_A = \gamma(t_B - t_A) - v(x_B - x_A)/c^2. \tag{1.17}$$

In parent's frame, as they are twins, one obviously has,

$$t_A = t_B. \tag{1.18}$$

Writing,

$$x_B - x_A = x_0, \tag{1.19}$$

the Eq.(1.17) turns out to be a time offset relation

$$\Delta t' = t'_B - t'_A = -\gamma v x_0/c^2, \tag{1.20}$$

implying that the birthday of B occurs before that of A. This means B is older than A after the trip. This is highly counter-intuitive since although both A and B have identical experiences their ages differ at the end of their journey! The paradox however can be explained by noting that for spatially separated clocks the change of relative synchronization cannot be unequivocally determined. The clocks can only be compared when they are in spatial coincidence. For example, when in S' either of the observers can slowly walk toward the other or both the observers can walk symmetrically (with respect to S') toward the other and compare their clocks (ages) when they meet[36]. However in that case one can show that they do not have identical local experiences–thus providing the resolution of the paradox[37]. While the paradoxical element of the problem goes away, the fact remains that the result (Eq. (1.20)) is correct and this time-offset remains unchanged even if they slowly walk toward each other and compare their clocks (ages) when they meet.

This temporal offset effect of identically accelerated clocks gives an important insight into the behavior of clocks in a uniform gravitational field, for, according to EP "...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system" [38]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. We shall see in chapter-III that the time-offset relation (Eq. (1.20)) of Boughn's paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins[37]. However the connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur [39], who derived the approximate formula for the "clock rate difference". We shall see in chapters (III-V), the importance of the time-offset relation (Eq. (1.20)) in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin.

Twin Paradox in a Closed Universe:

Cosmology Connection

An approach to explaining the differential aging of the twins which avoids acceleration has been to put the two paths onto a closed space-time. For example one may consider a cylindrical space-time- a two-dimensional universe in the shape of an infinitely long cylinder, in which time runs up the cylinder and space runs *around* it. In the cylinder, the time axis falls parallel to the axis of rotation of the cylinder and travelling twin departs and returns by going around the cylinder at constant velocity. The calculation of the proper times on the cylinder has been done by Dray[17] and Low[40]. In such a universe, the twin in the rocket can return to earth with constant speed without changing direction. In the standard twin paradox the "acceleration" of the traveller plays the role of an identifier as to the question who will be younger of the two. The absence of acceleration in the closed universe scenario apparently leaves no such identifier thus making the paradox more challenging.

Tevian Dray has resolved the issue by showing that there still exists an asymmetry between the two twins. He first provides a simple geometrical argument. It is believed that one should formulate the paradox in terms of "invariant concepts as opposed to observer dependent concepts". Although we do not agree with this observation in the context of the paradox, for the sake of completeness let us briefly reproduce the argument.

According to Dray the necessary invariant notion is that of proper time which is just the Lorentzian length of the path (but not Euclidean distance). The triangle inequality in the Minkowski space-time tells that one side of the triangle is greater than sum of the other two. Knowing this, the usual twin paradox is (as if!) easily solved. This we have already discussed (and criticized) in Sec.(1.2.4). For the present paradox the things are claimed to be "just as simple". The author remarks that a line parallel to the sides of the cylinder (time-axis) is clearly shorter than one that spirals around it. Thus the traveller twin travels along the "shorter" route and hence is found to be the older.

The author has correctly clarified that a family of observers in the closed universe "may be singled out by noting that they are the only stationary observers who cannot distinguish between "forward and backward"; e.g., by sending light rays in both directions around the cylinder and seeing which returns first". These observers should always be the oldest in any twin paradox calculation between different stationary observers.

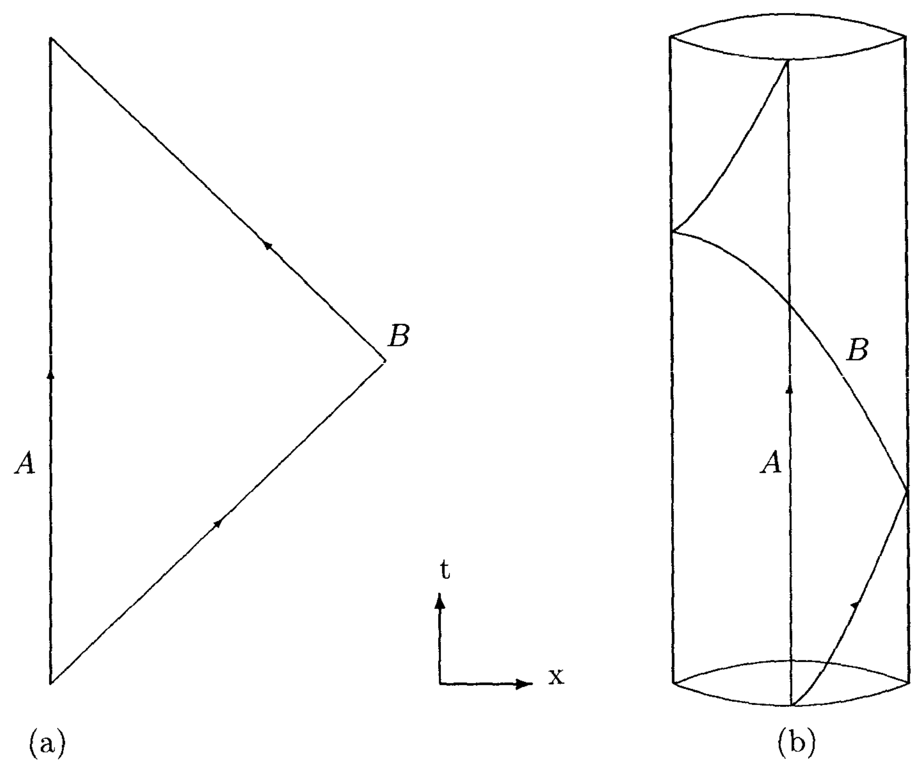

Figure 1.4: **The space-time diagram of the travelling twin *B* and stationary twin *A* (a) in a usual twin paradox (b) in a cylindrical universe.**

It may not be out of place to mention here that in order to observe the requirement of an asymmetry to get differential aging Hafale et.al performed one of the famous experiment called Hafele-Keating experiment[41]. In this experiment, differential aging was observed on two atomic clocks travelling on jets at the same speed around the earth in opposite directions. The rotation of earth provided the asymmetry that was necessary to produce the difference in proper times. These two paths without the rotation of the earth can be compared schematically to the two paths going around in the opposite directions on the cylindrical space-time.

In the context of closed universe Barrow and Levin argue that "the resolution of the twin paradox in the closed universe hinges on the existence of *preferred frame* singled out by the compact topology of space"[18]. They have shown that there is only one reference frame that can be at rest and all other inertial observers in relative motion live in the universe where both space and time coordinates are known. Thus the solution of twin paradox identifies a preferred place and preferred time at the center of the universe so that observer be able to synchronize their clocks and observe the smallest volume for the universe.

The question of existence (or the role) of preferred frame in the resolution of the twin paradox has recently been discussed by Unnikrishnan[27] and Kak[42]. Referring to Einstein's own resolution of the clock paradox in terms general relativity (gravitational slowing down of clocks) in 1918[20] and also to other standard resolutions, the former author remarks that they all suffer from a logical fallacy. Indeed Unnikrishnan has noted that "the time registered by two identical clocks that are synchronized initially can be different only if the rate of the clocks changes differently during motion, and one does not see any logical possibility of the required modification of the rate of either clock in any of the standard resolutions, including Einstein's resolution of the twin paradox....". According to the author (as has been discussed in Ref.[27]) the other alternative would have been to accept a "spooky" instantaneous action-at-a-distance which is not permissible in SR. As a logically consistent possibility Unnikrishnan then suggests that one has to accept that the "rate of a clock is modified according to the standard Lorentz factor with the velocity always relative to average rest frame of the universe or the frame in which the cosmic microwave background radiation (CMBR) is isotropic....". Obviously there does not exist a twin paradox in such a scenario. In fact in any clock comparison experiment (including the GPS timing and the Hafale-Keating experiment) the answer will always be unambiguous.

Clearly the foregoing analyses call for a "reassessment" of SR. In another paper the author has engaged himself to this task[43]. He holds that the novel theory has to be consistent with the "existence of the massive universe". He further maintains that all kinematical effects of SR which depends on relative motion in flat space-time should be viewed as due to the "gravitational effects of the nearly homogeneous and isotropic universe". According to him the correct theory should be the one with "a preferred cosmic rest frame". This is the one with respect to which, as the author advocates in the previous paper[27], the time dilation with the usual Lorentz factor is to be considered. However, the author has to admit that the theory (termed as Cosmic Relativity by the author) should preserve Lorentz invariance.

In our opinion the last requirement emphasizes that the new theory in essence is still Lorentzian and the difference if any may at best be structural. This reminds us again of the conventionality of simultaneity thesis of SR according to which one can use a plethora of transformation equations with different synchronization parameters (for example the Reichenbäch parameter ϵ or Selleri parameter[8, 44, 45]) to describe the same relativistic physics (see chapter II for details). One of the possible convention being that of absolute synchronization[8, 13, 44, 46, 47, 48]. For absolute synchronization scheme one needs to start with a preferred frame (and usually one considers it to be the rest frame of the CMBR) and observes that time dilation takes place with respect to that frame alone. Note however that according to the true CS- thesis it is not essential to choose a particular inertial frame (CMBR frame for example) to be the preferred one and any inertial frame for that matter may be considered as the first frame[48].

More recently Kak, in consonance with Unnikrishnan believes that "the special relativity principle formulated originally for physics in empty space is not valid in the matter filled universe." Further, according to him physical laws are a consequence of the large scale nature of the universe so that "there will be difference in the experience of two observers in relative uniform motion if isotropy of the universe is not maintained by them equally....". It is claimed that instead of considering the time dilation as a consequence of LT if it is viewed as a consequence of the experience of anisotropy (of the universe) by the moving observer, resolutions of the twin paradox and its variations may then become almost a trivial exercise. The basic idea of the author is much akin to the Cosmic Relativity of Unnikrishnan; however Kak in his paper[42] also suggests a means allowing one to infer (in principle) the speed of moving observer by measurements of the "distribution" of speeds of the receding distant objects.

Before we go to the next section, we would like to remark that the Cosmic Relativity theory to be truly a preferred frame one, the time dilation factor (or the length contraction factor or the both) need to differ (at least infinitesimally) from the usual relativistic expressions. In absence of this difference the theory continues to be relativistic. However there may be compelling reasons to think of a true preferred frame theory, coming from requirements elsewhere in physics. We discuss this issue in the next section in details, which will provide a background for the topics other than the twin paradox considered in this thesis.

1.3 Background-II

1.3.1 Greisen-Zatespin-Kuzim Limit and a Puzzle

After a century of successes, Einstein's special relativity (SR) appears to be violated by certain observations on ultra high energy cosmic rays (UHECR). These cosmic rays are high energy particles that are produced in distant galaxies and impose on the earths atmosphere generating showers of secondary particles. By detecting these secondary particles through the particle-cascade processes, the energy of the original cosmic rays can be constructed. These observations of UHECR seem to be in contradiction with SR. The aspect of such cosmic ray observations which is in conflict with the theory of relativity concerns a theoretical limit known as the Greisen-Zatespin-Kuzim (GZK) limit[49, 50, 51]. Cosmic rays with energies above this threshold should not reach earth since they are supposed to lose energy by interactions with relic radiation of the Big Bang. The limit was calculated by K. Greisen, G. Zatespin and V.Kuzim in 1966 based on interactions predicted between the cosmic ray nucleons and photons of cosmic microwave background radiation (CMBR):

$$p + \gamma \longrightarrow p + \pi^0 \tag{1.21}$$

$$p + \gamma \longrightarrow n + \pi^+. \tag{1.22}$$

The reaction will progress when the combined center of mass energy of the proton (p) and photon (γ) equals to or greater than sum of the pion (π^0 or π^+) and proton (p) or neutron (n) masses. This can be written as

$$m_p + m_\pi + m_\pi^2 \leq q((p^2 + m_p^2)^{1/2} - pcos\theta), \tag{1.23}$$

where q is the photon momentum along the x-axis and p is momentum of the proton hitting the photon at an angle θ in xy plane. As the pion mass is much smaller than proton or neutron mass, one may also write

$$E_p - pcos\theta \geq m_p m_\pi / q. \tag{1.24}$$

For a thermal gas of relativistic bosons $< q > \approx 2.7$T and with $T_{CMBR} \approx 2.7$K corresponds to an energy of 2.3×10^{-4}eV. On putting the pion and proton masses a cut-off energy (E_p) of $5 \times 10^{19} eV$ is calculated. This defines the GZK limit. Thus according to this theory the cosmic rays above the threshold energy (cut-off energy) of $5 \times 10^{19} eV$ would interact with cosmic microwave background photons to produce pions.

Because of the mean path associated with the interaction (this can be calculated from the photon density of CMBR and photo-pion reaction cross-section known from laboratory studies on γ-ray and proton collisions)[7], extragalactic cosmic rays with distances more than 50 Mpc from the earth with energies greater than this threshold energy should never be observed on earth and there are no known sources within this distance that could produce them[52]. However number of observations have been made[53, 54, 55] that appear to show cosmic ray energy spectrum from distant sources to extend well beyond GZK limit. The mechanism producing UHECR is unknown. Many models have been found in the literatures regarding the source of such high energy cosmic rays. There are different exotic origins that have been proposed, such as topological defects, active galactic nuclei and gamma-ray bursts[52]. These schemes are however ruled out by the GZK limit. Indeed the ground based detectors have detected a large number of events above 10^{20} eV, the highest energy of cosmic rays so far has been $3.2 \pm 0.9 \times 10^{20}$ eV detected by Fly's ye air shower detector in Utah[54]. In this detector the error box for the arrival direction in galactic coordinate had been centered on $b = 9.6^0$, $l = 163.4^0$ and the particle cascade reached a maximum size near a depth of $815 gm/cm^2$ in the atmosphere. However, if the sources of UHECR are believed to be extragalactic that one event at 10^{20} eV appears surprising. This is the UHECR paradox[56].

The aspect of GZK limit and its possible violation are concerned with relativity.

[7]Recall that the mean path is given by $\lambda = 1/n\sigma$ where σ is the relevant cross-section and n is the number density of the CMBR photons.

The value of the GZK threshold is purely a result of the relativistic kinematics. The observed conflict with this threshold value thus calls for a modification of special relativity or its postulates. In the literature one witness various attempts to modify the standard relativity theory in order to find an answer to the UHECR paradox which will be reviewed below. But before we do it let us note, in Amelino-Camelia's words "As with all emerging experimental paradoxes it is of course possible that the cosmic-ray paradox is the result of an incorrect analysis of the experiment, for example it is legitimate to speculate that the identification of these ultra-high-energy cosmic rays as protons produced by distant active galaxies might eventually turn out to be incorrect. But, in spite of its preliminary status, this cosmic-ray paradox provides encouragement for the study of new relativity postulates"[56].

1.3.2 Lorentz Invariance Violation: A Popular Scenario

There have been exotic proposals in the literature which try to explain trans-GZK cosmic ray events in the framework of Lorentz invariance violation (LIV) theories. For example Coleman and Glashow [57] have argued "that possible departures from strict Lorentz invariance, too small to have been detected otherwise, can affect elementary-particle kinematics so as to suppress or forbid inelastic collisions of cosmic-ray nucleons with background photons. Thereby can the GZK cutoff be relaxed or removed." The authors have argued elsewhere[58] that the velocity of light in vacuum c_0 in a moving frame relative to the rest frame of the universe can differ from the maximum attainable speed c of the material body by a small velocity parameter ϵ of the theory. The obvious consequences of this consideration is the existence of a preferred frame of reference. It is assumed that this preferred frame (the so-called ether frame) to be "the rest frame of the universe" with respect to which the CMBR is isotropic.

The precession tests for anisotropies in the velocity of light due to the CMBR frame have set a limit on this ϵ

$$|\epsilon| < 3 \times 10^{-22}. \tag{1.25}$$

However it is argued [52, 58] that stronger constraints on ϵ can be obtained from the observations on UHECR. If $c < c_0$ the photon 4-momentum becomes time like so that the energetic photon converts into an electron-positron pair. It has also been shown that the detection of primary proton energy up to 100 eV set the bound on ϵ

$$|\epsilon| < 5 \times 10^{-24}. \tag{1.26}$$

The physical basis on such a constraint on ϵ is that particle can be super luminal in vacuum. If $c_0 < c$ in such a case a proton being a charged particle will lose energy through vacuum Cerenkov radiation and will therefore fail to be detected with the super luminal speed. The limit on ϵ thus obtained does not require any postulates regarding the motion of the laboratory frame with respect to the preferred frame. According to one most popular scenarios [59], existence of different maximal speeds for different particle species is assumed and they are also assumed in general to differ from the speed of light in vacuo [see Ref.[52] and references therein]. In this way introduction of small LIV has been shown to have effects that increase rapidly with energy in such a manner that ultimately inelastic collisions with CMBR photons become kinematically forbidden.

To see briefly how the GZK cutoff is affected by Lorentz violation, consider the formation reaction yielding the first pion-nucleon resonance[57]

$$p + \gamma(CMBR) \longrightarrow \Delta(1232), \tag{1.27}$$

by which a nucleon with energy E collides inelastically with a CMBR photon of energy ω. The target photon energies are a thermal distribution with temperature

$T = 2.73$ K, or $KT \equiv \omega_0 = 2.35 \times 10^{-4}$eV. Energy conservation provides the condition under which the reaction (1.27) can proceed:

$$2\omega = M_p^2/2E \geq (c_\Delta - c_p)E + M_\Delta^2/2E, \quad (1.28)$$

where $c_\Delta - c_p$ is the relevant Lorentz-violating parameter. If $c_\Delta = c_p$ the above equation yields the usual threshold energy for this process $E_f = (M_\Delta^2 - M_p^2)/4\omega$. Otherwise it yields a quadratic inequality in E which can be satisfied if and only if $c_{(\Delta} - c_p) < \hat{\delta}(\omega) \equiv \omega/2E_f$. As $c_\Delta - c_p$ is increased toward $\hat{\delta}$, the threshold for the formation reaction grows toward $2E_f$. However, if it exceeds its critical value,

$$c_\Delta - c_p > 2\omega^2/(M_\Delta^2 - M_p^2) \approx 1.7 \times 10^{-25}[\omega^2/\omega_0^2]. \quad (1.29)$$

The reaction (1.27) becomes kinematically forbidden for all E. They have argued that the reaction (1.27) is the dominant process leading to the GZK cutoff as originally formulated. However, if $\Delta(1232)$ formation is not possible, a weakened version of the cutoff may result from non-resonant photo-production of one or more pion:

$$p + \gamma(CMBR) \longrightarrow p + N\pi. \quad (1.30)$$

If $c_\pi = c_p$, the threshold energy for pion production is $E_p = M_\pi(2M_p + M_\pi)/4\omega$. If Lorentz invariance is violated as $c_\pi - c_p$ is increased from zero, the threshold grows. for a fixed photon energy ω

$$c_\pi - c_p < \hat{\delta}(\omega) \equiv 2\omega^2/M_\pi^2 \approx 5 \times 10^{-24}[\omega^2/\omega_0^2], \quad (1.31)$$

Reaction (1.30) and multiple pion production are kinematically forbidden at all proton energies if $c_\pi - c_p > \hat{\delta}(\omega)$.

1.3.3 Doubly Special Relativistic Theories

There are others class of theories known as doubly special relativistic (DSR) theories which consider a generalization of SR to include one more invariant scale, in addition to that of the velocity of light. The theory considers a modified LT in momentum space (DSR 1)[60, 61] and it has later been shown to be nonlinear representation of the Lorentz group[62]. Besides its primary motivation coming from experimental side, this type of theory finds encouragement from quantum gravity considerations also, where the role of Planck's scale ($E_P \approx 10^{28} eV$) might be important. It is expected then E_p would define a transition *scale* beyond which classical space-time picture will not remain valid and the description of physics will change drastically. However if the principle of relativity is to be honored one should think that the scale should be observer independent. In other words one may assume that the relativity postulates are to be revised in such a way that the description of physical phenomena changes significantly at the *observer-independent* kinematical scale. Indeed the Planck's scale has a special role in effects to unify quantum mechanics and general relativity into quantum gravity since this scale is the combination of speed of light constant, the gravitational constant and quantum mechanical Planck's constant[63]. Quantum gravity predicts a new quantum picture of space-time for particles with momentum and energy above the Planck's scale, although the classical picture remains valid below this scale. In theory of SR, the consequences of quantum gravity would be paradoxical since the theory predicts that "same particle have energy higher than the Planck's scale according to some observers and energy lower than the Planck's scale according to other observers[56]." The doubly special relativity solves this problem. In particular the variant due to Magueijo and Smolin[62] (here after called DSR 2) holds that all observers at least agree whether a particle crosses the Planck's scale or not.

What are these modifications. According to Magueijo and Smolin theory the modification of SR is based upon four basic principles (1) the relativity of inertial frames: This predicts there is no preferred state of motion so that velocity is a purely relative quantity (2) the equivalence principle: Under the effect of gravity, freely falling observers are all equivalent to inertial observer (3) the modification: The observer independence of the Planck's energy and (4) the corresponding principle: At energy scales below Planck's energy the special relativity and general relativity both are true, i.e they will remain valid to first order in the ratio of energy scales to E_P.

Such modifications in turn deformed transformation equations which reduced to the usual ones at low energies, but keeps invariant the Planck's scale which marks the border line between classical and quantum gravity. That such DSR theories have the ability to explain high energy cosmic ray anomalies is due to the introduction of deformed dispersion relations of the form

$$E^2 = p^2 + m^2 + \lambda E^3 + ..., \tag{1.32}$$

where λ is of the order of Planck length. Note that this is the departure from the usual relativistic dispersion relations.

$$E^2 = p^2 + m^2, \tag{1.33}$$

here we have assumed c=1. Following detailed analysis one can indeed show that the interaction of proton and CMBR photons leads to the corrected threshold for pion production[64, 65]

$$E_{th} = U^{-1}(E_{th0}) = U^{-1}[((m_p + m_\pi)^2 - m_p^2)/E_{CMBR}], \tag{1.34}$$

where m_p and m_π are the proton and pion rest masses, E_{CMBR} is the photon CMBR energy in the cosmological frame and E_{th0} is the GZK threshold. Note that the U map can be chosen so as to implement various properties required from a phenomenological theory of quantum gravity.

The further development of non-linear relativity (DSR theories) is in progress. There are however many issues that are still unanswered. For example, there is the problem of how to modify the theory for composite system. With the loss of linearity, the kinematic relations valid for single particles need not be true for composite systems. It is however claimed that this is at least a desirable feature since *non-linearity builds, into the theory the concept of elementary particle*-a feature having the differentiating ability between fundamental particles and composites[65]. The solution to the problem does not come easily. For example, it can be shown that obvious straight forward covariant and composition law of energy and momentum leads to contradiction. One possible solution allows one to think that the map U use for a composite system need not be the same as that used for single particles. There are other solutions to this problem involving embedding the theory in higher dimensions however the details of which lies beyond the scope of this write up.

Another theoretical development concerns the position space picture of these theories which are usually constructed in momentum space. With the loss of linearity "duals no longer mimic one another"[62] or vectors no longer transform according to the inverse transformation of co-vectors. A number of solutions may be found either involving or avoiding non-commutative geometry[66] and quantum groups[67, 68]. One may however recover linearity by embedding the theory into a higher number of dimensions[69, 70]. The approach is elegant however alternative way to linearize the theory can also be found[71, 72]. A considerable amount of work is found in the formulating field theory[64] much of these are not relevant for what lie ahead. However one may point out that the most conceptual problem in front of DSR theory is the challenge of doing general relativity based on non-linear relativity[73].

In the context of what we are going to do in the next chapter regarding the UHECR issue let us summarize this brief review work by saying that although the UHECR paradox primarily provides encouragement for the DSR theories, the revision of dispersion relation as already stated is motivated from quantum-gravity considerations as well. These theories try to avoid the preferred frame issue prompted by the introduction of Planck's scale in the theory. The review is not exhaustive. Indeed there are many ramifications and there exist several DSR theories which try to deal with the UHECR paradox.

There are other approaches as well that may be found in the literature. Before we end the subsection we just give one such example. Booth[74] considers the reassessment of the GZK cutoff in the UHECR following his so called quasi static universe (QSU) model, according to which it is believed that the photon energy is an invariant in the cosmological reference frame so that the photon number density in the universe today is much less than (by a factor of 10^9) that of the standard model. Consequently, the mean free path of the cosmic rays (for the collision of photons and CMBR photons) will exceed the horizon distance of the universe, implying the latter to be essential transparent to the UHECRs. This QSU model therefore does not predict any cutoff for the cosmic ray spectrum. Thus it has been claimed that the reduced CMBR photon number density predicted by the model provides a natural explanation for the observed flux with energies greater than $10^{20}eV$. However the QSU model has not yet found a general acceptance and the idea although interesting can be regarded as a bit too speculative.

We follow a different approach. Referring to the most popular scenario we may recall that in an effort to look for new physics (in the theories involving LIV) one believes that the space-time is still governed by LT however other laws of physics might not remain covariant under LT. In this part of the thesis (Part II) we will deal with the issue rather differently. As will be explained we will consider the possibility of a deformed LT (not just a deformed dispersion relation as described in the DSR theories) to relate observations performed by different inertial observers. Clearly the predictions of deformed LT will be expected to be different from those of relativity theory. However the difference in the predictions should be undetectable in the domain where SR has been tested beyond doubt. We shall see in chapter-VI that the UHECR paradox can be explained in terms of a non-preferred frame effect of the laboratory frame which is moving with velocity ≈ 300 km/sec with respect to the preferred one, assumed to be at rest with CMBR frame. Unlike some earlier efforts as discussed in Sec.(1.3.2) (the Coleman-Glashow[58] scheme for example) which consider LIV but assume that the physical kinematics is still Lorentzian, we shall propose to modify the transformation equation itself. Deformed LT are generally discussed in connection with test theories like that of Robertson[75] or Mansouri and Sexl[76] on which improved tests of SR are often based (see for example[77]). But they are not usually considered to represent a new physics that may provide a solution for the UHECR paradox.

As we have seen, some authors find it troublesome giving up the principle of relativity. In the so called DSR theories, the particle dispersion relation is modified; but the introduction of an invariant length or energy scale in addition to the invariant velocity scale of SR, the "relativity of inertial frames" is still maintained. Such theories, often motivated by quantum-gravity considerations are interesting but are unable to resolve the UHECR paradox quantitatively at the moment.

We shall attempt to deform the relativistic kinematics using heuristic means. We will do it first by identifying the objective contents of the relativity principle and then will go in for modifying these contents *minimally* to obtain a new transformation that will be able to relate space-time of an arbitrary frame of reference with that of the universal rest frame of the cosmic substratum.

1.3.4 Variable Speed of Light

We have already pointed out the most challenging problem of the DSR proposal is that of modifying the general relativity (GR). Magueijo and Smolin[78] try to examine the question how the modification of SR proposed by the authors themselves[62, 64] can be extended to GR. The main reason of this endeavour is that there lies accessible modification of GR which is characterized by the features that the space-time geometry becomes energy dependent. For example "quanta of different energies see different classical geometries". One outcome of this novel picture is that the speed of light (and other massless quanta) naturally becomes energy dependent. If one now turns to the investigation of cosmological models one finds naturally the so called the variable speed of light (VSL) cosmologies. Indeed it has been speculated that there may be a connection between the DSR and VSL cosmologies[79, 80, 81]. The authors of Ref.[78] have shown that this connection does follow from their proposed GR.

We shall show in the concluding chapter that this "natural" connection between DSR and VSL cosmologies also holds good for a DSR of different genre (discussed in the next chapter). In this subsection therefore it will be worthwhile to briefly review the history of VSL from both theoretical and observation points of view. Hence as the previous section gives a necessary background for the next chapter, this subsection is intended to provide the same for the concluding chapter.

VSL Theories:

Historically in 1937 Paul Dirac and others[82] started investigating the outcome of the natural constants changing with time. As an example, Dirac thought about a change of only five part of 10^{11} per year of gravitation constant G to explain the relative meagerness of the gravitation force compared to other forces of nature. This proposal later become known as Dirac large number hypothesis.

In 1968, referring to wide speculations in cosmology by models makers without much hard facts to go on, Dirac wrote "One field of work in which there has been too much speculation is cosmology. There are very few hard facts to go on, but theoretical workers have been busy constructing various models for the universe, based on any assumptions that they fancy. These models are probably all wrong. It is usually assumed that the laws of nature have always been the same as they are now. There is no justification for this. The laws may be changing, and in particular quantities which are considered to be constants of nature may be varying with cosmological time. Such variations would completely upset the model makers"[65].

Although much has changed since Dirac wrote these words, we have now many observational inputs in cosmology, the views expressed by Dirac (regarding speculations in cosmology) are still applicable. Apart from Dirac, other physicist have also entertained the possibility of varying gravitational constant G[82, 83, 84], a varying electron charge e etc.. In contrast to G and e the constant c has remains sacred for a long time. It was perhaps thought that "varying c theories are expected to cause much more structural damage to physics' formalism than other varying constant theories".

A VSL cosmology has independently been proposed by Petit from 1998[85, 86, 87, 88] and Moffat in 1992[89]. In Petit's VSL model, the variation of c accompanies variation of all physical constants in such a way that all equations remain unchanged through the evolution of the universe.

One may still go to the past. Many VSL theories were considered since the advent of SR in 1905. In some context even Einstein considered such a theory long back in 1911[65]. In 1930's VSL was used as an alternative explanation of Hubble's discovery of recession of galaxies[90, 91, 92]. These theories are obviously in conflict with the fine structure observations. Let us not call them the true VSL theories. The first modern VSL theory is due to J. W. Moffat[89] whose "ground breaking" paper (involving spontaneous symmetry breaking of Lorentz symmetry) gives an elegant solution to the horizon problem of the big bang cosmology.

Since then the literature on the subject continues to grow starting from the work of Albrecht and Magueijo[93] who considers the cosmological implication of light travelling faster in the earlier universe. They proposed a prescription for deriving a set of new cosmological evolution equation while the speed of light c is changing. It has been shown by them how the flatness, horizon and cosmological constant problem may be solved. Later authors try to do some thing by improving upon the theories of their earlier authors. It will be outside the scope of the present thesis to review all these efforts. Rather it will be proper to categorize these endeavours from the considerations of their *departures* from SR, since one may note that all VSL theories must be in conflict with SR in some way. The classification below will therefore be based on the nature and depth of these conflicts with SR. This classification will also help put our own VSL approach (to be discussed) in the proper perspective.

Much of what follows have been taken from a detail and excellent review work of João Magueijo[65]. We briefly reproduce them for the sake of completeness. Magueijo recalls that the main corner stone of SR are the two independent postulates. One concerns the relative nature of motion and the asserts the constancy of the velocity of light (CVL). The author holds that the VSL theories do not need to violate the principle of relativity of motion. Although it may be difficult to think of relativity of motion if the CVL is tampered with. Consequently the first criterion for differentiating the various proposals is based on whether the theories honour the relativity of (inertial) motion.

As regards CVL there are various aspects of "constancy". These are explained in detail in Ref.[65]. For the present purpose it is enough to say that there is a large number of combination in which these different aspects can be violated. At the outset, I would like to point out that our proposal considers violation of both the postulates in an approximate manner and hence is a preferred frame theory altogether. When both the postulates of SR are violated one may say that the Lorentz symmetry is being broken in the "Hard way".

The main VSL mechanism proposed so far starting from Hard breaking of Lorentz symmetry are the following.

(1) Hard Breaking of Lorentz Symmetry

This model has been proposed by Albrecht and Magueijo[93] and Barrow[94]. In this model (like that of ours) both postulates of SR are violated. The authors postulate that there is a *preferred* frame in physics identified with cosmological frame. These theories describe a world where matter content of the universe as well as the laws of physics evolve in time (since the speed of light varies in time usually in the very early universe). The basic dynamical postulate is that Einstein's field equations are valid with minimal coupling in the particular form,

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = (8\pi G/c^4)T_{\mu\nu}. \tag{1.35}$$

Note that here c is to be interpreted as a field. The metric, connection, curvature and Einstein tensors are to be evaluated in a given frame at constant c where no extra term involving gradients of c will be present. Non-covariant extra terms in gradients of c will appear only in other frames. Minimum coupling at the level of Einstein's equations is at the heart of the model's ability to solve the cosmological problems.

(2) Bimetric VSL Theories

This model has been initially introduced by Moffat and Clayton[95] and by Drummond[96]. In these theories the speeds of the various massless species may be different but SR is still recognized within each vicinity. The speed of the graviton is taken to be different from that of massless matter particles. By introducing two metrics (one for gravity and one for matter) the model has been further developed in scaler-tensor model[97] and vector model[98, 99]. As for example, the scalar tensor model uses a scalar field ϕ which is minimally coupled to a gravitational field (the field variables being the usual metric $g_{\mu\nu}$). However it is assumed that the matter couples to a different metric;

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi. \tag{1.36}$$

Thus one talks of gravitational metric $g_{\mu\nu}$ and matter metric $\hat{g}_{\mu\nu}$. The total action is composed of three parts;

$$S = S_g + S_\phi + matter - action, \tag{1.37}$$

where the usual gravitational action

$$S_g = (-c^4/16\pi G)\int dx^4\sqrt{-g}(R(g) + 2\Lambda), \tag{1.38}$$

and scalar field action is

$$S_g = (c^4/16\pi G)\int dx^4\sqrt{-g}[(1/2)g_{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)]. \tag{1.39}$$

Note that the matter action is the usual one but uses the metric $\hat{g}_{\mu\nu}$. By varying $g_{\mu\nu}$ one obtains from Eq.(1.37) the gravitational field equation with source terms containing $T_\phi^{\mu\nu}$ (the stress-energy tensor of the scalar field action), $\hat{T}_{\mu\nu}$ (of the matter action) and finally the cosmological constant term Λ. Needless to say the left hand side of field equation is the usual Einstein tensor

$$G^{\mu\nu} = R^{\mu\nu} - (1/2)g^{\mu\nu}R. \tag{1.40}$$

The most important thing of the theory is that the speed of light is a dynamical variable predicted by a wave equation

$$\bar{g}^{\mu\nu}\hat{\Delta}_\mu\hat{\Delta}_\nu\phi + KV'[\phi] = 0, \tag{1.41}$$

where biscalar metric $\bar{g}^{\mu\nu}$ is defined in the original text [95]. This model predicts solutions with a de Sitter phase that provides sufficient inflation to solve the horizon and flatness problems. The model has also been used as an alternative explanation for dark matter and dark energy[98].

(3) Energy Dependent Speed of Light

This approach may or may not preserve the first postulate of SR (the relative nature of motion), however it violates the second one for certain. This model states that the speed of light is allowed to vary with the frequency (colour) close to Planck frequency. This is performed by deforming the photon dispersion relation ($E^2 - p^2 = m^2 = 0$). For example it was proposed that

$$E^2 = p^2 + m^2 + \lambda E^3 + ..., \tag{1.42}$$

where λ is of the order of Planck length. If the linear Lorentz transformations are still valid and if this dispersion relation is true in one frame then it cannot be true in any other frame so that this theory also violates the principle of relativity as described in model (1). The deformed dispersion relation above implies that the group velocity c of light ($c = dE/dp$) is energy dependent. As a consequence speed of light was faster in the past suggesting a VSL cosmology.

These theories are generally used to give phenomenological description of quantum gravity[62] and as an explanation of the dark energy in terms of energy trapped in very high momentum and low-energy quanta[65]. Finally the modified relation may lead to an explanation for the UHECR paradox[100], which is one of the main topics of this dissertation (See chapter VI).

(4) Lorentz Invariant VSL Theories

In this approach the essence of Lorentz invariance is preserved in its totality but space-time transformations relates with varying c. Moffat[89] proposed such VSL theory in which Lorentz invariance is spontaneously broken but the theory is endowed with exact local Lorentz symmetry. Typically in this scenario the speed of light undergoes *phase transition* to a value 30 order of magnitude smaller, corresponding to the currently known speed of light. In this model the entropy of the universe is reduced before the phase transition but increases afterward. This solves the enigma of the arrow of time and the second law of thermodynamics. Another model is proposed by Maguiejo[99] in which the covariant and local Lorentz invariance remain applicable when the speed of light is allowed to vary. Although elegant and interesting these theories at least suffer from the drawback that their implementation in cosmology is somewhat cumbersome.

(5) M-Theory

People often tend to consider the CVL postulate of SR as sacrosanct and try to cling to it by any means. An exotic effort to be described below is an example of this mindset. A type of VSL work has been initiated by Kiritis[101] and Alexander[102] and makes use of the brane-world picture, in which our universe is a three-dimensional brane embedded in a higher-dimensional (bulk) space-time. When a test brane is moving in the vicinity of a black-hole bulk space-time it is possible to have perfect Lorentz invariance i.e a perfect CVL in the "bulk" while one has a VSL on the "brane". It means that the Lorentz invariance of the full theory remains intact and VSL results, so to say, from a projection effect! More specifically, in this model the velocity of light can be found to vary depending on the distance between the brane and the black hole.

Several VSL theories in this scenario have used the Randall-Sundrum models[103] in which the extra dimensions are subject to warped compactification. The light signals in such space-time may travel faster through the extra dimensions. There are many ramifications of this sort of models and extensions, "mirage cosmology" and other exotic proposals, but the detailed description of such things are hardly necessary for the present purpose.

(6) Field Theory

This type of VSL theory describes that quantum field theory in curved space-time which predicts super luminal photon propagation. This was first proposed by Drummond[104], where one-loop vacuum polarization corrections to the photon propagation were computed in a variety of backgrounds. The phrase "super luminal light propagation" may at a first sight seem a contradiction in terms but one here typically distinguishes between c appearing in LT and the actual speed of propagation of light (which may be notified due to non-minimal coupling of gravity. A resolution of the horizon problem by means of this effect i.e in the situation where a light cone is distinct from the causal cone) was obtained in 1989, which can be found in[105]. The implications for optics and causality of "faster than light" motion are discussed by Shore[106]. The Casmir effect is an example where VSL has been discovered in fields theories. In which, vacuum quantum effects induce an anomalous speed of propagation for photons moving perpendicular to a pair of conducting plates[107].

As regards explicit breaking of Lorentz invariance one considers the possibility that LT might be a low energy phenomenon. For example Nielsen and his collaborators[108] have suggested that the "Lorentz invariance could be a stable infrared fixed point of the renormalization group flow of a quantum field theory". Neutrino oscillations are another example in this respect in which the endpoint of beta decay are currently being studied[109]. It is held that "high energy physics tests of CPT can also act as test of Lorentz invariance" and VSL may be studied in the framework of Lorentz violating extensions of the standard model[65].

Enough has been discussed so far although in a brief way, about the theoretical aspects of VSL motivation and scope and their different levels of departures from the basic tenets of SR but do we have enough hard facts about VSL to consider such things seriously? Perhaps yes. Below we review the observational status.

Observational status:

The most extraordinary observation with relevance for VSL is the work of Webb et.al[110], Murphy et.al[111] and Srianada et.al[112]. These authors have reported evidence for redshift dependence in the fine structure constant $\alpha = e^2/(\hbar c)$. The observations of Webb et.al[110] can be interpreted as supporting some nonstandard cosmological theories that considers VSL or the varying electron charge[113]. In the last reference it has been shown that the varying c cosmology, by transformation to standard unit can be rephrased as a varying e (electronic charge) theory. So far as the electromagnetic phenomena is concerned there has no difference if there is a c variation or e variation to account for the variation of α. If one introduces the gravitation in the discussions through the theory of black hole thermodynamics, as Paul Davies et.al[114] has shown that it is possible to test which constants might be actually varying. The authors arguments have indicated that the e variation with time are "at risk of violating both the second law of thermodynamics and the cosmic sensorship hypothesis". Although some later authors[115, 116] do not agree to this claim and the theory is not generally accepted, many other authors build up models where variation of α is considered to be as due to variation of c alone. For the present thesis as we shall see we will assume this view point and try to match the observational results of α variation with our theory. Keeping this in mind let us continue with the observational status of varying α which is the same thing as varying c according to the stipulation.

In the beginning of this subsection we have mentioned the work of an Australian-British group that it possibly has detected a varying α (which is about one part in 10^5).

However we have a long history of null measurements. In 1956 Savedoff used the so called alkali-doublet method to estimate changes in α from the measurements of the spectra of distant quasars producing essentially a null result. Spectra of quasars at cosmological distances provide natural laboratory for studying variations in α. Dark absorption lines of radiation are produced due to intervening gas clouds which are enriched with heavy elements. The fine structure constant is known to be associated with the doublet splitting of alkali spectra. Indeed the doublet splitting is proportional to α^2; therefore any variation in the wavelength separation will be approximately proportional to α. Since quasars spectra contain absorption lines at different redshifts (meaning different times of the evolving universe). It is therefore possible to study in principle the time variation of α simply by looking for changes in the doublet splitting of alkaline type ions with one outer electron (for example triply ionized carbon or silicon) as a function of time (redshift). Although this seems very simple, any change in α will be very small and so the measurement accuracy is required to be extremely high. It is therefore not surprising that the early measurements which uses this technique give null results. The most accurate measurement on the fractional change prior to the work of Webb et.al give a value of $3.5(\pm 5.5) \times 10^{-6}$[117, 118].

The study of certain isotropic abundances in the Oklo natural nuclear fission reactor in Gabon, west Africa gives a terrestrial limit on the *alpha*-variation. The analysis of the decay product of the Oklo uranium mine (discovered in 1972) gives a range for a fractional change ($\Delta\alpha/\alpha$) which is between 0.9×10^{-7} and 1.2×10^{-7} over the period of 1.8 billion years. If one assumes a linear scale the result becomes equivalent to one part to 10^6 over the life time of the universe.

Webb et.al improved on the alkaline doublet technique to introduce a new variant whereby one compares the absorption wavelengths of magnesium and iron atoms in the same absorbing cloud. In this way the sensitivity becomes one order of magnitude more than that of the alkaline doublet method. The trend of these results following the many-multiplet heavy element transition in quasar absorption method indicate the value of α was lower in the past

$$\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}, \tag{1.43}$$

for redshift $z \approx 0.5 - 3.5$.

This many-multiplet exploits the fact that energy of different transitions change differently for given change in the fine structure constant. The rest wavelengths of Mg II$\lambda\lambda$2797,2803 and Mg Iλ2852 transitions are insensitive to small changes in α whereby FeII multiplets are much sensitive to small variation in α hence the former transitions can be used as an anchor for measuring the systemic redshift. Thus measuring relative shifts between an anchor and different FeII lines can be used to measure accurately the α variation ($\Delta\alpha/\alpha$).

There are however methods other than the alkali doublet method or many multiplet method such as the one using OIII emission lines[119]. Although relativity robust, that technique is not sensitive enough to detect the small variations in $\Delta\alpha/\alpha$.

There are some investigations that relie on studies of molecular lines which have been detected in a couple of systems[120]. These studies give $\Delta\alpha/\alpha = (-0.10 \pm 0.22) \times 10^{-5}$ and $(-0.08 \pm 0.27) \times 10^{-5}$ at z values 0.2467 and 0.6847 respectively. Studies at higher z can not be obtained due to unavailability of molecular systems.

The later works of Webb et.al group continue to conform the initial claim. For example in 2001 the authors[121] claim there are no systematic effects which can explain the positive results. However there are other authors who dispute these results on the basis of their studies at much higher sensitivities[112, 122, 123]. While many-multiplet analysis of about 143 absorption spectra obtained from the Keck/HIRES instrument used by Webb et.al group (see Ref.[124]) indicates a variation of α, the many-multiplet analysis of data obtained from ultra-violate and visual Echelle spectrograph (UVES) on the very large telescope (VLT) in Chile apparently produces a null result[112] . Murphy, Webb and Flambaum[111] however have critically reviewed the null results of the group (in particular see Ref.[122] who reports a mean relative variation of $\Delta\alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5}$). Their analysis of same fits to the absorption profiles produces the very different $\Delta\alpha/\alpha$ values with uncertainties sometimes larger by the factor of 3. They attribute the discrepancies to flawed parameter estimation techniques in the original analysis.

One may thus conclude that a reliable comparison of HIRES and UVES constraints on varying α may take place after sufficient improvements in the analysis of UVES spectra. In the concluding chapter we will develop a simple minded VSL theory on the basis of HIRES measurements and claims by Webb et.al or Murphy et.al. The null results due to Srianad et.al however we will also be discussed in the context of our proposed approach. Some more background and relevant data will be provided in that chapter in order to make it self contained.

Before we end this section I would like to reproduce a table (Tab.(1.1)) from Ref.[111] providing the summary of many-multiplet constraints on $\Delta\alpha/\alpha$ in the literature. This is given for a ready reference. The table is self explanatory and hence no elaboration is being provided.

Table 1.1: Summary of the many-multiplet constraints on $\Delta\alpha/\alpha$ in the literature.

Instrument	N_{abs}	z_{abs}	$\Delta\alpha/\alpha[10^5]$	**Reference**
HIRES	30	0.5-1.6	-1.100 ± 0.400	Webb et.al (1999)[110]
HIRES	49	0.5-3.5	-0.720 ± 0.180	Murphy et.al (2001)[125]
HIRES	128	0.2-3.7	-0.543 ± 0.116	Murphy et.al (2003)[126]
HIRES	143	0.2-4.2	-0.573 ± 0.113	Murphy et.al (2004)[124]
UVES	23	0.4-2.3	-0.060 ± 0.060	Chand et.al (2004)[122]
UVES	1	1.151	-0.040 ± 0.190	Quast et.al (2004)[127]
UVES	1	1.839	$+0.240 \pm 0.380$	Levshakov et.al (2005)[128]
UVES	1	1.151	$+0.040 \pm 0.150$	Levshakov et.al (2005)[128]
UVES	1	1.151	$+0.100 \pm 0.220$	Chand et.al(2006)[129]
UVES	1	1.151	-0.007 ± 0.084	Levshakov et.al (2006)[130]
UVES	1	1.839	$+0.540 \pm 0.250$	Levshakov et.al (2007)[131]
UVES	23	0.4-2.3	-0.640 ± 0.360	Murphy et.al (2006)[111]

1.4 Topic Wise Summary

1.4.1 The Principle of Equivalence and the Twin Paradox

In chapter III the canonical twin paradox is explained by making a correct use of equivalence principle (EP) of general relativity (GR). Using EP the temporal offset effect of identically accelerated clocks is interpreted as the behavior of clocks in a uniform gravitational field. We follow an approach where such temporal offset effect is used in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin. In this chapter we have shown that the time offset effect and gravitational slowing down of clocks (GSDC) can be connected provided the world as purely relativistic in nature. To contrast this we have used Zahar transformations (following pseudo-standard synchrony in the classical world) and concluded that GSDC cannot be obtained from temporal offset effect in this world through EP. Thus in the relativistic world the temporal offset may be regarded as an integrated effect of GSDC while in the classical world (if it exists) is just an artifact of Einstein's synchrony. The experimental test of GSDC or the gravitational red shift is a test of a differential time offset effect in a way. It is worthwhile to note that the empirical verification of time offset provides empirical support for the relativity of simultaneity. Clearly because of EP the earth with its weak gravity has the ability to provide a convenient laboratory to test some special relativistic effects like relativity of simultaneity. Our approach in addition removes certain drawbacks of an earlier effort which claims to exploit EP in explaining the differential aging in the paradox.

1.4.2 Twin Paradox: A Classic Case of 'Like Cures Like'

In chapter IV, a novel approach to understanding the ordinary twin paradox based on a variation of the paradox (called the Boughn paradox), concerning the differential aging of two identically accelerated twins is presented. This time this is done without any reference to GR or EP. The genesis of the usual twin paradox lies in the incorrect use of the relativistic time-dilation formula by the traveller twin. The current approach explains how to take into account the Boughn paradox which provides automatically a uniform standard of simultaneity of the coordinate clocks of the traveller twin's *non-inertial* frame of reference so that the special relativistic time dilation formula can be used correctly in order to calculate the proper time of the stay-at-home sibling. There is no dearth of explanations of the twin paradox in the literature in the context of special relativity, but the present effort is a unique one where one paradox (the Boughn paradox) is used to explain another (the ordinary twin paradox), and hence may be looked upon in a lighter vein, as to justify the proverb *"like cures like"*.

1.4.3 Boughn Effect and Some Twin Paradoxes

A time offset effect between two identically accelerated twins is used to pose and resolve some interesting variations of the twin paradox in the chapter V. These novel paradoxes stem from the authors' attempt to isolate and expose the role of the aforesaid time offset effect from that of the time dilation effect of special relativity in the usual twin paradox. The resolution of these paradoxes provides some additional insight into the century old counter-intuitive problem. The present treatment once more makes it evident why any reference to general relativity in the context of the ordinary twin paradox is redundant.

1.4.4 Relativity in "Cosmic Substratum" and the UHECR Paradox

The second part of the thesis which deals with some observational puzzles in SR, starts with chapter VI. The special theory of relativity predicts an existence of the so called Greisen-Zatsepin-Kuzmin (GZK) phenomenon according to which cosmic ray protons coming from cosmological distances with energies above $5 \times 10^{19} eV$ should not be observed on earth. The cut-off value corresponds to the threshold energy of photo-pion production by protons colliding with soft CMBR photons pervading the universe. Experimentally a number of cosmic ray events have been detected above this GZK limit which is known as the ultra high energy cosmic rays (UHECR) paradox. We suggest a resolution of this paradox through a heuristic modification of the relativistic kinematics keeping in mind that it should not lead to predictions different from those of SR in the well tested domains. It is shown that the absence of GZK limit in UHECR spectrum can be explained in terms of a non-preferred frame effect of the solar system. It is remarked that the novel theory can also be called a "doubly special relativistic"(DSR) one but now in a sense different from that of the currently known DSR theories.

1.4.5 A Simple Minded VSL Cosmology

The concluding chapter (chapter VII) concerns VSL cosmology. The deformed relativistic kinematics developed in connection with our attempt to resolve the ultra high energy cosmic rays paradox has been found to go hand in hand with a variable speed of light cosmology. Some recent observational claims concerning the cosmological evolution of the fine structure constant α, substantiates our proposal. The spectroscopic data obtained from the detailed study of absorption lines from heavy elements in distant gas clouds along the line of sights of background quasars allow us to predict values for some parameters of the theory. A range of these values are tabulated as there is a considerable spread in the values for the observed variations of α for a given red-shift range. The present proposal although looks like a phenomenological one, has the ability to qualify itself as a "principle theory" like SR. The simple minded VSL theory is discussed in the context of theories of Albrect and Mageuijo, Barrow and Moffat and that of DSR ones.

1.5 List of Papers (Published/under Publication)

(1) **The Principle of Equivalence and the Twin Paradox**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, *Found. Phys. Lett.* **18**(7), 603-619 (2005).

(2) **A Twin Paradox for 'Clever' Students**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, *arXiv:gr-qc/0511126v1*, (2005).

(3) **Relativity in 'Cosmic Substratum' and the UHECR Paradox**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das,– *Proc. of Physical Interpretation of Relativity Theory*, held at Moscow, July 2-5, 2007.

(4) **Twin Paradox: A Classic Case of 'Like Cures Like'**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, appear in "Mathematics, Physics and Philosophy in the interpretations of Relativity theory" held in Budapest, September 7–9, 2007. [**WWW.phil-inst.hu/~szekely/PIRT_BUDAPEST/ft/Ghosal-ft.pdf**]

(5) **Boughn Effect and Some Twin Paradoxes**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das,(under publication).

(6) **Twin Paradox in the Classical World**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, (ready for communication).

(7) **A Simple Minded VSL Cosmology**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das,(ready for communication).

(8) *"Synchrony Guage in Classical and Relativistic Sagnac Effect and Related Issues"*– S. K. Ghosal, Biplab Raychaudhuri, Minakshi Sarkar and **Saroj Nepal**, *Proc of International Conference on Gravitation and Cosmology (ICGS-04)*, held at Kochi, 5–10 January, 2004 (Abstracted).

(9) *"Paradoxical Twins in Classical and Relativistic Worlds"*–S. K. Ghosal, **Saroj Nepal**, Debarchana Das, Biplab Raychaudhuri and Minakshi Sarkar, *Proc. of Seminar on Hundred Years of Three Seminal Papers of Albert Einstein and Con-*

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2.1 Conventionality of Simultaneity Thesis

In recent years some interesting (apparent) paradoxes in relativity theory such as the twin paradox, tippitop paradox, Selleri paradox and the likes have been successfully dealt with the conventionality of simultaneity (CS) thesis of SR. For example in one of the most cited paper on the twin paradox[1] a novel approach to understanding the twin paradox based on the conventionality of simultaneity has been presented providing a clearer way to settle the often discussed issue of twins relative aging. More recently some variants of this approach[2, 3] have been fruitfully used to resolve some other paradoxes found in the literature. The present dissertation also aims to discuss some counter-intuitive issues and their variants—from twin paradox to ultra high energy cosmic ray paradox and VSL and in its course, make use of the conventionality thesis of SR quite liberally, a brief review of the CS-thesis of SR therefore may be in order.

In Einstein's 1905 paper on the special theory of relativity it was indicated that the question of whether or not two spatially distant events were simultaneous did not necessarily have a definite answer, but instead depended on the adoption of a convention for its resolution. The convention in the definition of simultaneity is rooted in the conventionality of synchronization of clocks. The issue and role of conventionality concerning the synchronization of spatially distant clocks in a *given* inertial frame are much discussed in the literature[3, 4, 5]. The role of convention in the definition of the simultaneity of distant events (or the same thing in synchronizing spatially distant clocks) is one of the most debated issues in SR. The problem in synchronizing the distant clocks lies in the fact that in SR the spatially separated clocks in a *given* reference frame are synchronized by light signals, the one way speed (OWS) of light has to be known beforehand for the purpose. However to know the OWS one needs to have presynchronized clocks and the whole endeavour then ends up in a vicious circle which forces us to introduce some arbitrariness (within some limit) in assigning the value for the OWS of light. Einstein however chose to synchronize two spatially distant clocks by *stipulating* the equality of speed of the light in two opposite directions along the line joining the clock positions. This prescriptive assumption is known as the *standard synchronization* (or Einstein synchronization) convention in the literature. This standard synchronization procedure to synchronize clocks at different locations is but one of the several possible alternative conventions (termed as non-standard synchronization) and many of the results (or formulae) that he obtained depended on his special choice of synchronization. For example the issue of difference in judgments in regard to the simultaneity of two spatially distant events by different inertial observers in relative motion is also a matter of such a simultaneity convention.

Although Einstein gave indication of the problem, the role of convention in the procedure of the synchronization of clocks was exemplified specially by Reichenbach[6] in 1928 and later by Grünbaum[7]. These authors explained that the question of simultaneity of a pair of events within *one* inertial frame indeed contained an ineradicable element of convention which was linked to the assumption regarding the value for the OWS of light. To understand this point one may note that Einstein originally proposed that the criterion for the synchronization of any two spatially separated clocks be such that the time of arrival and the consequent reflection by a mirror at one clock position be determined by considering that the latter is halfway in time between the departure of the light signal and its arrival at the position of the other clock from where the light signal is sent out for synchronization. This criterion clearly is equivalent to the assumption that light has the same speed in all directions. Clearly because specifying a value for the OWS of light enables directly a simple light signal procedure for the synchronization of distant clocks, any prescription for OWS value(s) is equivalent to a convention for clock synchronization. It therefore follows that the specification of either distant simultaneity criterion or any assumption for the values of OWS of light can alike be referred to as a synchronization convention[8].

Einstein himself referred to the distant simultaneity criterion he proposed as a free stipulation for giving an empirical meaning of distant simultaneity[9], but the issue is whether other criteria leading to different one way speeds might not have been chosen without compromising on the empirical success of the theory. The conventionalist thesis holds that a range of choices are possible, all fully equivalent with respect to experimental outcome. According to the CS-thesis, any synchrony convention will be admissible so long as it is consistent with the round-trip principle, according to which the average speed of a light ray over any closed path has a constant value. It may not be out of place in this context to mention that one should restate the second relativity postulate (that is often found in text books) by replacing the phase "velocity of light" by the "TWS of light" or "round trip speed of light". A convention within the SR must be consistent with this round-trip principle since this principle is a consequence of the theory prior to adoption of any criterion for distant simultaneity and may in principle be tested with a single clock. According to CS-thesis the conventional ingredient of SR which logically cannot have any empirical content, gives rise to results that are often erroneously construed as the new philosophical imports of SR.

The CS-thesis has attracted a considerable amount of discussions in the literature. Possibility of using synchronization convention other than that adopted by Einstein has also been much discussed. John Winnie[10] first studied the consequences of SR when no assumption regarding the OWS of light was made and then developed the so called ϵ-Lorentz transformations (using Reichenbach's notation) adopting non-Einstein one-way velocity assumption or non-standard synchronization convention in general. To understand the meaning of ϵ we may recall from our previous discussion that when two spatially separated clocks are synchronized using light signals it is not necessary to divide the difference of transmission time t_1 and reception t_3 of a signal back by two, as adopted by Einstein, in order to fix the time t_2 of the other clock. One may assume in general that

$$t_2 = t_1 + \epsilon(t_3 - t_1), \tag{2.1}$$

so that $0 < \epsilon < 1$. Note that Einstein's convention is equivalent to the assumption $\epsilon = 1/2$. In developing the ϵ-Lorentz transformation Winnie assumed a principle called "principle equal passage time". This was used in addition to the "Linearity principle" and the "Round-trip light principle". These principles were then shown to be independent of one-way velocity assumptions and thus may form the basis of SR without distant simultaneity assumptions. Ungar[11] extended Winnie's idea by considering a generalized Lorentz transformation group that does not embody Einstein's isotropy convention. The approach seems to be well suited for establishing the results of Winnie as well as some new results. However these discussions were confined to one-dimension only. Later it has been noted by some authors that at least a two–dimensional analysis is necessary. Otherwise the isotropy of one-way speed of light which follows from the modified second relativity postulate cannot be used and therefore some subtleties and richness of the relativistic physics[12] will have to be sacrificed.

In a series of important papers Mansouri and Sexl[13] developed a test theory of SR and investigated the role of convention in various definitions of clock synchronization and simultaneity. They showed that two principal methods of synchronization could be considered: system internal and system external synchronization. Synchronization by the Einstein procedure (using the light signal) and that by slow clock transport (by collecting and synchronizing all clocks at a given locality and then after slowly transporting being them back at the respective space points of a given reference frame) turn out to be equivalent if and only if and only if the time dilation factor is given by Einstein result $(1-v^2/c^2)^{-1/2}$. The authors constructed an ether theory that maintains absolute simultaneity and was kinematically equivalent to SR.

Sjödin[14] developed the CS-thesis by considering the whole issue more generally and also by assuming the role of synchronization in SR and some related theories. Sjödin presented all logically possible linear transformations between inertial frames depending on physical behavior of scales and clocks in motion with respect to the so called "physical vacuum" and then examined Lorentz transformation in the light of true length contraction and time dilation. In his article Sjödin tried to separate the true effects and the effects due to synchronization convention. For this, the author considered two special cases: The Newtonian world– without any contraction of moving bodies and slowing down of moving clocks and Lorentzian world– with longitudinal contraction of moving bodies and slowing down of clocks. The author then used *standard synchrony* in the Newtonian world (This was later termed as Pseudo-standard synchrony by Ghosal, Mukhopadhyay and Chakraborty[12]) and got the transformations which were already derived by Zahar[15][1]. These transformations show that the (apparent) relativistic effects in the Newtonian world are only due to choice of special synchrony. But when Sjödin used absolute synchronization in the Lorentzian world, the relevant transformations were due to Tangherlini[16] which

[1]We shall later find the importance of this transformation in clarifying some counter-intuitive issues in SR.

showed the "real" effects. In this way Sjödin came to the conclusion that the confusion regarding the existence of the ether and the reality of length contraction/time dilation effects was mainly due to the mixing up of the effects arising out of synchronization and the real contraction of moving bodies and retardation of moving clocks.

We have already discussed that the conventionality thesis asserts that there can be a number of choices on the value of the OWS of light of which Einstein convention is just one. It is well known that in the relativistic world the transformation equations that follows from this choice is nothing but LT. Clearly, in a given kinematical world, different choices of the OWS of light may be made which will lead to different transformation equations. These equations although may be different outwardly, will predict the same kinematical world. In recent years these structurally different transformation equations have been found to give much insight into many conceptual issues including some interesting paradoxes in SR. (We have used some of these for the present investigation.) We give below some important transformation equations which explicitly incorporate the CS-thesis. These equations relate coordinates x, y, z and time t in an inertial frame Σ with those (x',y',z',t') in another inertial frame Σ'.

Winnie transformations:

Based on three synchrony independent principles "the round trip light principle, the principle of equal passage times and the linearity principle Winnie arrived at his following ϵ-Lorentz transformations (see Ref. [10] for the interesting derivation of these transformation equations).

$$x' = \alpha^{-1}(x - \vec{v_\epsilon} t),$$
$$t' = \alpha^{-1} t[2\vec{v_\epsilon} c^{-1}(1 - \epsilon - \epsilon') + 1] - xc^{-2}[2c(\epsilon - \epsilon') + 4\vec{v_\epsilon}(\epsilon)(1 - \epsilon)], \tag{2.2}$$

where

$$\alpha = [(c - \vec{v_\epsilon}(2\epsilon - 1))^2 - \vec{v_\epsilon}^2]^{1/2}/\epsilon, \tag{2.3}$$

and ϵ and ϵ' are Reichenbach parameters in the two frames which are in relative motions. Recall that ϵ parameter(s) have already been defined by Eq.(2.1).

Note that the symbol $\vec{v_\epsilon}$ denotes the relative speed of Σ' with respect to Σ. There is a word of caution however; the vector sign does not imply that the transformation equations involve more than one dimension, the arrow sign only emphasizes the non-reciprocity of relative velocity when $\epsilon \neq 1/2$ (for non-standard synchronization). The equation could also have been written in terms of $\overleftarrow{v_\epsilon}$ which denotes the relative speed of Σ to Σ' and in general $\vec{v_\epsilon} \neq \overleftarrow{v_\epsilon}$.

Selleri transformations:

The general form of the transformation obtained by Selleri[17] following the CS-thesis approach is given by

$$\begin{aligned} x' &= (x - \beta ct)/R(\beta) \\ y' &= y \\ z' &= z \\ t' &= R(\beta)t + \epsilon(x - \beta ct) + e(y + z), \end{aligned} \tag{2.4}$$

where ϵ and e are two undetermined functions of relative velocity v and $\beta = v/c$ and also $R(\beta) = (1 - \beta)^{1/2}$. The demand of rotational invariance around x-axis gives $e = 0$, giving the final form of these transformation equations as

$$\begin{aligned} x' &= (x - \beta ct)/R(\beta) \\ y' &= y \\ z' &= z \\ t' &= R(\beta)t + \epsilon(x - \beta ct). \end{aligned} \tag{2.5}$$

The transformation Eqs.(2.2) and (2.5) represent the relativistic world.

An interesting consequence of these equations is that it allows for absolute synchronization ($\epsilon = 0$) and the consequent transformation equations for $\epsilon = 0$ are obtained as

$$x' = \gamma(x - vt), \qquad t' = \gamma^{-1}t, \tag{2.6}$$

which are known as Tangherlini transformations[16] or inertial transformations[17, 18, 19]. Note that although the above equations represent the relativistic world, simultaneity is not relative in character i.e it is absolute.

Zahar Transformation:

In the classical or Galilean world the question of clock synchronization by light signals is not an issue. Since there is no time dilation, clock transport synchronization holds without any ambiguity hence the transformation equations are the well known Galilean ones. However, if one tries to incorporate the light signal synchronization following Einstein's procedure (playfully say) one observes that the Galilean transformations are replaced by the Zahar transformation (named after E. Zahar who obtained these transformation equations originally in 1977[3, 12, 14, 15]).

$$x' = x - vt, \qquad t' = \gamma^2(t - vx/c^2). \tag{2.7}$$

There are some other interesting transformation equations as an outcome of the CS approach where synchronization is achieved by non-luminal signal (in general) following the standard synchronization procedure. The equations are quite general in nature in the sense that the world (classical or relativistic) is not specified beforehand. These transformations have been much helpful for our investigations reported in this thesis (specially in second part) hence the derivations will be given rather in some details at the end of this section.

2.2 Dealing with Myths and Paradoxes

There are many myths and paradoxes that still exist in SR. Much of these misconceptions concerning the relativity theory stems from overlooking of the role of conventionality in gradients of SR. Thus the CS-thesis often comes as an aid to understanding of these myths and counter-intuitive issues. In recent years some of them have been dealt with efficiently. Some of them as mentioned before, being Selleri paradox[3], Tippie Top paradox[2] and Twin paradox[1]. One myth most relevant for the present report will now be discussed in some detail, since this background will also prepare the reader for the material given in Chapter V.

In a recent paper Ralph Bairlein[20] addressed one myth or misconception concerning the low speed behavior of the Lorentz transformation. Much before, Ghosal et.al[21] discussed the same issue in the light of the CS-thesis[2]. The question is "Does SR goes over to Galilean relativity for relative speeds small compared to the speed of light in vacuum?". The myth is "yes" but this is not correct. In fact it can be shown that if the belief is taken to be true it would have led to an interesting fallacy which we shall discuss below. It will be argued that Galilean synchrony and Einstein Synchrony are different and we will show that small velocity approximation cannot alter the convention of distant simultaneity[21].

Consider two events $E_1 : (x_1, t_1)$ and $E_2 : (x_2, t_2)$ in an inertial frame S. Representing in a Minkoski diagram, the invariant interval between these two events

[2]In a private communication to my supervisor referring the myth Prof. Bairlein writes "... it seems that the physics community needs a reminder every twenty years or so that LTs do not reduce to GTs when the relative speeed of frames is small relative to c....".

is

$$\Delta s^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t)^2 = (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2 - c^2(\Delta \bar{t})^2, \tag{2.8}$$

where $(\Delta x_i)^2 = x_{i2} - x_{i1}$, $\Delta t = t_2 - t_1$ and bars represent the corresponding quantities in another references frame $\bar{S}$ moving relative to S with the uniform non-zero speed v. If v^2/c^2 is neglected and if it were true that LT goes over GT for $v^2/c^2 \to 0$, then one would usually expect the time to be absolute i.e

$$(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 = (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2. \tag{2.9}$$

This appears to be all very fine since it looks as if we are merely going from Minkowski metric to Euclidean metric. But this is only an illusion and students often make such a mistake. We will see that this leads to a contradiction since, according to GT

$$\bar{x} = x - vt, \quad \bar{y} = y, \quad \bar{z} = z, \quad \bar{t} = t, \tag{2.10}$$

so that

$$\Delta \bar{x} = \Delta x - v\Delta t, \quad \Delta \bar{y} = \Delta y, \quad \Delta \bar{z} = \Delta z, \quad \Delta \bar{t} = \Delta t, \tag{2.11}$$

and clearly, for any two non-simultaneous ($\Delta t \neq 0$) events, $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is not an invariant. The above fallacious situation can not be resolved unless one rejects the notion that alone the neglect of v^2/c^2 in LT leads to GT. Indeed, if v^2/c^2 is neglected in the Lorentz factor., the LT reduces to the approximate Lorentz transformation (ALT)[22]

$$\begin{aligned} \bar{x} &= x - vt, \\ \bar{t} &= t - vx/c^2, \end{aligned} \tag{2.12}$$

thus, for any pair of events

$$\begin{aligned} \Delta \bar{x} &= \Delta x - v\Delta t, \\ \Delta \bar{t} &= \Delta t - (v/c^2)\Delta x. \end{aligned} \tag{2.13}$$

Notice here that for any chosen spatial separation Δx between two events, we can take v sufficiently small, so that Δt becomes very large compared to $(v/c^2)\Delta x$ and hence the latter may be neglected implying $\Delta \bar{t} = \Delta t$. On the other hand, the approximation $v^2/c^2 << 1$ is certainly not dependent on the space time separation of two arbitrary and independent events. In fact, for any preassigned small value of v, one is free to consider a pair of sufficiently distant events so that one cannot ignore the $(v/c^2)\Delta x$ term in Eq.(2.13). Therefore absolute nature of distant simultaneity ($\Delta \bar{t} = \Delta t$) can never be retrieved. That is, simultaneity is still relative. This is not surprising since we should realize that the relative character of distant simultaneity is the result of a synchronizing convention[3, 6, 7, 21]. A convention once chosen a priori is unlikely to change into a different one merely due to approximative assumption on the relative velocity alone.

Let us recall that the standard Einstein synchronization procedure requires spatially distant clocks to be so adjusted that in any given inertial frame the *to and fro* speeds of light appear to be the same and equal to the round trip speed of light. In this context it is now worthwhile to examine the nature of ALT (Eq.(2.12)) for all v. To do this, the velocity addition laws can be obtained from Eq.(2.12) as

$$\begin{aligned} \bar{W}_x &= (W_x - v)/(1 - vW_x/c^2), \\ \bar{W}_y &= W_y/(1 - vW_x/c^2), \\ \bar{W}_z &= W_z/(1 - vW_x/c^2). \end{aligned} \tag{2.14}$$

As expected, W_y and W_z do not transform as in SR. Now, if a light pulse is sent back and forth along the x-direction alone, that is,

$$\begin{aligned} W_x &= \pm c, \\ W_y &= W_z = 0, \end{aligned} \tag{2.15}$$

then the to and fro speed of light in $\bar{S}$, parallel to the direction of motion, is given by

$$c_{||} = \pm c. \tag{2.16}$$

If, on the other hand, a light pulse is sent back and forth in S in such a direction that the signals travel back and forth only in the y-direction in $\bar{S}$, then

$$W_x = W_z = 0. \tag{2.17}$$

Now using the fact that $W_x^2 + W_y^2 = c^2$ in S, one obtains the speed of light in $\bar{S}$, perpendicular to the direction of motion, the value

$$c_{\perp} = \pm c/(1 - v^2/c^2)^{1/2}. \tag{2.18}$$

These results, i.e Eqs(2.16) and (2.18) certainly do not agree with the corresponding classical results unless $v = 0$ strictly. Furthermore, from Eqs(2.16) and (2.18) we see that the to and fro speeds are individually equal both in the longitudinal direction and in the transverse direction. In fact, it can be shown that the same conclusion holds also for any arbitrary direction in $\bar{S}$. This is precisely the *standard synchronization convention.* Thus Einsteinian synchrony inherent in LT is preserved (even under the approximation $v^2/c^2 << 1$). This is exactly in accordance with our earlier assertion.

However, one may still suspect whether the transformation Eq.(2.12) represents a Galilean world in essence, save the synchronization convention. In order to decide this, one must compare synchrony independent quantities obtained from Eq.(2.12) with those obtained from the usual Galilean transformations. One such quantity is the round-trip speed of any signal. In fact, two sets of transformations may appear structurally very different depending on the choice of synchrony, but when synchrony independent quantities are compared one might discover that they are essential same. In that case we say that these two transformations represent the same kinematical "World". From the Galilean transformation, it follows that two-way average speed of light in the direction parallel and perpendicular to the direction of relative motion are given respectively by

$$\overleftrightarrow{c}_{\|} = c(1 - v^2/c^2),$$
$$\overleftrightarrow{c}_{\perp} = c(1 - v^2/c^2)^{1/2}, \tag{2.19}$$

whereas we see from Eqs.(2.16) and (2.18) that they are given by

$$\overleftrightarrow{c}_{\|} = c,$$
$$\overleftrightarrow{c}_{\perp} = c(1 - v^2/c^2)^{1/2}, \tag{2.20}$$

Thus Eq.(2.12) for all v in general, does not represent a Galilean World (GW). Of course one may choose $v^2/c^2 << 1$ again in Eqs.(2.19) and (2.20), and it becomes clear that Eq.(2.12) represents GW approximately. But then there is a subtle point that must be carefully noted. The resulting GW is not a GW in totality but it is limited by the very approximation. To exemplify this point, consider the Tangherlini Transformations (TT), which represents an Einstein World (EW) with absolute (Galilean) synchrony[16]:

$$\bar{x} = (x - vt)/(1 - \beta^2)^{1/2},$$
$$\bar{t} = t(1 - \beta^2)^2, \tag{2.21}$$

with $\beta = v/c$.

Note that if $v^2/c^2 << 1$, the resulting transformations represent a GT in totality. This is expected because we mentioned before that any set of transformations depends structurally on the choice of synchrony. Since here we consider Galilean synchrony it is natural that under the condition $\beta^2 << 1$ it gives GT in totality. Obviously, this fact is absent in Eq.(2.12). Hence it proves again that a convention once chosen does not change into a different one due to an approximate assumption on the relativity velocity alone.

Thus we have demonstrated that the LT does not lead under the small velocity approximation to Galilean (absolute) synchrony. As a result, the GT for one-way velocities could not be obtained unless $v = 0$ strictly. However, Eq.(2.12) represents a GW only for small velocities but not for the entire velocity range, in contrast to the Tangherlini case just mentioned above.

Finally, one may raise the question whether it is at all possible to construct a transformation which represents a GW in totality having standard synchrony. Indeed, one may verify that the transformations due to Zahar and Sjodin[12, 14, 15] satisfies the above characteristics which are just complementary to those of TT.

$$\begin{aligned} \bar{x} &= (x - vt), \\ \bar{t} &= [t - (vx/c^2)]/(1 - v^2/c^2). \end{aligned} \tag{2.22}$$

It is evident that this transformations (ZT) reduces to ALT from Eq.(2.12) if the v^2/c^2 term is neglected. Note that here again the Poincare-Einstein synchrony is preserved.

Thus we see that LT under the small velocity approximation does not go over to GT but instead, it becomes, as it should be equivalent to ZT from Eq.(2.22) under the same approximation. In contrast, TT from Eq.(2.21) directly goes over to GT. Therefore, in order to fully comprehend the passage of SR to GR one should examine LT vis-a-vis ZT and GT vis-a-vis TT in the context of small speed approximation.

2.3 CS-thesis and Preferred Frame

In an interesting paper Ghosal et.al [12] dealt with the CS issue in a novel way by considering "Relativity in a substrate". Later Chakraborty[23] while putting it in the context of ether wrote "Sometimes in connection with the CS-thesis, the debatable issue of ether (as a *hypothetical* substrate providing a preferred inertial frame) often crops up[13, 14, 24]. But question have been raised whether considerations of synchronization alone can distinquish an ether frame or not[24, 25, 26]. As it stands now, as if the existence of a real physical ether as a preferred frame would have placed the CS-thesis on a stronger footing. In fact efforts are still on to give a physical support to this preferred frame of ether. (However for the understanding of CS-thesis at least, one can bypass the debate concerning the existence of ether by introducing at the out-set a real physical substrate (water for example) through which different inertial frames may be considered to be in relative motion). Given this perspective of confusion, misconstruction and polemics regarding the CS-thesis or SR for that matter, we are led to conclude that everything of SR is still not well understood. We therefore feel that it is necessary to provide some additional clarifications in this regard". It is to this task that the aforesaid paper addresses itself.

Before we discuss the context of the paper let us start with the following observations. In the standard formulation of SR light has two different roles to play. On the one hand it acts as a synchronizing agent, on the other hand it has invariant two-way-speed (TWS) in vacuum. The second role has a basis in the empirically verifiable property, but the first one is purely perspective in origin. In the derivation of the LT in the standard SR, these two roles are mixed up. The inseparability contributes to several misconceptions and prejudices in relativity theory. In order to separate these roles one may introduce non-luminal signal to synchronize clocks and re derive transformation transformation equations. The authors[12] consider reference frames submerged in a substrate. In order to derive the transformation equations, they propose to synchronize the clocks by some other signal (acoustic signal (AS)) which is a characteristic of the substratum. The authors first consider an acoustic wave generated at $t = 0$ at the common origin of the frames S_i and

S_k. In all other frames except for the frame S_0 which is at rest relativity to the substratum, the velocity of AS in the positive x-direction and negative x-direction will not be the same. Using the CS-thesis they define the synchronization of clocks so that these two velocities are equal in all frames although their values vary from frame to frame. This synchrony is called the *pseudo-standard synchrony* other than Einstein's standard synchrony. According to pseudo-standard synchrony, the one dimensional wave front equation will be

$$x_k^2 = a_{kx}^2 t_k^2, \tag{2.23}$$

where x_k's are co-ordinates of a frame S_k which is moving with respect to S_0 frame which is fixed in the substrate and a_{kx} is the TWS of the AS in the x- direction.

The acoustic wave front will not be spherical in frames other than in S_0 frame. Two-way-speed (TWS) of AS will not be the same in all directions, for example along the y-direction the wave front equation will be

$$y_k^2 = a_{ky}^2 t_k^2, \tag{2.24}$$

where a_{ky} is the TWS of AS along y-direction and may have different value from a_{kx}.

The Derivation of Transformation Equations:

In order to derive the transformation equations (TE) between two general inertial frame S_i and S_k one can use TE in the linear form as,

$$\begin{aligned} x_k &= \alpha_{ik}(x_i - v_{ik}t_i), \\ y_k &= y_i, \\ t_k &= \xi_{ik}x_i + \beta_{ik}t_{ik}. \end{aligned} \tag{2.25}$$

In the above equation v_{ik} is the velocity of S_k with respect to S_i and α_{ik}, ξ_{ik} and β_{ik} are constant that are to be determined by using pseudo-standard synchrony. Hence according to the chosen synchrony, one can set the condition

$$x_k^2 - a_{kx}^2 t_k^2 = \lambda_{ik}^2 (x_i^2 - a_{ix}^2 t_i^2), \tag{2.26}$$

where λ_{ik} is a sale factor that is independent of the space and time coordinates. Using Eqs.(2.25) and (2.26) one can obtain the transformation coefficients as

$$\alpha ik = \lambda_{ik}\gamma_{ik}, \tag{2.27}$$

$$\beta_{ik} = \alpha ik/\rho_{ik}, \tag{2.28}$$

$$\xi_{ik} = -\frac{\alpha_{ik}/\rho_{ik}}{v_{ik}/a_{ix}^2}, \tag{2.29}$$

with

$$\gamma_{ik}(1 - v_{ik}^2/a_{ix}^2)^{-1/2}. \tag{2.30}$$

and

$$\rho_{ik} = a_{kx}/a_{ix}. \tag{2.31}$$

Thus the transformation Eqs.(2.25) can be written as,

$$\begin{aligned} x_k &= \lambda_{ik}\gamma_{ik}(x_i - v_{ik}t_i), \\ t_k &= (\lambda_{ik}/\rho_{ik})\gamma_{ik}(t_i - v_{ik}x_i/a_{ix}^2). \end{aligned} \tag{2.32}$$

According to adopted synchrony the TWS of AS is isotropic in the preferred frame S_0 which is stationary with respect to the medium. In the general frame S_k it will not be isotropic. If the isotropic signal speed is a_0, one can write

$$a_x^2 + a_y^2 = a_0^2, \tag{2.33}$$

where a_x and a_y are the x and y components of the velocity of the wavefront along the direction.

The TWS in x- direction in S_k is given by

$$a_{kx} = \frac{\alpha_{0k}a_0(1 - v_{0k}^2/a_0^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.34}$$

The TWS in y- direction in S_k is given by

$$a_{ky} = \frac{a_0(1 - v_{0k}^2/a_0^2)}{\beta_{0k} + \xi_{0k}v_{0k}}. \tag{2.35}$$

Also, the general transformation laws for any other signal whose isotropic TWS (equal to its OWS) in S_0 is a'_0 (which may differ from a_0) can be written as

$$a'_{kx} = \frac{\alpha_{0k}a'_0(1 - v_{0k}^2/a_0'^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.36}$$

The TWS in y- direction in S_k is given by

$$a'_{ky} = \frac{a'_0(1 - v_{0k}^2/a_0'^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.37}$$

where a'_{kx} and a'_{ky} are the TWS of the signal as measured from S_k in the longitudinal and in the transverse directions respectively. However it is clear that to arrive at these relations one assumes that with respect to S_0 under the chosen synchrony, the OWS of "other" signal is isotropic and hence is equal to its TWS. In other words it has been tacitly assumed that in S_0 the pseudo-standard synchrony with AS and with the "other" signal are equivalent.

Now using Eqs.(2.27), (2.34) and (2.35), after simplification reads

$$\lambda_{0k} = a_{kx}/a_{ky}. \tag{2.38}$$

Also

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}}\frac{a_{iy}}{a_{ix}}. \tag{2.39}$$

On putting the value of λ_{ik} the TE of Eq. (2.32) becomes

$$\begin{aligned} x_k &= (a_{kx}/a_{ky})(a_{iy}/a_{ix})[(x_i - v_{ik}t_i)/(1 - v_{ik}^2/a_{ix}^2)^{1/2}], \\ t_k &= (a_{iy}/a_{ky})[(t_i - (v_{ik}/a_{ix}^2)x_i)/(1 - v_{ik}^2/a_{ix}^2)^{1/2}]. \end{aligned} \tag{2.40}$$

With respect to preferred frame S_0 (where $a_{0x} = a_{0y} = a_0$) the TE from S_0 to any other inertial frame S_k is given by

$$\begin{aligned} x_k &= (a_{kx}/a_{ky})[(x_0 - v_{0k}t_0)/(1 - v_{0k}^2/a_0^2)^{1/2}], \\ t_k &= (a_0/a_{ky})[(t_0 - (v_{0k}/a_0^2)x_0)/(1 - v_{0k}^2/a_{0x}^2)^{1/2}]. \end{aligned} \tag{2.41}$$

In a lighter vein the authors term this set of transformation equations dolphin transformations (DT) as these TE perceived by intelligent dolphins. The DT is usable the space-time relations between two frames provided one knows the TWS of AS in these two frames. If one chooses light signal (vacuum) for synchronization of clocks instead of AS, by virtue of CVL postulate in SR

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c, \tag{2.42}$$

so that one would obtain the familiar LT. However in absence of any communication with the outside world, *apparently* c does not play any role in DT even though the dolphins live in the relativistic world where we know c plays a fundamental role! Indeed in the DT, c will appear as a *physical constant* through a_{kx} and a_{ky}. In order to make use of DT, the dolphins will have to measure the TWS of AS in S_k as a function of velocity v_{0k} and one can anticipate that they will eventually find that

$$\begin{aligned} a_{kx} &= a_{kx}(v_{0k}, c), \\ a_{ky} &= a_{ky}(v_{0k}, c), \end{aligned} \tag{2.43}$$

where c appears not as the speed of light but as some physical constant. If now the dolphins are able to communicate with the outside world and discover that their world admits an invariant speed c. Recall the formulas for two-way velocity transformation Eqs.(2.36) and (2.37) and put $a'_{kx} = a'_{ky} = a'_0 = c$. Now using Eqs(2.27-2.30) one may easily demonstrate that

$$\rho_{0k} = \frac{(1 - v_{0k}^2/a_0^2)}{(1 - v_{0k}^2/c^2)}, \tag{2.44}$$

and

$$\lambda_{0k} = \frac{(1 - v_{0k}^2/a_0^2)^{1/2}}{(1 - v_{0k}^2/c^2)^{1/2}}. \tag{2.45}$$

or by Eqs.(2.31) and (2.38)

$$a_{kx} = a_0 \frac{(1 - v_{0k}^2/a_0^2)}{(1 - v_{0k}^2/c^2)}, \tag{2.46}$$

and

$$a_{ky} = a_0 \frac{(1 - v_{0k}^2/a_0^2)^{1/2}}{(1 - v_{0k}^2/c^2)^{1/2}}. \tag{2.47}$$

Now inserting Eqs.(2.45) and (2.46) in Eq.(2.41) gives the DT for the relativistic world

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0)/(1 - v_{0k}^2/a_0^2)^{1/2}, \\ t_k &= (1 - v_{0k}^2/c^2)^{1/2}(1 - v_{0k}^2/a_0^2)^{-1}[t_0 - (v_{0k}/a_0^2)x_0]. \end{aligned} \tag{2.48}$$

There are important consequences of DT. These are the following:

1. The transformation equations contain TWS of synchronizing signal. The simultaneity is relative. Under this synchrony relative speeds are not symmetric in general.

2. a_0 is the speed of AS that is conventional. c appears as a physical constant - the TWS of light - and is not based on any convention. The factor $(1 - v_{0k}^2/c^2)^{1/2}$ is due to real effects. The other factor, $(1 - v_{0k}^2/a_0^2)$ arises from the synchronization procedure which is evident from the presence of the term a_0. Thus this clarifies that different synchronization procedure may not have relativity of simultaneity but they can predict length contraction and time dilation effects. From the DT, the length contraction factor (LCF) and time dilation factor (TDF) comes out to be

$$\begin{aligned} LCF &= (1 - v_{0k}^2/a_0^2)/(1 - v_{0k}^2/c^2)^{1/2}, \\ TDF &= (1 - v_{0k}^2/c^2)^{1/2}/(1 - v_{0k}^2/a_0^2). \end{aligned} \tag{2.49}$$

3. As we have mentioned earlier that light has two roles to play in SR. One is that its TWS in vacuum is constant and the other is that it is the synchronization agent in SR. These two roles are mixed up in standard SR. In the derivation of DT we see that these two roles are clearly split up.

Some important transformation equations in relativistic and classical worlds obtains by others can be obtained from DT by the choice and making use of the properties of the synchronization signal:

Lorentz transformation ***(Einstein synchrony and relativistic world)*****:** In the standard synchrony the synchronization agent is light. Putting $a_0 = c$ in DT one may obtain Lorentz transformation.

Tangherlini transformation ***(absolute synchrony and relativistic world)*****:** If in the *preferred frame* the speed of synchronization signals $a_0 \to \infty$ then we obtains (for $S_0 \to S_k$) the Tangherlini transformation

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma^{-1}t, \end{aligned} \tag{2.50}$$

Zahar transformation ***(Einstein synchrony and classical world)*****:** In the classical world the velocity addition law is the Galilean one. Then the TWS of AS is obtained to be

$$\begin{aligned} a_{kx} &= a_0(1 - v_{0k}^2/a_0^2), \\ a_{ky} &= a_0(1 - v_{0k}^2/a_0^2)^{1/2}. \end{aligned} \tag{2.51}$$

Inserting these expressions for a_{kx} and a_{ky} in the DT (and in particular in Eq.(2.41) we obtain DT in in classical world

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0), \\ t_k &= [t_0 - (v_{0k}/a_0^2)x_0]/(1 - v_{0k}^2/a_0^2). \end{aligned} \tag{2.52}$$

In the standard synchrony, ($a_0 = c$) DT becomes Zahar transformation (ZT) as we have discussed earlier

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0), \\ t_k &= [t_0 - (v_{0k}/c^2)x_0]/(1 - v_{0k}^2/c^2). \end{aligned} \tag{2.53}$$

Galilean transformation ***(absolute synchrony and classical world)*****:** In this classical world if the synchronizing signal's speed is assumed to be arbitrarily large (hypothetically) so that one may put $a_0 \to \infty$ in Eq.(2.52), one retrieves the familiar form of GT.

Before we conclude this section it may be mentioned that the Dolphin transformation will be found to serve as a spring board for developing preferred frame theories which may be prompted by the possible violation of GZK limit by the ultra high energy cosmic rays or by the considerations of the variable speed of light in the context of cosmology (vide chapters VI and VII for details). Here I would like to point out that DT has been used earlier (by considering the cosmic microwave background as the substratum) to deal with the question of existence of non-zero photon rest mass (advocated by Narlikar, Pecker and Vigier[27]) by Ghosal and Karmakar[28].

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Chapter III

The Principle of Equivalence and the Twin Paradox

3.1 Introduction

The principle of equivalence between acceleration and gravity is considered as a *cornerstone* of Einstein's theory of gravitation or that of general relativity (GR). According to Einstein, the principle states that: "A system in a uniform acceleration is equivalent to a system at rest immersed in a uniform gravitational field"[1]. Text books often introduce GR by first demonstrating that the Equivalence Principle (EP) predicts gravitational redshift, which Einstein viewed as a test of general relativity. However we now regard it as a more basic test of EP and the existence of curved space-time[2]. The phenomenon of gravitational red-shift, which has been tested by precision experiments by Pound-Rebka and Snider in the sixties[3, 4] is also interpreted as that of gravitational slowing down of clocks (GSDC). The GSDC has now been tested with much accuracy by using a hydrogen maser clock with extraordinary frequency stability flown on a rocket to an altitude of about 10,000 Km[2]. In the literature GSDC phenomenon has been found to play an important role in resolving the notorious twin paradox[5].

In the canonical version of the twin paradox, of the two twins initially living on earth (assumed to be an inertial frame), one leaves the earth by a fast rocket to a distant star and then returns to meet her stay-at-home brother to discover that they age differently. This as such is not a paradox since the rocket-bound sibling, on account of her high velocity will suffer relativistic time dilation of her (biological) clock throughout her journey and will therefore return younger with respect to her brother. Indeed with respect to the inertial frame of the stay-at-home twin, the world lines of the twins in the Minkowski diagram are different (although from the description of the problem the end points of these lines i.e the time and the place of departure and that of their reunion, meet) and hence the asymmetry in the aging can be attributed to the fact that proper time is not integrable[6]. The paradox arises if one näively treats the perspectives of the twins symmetrically. For example if the traveller twin considers herself to remain stationary and relate the motion to her brother, she would (erroneously) expect her brother to stay younger by believing that the Lorentz transformation (LT) predicts reciprocal time dilation of moving clocks. Qualitatively the resolution lies in the observation that one of the twins is in an accelerated (non-inertial) frame of reference and hence the postulates of Special Relativity (SR) are not applicable to it and therefore the claim of reciprocity of time dilation between the frames of reference of the twins falls through. Indeed Einstein himself found this sort of argument preferable in dismissing the paradoxical element in the twin problem[7]. However this suggestion should not be construed as a statement that the resolution of the paradox falls outside the purview of SR . On the contrary much of the expositions found in the literature on the subject deal with the problem in the frame work of SR alone[1], although many tend to believe that the introduction of GR and a gravitational field at the point of acceleration is the right way to understand the asymmetry in the perspectives of the twins. Bohm notes in the context that " two clocks running at places of different gravitational potential will have different rates"[10]. This suggests that EP can directly be used to explain the asymmetry (difference between the experiences of the rocket-bound and the stay-at-home twin). However, as pointed out by Debs and Redhead[6] and also others[11], that since in the twin problems one deals with flat space-time, any reference of GR in this context is quite confusing.

Coming back to the issue of acceleration, one finds often that the direct role of acceleration of the rocket-bound twin in causing the differential aging has been

[1]Very extensive treatment is available in Special Relativity Theory-Selected Reprints[8], (see also Ref.[9]). For newer expositions see for example Ref.[6] and references therein.

much criticized although it is quite clear that in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of the twins have to be compared at the same space-time events) one cannot avoid acceleration.

In an interesting article Gruber and Price[12] dispel the idea of any direct connection between acceleration and asymmetric aging by presenting a variation of the paradox where although one twin is subjected to undergo an arbitrarily large acceleration, no differential aging occurs. That the acceleration per se cannot play a role is also evident from the usual calculation of the age difference from the perspective of the inertial frame of the stay-at-home twin if one notes that the duration of the turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be understood in terms of the usual relativistic time dilation of the traveller twin during essentially the unaccelerated segment of her journey [2]. One is thus caught in an ambivalent situation that, on the one hand the acceleration does not play any role, on the other hand the paradox is not well posed unless there is a turn-around (acceleration) of the traveller twin[3].

In order to get out of this dichotomy it is enough to note that from the point of view of the traveller twin, the acceleration (or the change of reference frame in the abrupt turn-around scenario) is important. The consideration of this acceleration only has the ability to explain that the expectation of symmetrical time dilation of the stationary twin from the point of view of the rocket-bound twin is incorrect.

In an interesting paper A.Harpaz[5] tries to explain the twin paradox by calculat-

[2]In such a calculation the time dilation is also calculated during the acceleration phase (assuming the clock hypothesis to be true[6]) and is shown to contribute arbitrarily small value in the age offset if the duration of the acceleration phase is assumed to tend to zero.

[3]Here we are considering the standard version of the paradox and the variation where the twins live in a cylindrical universe[13, 14] has been kept out of the present scope.

ing the age difference from the perspective of the traveller twin directly by applying EP i.e by introducing GSDC. From the previous discussions it may seem unnecessary (or even confusing) to invoke gravity in the essentially special relativistic problem. However the fact is, Harpaz's approach apparently provides an alternate explanation for the differential aging from the traveller's perspective.

The author of the pedagogical article observes that although the special relativistic approach can correctly account for the age difference between the twins, " it does not manifest the 'physical agent' responsible for the creation of such a difference"[5]. It is held that EP provides such an agent and that is gravity. But how does gravity find way into the problem? Gravity enters through EP and its connection with the resolution of the paradox can symbolically be written as

Acceleration $\stackrel{EP}{\rightarrow}$ *Gravity* $\rightarrow$ *Gravitational red-shift* $\rightarrow$ *GSDC* $\rightarrow$ *Extra aging*,

where the last item of the flow diagram indicates that with respect to the rocket-bound twin, GSDC provides the extra aging of the stay-at-home one, explaining the asymmetrical aging of the problem.

However while there is as such no harm in understanding the twin problem from a different perspective (here, this is in terms of GSDC), Harpaz's approach suffer from two fold conceptual difficulties which we will elaborate in the next section. These difficulties include the fact that the calculations are only approximate. The other difficulty will be seen to be of more fundamental in nature. The aim of the present study (reported in this chapter) is to remove these difficulties and give an *accurate* account of the asymmetric aging from the perspective of the rocket-bound twin directly in terms of a time-offset between the siblings which is introduced due to the pseudo-gravity experienced by the traveller twin.

3.2 GSDC and Extra Aging

In the standard version of the twin paradox the differential aging from the perspective of the stay-at-home (inertial) observer A can easily be calculated assuming that for the most parts of the journey of the traveller twin B, the motion remains uniform except that there is a turn-around acceleration of the rocket so that finally the siblings are able to meet and compare their ages. In the Minkowski diagram the whole scenario is characterized primarily by three events: (1) Meeting of the world lines of A and B when the voyage starts taking place, (2) the turn around of B and (3) meeting of the world lines when A and B reunite. For the paradox it is not necessary that at events (1) and (2), the relative velocity between A and B has to be zero, since ages or clocks can be compared at a point even if the observers are in relative motion, therefore the analysis of the problem can be done by considering the acceleration only during the turn-around. The duration of the acceleration phase can be considered to be arbitrarily small compared to the time it takes during its forward and return journeys and hence the age difference occurs due to the usual relativistic time dilation of a clock for its uniform motion. This is clearly given by

$$Age\ \ difference = 2t_A(1-\gamma^{-1}) \approx 2t_A v^2/c^2, \tag{3.1}$$

where $2t_A$ is the time the rocket takes for its entire journey (up and down) in uniform speed v and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the usual Lorentz factor.

The paradox is resolved if one can show that B also predicts the same difference in spite of the fact that the time dilation effect is reciprocal. Clearly some new considerations (that were absent in arriving at Eq.(3.1)) must offset this reciprocal time dilation and also this must provide some extra aging to A from the point of view of B so that the age difference remains independent of the two perspectives.

One of these new considerations, as has already been pointed out, is the one of a synchronization gap that B discovers due to her change of inertial frame during her entire voyage. This has been clearly demonstrated by Bondi[15] in the context of Lord Hulsbery's three brother approach[6] to understanding the twin paradox.

The other way of understanding the same thing is the consideration of pseudo-gravity experienced by B because of its turn-around. In order to demonstrate how EP plays the role in the analysis, Harpaz uses the gravitational red-shift formula, which can be obtained heuristically (using the EP) as

$$\Delta\nu = \nu_0(1 + gh/c^2), \tag{3.2}$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed from a distance h from the source where the frequency of the same light is seen to be ν_0. Interpreting this red-shift effect in terms of GSDC, the formula can be written as

$$t_1 = t_2(1 + \Delta\Phi/c^2), \tag{3.3}$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\Phi = gh$, is the potential difference between these points. It has been shown that with respect to B the acceleration plays a role by providing an extra time difference between B and A, because of the integrated effect of GSDC during the (arbitrarily) short duration of B's acceleration. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount. As pointed out earlier there are two conceptual difficulties in understanding the treatment. First, in an effort to find a "physical agent" responsible for the extra aging, Harpaz relies on some approximate formulae including that of the gravitational red-shift because of his assumption, $v^2/c^2 \ll 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make any such small velocity approximation for the problem. One might of course argue that for the author's stated purpose it would be enough to show that the " physical agent" i.e. gravity is at work when B's point of view is considered. However, it will be shown that such an argument would also not hold good and the reason for it concerns the second difficulty. The explanations based on SR relies on the fact that during the direction reversing acceleration, the travelling twin changes from one reference frame to another and the lack of simultaneity of one reference frame with respect to the other provides the "missing time" which constitutes the reason for the differential aging[6]. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula (see for example[16, 17, 18]) which is also followed by the author of Ref.[5], one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $Z = gh/c^2$ is only approximate and is derived by making use of the classical Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator[5]. According to EP an observer within the elevator will " attribute his observations in the elevator, to the existence of a uniform gravitational field in a rest system of reference"[5]. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation! How then is GSDC able to account for an effect, viz. the lack of simultaneity which is essentially a standard relativistic phenomenon?

In the next section we will show that indeed the EP can explain the twin paradox exactly provided the connection of EP and GSDC is obtained using the full machinery of SR.

3.3 EP and the Gravitational Time Offset

In an interesting paper Boughn[19] presents a variation of the twin paradox where two twins A and B on board two identical rockets (with equal amount of fuel), initially at rest a distance x_0 apart in an inertial frame S, get identical accelerations for some time in the direction AB (x-direction say), and eventually come to rest (when all their fuel has been expended) with respect to another inertial frame S' moving with velocity v along the positive x-direction with respect to S. From the simple application of Lorentz transformation Boughn obtains a very surprising result that after the acceleration phase is over, the age of A becomes less than that of B.

The result is counter-intuitive by virtue of the fact that the twins throughout have identical local experiences but their presynchronised (biological) clocks go out of synchrony. The amount of this time offset turns out to be

$$\Delta t' = -\gamma v x_0/c^2. \tag{3.4}$$

The result follows from the simple application of LT which one may write for time as

$$t_k{}' = \gamma(t_k - v x_k/c^2), \tag{3.5}$$

where t_k and x_k denote the time and space coordinates of the observer k (k stands for A or B) with respect to S and the prime refers to the corresponding coordinates in S'.

From Eq.(3.5) it follows that

$$t_B' - t_A' = \gamma[(t_B - t_A) - v(x_B - x_A)/c^2]. \tag{3.6}$$

Assuming the clocks of the observers A and B are initially synchronized in S, i.e assuming $t_B - t_A = 0$ and also noting that $x_B - x_A = x_0$ remains constant throughout their journeys, the time offset between these clocks is given by the expression (3.4) provided $\Delta t'$ is substituted for $t_B' - t_A'$.

The paradox however can be explained by noting that for spatially separated clocks the change of relative synchronization cannot be unequivocally determined. The clocks can only be compared when they are in spatial coincidence. For example, when in S' either of the observers can slowly walk towards the other or both the observers can walk symmetrically (with respect to S') towards the other and compare their clocks (ages) when they meet. However in that case one can show[20] that they do not have identical local experiences— thus providing the resolution of the paradox.

While the paradoxical element of the problem goes away, the fact remains that the result (3.4) is correct and this time offset remains unchanged even if they slowly walk towards each other and compare their clocks (ages) when they meet[21].

This temporal offset effect of identically accelerated clocks gives an important insight into the behaviour of clocks in a uniform gravitational field, for, according to EP "...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system"[17]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. With this insight we write Eq.(3.4) as

$$t - t_0 = -\gamma(t)v(t)x_0/c^2 = -f(t), \tag{3.7}$$

where now t and t_0 are the readings of two clocks at higher and lower potentials respectively and also $f(t)$ stands for the right hand side of Eq.(3.4) without the minus sign

$$f(t) = \gamma(t)v(t)x_0/c^2. \tag{3.8}$$

In terms of differentials one may write Eq.(3.7) as

$$\delta t - \delta t_0 = -\dot{f}(t)\delta t, \tag{3.9}$$

where the time derivative $\dot{f}(t) = gx_0/c^2$, with $g = d(\gamma v)/dt$ is the *proper acceleration*.

We may now replace δt and δt_0 by n and n_0, where the later quantities corresponds to the number of ticks (second) of the clocks at their two positions. We therefore have,

$$(n - n_0)/n_0 = -\dot{f}(t), \tag{3.10}$$

or in terms of frequency of the clocks

$$-\delta\nu/\nu_0 = \dot{f}(t), \tag{3.11}$$

where $\delta\nu$ refers to the frequency shift of an oscillator of frequency ν_0. The slowing down parameter for clocks, $-\delta\nu/\nu_0$ in Eq.(3.11) is nothing but the so called red-shift parameter Z for which we obtain the well-known formula[4]

$$Z = gx_0/c^2. \tag{3.12}$$

[4]In terms of *ordinary* acceleration $\overline{g} = dv/dt$, measured with respect to S the formula comes out to be $Z = (\overline{g}\gamma x_0/c^2)(1 - v^2\gamma^2/c^2)$ which for small velocities can also be written as $Z = \overline{g}x_0/c^2$.

One thus observes that the time-offset relation (3.7) of Boughn's paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins.[5] We shall see the importance of the time-offset relation (3.7) in accounting for the assymetrical aging of the standard twin paradox from the perspective of the traveller twin. However before that, in the next section we show that the connection of the time-offset and GSDC is purely relativistic in nature.

3.4 Boughn's Paradox in the Classical World

The origin of Boughn's paradox can be traced to the space dependent part in the time transformation of LT. The existence of this term is indeed the cause of relativity of simultaneity in SR.

The notion of relativity of simultaneity however can also be imported to the classical world. By classical or Galilean world we mean a kinematical world endowed with a preferred frame (of ether) S with respect to which the speed of light c is isotropic and moving rods and clocks do not show any length contraction and time dilation effects. However the speed of light measured in any other inertial frame S' moving with velocity v with respect to S will change and will depend on direction. The synchronization of spatially separated clocks is generally not an issue in this world as clocks can be transported freely without having to worry about time

[5]The connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur[22], who derived the approximate formula for the "clock rate difference" mentioned in the previous foot-note.

dilation, therefore all clocks can be synchronized at one spatial point and then may be transported with arbitrary speed to different locations. (The process is generally forbidden in SR). Clearly one uses the Galilean transformation (GT) to compare events in different inertial frames. Using GT one can show that the two way speed (TWS) of light $\overleftrightarrow{c}$ in S' along any direction θ with respect to the x-axis (direction of relative velocity between S and S') is given by

$$\overleftrightarrow{c}\ (\theta) = c(1-\beta^2)/(1-\beta\sin^2\theta)^{1/2}. \tag{3.13}$$

According to GT this TWS is not the same as the one-way speed (OWS) of light, for example, along the x-axis it is $c-v$ and $c+v$ in the positive and negative x-directions respectively, while the two way speed, i.e the average round-trip speed of light along the x-direction is given by $c(1-v^2/c^2)$. However, in a playful spirit one may choose to synchronize the clocks in S' such that the one way speeds, to and fro are, the same as $\overleftrightarrow{c}$. This is similar to Einstein's stipulation in SR which is commonly known as the standard synchrony. In the Galilean world the synchrony is somewhat an awkward one but none can prevent one in adopting such a method. For this synchrony GT changes to the following transformations[6]

$$\begin{aligned} x' &= (x - vt), \\ t' &= \gamma^2(t - vx/c^2), \end{aligned} \tag{3.14}$$

which was first obtained by E. Zahar and is therefore known as the Zahar transformation (ZT)[23, 24, 25, 26]. The transformations have been successfully used to clarify some recently posed counter-intuitive problems in SR[27, 28]. The presence of the phase term and γ^2 in Eq.(3.14) distinguishes the ZT from GT. Clearly the appearance of these terms is just an artifact of this synchrony.

[6]See chapter II for a derivation of the transformation equations following conventionality of simultaneity thesis in the classical world.

One is thus able to recast Boughn's paradox using the above transformations and extending the arguments leading to the Eq.(3.4), one obtains for the differential aging,

$$\Delta t' = -\frac{\gamma^2 v x_0}{c^2}. \tag{3.15}$$

The above expression for the differential aging between two spatially separated twins is also therefore an artifact of the synchrony.

Let us note that ZT has many interesting features which include the existence of apparent time dilation and length contraction effects as observed from an arbitrary reference frame S'. (With respect to the preferred frame however there are no such effects). We have already pointed out that the temporal offset between clocks cannot have any unequivocal meaning unless it corresponds to measurement at one spatial point.

One may therefore define without much ado the reality of the temporal offset effect due to Boughn (hereafter referred to as Boughn-effect), provided the clocks are finally compared when they are brought together. In the relativistic world a clock is slowly transported towards the other in order to minimize the time dilation effect in the process. In this world if one of the presynchronized spatially separated clocks is brought to the other in an arbitrarily slow motion, it can be seen that when they are compared at the position of the second clock, they remain synchronized. In other words if two clocks have an initial temporal offset between them (due to Boughn-effect or otherwise) when separated, the value for this offset will remain unchanged when they are brought together for comparison. Boughn-effect is thus a real effect (according to the definition) in the relativistic world. In the classical world the situation is different. Below we calculate the effect of clock transport from ZT.

From ZT between a preferred frame S_0 and an arbitrary frame S, one may write the transformation equation between any inertial frames S_i and S_k as,

$$x_i = {\gamma_k}^2(1 - \frac{v_i v_k}{c^2})x_k - (v_i - v_k)t_k, \tag{3.16}$$

$$t_i = {\gamma_i}^2[(1 - \frac{v_i v_k}{c^2})t_k - \frac{\gamma_k^2}{c^2}(v_i - v_k)x_k], \tag{3.17}$$

where the suffixes i and k of coordinates x, t and v refer to the coordinates in S_i and S_k and velocities of the concerned frames with respect to S_0 respectively. Also $\gamma_i = (1 - \frac{v_i^2}{c^2})^{-1/2}$ and $\gamma_k = (1 - \frac{v_k^2}{c^2})^{-1/2}$.

Clearly a clock stationary with respect to S_k will suffer a time "dilation" according to

$$\Delta t_i = \frac{1 - v_i v_k / c^2}{1 - v_i^2 / c^2} \Delta t_k, \tag{3.18}$$

where Δt_k refers to the proper time between two events at the same point of S_k and Δt_i is the corresponding time measured by observers in S_i.

Consider now two synchronized clocks are spatially separated by a distance x in S_i and a third clock attached to S_k slowly covers the distance. The time taken by the clock to cover this distance in S_i is given by

$$\Delta t_i = \frac{x}{w}, \tag{3.19}$$

where w is the relative velocity of S_k with respect to S_i. The corresponding time measured by the third clock (S_k - clock) may be obtained from Eq.(3.18).

From ZT the relative velocity formula is obtained as

$$w = \frac{(1 - \frac{v_i^2}{c^2})(v_k - v_i)}{1 - \frac{v_i v_k}{c^2}}. \tag{3.20}$$

Using Eqs.(3.18), (3.19) and (3.20) one obtains for the difference of these two times

$$\delta t' = \Delta t_k - \Delta t_i = \frac{v_i x}{c^2} \gamma_i^2. \tag{3.21}$$

This non-vanishing integrated effect of the time dilation in the classical world due to clock transport is independent of the speed (v_k) at which the clock is transported.

In contrast, in the relativistic world one finds different values for the effect for different velocities and in particular the value is zero when the speed is vanishingly small.

If now the two stationary (with respect to S_i) clocks refer to two Boughn's observers A and B, they have precisely this amount (Eq.(3.21)) of temporal offset with a negative sign and hence if the observer A walks towards B no matter whether slow or fast, the result will be the zero time difference between the clocks when compared at one spatial point. This observation demonstrates that although Boughn's paradox can be recast in the Galilean world the time-offset effect is just an artifact and not real according to our definition of "reality" of the effect. Thus GSDC cannot be obtained from this Boughn's effect in the classical world via EP. Conversely Boughn's temporal offset may be regarded as an integrated effect of GSDC while in the classical world if it exists is just an artifact of the synchrony.

3.5 Resolution

Let us now move on to the details of the arguments leading to Eq.(3.1). The outward trip of the traveler twin B from the point of view of the earth twin is composed of two phases. In the first phase, the rocket moves a distance L_A in time t_{A1} with uniform velocity v which is given by

$$t_{A1} = \frac{L_A}{v}, \tag{3.22}$$

and in the second phase, which corresponds to the deceleration phase of the rocket which finally stops before it takes the turn-around, the time t_{A2} taken by B is given by

$$t_{A2} = \frac{\gamma v}{g}, \tag{3.23}$$

where the proper acceleration g has been assumed to be uniform with respect to the earth frame. In the present analysis this term does not contribute since we consider the abrupt turn-around scenario where t_{A2} tends to zero as $g \to \infty$; however for the time being we keep it. Therefore the total time elapsed in S for the entire journey is given by

$$T_A = \frac{2L_A}{v} + 2t_{A2}. \tag{3.24}$$

Now we compute this time as measured in B's clock by taking the time dilation effect from the point of view of A. For phase 1 this time t_{B1} may be computed as

$$t_{B1} = \gamma^{-1} t_{A1} = \frac{\gamma^{-1} L_A}{v}, \tag{3.25}$$

where we have applied the simple time dilation formula. For phase 2 however this time-dilation formula is differentially true as the speed is not a constant i. e one may write

$$dt_{B2} = (1 - \frac{v^2}{c^2})^{1/2} dt_{A2} = (1 - \frac{v^2}{c^2})^{1/2} \frac{1}{g} d(\gamma v). \tag{3.26}$$

Hence after integration one obtains[29]

$$t_{B2} = \frac{c}{2g} ln(\frac{1 + v/c}{1 - v/c}). \tag{3.27}$$

However once again this tends to zero as $g \to \infty$. In any case we shall however not need this expression any more. Therefore the total elapsed time measured in B's clock for the complete journey is given by

$$T_B = \frac{2\gamma^{-1} L_A}{v} + 2t_{B2}. \tag{3.28}$$

The differential aging from the point of view of A is thus

$$\delta T_A = T_A - T_B = \frac{2L_A}{v}(1 - \gamma^{-1}) + 2(t_{A2} - t_{B2}). \tag{3.29}$$

From the point of view of B the stay-at-home observer A is moving in the opposite direction and as before one may divide the relative motion of A into two phases, phase I and phase II, where the later corresponds to the acceleration phase. The phase II may be interpreted as turning on of a gravitational field. When this field is switched off (marking the end of the acceleration phase), the phase I starts where the stay-at-home observer A moves with a velocity $-v$ up to a distance L_B which on account of the Lorentz contraction of L_A is given by,

$$L_B = L_A(1 - \frac{v^2}{c^2})^{\frac{1}{2}}, \tag{3.30}$$

and the corresponding elapsed time t_{B1} is given by,

$$t_{BI} = \frac{L_B}{v} = \frac{\gamma^{-1} L_A}{v}. \tag{3.31}$$

This obviously comes out to be the same as t_{B1} since the result is obtained from considerations with respect to the inertial observer A. Similarly t_{BII} i.e. B-clock's time during phase II should be the same as t_{B2} during which the gravitational field is turned on, i.e

$$t_{BII} = t_{B2}, \tag{3.32}$$

and hence the total time

$$\tau_B = 2t_{BI} + 2t_{BII} = \frac{2\gamma^{-1} L_A}{v} + 2t_{BII} = T_B. \tag{3.33}$$

The corresponding time of A's clock by taking into account the time dilation effect is

$$t_{AI} = \gamma^{-1} t_{BI} = \frac{\gamma^{-2} L_A}{v}. \tag{3.34}$$

Writing A-clock's time during phase II from B's perspective as t_{AII}, one may write for A's clock time for the entire journey as

$$\tau_A = 2t_{AI} + 2t_{AII} = \frac{2\gamma^{-2} L_A}{v} + 2t_{AII}. \tag{3.35}$$

The difference of these times of clocks A and B as interpreted by the observer B, is given by,

$$\delta T_B = \tau_A - \tau_B = \frac{2\gamma^{-1}L_A}{v}(\gamma^{-1} - 1) + 2(t_{AII} - t_{BII}). \tag{3.36}$$

Note that at the moment we do not know the value of t_{AII}, since it refers to the time measured by A as interpreted by B when it is in its acceleration phase. The paradox is resolved if

$$\delta T_A = \delta T_B. \tag{3.37}$$

In other words using Eqs.(3.29) and (3.36) one is required to have,

$$t_{AII} = \frac{L_A}{v}(1 - \gamma^{-2}) + t_{A2} = \frac{L_A v}{c^2} + t_{A2}. \tag{3.38}$$

In the abrupt turn-around scenario, as we have already observed $t_{A2} = 0$, one therefore must have

$$t_{AII} = \frac{L_A v}{c^2} = \frac{\gamma L_B v}{c^2}. \tag{3.39}$$

The resolution of the twin paradox therefore lies in accounting for this term. It is interesting to note that the term is independent of the acceleration in phase II. This is possibly the implicit reason why the role of acceleration in the explanation of the twin paradox is often criticized in the literature. However we shall now see how, we can interpret this term as an effect of the direction reversing acceleration (or the pseudo-gravity) experienced by the traveller twin.

Now recall the Boughn-effect of temporal offset between two identically accelerated observers. To be specific, consider an inertial frame of reference S attached to the observer B when it is in the uniform motion phase (phase I). Suppose now there is another observer B' at rest in S at a distance L_B behind B and both of them get identical deceleration and eventually come to rest with respect to A in the frame of reference S', which is moving with velocity $-v$ in the x-direction with respect to S. According to Boughn-effect then the clocks of these two observers get desynchronized and the amount of this desynchronization is given by the expression (4) only with the sign changed, that means

$$desync = \frac{\gamma v L_B}{c^2}, \tag{3.40}$$

which is nothing but t_{AII}. It has already been pointed out that this Boughn-effect may be interpreted as the effect of pseudo-gravity (in this case as experienced by the observer B) according to EP. In terms of the pseudo acceleration due to gravity the above expression can also be obtained as

$$desync = \frac{g \Delta t_B L_B}{c^2}. \tag{3.41}$$

Note that $g\Delta t_B$ is finite (equal to γv) even if $g \to \infty$.

The observer B' which is L_B distance away from B is spatially coincident with A, hence, in calculating the clock time of A from B's perspective this time-offset due to Boughn-effect must be taken into account. This effect is ignored when the twin paradox is posed by näively asserting the reciprocal time-dilation effect for the stay-at-home and the rocket-bound observers. Clearly the paradox is resolved if the Boughn-effect or the pseudo gravitational effect is taken into consideration.

3.6 Concluding Remarks: Test of Boughn-Effect

We have seen that the Boughn-effect can be interpreted as the integrated effect of GSDC. The experimental test of GSDC or the gravitational red-shift is therefore a test of a differential Boughn-effect in a way. On the contrary one may directly measure the integrated effect by the following means:

First two atomic clocks may be compared (synchronized) at the sea level, then one of the clocks may be slowly transported to a hill station of altitude h and then kept there for some time T. In this time these two atomic clocks according to Boughn scenario are perpetually accelerated from a rest frame S to a hypothetical inertial frame S' moving with velocity v, with proper acceleration g so that $\gamma v = gT$. Boughn-effect therefore predicts a temporal offset (see Eqs.(3.40) and (3.41)),

$$\Delta t_{offset} = \frac{ghT}{c^2}. \tag{3.42}$$

This offset can be checked by bringing the hill station clock down and then comparing its time with the sea level one. Any error introduced in the measurement due to transport of clocks can be made arbitrarily small compared to Δt_{offset} by increasing T. As a realistic example for h =7000ft (altitude of a typical hill station in India), and T =1 year and taking the average g to be about $9.8m/sec^2$, the Boughn-effect comes out to be in the micro-second order:

$$\Delta t_{offset} = 7.3\mu s, \tag{3.43}$$

which is easily measurable without requiring sophisticated equipments, such as those used in Pound-Rebka type experiments.

It is interesting to note that from the empirical point of view the effect is not entirely unknown. For example Rindler[16], in seeking to cite an evidence for the GSDC effect, remarks: "Indeed, owing to this effect, the US standard atomic clock kept since 1969 at the National Bureau of standards at Boulder, Colorado, at an altitude of 5400ft. gains about five microseconds each year relative to a similar clock kept at the Royal Greenwich Observatory, England, ...". However one can consciously undertake the project with all seriousness, for the accurate determination of the time-offset (with the error bars and all that), not merely to prove GSDC but to verify the Boughn-effect of SR. It is worth while to note that the empirical verification of this time-offset as a function of T would not only test the Boughn-effect and the integral effect of GSDC but it would also provide empirical support for the relativity of simultaneity [7] of SR. So far no experimental test has been claimed to be the one verifying the relativity of simultaneity. Indeed SR is applicable in the weak gravity condition of the earth so that gravity can be thought of as a field operating in the flat (Minkowskian) background of the space-time[30]. Clearly because of EP, the earth with its weak gravity has the ability to provide a convenient Laboratory to test some special relativistic effects like the relativity of simultaneity or the Boughn-effect.

[7] In the light of the CS-thesis however " relativity of simultaneity" loses its absolute meaning, since for example if absolute synchrony is used, there is no lack of synchrony between two spatially separated events as observed from different inertial frames, however, the differential aging or the temporal offset will pop up as a time dilation effect in the absolute synchrony set-up when the clocks are brought together by slow transport. The details of this issue is a subject matter of another paper by the authors in preparation.

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Chapter IV

Twin Paradox: A Classic Case of 'Like Cures Like'

4.1 Introduction

In an interesting article, S. P. Boughn[1] discussed a variation of the twin paradox parable where twins (P and Q say) on board two identical rockets (with equal amount of fuel), initially at rest a distance L apart in an inertial frame Σ, underwent identical accelerations for some time in the direction $\overrightarrow{PQ}$, and eventually came to rest (when all their fuels had expended) with another inertial frame Σ' moving with non-zero relative velocity v with respect to Σ. From the simple application of Lorentz transformation (LT) Boughn obtained a rather surprising result that in the new abode (Σ') the age of P became less than that of Q! Viewed differently, if the twins would carry presynchronized clocks, the outcome would have been a net time-offset effect between these clocks in Σ'.

The result is counter-intuitive by virtue of the fact that the twins of the parable throughout had identical local experiences yet their presynchronized clocks (also their own biological clocks) went out of synchrony!

Quantitatively this time-offset or desynchronization turns out to be

$$\delta t'_{desync} = -\gamma_v v L/c^2, \tag{4.1}$$

where the Lorentz factor

$$\gamma_v = (1 - v^2/c^2)^{-1/2}, \tag{4.2}$$

and c is the speed of light in free space.

The result can be seen to follow from the simple application of LT:

$$\begin{aligned} x_k' &= \gamma_v(x_k - vt_k), \\ t_k' &= \gamma_v(t_k - vx_k/c^2), \end{aligned} \tag{4.3}$$

where in the current context, t_k and x_k denote the time and space coordinates of the observer k (k stands for P or Q) with respect to Σ and the prime refers to the corresponding coordinates of the observers when they arrive and settle stationary in Σ' after their acceleration phases are over.

From the time transformation of Eq.(4.3) one obtains

$$t_Q{}' - t_P{}' = \gamma_v[(t_Q - t_P) - v(x_Q - x_P)/c^2]. \tag{4.4}$$

Assuming the clocks of the observers P and Q are initially synchronized i.e assuming $t_Q - t_P = 0$ (since the relative clock readings of P and Q should not change with respect ton Σ as they get identical acceleration for equal amount of time) and also substituting $x_Q - x_P = L$, which remains constant throughout their journeys, the desynchronization $\delta t'_{desync} = t'_Q - t'_P$ between these clocks (when they are at rest in Σ') is given by the expression (4.1). Obviously the above desynchronization corresponds to a differential aging of the twins in their new abode.

The apparently paradoxical result that the twins age differently in spite of their identical history of acceleration is readily explained if one notes that for spatially separated (biological) clocks the change of relative synchronization cannot have any unequivocal meaning. They can only be compared unambiguously when they are in spatial coincidence. For instance in Σ', one of the observers can slowly walk towards the other (or both of them can do the walking) and compare their ages (or their clock readings) when they meet. Since in the relativistic world the so called "slow transport synchronization" is equivalent to the Einstein synchronization[2, 3], the calculated differential aging or time-offset between their clocks when they were in spatial separation would continue to hold even when the twins meet after their slow walk. However in that case it can easily be seen[4] that they do not have symmetrical experiences, and hence the paradox gets resolved.

While the paradoxical element of the counter-intuitive outcome melts away, the fact remains that the differential aging for the "case of identically accelerated twins" given by Eq.(4.1) is *correct* and the time-offset can be verified at one spatial point if they slowly walk towards each other and compare their clocks (ages) when they meet[3]. Boughn in his paper claimed that the ordinary twin paradox could be explained in terms of this effect (which hereafter will be referred to as the Boughn effect (BE)). According to the parable of the ordinary twin paradox, Adam (A) stays at home on earth in a frame of reference Σ_0, while his traveller twin sister Beatrice (B) on board a fast rocket leaves earth with velocity v for a voyage to a distant star and subsequently turns around and then returns with the same speed v to meet her stay-at-home sibling to discover that they age differently. By applying time dilation formula (TDF) of SR on B's (biological) clock, A predicts that B should be younger on her return. The apparent paradox arises if B tries to apply the special relativistic TDF on A's clock (pretending that A is doing all the moving) and makes the contradictory claim that it is B who should be younger after the round-trip.

In this context Boughn observed that according to twin B, twin A would age less rapidly by a factor $1/\gamma$ during the entire trip. However, with obvious reference to the time-offset effect discussed earlier, Boughn further argued that because of acceleration at turn around, there would be a change in synchronization between the two twins' clocks. This change would overcompensate for the apparent slowdown in twin A's aging and finally twin A would be the older of the two. This was how both the twins could finally agree on their predictions.

Although there is no dearth of explanations of the canonical twin paradox in the literature (already about three hundred articles have been written on the subject[5]), it is still an interesting prospect to find a novel one where the pedagogical power of Boughn's paradox can be used to explain the usual twin paradox. However as outlined in the previous paragraph, the brief account given by Boughn himself to this end is only a qualitative one. Besides, Boughn's paradox refers to the time-offset between two twins whose spatial separation has been maintained constant throughout with respect to Σ. One may therefore wonder how this can be related to the desynchronization of twins (of the ordinary twin paradox), one of whom remains stationary while the other makes the round-trip? In this chapter we explain this, and show how Boughn paradox can be fruitfully used to resolve the usual twin paradox *quantitatively.* Indeed the actual demonstration of unequivocal prediction for differential aging from both the twins' perspectives by employing BE will be found to be a non-trivial exercise. As in the following one paradox is to be used to explain another, the present effort to explain the usual twin paradox may be looked upon as to correspond to a classic case of the proverbial "like cures like".

4.2 Coordinate Clocks and Time Dilation

The relativistic time dilation effect relates times of two different nature. One concerns the rate of ticking of a moving clock at its position and the corresponding time is known as the proper time (often denoted by τ) of the clock. The other refers to readings of spatially separated coordinate clocks (at rest with respect to some inertial frame of reference), as the concerned clock moves past these coordinate clocks. Time recorded by the coordinate clocks are therefore known as coordinate time which may be denoted by t. Note that, since the coordinate clocks are spatially separated, the coordinate time for a given pair of events depends on the synchronization convention (or the standard of simultaneity) adopted to synchronize these coordinate clocks. In SR we adopt the standard synchrony or Einstein synchrony according to which the one-way-speed of light is *stipulated* to be equal to its round trip speed. The proper time τ of a clock however is independent of any synchronization convention.

The standard relativistic TDF which connects τ and t is therefore valid provided the coordinate clocks are synchronized following Einstein's convention. According to the conventionality of simultaneity thesis[1] however, other quite equally valid synchronization schemes can be adopted but in that case the relativistic TDF will not be valid. Since in the twin paradox thought experiment, τ of one twin (clock) is "calculated" from the "knowledge" of the coordinate time elapsed in the other twin's frame of reference, one must ascertain the latter with great caution.

The genesis of the twin paradox lies in the failure to do so in the frame of reference attached to the traveller twin. Let us now clarify this. Consider the abrupt turn around scenario of the standard twin parable. Assume that the turn around of B takes place when the distance between the twins (with respect to Σ) measures L say. Now, just before the deceleration phase starts, one may consider another observer Alfred ($\bar{A}$) of the same age as that of Beatrice (i.e it is assumed that $\bar{A}$'s clock is synchronized with B's in Σ) and at same location of A comoving with respect to B such that, like in Boughn's scenario, $\bar{A}$ and B both undergo the same but arbitrarily large negative acceleration with respect to Σ, which moves with constant velocity v with respect to Σ_0. From Σ frame, B and $\bar{A}$ may be considered as Boughn's twins accelerating from rest along the negative x-direction (i.e now $\bar{A}$ is forwardly placed with respect to B) and settles in some inertial frame Σ' moving with velocity $-w$ (say) with respect to Σ (and $-v$ with respect to Σ_0).

BE therefore tells us that with respect to Einstein synchronized clocks in Σ', there is a desynchronization effect between the clocks (or ages) of Alfred and Beatrice,

$$t'_B - t'_{\bar{A}} = \delta t'_{desync} = \gamma_w w L/c^2, \tag{4.5}$$

[1] See for example[6, 7, 8]. For a more comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman[9].

which has been obtained from Eq.(4.1) replacing γ_v and v by γ_w and $-w$ respectively. Note that here

$$\gamma_w = (1 - w^2/c^2)^{-1/2} = (1 - v^2/c^2)/(1 + v^2/c^2), \tag{4.6}$$

where we have used

$$w = 2v/(1 + v^2/c^2). \tag{4.7}$$

The last relation of course follows from the relevant relativistic velocity addition law. Using the last two expressions in Eq.(4.5) one obtains

$$\delta t'_{desync} = 2v\gamma_v^2 L/c^2. \tag{4.8}$$

The above desynchronization also corresponds to a synchronization gap between the Einstein synchronized reference frames Σ and Σ'. The presence of this synchronization gap between instantaneously comoving inertial frames for an accelerated observer is the reason why such frames cannot be meshed together. Because of the (instantaneous) turn-around Beatrice switches her inertial frame and because of desynchronization, the clocks of Alfred and Beatrice no longer represent the Einstein synchronized coordinate clocks of Σ'. Instead of not turning around if Beatrice would continue to move forward covering the same length of journey with uniform speed as she would do after the turn-around, coordinate clocks (Einstein synchronized) of Σ frame of Beatrice could be used to measure the coordinate time and connect the same with the proper time of Albert through TDF for the entire trip. However if during the second phase of the trip someone playfully tamper with the synchronization, any coordinate time measurement following it will then be erroneous and hence a calculation to obtain the proper time τ_A of A from this measurement (by applying TDF on it) will give wrong result. In order to get the correct answer the remedy is to first undo the mischief by getting back to the Einstein synchronization that was adopted before and then one is free to use TDF in order to obtain proper time from the coordinate time. Let us now see what is the corresponding situation if we consider Beatrice's turn-around. In this case the second leg of Beatrice's journey corresponds to the inertial frame Σ'. The adoption of Einstein synchronization in this frame can be equated with the deliberate alteration of synchronization just discussed in connection with the uniform motion scenario of Beatrice, since the standard of simultaneity in Σ' is thus made different from that in Σ which corresponds to the earlier leg of Beatrice's trip.

It is clear that the proper time and coordinate time of a clock are connected by TDF provided the latter refers to a *uniform* synchronization. We then ask if there is any way so that one can continue with the standard of simultaneity (synchrony) of Σ in Σ'. The answer is in the affirmative and is provided by Boughn's thought experiment. From the symmetry of the problem it is evident that clocks of Alfred and Beatrice initially synchronized in Σ continue to remain synchronized with respect to Σ even when they arrive stationary in Σ' after the turn-around acceleration[2]. From B's perspective one can easily obtain the round-trip time τ_B in B-clock for A's journey (see later), but this does not correspond to the coordinate time for the same in Σ. Clearly a correction term $\delta t'_{desync}$, is to be added to τ_B to obtain the said coordinate time. This correction is equivalent to the process of restoration of the synchronization mentioned in Beatrice's non turn-around example.

[2]In other words as if the clocks A and B behave in an obstinate manner and refuse to be synchronized in the new frame according to standard synchrony automatically. The clock readings are to be tampered with in order to resynchronize them in Σ' according to the Einstein synchrony. If instead the clocks are left alone, these coordinate clocks then define absolute synchronization (see chapter II for discussions on CS-thesis and absolute synchronization) which Selleri refers to as "nature's choice"[10, 11].

4.3 Resolution

Before we proceed to provide the quantitative resolution of the twin paradox using BE , let us for convenience, remove the inconsequential initial and final accelerations from the problem. We thus assume that B makes a flying start and also after the return trip it flies past A. The only unavoidable acceleration that we keep is the one associated with B's turn-around without which A and B cannot compare their clocks (or ages) at one spatial point after the latter's round-trip. The resolution can now be laid down in the following steps:

Perspective of A:

Step 1:

The reciprocity of the relativistic TDF from the perspective of A and B can symbolically be expressed as,

$$TDF1: \qquad \Delta\tau_B(A) = \gamma_v^{-1}\Delta t_A(A), \tag{4.9}$$

$$TDF2: \qquad \Delta\tau_A(B) = \gamma_v^{-1}\Delta t_B(B). \tag{4.10}$$

In the above we follow a notation scheme, where $\Delta\tau_B(A)$ $[\Delta\tau_A(B)]$ denotes the B $[A]$-clock reading for a time interval between two events occurred at its position as inferred by the observer A $[B]$ drawn from its own coordinate clocks' records for the interval, $\Delta t_A(A)$ $[\Delta t_B(B)]$ and its knowledge of the relevant time dilation effect. Indeed the time intervals $\Delta\tau_B(A)$ or $\Delta\tau_A(B)$ are based on one clock measurements and hence they refer to proper times of B and A respectively.

Regarding the notations $\Delta t_B(B)$ or $\Delta t_A(A)$, a clarification is needed. While, for example $\Delta\tau_A(B)$ refers to the difference between one clock (A) reading for two events, $\Delta t_B(B)$ refers to in general, the observed difference in readings (for the same events) recorded in two spatially separated (synchronized) clocks stationary with respect to the frame of reference attached to B. However when $\Delta t_B(B)$ concerns measurement of the round trip time of an object or a clock (A say), it also refers to a single clock (B) measurement. Although τ-symbol would have been more appropriate in the later case but we shall continue to use the symbol 't' to emphasize that the corresponding time is supposed to be the coordinate time.

We now quote the relevant length contraction formula (LCF),

$$LCF: \qquad L = \gamma_v^{-1} L_0, \tag{4.11}$$

where L_0 is the distance of the distant star from the earth (measured in Σ_0) and L is the corresponding distance measured in Σ.

Step 2:

A-clock time for B's up and down travel of distance $2L_0$ is

$$\Delta t_A(A) = 2L_0/v, \tag{4.12}$$

and using the above result, the B-clock time for the same as calculated by A using TDF 1 (Eq.(4.9)) is

$$\Delta \tau_B(A) = \gamma_v^{-1} 2L_0/v. \tag{4.13}$$

Step 3:

Differential aging with respect to A is therefore given by

$$\delta t(A) = \Delta t_A(A) - \Delta \tau_B(A) = (1 - \gamma_v^{-1}) 2L_0/v. \tag{4.14}$$

Perspective of B:

Step 4:

From B's point of view, A makes the round trip and B measures the time for this trip as $\Delta t_B(B)$. This is nothing but the B-clock time as calculated by A, $\Delta\tau_B(A)$ which is given by Eq.(4.13). Hence

$$\Delta t_B(B) = \gamma_v^{-1} 2L_0/v. \tag{4.15}$$

This can also be seen in the following way. According to B, A travels a distance $\gamma_v^{-1}2L_0$ (using LCF Eq.(4.11)) for the round trip . The speed of A with respect to B is also v as LT honours the reciprocity of relative velocity. Hence the travel time $\Delta t_B(B)$ is again calculated as $\gamma_v^{-1}2L_0/v$.

Step 5:

The same time interval in A-clock as calculated by B by the *näive* application of TDF2 (Eq.(4.10) alone on $\Delta t_B(B)$) is obtained as,

$$\Delta\bar{\tau}_A(B) = \gamma_v^{-2} 2L_0/v. \tag{4.16}$$

This is however incorrect since desynchronization of distant clocks due to BE has not been taken into account and hence we have put a bar sign on τ, to be removed later after correction.

Step 6:

The above expression must be corrected by taking into account the BE. To calculate this effect we first split the frame of reference (K) attached to B into two inertial frames Σ and Σ' which move with velocities v and $-v$ respectively with respect to Σ_0. As discussed in Sec.(4.2), $\bar{A}$ and B separated by a length L in Σ after deceleration arrives in the final frame of reference Σ' producing a temporal offset (desynchronization) between their clocks which is given by Eq.(4.8)

$$\delta t'_{desync} = 2v\gamma_v^2 L/c^2 = 2v\gamma_v L_0/c^2, \tag{4.17}$$

where for the last equality we have made use of Eq.(4.11). Going back to Eq.(4.15), leading to Eq.(4.16) one now discovers that the application of Eq.(4.10) on $\Delta t_B(B)$

to obtain $\Delta\tau_A(B)$ is a mistake since, as has been explained in Sec.(4.2), the former does not represent the coordinate time as the coordinate clocks in K fail to remain synchronized according to the standard uniform synchronization scheme as B changes her inertial frame from Σ to Σ' for her turn around acceleration. This is the lesson we learn from BE. One therefore needs to add this desynchronization effect (Eq.(4.17)) to $\Delta t_B(B)$ before the application of TDF2 (given in Eq.(4.10))[3].

Adding $\delta t'_{desync}$ to $\Delta t_B(B)$ will undo the "resynchronization" (see footnote(2)) of clocks in Σ'[4] and hence the standard synchronization of coordinate clocks in Σ will be carried over in Σ' as well. This is indeed the precondition that ensures the applicability of the relativistic TDF.

Therefore the true coordinate time is obtained as

$$\Delta t_B^{coord}(B) = \Delta t_B(B) + \delta t'_{desync} = 2\gamma_v^{-1} L_0/v + 2v\gamma_v L_0/c^2. \tag{4.18}$$

Now applying TDF2 on the true coordinate time $\Delta t_B^{coord}(B)$, B calculates the round-trip time (proper) measured in A-clock as

$$\Delta\tau_A(B) = \gamma_v^{-1}(\gamma_v^{-1} 2L_0/v + 2v\gamma_v L_0/c^2). \tag{4.19}$$

Step 7:

Thus the differential aging from the perspective of B turns out to be,

$$\delta t(B) = \Delta\tau_A(B) - \Delta t_B(B) = (1-\gamma_v^{-1})2L_0/v, \tag{4.20}$$

which agrees with Eq.(4.14).

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[3]Whether one should add or subtract this desynchronization effect depends on its definition as $\delta t'_{desync}$ could have been defined as $t'_A - t'_B$, in which case one would need to subtract the effect for undoing the resynchronization.

[4]Resynchronization has been tacitly assumed in calculating $\Delta t_B(B)$ (see arguments following Eq.(4.15)) when reciprocity of relative velocity and LCF has been assumed to be valid in the frame of reference of B in her return journey.

4.4 Summary

Boughn has shown that two identically accelerated twins initially at rest with some inertial frame ages differently when they arrived stationary in another inertial frame (after their acceleration phases are over). Although the outcome is counter-intuitive (since in spite of the twins' accelerations being symmetric in every respect they age differently), the effect is an undeniable fact since it follows from SR. It has been remarked in the literature that ordinary twin paradox can be explained in terms of the paradox of the identically accelerated twins due to Boughn. Here we have taken up the issue and solved the usual twin paradox *quantitatively* using the Boughn paradox. We have considered the abrupt turn around scenario and the essence of the present approach to resolve the issue lies in recognizing the fact that the coordinate clocks (that of Alfred and Beatrice say) of Σ no longer represent the Einstein-synchronized coordinate clocks in Σ' after the turn around. Indeed these coordinate clocks of Σ carry over their synchronization convention in Σ', a lesson we learn from Boughn paradox. With these clocks the standard of simultaneity in Σ, according to Einstein convention (discussed in chapter II in the context of CS-thesis) is preserved in spite of their acceleration; in Σ' though, these clocks are not Einstein-synchronized. This departure from Einstein synchronization of the clocks is reflected in the Boughn effect.

Relativistic TDF can be used to calculate the proper time of A-clock from the coordinate time in the frame of reference attached to B provided the coordinate clocks represent uniform synchronization according to Einstein's scheme. It has been shown that the round trip time $\Delta t_B(B)$ of Adam as recorded by Beatrice's clock cannot represent the readings of the coordinate clocks of Beatrice's frame of reference having uniform synchronization and it has been explained how this can be corrected using the Boughn effect. Thus the genesis of the paradox lies in the mistake in the reasoning by Beatrice who näively use the TDF on $\Delta t_B(B)$ to draw inference regarding the proper time of Adam's clock. Once this is recognized the problem gets dissolved fully in the context of SR.

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Chapter V

Boughn Effect and Some Twin Paradoxes

5.1 Introduction

The relativistic time dilation (RTD) of a clock is governed by the so called Lorentz factor γ of the Lorentz transformation (LT), where $\gamma = (1 - v^2/c^2)^{-1/2}$ with v and c being the speeds of the clock and light in free space respectively. A counter-intuitive aspect of the RTD lies in its reciprocity. Simply stated, two observers in relative motion, will observe each other's clock to run equally slow compared to their own. This enigmatic feature of RTD is seen to be best exemplified in the posing of the so called twin paradox. In the standard twin parable, one twin Albert (A) stays on earth while the other Barbara (B) leaves the earth and travels in a fast rocket in uniform speed to a distant star. Subsequently she turns around and returns with the same speed to meet her stay-at-home sibling to find that Albert has aged more. This will happen since B's (biological) clock runs at a slower rate due to RTD caused by her speed required for the trip. The paradox comes about when B considers herself to be at rest and pretends that A is doing all the moving. Hence B can claim that A's clock should run slow and therefore expects that it is A who should be younger at their reunion after the trip. In order to resolve the paradox, it is often argued that the conditions experienced by the twins are asymmetric. While the earth bound twin may be considered to lie in an inertial frame[1], the rocket-bound one is a non-inertial observer by virtue of her direction reversing deceleration during the turn around. Thus since the rules of special relativity (SR) holds good only for an inertial observer, A's conclusion must be correct and therefore the traveller twin B will indeed be younger. Besides since there is a basic asymmetry between the motion of the siblings, one should not be surprised by the asymmetric outcome

[1] One ignores here the non-inertiality of the earth's frame due to the motion of the earth about the sun and also the former's spin about its axis.

concerning their ages after the trip. However, although correct, this qualitative argument explaining away the paradoxical element (asymmetric aging) of the twin problem may lead to a misconception that the turn around acceleration (or the force causing the acceleration) is the direct cause of the differential aging. In an interesting article Gruber and Price[1] have clarified the relationship of acceleration of the rocket twin and time dilation by giving an example of "no asymmetric aging" in spite of one of the twins arbitrarily large acceleration. There is also a converse situation as discussed by Boughn[2] in connection with an interesting variation on the twin paradox. It is shown therein that spatially separated twins can age differently although their history of acceleration remains the same. (In what follows, this counter-intuitive effect of SR first elaborately discussed by Boughn, will play one of the key roles in various issues to be addressed in this context. We here after will refer it to as the Boughn effect (BE)).

It follows therefore that the acceleration *per se* cannot be the direct cause of the asymmetric aging in the usual version of the paradox even as the turn-around acceleration of B is necessary for an unambiguous comparison of the ages of the twins at one location. (In some variation of the paradox even the turn-around acceleration is avoided by posing the problem in a closed universe setting[3, 4]).

It is now generally understood that at the heart of the paradox lies the question of relativity of distant simultaneity of SR. Indeed in a paradox, incorrect arguments are given at the time of posing the problem. The incorrect arguments come about when the time dilation formula of SR is freely used from the perspectives of both the twins and the question of applicability of the formulas with respect to the traveller twin who changes inertial frames (because of her turn around) is ignored. Indeed while standard relativistic time dilation formula (TDF) is correct within one inertial frame of Albert, the same formula does not hold with respect to the non-inertial frame attached to Barbara. In the abrupt turn-around scenario however, relativistic TDF is valid separately in the inertial frames of B in its outward and return journeys. But change of inertial frames by B produces another effect linked with the "relativity of simultaneity" of SR, in which the key to the resolution of the paradox lies. What happens when a change of reference frame takes place is best exemplified in the BE mentioned earlier.

It is clear that any resolution of the twin paradox must involve a demonstration of equal differential aging from the perspectives of A and B . From A's point of view there is only RTD, while from B's perspective, in addition to the time dilation of A's clock[2] this BE comes into play. The latter then more than compensates (to be discussed) for the time dilation of A's clock to produce the differential aging predicted from A's perspective, hence dissolving the paradox.

In this chapter we show that apparently situations can be created (or kinematical worlds can be constructed) where the usual time dilation effect of relativity is absent but BE exists. Paradoxes of different nature then comes into existence in the canonical twin problem scenario. All this along with its resolution and some more ramifications of the issue are discussed in sections(5.3), (5.4) and (5.5). As a bonus the reader will find in it a *quantitative* explanation of the ordinary twin paradox in terms of BE which to our knowledge has not yet been available in the literature. The last section (Sec.(5.6)) will provide the summary followed by some concluding remarks. In order to set the stage we will briefly reproduce in Sec.(5.2), the arguments due to Boughn, that will explain BE.

[2]In the text we have implicitly assumed that each observer (A, B, etc.) carries a clock so that the age of an observer also means the clock reading and hence terms like differential aging or time-offset between observers' clocks can be used interchangeably.

5.2 Boughn Effect and Twin Paradox

Boughn, as mentioned in the previous section, presents a form of twin paradox in SR where although the twins experience equal amount of acceleration for the same time, they age differently. According to Boughn's parable, twins P and Q on board two identical rockets, initially at rest at a distance x apart in an inertial frame S, start travelling with equal amount of acceleration for some time and eventually come to rest (when all their fuel has been expended) with respect to another inertial frame S' moving with a constant velocity v with respect to S along the positive x-direction.

Using the Lorentz transformation (LT), Boughn has obtained a counter-intuitive result that after the acceleration phase is over, the age of the forwardly placed (with respect to direction of acceleration) twin Q becomes more than that of P. The result is surprising since in the problem it has been assumed that P and Q throughout have identical experiences, but their presynchronized (biological) clocks go out of synchrony.

The amount of this desynchronization or the age difference can be quantified as follows:

Let us first write down LT

$$\begin{aligned} x_k' &= \gamma(x_k - vt_k), \\ t_k' &= \gamma(t_k - vx_k/c^2), \end{aligned} \tag{5.1}$$

where x_k and t_k refer to the space-time coordinates of the observer k (k stands for P and Q) with respect to a frame S and the prime refers to the corresponding coordinates in S'.

From the time transformation of Eq.(5.1), it follows that

$$t_Q' - t_P' = \gamma[(t_Q - t_P) - v(x_Q - x_P)/c^2]. \tag{5.2}$$

Consider an event simultaneous to P and Q, say a common birthday in S of P and Q so that $t_Q = t_P$. For this event putting $t_Q - t_P = 0$ in equation (5.2) one obtains,

$$t'_Q - t'_P = -\gamma vx/c^2, \tag{5.3}$$

implying in S', birthday of Q occurs before that of P and hence Q becomes older than P in their new abode (S') by the above precise amount. Hence the two clocks (twins) separated by a proper distance x that are synchronized in their rest frame S becomes unsynchronized by an amount

$$\delta t_{desync} = -\gamma vx/c^2, \tag{5.4}$$

after their arrival in S'.

The paradox associated with the fact that spatially separated twins, in spite of their identical history of accelerations with respect to S, age differently, can be solved as soon as one recognizes that age difference of two observers at different locations or time offset between two spatially separated clocks does not have any unequivocal meaning. Two clocks can only be compared unambiguously only at one spatial point; once that is done in Boughn's scenario after P and Q arrive in S' (for example P may walk towards Q for comparing their watches), the histories of their acceleration with respect to S then fail to be identical[5, 6, 7].

However the desynchronization δt_{desync} is real in the sense that it refers to desynchronization in relation to the Einstein convention of synchronization[3]. We have already called this departure from Einstein synchrony as BE.

Incidentally in a recent paper[6] we have shown how BE can account for the differential aging from the traveller's point of view resolving the paradox. That

[3] Incidentally Selleri[8] has noted in different words in this context that after identical acceleration, the two clocks readings define a natural (absolute) synchronization, which is different from Einstein's synchrony; the latter can only be established by resynchronizing them artificially (see also[9]).

has however been done in an indirect way by making an appeal to the principle of equivalence of general relativity. However, this is not necessary. Boughn paradox can directly be used to understand how BE can offset the reciprocal time dilation effect of A-clock with respect to the observer B, providing the correct asymmetric aging (that A is older than B) that one obtains from A's point of view. In this chapter we have just stopped short of showing this for the usual twin paradox, however we have provided a couple of templates for solutions in some other worlds, which the readers can make use of to complete the exercise.

We now proceed by noting that whenever the standard twin paradox is posed, the symmetrical time dilation effect of SR is only highlighted and a possible role of BE in contributing to the differential aging of the twins is suppressed. To understand the importance of BE more clearly we look for some possible contrasting scenarios where BE is readily evident while the other effect viz RTD of SR is either hidden or is truly absent. Surely as we shall see, this converse situation will again invite interesting paradoxes. All this will be the subject matter of the next section. These novel paradoxes and their resolutions will hopefully provide more clarity and insight into the century old twin paradox.

5.3 Separating RTD from BE and a Paradox

As we have indicated in the last section, the best way to highlight the importance of the role of BE in resolving the twin paradox is to remove the other effect viz RTD altogether from the twin problem. This can be conceived in the following way. We consider LT under small velocity approximation such that the world is essentially classical. When the relative velocity v between S and S' is very small compared to the speed of light c, such that v^2/c^2 terms can be neglected in comparison to unity, the so called Lorentz factor γ can be assumed to be 1. In this approximation the well-known relativistic effects like length contraction and time dilation are absent. This is in conformity with classical kinematics and hence in this small velocity regime the world is expected to be classical or non-relativistic. However contrary to the common belief (see for example[10, 11, 12]), for such small velocities, LT does not go over to Galilean transformation (GT), it becomes instead the so called approximate Lorentz transformation (ALT)[13, 14, 15, 16]:

$$\begin{aligned} x' &= (x - vt), \\ t' &= (t - vx/c^2), \end{aligned} \tag{5.5}$$

which expresses coordinates (x') and time (t') of S' in terms of those (x and t) of S. The inverse transformation under the same approximation is given by

$$\begin{aligned} x &= (x' + vt'), \\ t &= (t' + vx'/c^2). \end{aligned} \tag{5.6}$$

The above transformation equations, that have been obtained by putting $\gamma = 1$ in LT or in its inverse, represent the Galilean (classical) world since one may verify from the transformations (5.5) and (5.6) that moving rods do not contract and also there is no RTD effect for moving clocks. Relative velocity or velocity transformation formulas for small velocities are also Galilean in character. The only apparent non-Galilean feature of ALT lies in the space-dependent terms in the time transformations of (5.5) and (5.6)[4]. This is expected since in the relativity theory, synchronization of distant clocks in a given inertial frame is performed using light signal following the convention of standard synchrony (or the Einstein synchrony) according to which the one-way-speed (OWS) of light is assumed to be the same as its two-way-speed (TWS) along a given direction. It is well known that the so-called

[4]Note that these space-dependent terms cannot be dropped since for any preassigned small velocity, x or x' can be taken to be arbitrarily large so that vx/c^2 or vx'/c^2 may not be neglected.

"relativity of simultaneity" of special relativity is the direct result of this "stipulation" of equality of TWS and OWS of light. It is understandable that a mere small velocity approximation cannot alter this conventional ingredient embedded in LT. Indeed one can verify by simple kinematical calculation that ALT still represents Einstein synchrony[14].

One recalls that when the time dilation effect of relativity is obtained from LT, the moving clock is assumed to be at a fixed position (say at $x' = 0$) in some inertial frame S', which moves with velocity v with respect to the observer's frame S and hence the term vx'/c^2 of LT does not have any contribution to the time dilation effect. However when it comes to the question of resolution of the paradox the role of this term, which is linked to clock synchronization issue, has to be brought into fore.

Now the world described by ALT, is clearly a non-relativistic one and if one ignores its history[5], there is no trace of time dilation of clocks-in-motion and hence apparently the twin paradox should not exist. However one who is aware of BE which is linked to the phase terms in ALT, discovers a differential aging from the perspective of Barbara although Albert does not expect Barbara's (biological) clock to go slow. Let us see how this contradiction comes about a bit more clearly.

Consider the abrupt turn around scenario of the standard twin parable. Assume that the turn around of B takes place when the distance between the twins (with respect to S') measures L' say. Now, just before the deceleration phase starts, one may consider another observer Alfred ($\bar{A}$) of the same age as that of Barbara (i.e it is assumed that $\bar{A}$'s clock is synchronized with B's in S') and at same location

[5] Here the equations have been obtained by putting $\gamma = 1$ in LT or its inverse. This could have been obtained under the same approximation from Zahar transformation[17] (see later) which describes a classical world with Einstein synchrony[9, 18, 19, 20]

of A comoving with respect to B such that, like in Boughn's scenario, $\tilde{A}$ and B both undergo the same but arbitrarily large negative acceleration with respect to S', which moves with constant velocity v with respect to S. From S' frame, B and $\tilde{A}$ may be considered as Boughn's twins accelerating from rest along the negative x-direction (i.e now $\tilde{A}$ is forwardly placed with respect to B) and settles in some inertial frame S'' moving with velocity $-w$ (say) with respect to S' (and $-v$ with respect to S).

BE therefore tells us that with respect to Einstein synchronized clocks in S'', there is a desynchronization effect between the clocks (or ages) of Alfred and Barbara,

$$t''_B - t''_{\tilde{A}} = \delta t_{desync} = \gamma_w w L'/c^2, \tag{5.7}$$

which has been obtained from Eq.(5.4) replacing γ, v and x by γ_w, $-w$ and L' respectively. Note that here $\gamma_w = (1 - w^2/c^2)^{-1/2} \approx 1$ and $w = 2v/(1 + v^2/c^2) \approx 2v$ in the classical regime. Hence

$$\delta t_{desync} \approx 2vL/c^2, \tag{5.8}$$

since there is no length contraction effect.

The above desynchronization also corresponds to a synchronization gap between the Einstein synchronized reference frames S' and S''. The presence of this synchronization gap between instantaneously comoving inertial frames for an accelerated observer is the reason why such frames cannot be meshed together. Because of the (instantaneous) turn-around Barbara switches her inertial frame and because of desynchronization, the clocks of Alfred and Barbara no longer represent the Einstein synchronized coordinate clocks of S''. Instead of not turning around if Barbara would continue to move forward covering the same length of journey with uniform speed as she would do after the turn-around, coordinate clocks (Einstein synchronized) of S' frame of Barbara could be used to measure the coordinate time and connect the same with the proper time of Albert through TDF for the entire trip. However if during the second phase of the trip someone playfully tamper with the synchronization, any coordinate time measurement following it will then be erroneous and hence a calculation to obtain the proper time τ_A of A from this measurement (by applying TDF on it) will give wrong result. In order to get the correct answer the remedy is to first undo the mischief by getting back to the Einstein synchronization that was adopted before and then one is free to use TDF in order to obtain proper time from the coordinate time. Let us now see what is the corresponding situation if we consider Barbara's turn-around. In this case the second leg of Barbara's journey corresponds to the inertial frame S''. The adoption of Einstein synchronization in this frame can be equated with the deliberate alteration of synchronization just discussed in connection with the uniform motion scenario of Barbara, since the standard of simultaneity in S'' is thus made different from that in S' which corresponds to the earlier leg of Barbara's trip.

It is clear that the proper time and coordinate time of a clock are connected by TDF provided the latter refers to a *uniform* synchronization. We then ask if there is any way so that one can continue with the standard of simultaneity (synchrony) of S' in S''. The answer is in the affirmative and is provided by Boughn's thought experiment. From the symmetry of the problem it is evident that clocks of Alfred and Barbara initially synchronized in S' continue to remain synchronized with respect to S' even when they arrive stationary in S'' after the turn-around acceleration. From B's perspective one can easily obtain the round-trip time τ_B in B-clock for A's journey (see later), but this does not correspond to the coordinate time for the same in S'. Clearly a correction term δt_{desync}, is to be added to τ_B to obtain the said coordinate time. This correction is equivalent to the process of restoration of the synchronization mentioned in Barbara's non turn-around example. Assuming the relevant time dilation factor to be unity, the proper time of A-clock obtainable from the coordinate time, automatically gets modified by the same amount. Thus Barbara should predict the proper time τ_A of A after the round trip to be $\tau_B + \delta t_{desync}$ instead of just τ_B. The correction term to τ_B is the result of BE, which exists even if RTD is ignored. Note that this is a "distance" effect[6] and takes place just after the turn around and this is happening since we have assumed that the clocks of both S' and S'' are synchronized following Einstein synchrony. On the other hand if we had adopted absolute synchrony in the Galilean world directly (and not by taking the small velocity approximation of LT preserving the Einstein synchrony) one does not have to deal with the space-dependent term in the time transformation (since in this case $t' = t$). Summarizing one observes that since under the small velocity approximation, one essentially deals with a classical world, B's clock does not run slow compared to A and hence A does not predict differential aging. But since relativity of simultaneity is preserved in the approximation, although A's clock does not run slow with respect to B's clock (as we have put $\gamma = 1$) B predicts a differential aging due to BE of amount $2vL/c^2$ which can be made arbitrarily large by increasing B's length of journey for any preassigned small uniform velocity v of B. Time dilation of clocks in SR refers to the *rate* of ticking of clocks in relative motion, on the other hand BE is the result of an offset of the initial setting of spatially separated clocks when an observer (in this case Barbara) changes from one inertial frame to another. Resolution of the standard twin paradox depends on a beautiful interplay of these two relativistic effects — one overcompensating the other, so that both the twins finally agree on their age

[6]For this reason the initial acceleration of Barbara at the time of departing from Albert and the final deceleration required to reunite with Albert do not have any BE since in those phases of Barbara's trip the distance of separation between them tends to zero.

difference.

In the present setting apparently there is no time dilation effect that the differential aging due to BE predicted by Barbara is required to be balanced! This results in contradicting claims by Albert and Barbara regarding their ages (Albert does not predict any age difference, which Barbara disagrees) signifying a paradox.

The fallacy hinges on the fact that although the time dilation factor γ can be assumed to be arbitrarily close to unity, BE can be made arbitrarily large by increasing the length of the trip. The resolution of the paradox however is not a difficult job. We should first recognize that the problem arises as we are comparing two relativistic effects of different nature. While, as we have observed the RTD effect refers to clock rate, BE refers to the time-offset, which is an integrated effect. Indeed if one increases L arbitrarily the integrated effect of time dilation of B's (biological) clock leading to differential aging with respect to A also becomes arbitrarily large. Hence although $\gamma \approx 1$ in the approximation, the accumulated effect of time dilation of B's clock cannot be neglected. The remedy of the problem therefore lies in not neglecting the time dilation effect in the first place. All this therefore suggests that in order to ascertain unequivocal differential aging, ALT will not work and hence one should get back to the full LT; then only A also predicts a differential aging (for large L) in spite of arbitrarily slow trip of B.

5.4 Yet Another!

Surprisingly the problem does not end here with the suggested remedy. There are deeper questions and the fallacy seems to persist. To understand this we begin by asking what happens if ALT is obtained via a different route? For example consider the Zahar transformation (ZT),

$$x' = (x - vt),$$
$$t' = \gamma^2(t - vx/c^2). \tag{5.9}$$

As indicated in the footnote number(5), the above transformation represent a classical (Galilean) world with Einstein's synchrony[7]. This Galilean or classical world is supposed to be endowed with a preferred (ether) frame S, where light propagation is assumed to be isotropic. In any other frame S', however it will not be so. The TWS of light will be different in different directions in S' as one would expect in a classical world. Note that with respect to S, as expected, the moving rods do not contract and clocks in motion do not run slow. The effect of Einstein synchrony however is manifested through the phase term of the time transformation of Eq.(5.9) with consequent apparent length contraction and time dilation effects with respect to S'.

In the small velocity regime even this apparent length contraction and time dilation effects which are artifacts of the Einstein synchrony go away and one obtains the approximate Zahar transformation (AZT)[14, 20] which is the same as ALT.

Now, as before, in this classical world there is BE with respect to Barbara but there is no time dilation effect with respect to Albert. But the problem here is that there is no time dilation of moving clocks with respect to Albert is not an approximate result, hence there is no possibility of non-null differential aging from A's perspective, that can compete (as has been the case in the relativistic world) with BE. It therefore appears that the resolution of the problem for the world described by ALT, as discussed in the preceding paragraph, falls through in the case of AZT although algebraically the latter is the same as ALT, only their histories

[7] See chapter II for detailed discussions of the CS-thesis in both classical (due to Zahar) and relativistic worlds.

are different.

The answer to this paradox lies in the details of the workings of some effects similar to the relativistic ones (such as time dilation and length contraction effects and BE) from the perspectives of both the twins. If done properly (using the full transformation equation) both A and B will agree on their predictions, no matter whether the world is classical or relativistic. This we will not do here. However one will be able to verify it by following the steps outlined in the next section.

5.5 What is Wrong?

We begin this section by asking what is wrong with the transformation Eq.(5.5) representing ALT (i.e the same as AZT). Cannot it by itself (not as an approximation of LT) represent even a hypothetical kinematic world with its characteristic (whatever) behavior of moving rods and clocks and synchronization scheme? A mathematical transformation can lead to results, which may not be supported by the empirical world, but here the mathematical consistency of the Eqs.(5.5) and (5.6) seems to be at stake.

To understand this, it is enough to note that Eq.(5.5) itself may represent a hypothetical world (kinematical) but Eq.(5.6) is not its inverse, although the latter has been obtained as an approximation of LT representing transformation of coordinates of S' in term of those of S.

The inverse of Eq.(5.5) instead is given by

$$\begin{aligned} x &= \gamma^2(x' + vt'), \\ t &= \gamma^2(t' + vx'/c^2). \end{aligned} \tag{5.10}$$

We shall show below that not only algebraically, but also from the twin paradox point of view Eq.(5.5) and Eq.(5.10) represent a consistent kinematical world (World

1).

Similarly if one starts with Eq.(5.6), the corresponding inverse transformation would have been

$$x' = \gamma^2(x - vt),$$
$$t' = \gamma^2(t - vx/c^2). \tag{5.11}$$

Clearly the pair of transformations (5.11) and (5.6) represent another kinematical world (World 2) different from world 1. In this case also observers A (stationary in S) and B can be shown to agree in their predictions of the differential aging.

We show below step by step how do the twins living in world 1 and 2 make unequivocal predictions regarding their age differences. As mentioned in the last section, the reader may follow these steps as a template to resolve the twin paradox put in different worlds including the usual relativistic one. We start by considering world 2 first.

World 2

Step 1:

The transformation equations (5.6) and (5.11) suggest that A (an observer in S) does not predict any time dilation effect, however B predicts a time dilation for a clock stationary in S. These two observations may be summarized as follows:

Time Dilation Formulas:

$$TDF1: \qquad \Delta t_B(A) = \Delta t_A(A), \tag{5.12}$$

$$TDF2: \qquad \Delta t_A(B) = \gamma^{-2}\Delta t_B(B). \tag{5.13}$$

In the above we have used a notation scheme where $\Delta t_B(A)$ $[\Delta t_A(B)]$ denotes the B $[A]$-clock reading for a time interval between two events occurred at its position as inferred by the observer A $[B]$ drawn from its own coordinate clocks' records for the interval, $\Delta t_A(A)$ $[\Delta t_B(B)]$ and its knowledge of the relevant time dilation effect. Indeed the time intervals $\Delta t_B(A)$ or $\Delta t_A(B)$ are based on one clock measurements and hence they refer to proper times of B and A respectively.

As regards the notations $\Delta t_B(B)$ or $\Delta t_A(A)$, a clarification is needed. While, for example $\Delta t_A(B)$ refers to the difference between one clock (A) reading for two events, $\Delta t_B(B)$ refers to in general, the observed difference in readings (for the same events) recorded in two spatially separated (synchronized) clocks stationary with respect to the frame of reference attached to B. However when $\Delta t_B(B)$ concerns measurement of the round trip time of an object or a clock (A say), it also refers to a single clock (B) measurement.

Similarly one has two length contraction formulas (LCF) from the perspective of A and B. These formulas which follow from the transformation equations (for space) (5.6) and (5.11) may be written as

$$LCF1: \qquad L_B(A) = \gamma^{-2} L_B(B), \tag{5.14}$$

$$LCF2: \qquad L_A(B) = L_A(A). \tag{5.15}$$

Where $L_A(A)$ and $L_B(B)$ are the rest lengths of rods in S and S' respectively and $L_B(A)$ and $L_A(B)$ are the corresponding observed lengths from the other frames (A and B respectively). However we shall have no occasion to use Eq.(5.14) since the only distance of interest is that of the distant star from A which is clearly a rest length in A i.e. $L_A(A)$. Hence

$$L_A(A) = L, \qquad L_A(B) = L', \tag{5.16}$$

according to our definitions of L and L'.

Step 2:

A-clock time for B's up and down travel of distance $2L$ is

$$\Delta t_A(A) = 2L/v, \tag{5.17}$$

and using the above value for $\Delta t_A(A)$ the B-clock time for the same as calculated by Albert using relevant time dilation formula (Eq.(5.12)) is obtained as

$$\Delta t_B(A) = 2L/v. \tag{5.18}$$

Step 3:

Differential aging with respect to A is therefore given by

$$\delta t(A) = \Delta t_A(A) - \Delta t_B(A) = 0. \tag{5.19}$$

The following steps will lead to the same (null) differential aging from B's perspective.

Step 4:

From Barbara's point of view, A makes the round trip and Barbara measures the time for this trip as $\Delta t_B(B)$. This is nothing but the B-clock time as calculated by Albert, $\Delta t_B(A)$ which is given by Eq.(5.18) Hence

$$\Delta t_B(B) = 2L/v. \tag{5.20}$$

This can also be seen in the following way. According to B, A also travels a distance $2L$ for the latter's round trip as there is no length contraction effect with respect to B (see Eq.(5.15)). The speed of A with respect to B is also v as the transformation equations (5.6) and (5.11) honour the reciprocity of relative velocity. Hence the travel time $\Delta t_B(B)$ is again calculated as $2L/v$.

Step 5:

The same time interval in A-clock as calculated by B by the *näive* application of TDF2 (Eq.(5.13) alone on $\Delta t_B(B)$) is obtained as,

$$\Delta \bar{t}_A(B) = \gamma^{-2} 2L/v. \tag{5.21}$$

This is however incorrect since desynchronization of distant clocks due to BE has not been taken into account and hence we have put a bar sign on t, to be removed later after correction.

Step 6:

The above expression must be corrected by taking into account the BE. To calculate this effect we first split the frame of reference (K) attached to B into two inertial frames S' and S'' which move with velocities v and $-v$ respectively with respect to S. Clearly B is at rest with these frames in its onward and return journeys.

Writing the transformation equations connecting the space-time coordinates of S and S'' as

$$\begin{aligned} x'' &= \gamma^2(x + vt), \\ t'' &= \gamma^2(t + vx/c^2), \end{aligned} \tag{5.22}$$

one readily obtains transformation equations between S' and S'' as

$$\begin{aligned} x'' &= \gamma_w(x' + wt'), \\ t'' &= \gamma_w(t' + wx'/c^2), \end{aligned} \tag{5.23}$$

where

$$w = 2v/(1 + v^2/c^2), \tag{5.24}$$

represents the relative speed of S'' with respect to S' and

$$\gamma_w = (1 - w^2/c^2)^{-1/2} = (1 + v^2/c^2)/(1 - v^2/c^2). \tag{5.25}$$

As discussed earlier, Alfred ($\bar{A}$) and Barbara separated by a length L' in S' after deceleration arrives in the final frame of reference S'' producing a temporal offset (desynchronization) between their clocks which is given by (obtained from transformation for time in Eq.(5.23))

$$\delta t_{desync} = \gamma_w w L'/c^2 = \gamma_w w L/c^2, \tag{5.26}$$

The last equality follows from Eqs.(5.15) and (5.16) which states that $L' = L$.

Step 7:

Going back to Eq.(5.20), leading to Eq.(5.21) one now discovers that the application of Eq.(5.13) on $\Delta t_B(B)$ to obtain $\Delta t_A(B)$ is a mistake (which has been pointed out earlier, see Sec.(5.3)) and one needs to add this desynchronization effect (Eq.(5.26)) to $\Delta t_B(B)$ before the application of TDF2 (Eq.(5.13)). Having done so and reapplying TDF2, one should obtain, removing the bar sign on Δt in Eq.(5.21),

$$\Delta t_A(B) = \gamma^{-2}(2L/v + \gamma_w w L/c^2). \tag{5.27}$$

Which by using Eqs.(5.24) and (5.25) gives

$$\Delta t_A(B) = 2L/v. \tag{5.28}$$

This again leads to the null differential aging from B's perspective as well,

$$\delta t(B) = \Delta t_A(B) - \Delta t_B(B) = 0. \tag{5.29}$$

thus resolving the paradox.

World 1

Step 1:

Transformation equations (5.5) and (5.10) representing World 1 gives the following results for time dilation and length contraction:

$$TDF1: \qquad \Delta t_B(A) = \gamma^{-2}\Delta t_A(A), \tag{5.30}$$

$$TDF2: \qquad \Delta t_A(B) = \Delta t_B(B), \tag{5.31}$$

$$LCF1: \qquad L_B(A) = L_B(B), \tag{5.32}$$

$$LCF2: \qquad L_A(B) = \gamma^{-2}L_A(A). \tag{5.33}$$

According to our definitions of L and L' one can write the last equation as

$$L' = \gamma^{-2}L. \tag{5.34}$$

Step 2:

A-clock time for B's up and down travel of distance $2L$ is as before

$$\Delta t_A(A) = 2L/v, \tag{5.35}$$

and the same recorded in B's clock as interpreted by A can be obtained by applying TDF1 (Eq.(5.30)) on $\Delta t_A(A)$. Hence

$$\Delta t_B(A) = \gamma^{-2}2L/v. \tag{5.36}$$

Step 3:

From the last two relations, the differential aging with respect to A now comes out to be

$$\delta t(A) = \Delta t_A(A) - \Delta t_B(A) = 2Lv/c^2. \tag{5.37}$$

Step 4:

With respect to B, A travels a distance $2L'$ for its round-trip with speed v hence, the time recorded in B's clock for A's round-trip is given by

$$\Delta t_B(B) = 2L'/v = \gamma^{-2}2L/v. \tag{5.38}$$

where we have made use of Eq.(5.34).

Step 5:

B may try to calculate the corresponding time as recorded by A by näively applying only TDF2 given by Eq.(5.31) on $\Delta t_B(B)$ and obtains,

$$\Delta \bar{t}_A(B) = \gamma^{-2}2L/v. \tag{5.39}$$

Step 6:

As mentioned before the above expression is incorrect (hence we have put the bar sign on Δt) since BE has not been taken care of. To calculate this effect we again split the reference frame K attached to B into two inertial frames S' and S'' representing the inertial frames of B in its forward and return journeys. The transformation equations between coordinates of S' and S'' remain the same as that in world 2.

$$\begin{aligned} x'' &= \gamma_w(x' + wt'), \\ t'' &= \gamma_w(t' + wx'/c^2), \end{aligned} \tag{5.23}$$

where w and γ_w are as defined by Eqs.(5.24) and (5.25).

Hence following the previous arguments leading to Eq.(5.26), we have

$$\delta t_{desync} = \gamma_w w L'/c^2, \tag{5.40}$$

which after using Eq.(34) can be written as

$$\delta t_{desync} = \gamma_w w \gamma^{-2} L/c^2. \tag{5.41}$$

Step 7:

Following arguments given in step 7 (world 2) but now using Eq.(5.31) after making the correction due to Boughn effect, we find that

$$\Delta t_A(B) = \Delta \bar{t}_A(B) + \gamma_w w \gamma^{-2} L/c^2 = \gamma^{-2}(2L/v + \gamma_w w L/c^2) = 2L/v. \tag{5.42}$$

Hence

$$\delta t(B) = \Delta t_A(B) - \Delta t_B(B) = 2L/v - \gamma^{-2} 2L/v = 2Lv/c^2, \tag{5.43}$$

which is the same as $\delta t(A)$. Hence in this case also the paradox does not exist.

5.6 Summary and Concluding Remarks

According to BE two presynchronized clocks that accelerate identically from one inertial frame to another along the direction of their spatial separation should get desynchronized. It has been remarked in the literature that the standard twin paradox can be explained in terms of this effect which, if taken care of properly, may be seen to overcompensate for the apparent slowing down of clocks of the stay-at-home twin with respect to that of the traveller one[2]. However the actual demonstration of unequivocal prediction for differential aging by both the twins using BE seems to be a non-trivial exercise. It is however clear that the answer to the twin paradox depends on an interesting interplay of two special relativistic effects viz RTD and BE.

In a bid to isolate the role of BE in the standard twin paradox one may try to study it in the classical (small velocity) regime where γ, hence the time dilation factor is assumed to be unity. In this regime, since relativity of distant simultaneity persists, BE continues to take part in the twin problem. But since one does not expect any time dilation, one ends up with a new fallacy. Since now as if Albert predicts no differential aging (in absence of RTD) which Barbara contradicts because of her knowledge of BE. We thus have a converse situation here. Recall that in posing the usual twin paradox, one emphasizes on the RTD effect only but BE is overlooked. But now in the new problem, one highlights BE and RTD is ignored. The paradox however is a mild one which gets resolved as soon as one takes into account the integrated effect of time dilation which was previously ignored under the small velocity approximation ($v^2/c^2 << 1$). However the fallacy reappears if one considers ZT which describes a classical world with Einstein synchrony. In this world there is no time dilation and length contraction effects with respect to the preferred frame S from the beginning. In the small velocity regime though, the transformation equations are the same as ALT. The question then arises as to what, in absence of RTD (from Albert's perspective) can compensate BE which still exists in the classical world because of adopted synchrony.

The answer lies in the details of the workings of the transformation equations in producing time dilation effects and BE from *both the twins'* perspectives. Here we have worked out these details for two mathematically consistent hypothetical kinematical worlds to show that the twins in any case make unequivocal predictions.

We end chapter by briefly addressing a much talked about issue regarding the often made claim in the literature that the full solution of the twin paradox lies in the realm of general relativity (GR)[21, 22, 23, 24, 25, 26, 27]. As correctly pointed out by Builder[28], it is indeed strange to first deny by some authors the applicability of SR in the resolution of the twin paradox and then use conclusions derived from SR itself by means of the principle of equivalence of GR. The essence of any general relativistic solution of the problem lies in introducing an equivalent pseudo gravitational potential to be experienced by the traveller twin at the time of her direction reversing acceleration. A consequent gravitational time offset effect then provides the extra aging of the stay-at-home twin required to make the correct prediction by Barbara. Now, since as we have mentioned that BE can directly be used to resolve the paradox, the use of pseudo-gravitational field, to explain the problem of equivocal prediction of differential aging by the twins must be a trivial exercise. After all no true gravitational field exists in the problem; hence in order to resolve the issue, introduction of GR in the essentially flat space-time (with vanishing Riemann tensor) is utterly misleading[5, 28, 29, 30]. This conclusion now is strengthened by the fact that twin paradoxes can be devised, as has been shown earlier, in some hypothetical kinematical worlds characterized by the existence of BE, which in turn is an outcome of relativity of distant simultaneity. The unequivocal predictions of differential aging (or its absence) by the twins of these worlds can be explained by appropriate use of this non-special relativistic BE in addition to the time dilation effects. It has been shown elsewhere by the present authors[6] that the gravitational time offset-effect of GR (in the case of uniform gravity) follows from the equivalence principle provided one uses the full machinery of SR and is therefore essentially special relativistic in origin. On the other hand the two worlds discussed in the last section are only theoretical constructs hence it is not possible to replace BE of these hypothetical worlds by equivalent gravitational fields which may act as a "physical agent" responsible for producing the extra aging of the stationary sibling. Yet we have seen how the resolution of the paradox comes about from purely kinematical considerations.

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Part II

Chapter VI

Relativity in "Cosmic Substratum" and the UHECR Paradox

6.1 Introduction

The possibility that Lorentz invariance can be violated in nature has currently become a subject of interest. People often doubt if the special relativity (SR) is only an approximate symmetry of nature[1, 2]. To give a quantitative measure of Lorentz-invariance violation (LIV), one can build up a test theory where the Lagrangian of electrodynamics can be slightly deformed by adding to it a tiny Lorentz violating term. One such deformation considered by the authors of Ref.[1] (see also[3]) following standard practice causes the speed of light c to differ from the maximum attainable speed c_0 (which hereafter, unless stated otherwise, will be assumed to be equal to 1) by a small velocity parameter ϵ of the theory. The obvious consequence of this consideration is the existence of a preferred inertial frame of reference.

It is a common practice and also reasonable to assume this preferred frame to be "the rest frame of the universe" (Σ_0) with respect to which the cosmic microwave background radiation (CMBR) is isotropic. Let us call it the rest frame of the cosmic substratum (RFCS).

Precision tests for anisotropies in velocity of light due to the motion of the solar system relative to the CMBR frame have set a limit on this ϵ[1, 4],

$$|1 - c| = |\epsilon| < 3 \times 10^{-22}. \tag{6.1}$$

However it has been argued[1, 2, 3] that stronger constraints on ϵ can be obtained, not from precision tests, but from observations on ultra high energy cosmic rays (UHECR). For example, if $c < 1$ it has been shown that the mere detection of primary proton energy up to 100 EeV set the bound on ϵ more than one order of magnitude stronger:

$$|\epsilon| < 5 \times 10^{-24}. \tag{6.2}$$

The physical basis for obtaining such a bound is that a particle can be super luminal in vacuum (if $c < 1$) in which case, a proton being a charged particle will in its passage, quickly lose energy through the so called "vacuum Cerenkov radiation" and will therefore fail to be detected with the super luminal speed. The last bound on $|\epsilon|$ is obtained by equating the speed of proton at 100 EeV with the speed of light c, then subtracting it from unity, the latter being the limiting speed of SR. The limit on ϵ thus obtained does not require (unlike the way it is obtained through precision test mentioned before) any assumption regarding the motion of the laboratory frame with respect to Σ_0.

LIV is also much discussed in connection with one of the most puzzling paradoxes in physics concerning UHECRs. One quite robust predictions of special relativity is the existence of the so called Greisen-Zatsepin-Kuzmin (GZK) phenomenon, which tells us that cosmic ray protons coming from cosmological distances with energies above certain limiting value (GZK cutoff), should not be observed on Earth. The predicted value for this catastrophic cutoff is $5 \times 10^{19} eV$. This value corresponds to the threshold energy for photo-pion production by cosmic ray protons interacting with soft CMBR photons which pervades the universe. However some recent experiments have shown that this relativistically calculated threshold energy seems to be too low. Indeed recently ground based detectors have detected over about a hundred events near and above the GZK cutoff and a double digit number of events with energies at or above $10^{20} eV$. The highest energy cosmic ray so far has been the $3.2 \times 10^{20} eV$ detected by the Fly's Eye air shower detector in Utah[5]. However if the sources of UHECRs are really extragalactic (there are ample reasons to believe so[6]) and since the calculation of GZK limit is so robust that even one event at $10^{20} eV$ "appears surprising"[7]. The arrival of UHECR on Earth with energies above the GZK threshold is known as the UHECR paradox[8, 9, 10] mentioned in the beginning of this paragraph.

There have been exotic proposals in the literature which try to explain the trans-GZK cosmic ray events in the framework of LIV theories which assume the existence of a preferred frame[2, 9, 11]. Let us call them preferred frame theories. As an example, according to one most popular scenarios[12], existence of different maximal speeds for different particle species is assumed and they are also assumed in general to differ from the speed of light in vacuo [see Ref.[2] and references therein]. In this way, introduction of small LIV has been shown to have effects that increase rapidly with energy in such a manner that ultimately inelastic collisions with CMBR photons become kinematically forbidden[2].

However there are other class of theories known as the doubly special relativistic (DSR) theories which consider deformation of relativistic dispersion relations for photons and massive particles. Although cosmic ray paradox primarily provides encouragement for such theories, the revision of dispersion relation is often motivated from quantum-gravity considerations, according to which a fundamental length or energy scale (plank length or plank energy) should play a role[8]. DSR theories try to avoid the preferred frame issue prompted by the introduction of such scales in the theory (since length and energy are frame dependent quantities) by introducing the notion of an invariant length or energy scale in addition to the constant c of the usual relativity theory. DSR theories therefore formulates the postulates of SR in ways in order to introduce observer independent length or energy scales. Although one[13] or the other[14] forms of DSR theories are interesting and intellectually satisfying, these are still in a preliminary stage, in so far as their efficacy in solving the threshold anomaly is concerned[10]. In any case, the cosmic ray paradox provides ample reasons for new alternatives to the standard relativity theory. Indeed if the identification of UHECR as protons produced by distant active galaxies eventually turns out to be absolutely correct, one of the varieties of DSR theories or that of the preferred frame ones mentioned earlier can be strong contenders as the candidates describing new physics. The present chapter proposes a theory of the latter variety with a very different flavor. It will be shown that the velocity of the solar system with respect to the rest frame of the universe might play a role in explaining the paradox.

In an effort to look for new physics, when one considers theories involving LIV one still believes that behavior of moving rods and clocks is still governed by the Lorentz transformation (LT) however other laws of physics might not strictly remain covariant under LT. For example one may consider the possibility that causal cone need not coincide with the light cone[15], i.e the speed of light may not be the same as the invariant speed "c" of LT.

However if one is prepared to do away with the principle of relativity, or in otherwards if one believes in the existence of a preferred inertial frame, there is no point in holding on to the belief that standard rods and clocks of different inertial frames behave strictly according to LT. Note that after all LT is a consequence of the relativity principle[1].

Hence in search for a new physics one may consider the possibility of a deformed LT (not just a deformed dispersion relation) to relate observations performed by different inertial observers.

Once such a transformation is guessed, other aspects of kinematics such as expressions for momentum $\mathbf{p}$ and energy E of a particle or the dispersion relation can

[1]The relativity principle turns out to be a sufficient condition for LT (if coordinate clocks are synchronized by light signal following Einstein's convention), however it is not a necessary condition. In other words LT may describe the kinematical world even if the principle is seen to be violated in other realms of physics. But here we emphasize that if the relativity principle is sacrificed, LT loses its very foundation.

be obtained through a kind of 4-vector formulation (see below).

Clearly the predictions of the deformed LT will be different from those of the relativity theory. However the difference in the predictions must be undetectable in the domain where special relativity has been tested beyond doubt.

In the following we shall look for such a transformation that will be capable to explain the UHECR paradox and at the same time will be able to reproduce the standard relativistic results. We know that Einstein obtained his transformations deductively from his relativity and the "constancy of velocity of light" (CVL) postulates. If the relativity postulate is sacrificed what guidelines should one follow in order to guess the transformation equation? The next section will provide an answer to this question.

6.2 Transformation Equations

Although the kinematics of relativity theory was obtained by Einstein from a general principle like the relativity of motion and a principle concerning the speed of light, the operative aspects of these postulates used in the derivation can be laid down in more concrete terms. Indeed if one consults a standard text book on relativity, one finds that the derivation of LT starts from the assumption of a linear transformation with unknown coefficients which are determined using essentially the following operative inputs:

(1) The coordinate clocks in any inertial frame are assumed to be synchronized by light signal following the Einstein synchrony or the standard synchrony, according to which the one-way-speed (OWS) of light is assumed to be the same as its two-way-speed (TWS) in any direction[16, 17].

(2) The speed of light[2] is the (i) same and (ii) isotropic with respect to all inertial observers.

(3) Measuring rods placed perpendicular to its direction of motion do not undergo any contraction or elongation with respect to its rest length.

The first of the above is just a synchronization convention but the other two items are the consequences of the relativity principle[3]. A little amplification of this statement in respect of item (2) may be in order. One might think that (2) is equivalent to Einstein's CVL postulate. This is indeed a misconception[19]. The CVL postulate of Einstein refers to constancy with respect to change in the velocity of light source. In effect this postulate emphasizes the wave character of light. Once wave is launched it is no longer linked to the source. Indeed Einstein's second postulate concerning the speed of light in conjunction with the principle of relativity only imply the constancy with respect to the change of the inertial observer as well[19].

In a preferred frame theory where the principle of relativity is expected to be violated, the transformation equations cannot be obtained with item (2) as an input which, as explained, depends on the relativity principle although CVL can be used in the stationary frame. As regards input (1) however there is no difficulty but there is no special advantage in synchronizing coordinate clocks using light signal. One may then ask what if the clocks were synchronized by some other signal say an "acoustic signal" for example[4]. One may consider a substratum which can support

[2]If standard synchrony is not used, the phrase "speed of light" then means TWS of the same, which is a synchrony independent quantity.

[3]See any standard text book derivation of LT (for example see[18]) which explains and uses item (3) as a consequence of the relativity principle.

[4]The choice of the phrase "acoustic signal" is just symbolic. We only emphasize here that the transformation equations can be obtained without any reference to the physical nature of the synchronizing agent. However soon we will resort to optical synchronization (see below)

such a signal and through which different inertial frames are supposed to move. To effect the synchronization, like the standard synchrony we shall stipulate the OWS of the signal along a straight line be equal to its TWS along the line in any frame Σ_k. It has been shown elsewhere[16] that if input (2) is withheld, and the coordinate clocks of any inertial frame is synchronized by "acoustic signal", the transformation equation between a preferred frame Σ_0 and an arbitrary inertial frame Σ_k can be obtained as,

$$x_k = (a_{kx}/a_{ky})(1 - u_{0k}^2/a_0^2)^{-1/2}(x_0 - u_{0k}t_0), \tag{6.3}$$

$$t_k = (a_0/a_{ky})(1 - u_{0k}^2/a_0^2)^{-1/2}(t_0 - u_{0k}x_0/a_0^2), \tag{6.4}$$

where x_0, t_0 and x_k, t_k refer to space-time coordinates as measured with respect to the stationary (Σ_0) and moving frame (Σ_k) respectively. The relative velocity of Σ_k with respect to Σ_0 has been denoted by u_{0k}. As regards other terms, a_0 denotes the isotropic "acoustic speed" (two way or one way) in the stationary substratum, whereas a_{kx} and a_{ky} are the TWS' of the synchronizing signal in Σ_k parallel (along the x-direction) and perpendicular (along the y-direction) to its direction of motion respectively. Note that in general a_{kx} and a_{ky} are expected to be functions of u_{0k} and hence the above equations are only formal and not usable unless some phenomenological assumptions are made regarding these functions. For optical signal synchronization we replace the terms a_{kx}, a_{ky} and a_0 in Eqs.(6.3) and (6.4) by c_{kx}, c_{ky} and c_0 respectively where the latter three terms represent the respective speeds of the light signal. In the relativistic world, by input (2), one finds in any Σ_k

$$c_{kx}(u_{0k}) = c_{ky}(u_{0k}) = c_0, \tag{6.5}$$

and the above equations (Eqs.(6.3) and (6.4)) turns out to be LT under optical synchronization.

We now ask what if Eq.(6.5) is approximately valid, so that the speed of light is almost and not quite independent of the speed of the reference frame with respect to a "preferred" one. Note that the transformation equations (6.3) and (6.4) are now most appropriate to deal with such questions. We now wish to use input (2) in these equations by modifying the former minimally. We try this by preserving the isotropy component (2(ii)) and relaxing the constancy component (2(i)) of the said input. Thus TWS of light is assumed to be isotropic in any frame Σ_k and now we conjecture that this isotropic speed depends on u_{0k} in following way,

$$c_{kx} = c_{ky} = c_k = c_0(1 + \alpha u_{0k}^2/c_0^2)^{1/2}, \tag{6.6}$$

where we have introduced a dimensionless constant α which is assumed to have such a small value that the proposed theory does not differ in its predictions from that so far tested relativistically. Clearly Eq.(6.5) is now replaced by Eq.(6.6) which approximately reduces to Eq.(6.5) for $\alpha u_{0k}^2/c_0^2 << 1$. Note that, depending on the smallness of α, u_{0k} can be very close to c_0 and yet the last condition can still remain valid. We shall show below that if the phenomenological assumption described by Eq.(6.6) is believed to be true, the UHECR paradox can be explained in terms of the motion of the solar system with respect to the RFCS.

We conclude this section by quoting the relevant transformation equations which are obtained by plugging in Eq.(6.6) in Eqs.(6.3) and (6.4):

$$x_k = (1 - u_{0k}^2/c_0^2)^{-1/2}(x_0 - u_{0k}t_0), \tag{6.7}$$

$$t_k = (1 + \alpha u_{0k}^2/c_0^2)^{-1/2}(1 - u_{0k}^2/c_0^2)^{-1/2}(t_0 - u_{0k}x_0/c_0^2). \tag{6.8}$$

6.3 Metric and 4-Vectors

In SR classical expressions for momentum and energy had to be altered in order for the conservation principles to be Lorentz covariant. These expressions can easily be obtained by writing the energy momentum conservation in terms of a 4-vector relation. The energy momentum 4-vectors are obtained in terms of the invariant interval of SR.

In the present situation, such a thing cannot be obtained easily since one recalls that the notion of invariant interval of SR is an outcome of the existence of an invariant speed c (c_0) of the theory. In the present context in absence of such a speed the invariant interval does not exist in the way it existed in SR. Besides, since there should exist a preferred frame, in order to obtain the correct conservation principle (or to obtain definition of energy and momentum) an appeal to covariance of physical laws cannot be made. In the following we suggest a way out. From the transformations (6.3) and (6.4) together with

$$y_k = y_0,\ z_k = z_0, \tag{6.9}$$

it is evident that

$$(c_{ky}/c_{kx})^2 x_k^2 + y_k^2 + z_k^2 - c_{ky}^2 t_k^2 = x_0^2 + y_0^2 + z_0^2 - c_0^2 t_0^2. \tag{6.10}$$

Recalling Eq.(6.6) the above relation reads

$$x_k^2 + y_k^2 + z_k^2 - c_k^2 t_k^2 = x_0^2 + y_0^2 + z_0^2 - c_0^2 t_0^2, \tag{6.11}$$

and in terms of the differential intervals one obtains the following invariant interval

$$d\tau^2 = dt_k^2 - (1/c_k^2)(dx_k^2 + dy_k^2 + dz_k^2), \tag{6.12}$$

and by analogy with SR we call $d\tau$ as the proper time interval.

Note that the *expression* for the above invariant interval is frame dependent unlike the case in SR because of the presence of $c_k(u_{0k})$ in the last expression. However one can easily develop a 4-vector formulation like that in SR by defining the 4-momentum of a particle of mass m as

$$\mathcal{P} = (m\gamma_k, m\gamma_k(\mathbf{v_k})_i), \tag{6.13}$$

with

$$\gamma_k = (1 - (v_k)^2/c_k^2)^{-1/2}, \tag{6.14}$$

where $(\mathbf{v_k})_i$ represents the i^{th} component of the three velocity $\mathbf{v_k}$ of the particle in Σ_k. Imposing

$$\mathcal{P}.\mathcal{P} = \mathcal{P}^2 = \eta_{\mu\nu}p^\mu p^\nu = invariant, \tag{6.15}$$

where

$$\eta_{\mu\nu} = (1, -1/c_k^2, -1/c_k^2, -1/c_k^2), \tag{6.16}$$

one obtain the dispersion relation for the particle in any frame Σ_k as

$$E_k^2 = p_k^2 c_k^2 + m^2 c_k^4, \tag{6.17}$$

where

$$\mathbf{p_k} = m\mathbf{v_k}/(1 - v_k^2/c_k^2)^{1/2} = m\gamma_k \mathbf{v_k}, \tag{6.18}$$

and

$$E_k = m\gamma_k c_k^2. \tag{6.19}$$

Although Eqs.(6.17), (6.18) and (6.19) look like the corresponding equations in SR, they are different since the relations are dependent on the frame considered, since now $c_k = c_k(u_{0k})$.

Note that expressions for energy, momentum and the dispersion relation reduce to the usual relativistic ones in the preferred frame Σ_0.

6.4 Velocity Transformations

Our theory therefore does not predict outcomes which are different from those in SR in Σ_0. The question now arises as to whether it is possible to predict a result significantly different from that of SR in a frame of reference (solar system) which is moving with a non-relativistic speed ($u_{0k} \approx 10^{-3}$), with respect to RFCS (Σ_0). The answer seems to be affirmative and we suspect that the resolution of the cosmic ray paradox lies in such a non-preferred frame effect of the theory. To understand this question let us first quote the velocity transformation laws that follow from the transformation relations. We first consider a particle (say a proton) travelling along the x-direction with speed v_0 with respect to Σ_0. The corresponding speed in Σ_k will be obtained from the transformations (6.7) and (6.8) as

$$v_k = (1 + \alpha u_k^2/c_0^2)^{1/2}(v_0 \pm u_k)/(1 \pm v_0 u_k/c_0^2), \tag{6.20}$$

where we have put u_k for u_{0k} for brevity. We shall consider the speeds of the cosmic ray protons in Σ_0 to be very close to unity,

$$v_0 = 1 - \epsilon_0, \tag{6.21}$$

where ϵ_0 is of the order of 10^{-22} (see below). With this range of values for v_0 and recalling $u_k \approx 10^{-3}$, the velocity transformation formula (Eq.(6.20)) can be approximated as

$$v_k = (1 + \alpha u_k^2)^{1/2} v_0, \tag{6.22}$$

where the terms of the order of ${\epsilon_0}^2$ and $\epsilon_0 u_k$ have been neglected in comparison to unity. Although in obtaining Eq.(6.22) we have assumed the motion of the particles to be along the x-direction, interestingly it can be shown that the above relation holds even for particle travelling along *any* direction under the above mentioned approximation.

6.5 Velocity Threshold and the Resolution of the Paradox

Using the usual relativistic energy formula valid in Σ_0

$$E_0 = m/(1 - {v_0}^2)^{1/2}, \tag{6.23}$$

the velocity threshold for proton in Σ_0 corresponding to the GZK threshold energy $E_{0th} = 5 \times 10^{19}$eV speed can be calculated as

$$v_{0th} = 1 - 1.76 \times 10^{-22}. \tag{6.24}$$

Now we will provide a possible explanation for the apparent detection of the trans-GZK events in terms of the motion of the solar system with respect to the CMBR frame. A surprisingly small value of the parameter α of the theory will be found to do this job. In order to demonstrate this we first anticipate (see below) this value for α:

$$\alpha = 3.42 \times 10^{-16}. \tag{6.25}$$

From Eq.(6.6) the speed of light in the laboratory frame Σ_k (for which $u_k \approx 10^{-3}$) can approximately be written as

$$c_k = 1 + \eta_k \approx (1 + \alpha u_k^2/2), \tag{6.26}$$

where η_k measures the departure of the light speed value in Σ_k from unity. Clearly

$$\eta_k \approx \alpha u_k^2/2 = 1.71 \times 10^{-22}. \tag{6.27}$$

However this term is absent in the preferred frame and as we have seen, the special relativistic results (formulas for energy, momentum, dispersion relation etc) hold in Σ_0 and hence GZK cut off value for proton energy obtained from SR is still valid in the CMBR frame. We shall see how this threshold value may appear to be about $3 \times 10^{20} eV$ in Σ_k as detected by Fly's eye air shower detector. Without going into the details of the experimental analysis we now *speculate* that the observed energy of a cosmic ray particle is its *relativistic* energy. We denote it byE_k^{rel} which is given by,

$$E_k^{rel} = mc_0^2/(1 - v_k^2/c_0^2)^{1/2}, \tag{6.28}$$

where we have explicitly retained c_0 for clarity.

Returning to the energy formula for a particle in our frame Σ_k one notes that its value in the solar system (laboratory) practically does not differ from its relativistic value in Σ_0, as

$$E_k = mc_k^2/(1 - v_k^2/c_k^2)^{1/2} \approx mc_0^2/(1 - v_0^2/c_0^2)^{1/2} = E_0, \tag{6.29}$$

where we have used

$$v_k^2/c_k^2 = v_0^2/c_0^2, \tag{6.30}$$

that follows from Eqs.(6.6) and (6.22).We have also assumed in Eq.(6.29), $c_k^2 \approx c_0^2$, since the error involved in such an approximation is only about 1 part in 10^{22}, which can be disregarded since ultimately we will have to explain a discrepancy much bigger than this error (3×10^{20} eV against $5 \times 10^{19} eV$).

The above energy formula (Eq.(6.29)) can also be expressed as

$$\begin{aligned} E_k &= mc_k^2/[1 - v_k^2/c_0^2(1 + \alpha u_k^2/c_0^2)]^{1/2} \\ &\approx m/[(1 - v_k^2)^{1/2}(1 + \alpha u_k^2/2\epsilon_k)^{1/2}], \end{aligned} \tag{6.31}$$

where in arriving at the last approximate expression we have put $c_0 = 1$ again and defined,

$$\epsilon_k = 1 - v_k. \tag{6.32}$$

Using Eq.(6.28), one obtains from Eq.(6.31)

$$E_k^{rel} = E_k(1 + \alpha u_k^2/2\epsilon_k)^{1/2}, \tag{6.33}$$

which by Eq.(6.29) can be written as

$$E_k^{rel} \approx E_0(1 + \alpha u_k^2/2\epsilon_k)^{1/2}. \tag{6.34}$$

Note that this is the relativistic energy of a particle moving with speed v_k. We now calculate this relativistic value of energy for a proton having the GZK threshold energy E_{0th}. Using the transformation (6.22) and assuming $v_0 = v_{0th}$, where the later is given by Eq.(6.24), one obtains the corresponding v_k as

$$v_k = 1 - \epsilon_k = 1 - 5 \times 10^{-24}, \tag{6.35}$$

giving

$$\epsilon_k = 5 \times 10^{-24}. \tag{6.36}$$

Using this value for ϵ_k and assumed value for α (Eq.(6.25)) and finally putting $E_0 = E_{0th}$, we find from Eq.(6.34)

$$E_k^{rel} \approx 3 \times 10^{20} eV, \tag{6.37}$$

which is nothing but the energy of the 300 EeV event detected by the Fly's Eye.

Therefore we conclude that the value of the parameter $\alpha \approx 3.42 \times 10^{-16}$ can explain the apparent detection of trans-GZK events. Note that the above calculation (or the choice of the value for α) depends on the assumption that the 300 EeV event corresponds to the cut off value. However, it may not be so, indeed in future, a bit higher energy event may be detected, in which case the value of α will slightly go up. But this will not pose much problem since the assumed value of α is so small, it has enough flexibility to increase even substantially without contradicting SR in the tested domain.

6.6 Doubly Relativistic?

Velocity transformation formulas in the Galilean (classical) world do not contain a constant velocity parameter whereas the same in the relativistic world contains one constant velocity parameter c (c_0 according to the present notation).

Returning to the expression given by Eq.(6.6),

$c_k = c_0(1 + \alpha u_k^2/c_0^2)^{1/2}$ One may note that instead of expressing c_k in terms of a dimensionless constant α, one may also write the same as

$$c_k = c_0(1 + u_k^2/\xi^2)^{1/2}, \tag{6.38}$$

where $\xi = \alpha^{-1/2}c_0$ is a constant velocity parameter of the theory. The consequent velocity transformation laws (Eqs.(6.22)) therefore are governed by *two* constant velocity parameters (instead of *one* as in SR), c_0 and ξ and hence the present theory can also be called a doubly relativistic one in a sense different from that of currently known doubly special relativistic theories advocated by Amelino-Camelia and others.

6.7 Discussions

In this chapter we have shown that the UHECR paradox can be explained in terms of a non-preferred frame effect of the laboratory frame which is moving with velocity $\approx$ 300 km/sec with respect to the preferred one, assumed to be at rest with CMBR frame. Unlike some earlier efforts (the Coleman Glashow scheme for example) which consider LIV but assume that the physical kinematics is still Lorentzian, we propose to modify the transformation equation itself. Deformed LT are generally discussed in connection with test theories like that of Robertson[20] or Mansouri and Sexl[21] on which improved tests of SR are often based (see for example[22]). But they are not usually considered to represent a new physics that may provide a solution for the UHECR paradox.

Some authors find it troublesome giving up the principle of relativity. In the so called "doubly special relativistic" theories, the particle dispersion relation is modified but the introduction of an invariant length or energy scale in addition to the invariant velocity scale of SR, the "relativity of inertial frames" is still maintained. Such theories, often motivated by quantum-gravity considerations are interesting but are unable to resolve the UHECR paradox quantitatively at the moment.

We here attempt to deform the relativistic kinematics using heuristic means. We do it first by identifying the objective contents of the relativity principle and then go in for modifying these contents minimally to obtain a new transformation that relates space-time of an arbitrary frame of reference with that of the universal rest frame of the cosmic substratum. The only phenomenological assumption regarding the speed of light in Σ_k, $c_k = c_0(1 + \alpha u_k^2/c_0^2)^{1/2}$ (in contrast to the assumption, $c_k = c_0$ in SR) for which $c_k - c_0 = \eta_k \approx 1.71 \times 10^{-22}$ in the laboratory frame ($u_k \approx 300$ km/sec), is the only speculative aspect that has been used to derive the new kinematics. Since the isotropy ingredient of the second relativity postulates has not been disturbed, Michelson-Morley type experiments cannot distinguish the proposed kinematics with that of the relativistic one. Also the limit on ϵ given in Eq.(6.1) as a result of precision test becomes inconsequential, since the expected result in the present case would be zero. The recent improved test of time dilation in SR using laser spectroscopy sets a new limit of 2.2×10^{-7} for deviation of time dilation factor[22]. This even does not match with the smallness of η_k which is also the measure of this deviation according to the new kinematics. Hence the precision tests possibly will be unable to discern any deviation from SR in the near future, yet one may find an explanation of the cosmic ray paradox in the proposed deformed relativistic kinematics.

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Chapter VII

A Simple Minded VSL Cosmology

7.1 Introduction

In recent years a large number of papers[1, 2, 3, 4, 5, 6] have appeared in the literature in which the possibility of variation of speed of light with respect to time (and often with respect to space) has been investigated in the context of cosmology. The idea of variable speed of light (VSL) has gained a considerable popularity since it has been argued that cosmological models which assume large increase in the speed of light in the early universe might solve some cosmological problems[2, 3, 4, 5, 7, 8, 9, 10, 11, 12] and therefore they can be considered as an alternative to inflationary models[13, 14, 15].

On the empirical side, from the analyses of absorption systems in the spectra of distant quasars, the cosmological evolution of the fine structure constant α seems to be a distinct possibility. The many multiplet technique of Webb et.al[16, 17] involves studying relativistic transitions to different ground states using absorption lines in the spectra of quasi stellar objects (QSO) at medium red-shifts. These absorption lines are obtained from heavy elements in distant gas cloud (absorbed system) along the sight-lines of background QSOs. These studies offer one of the earliest evidences that α might change with cosmological time[16, 17, 18]. The group continues to study the possible variation of α with improved precision[19, 20, 21] and the trend of results indicate that the value of α was lower in the past. Many authors tend to relate this varying α to a varying speed of light[1][2][1, 2, 3, 22, 23].

[1]Indeed Joao Magueijo has clarified the meaning of a varying c by dispelling the myth that the constancy of the speed of light is a logical necessity. For a meaningful definition and discussion of variation of the dimensional constant c the reader is referred to the interesting and spirited article by the author[22].

[2]Another group (Srianand et.al)[24] who worked on UVES on the very large telescope in Chile however claimed a "null result". According to the team the relative variation of α ($\Delta\alpha/\alpha$) must be less than 0.6 part per million. Murphy et.al[25] however revised the results of Srianand et.al

Coming back to the theory, several authors have found a connection between VSL and theories of quantum gravity. "Doubly special relativity" (DSR) seems to have emerged as a VSL effective model of quantum space-time with observational implications for ultra high energy cosmic rays (UHECR) paradox[22, 26, 27, 28]. In the last chapter we have presented a heuristic modification of the relativistic kinematics. It has been shown that the absence of Greisen-zatespin-Kuzim (GZK) limit[29] in the UHECR paradox can be resolved in terms of a non-preferred frame effect of the solar system for its motion with respect to the rest frame of the universe (the cosmic substratum)[30]. We have dubbed this novel theory as also a DSR one from a sense different from that of the currently known DSR theories. Interestingly we find that this neo-DSR (NDSR) theory can also lead to a VSL cosmology after proper interpretation. In the present chapter we will develop a VSL theory on the basis of the new kinematics proposed earlier. The purpose of this endeavour will however be limited to show that the theory can accommodate the Webb et.al's and other's result concerning the cosmological variation of the fine structure constant.

At present no attempt will be made to suggest a direction along which the gravitational field equation of GR is to be modified as a consequence of the deformed Lorentzian kinematics proposed. Neither we will attempt for now how the consequent VSL scenario would be able to solve the cosmological problems usually tackled by inflation. The present exercise will help clarify how a modification of the relativistic kinematics prompted by the UHECR paradox can go hand in hand with the idea of VSL. Indeed the later emerges naturally from the modified "second relativity postulate"[3] (MSRP) proposed in the earlier chapter in connection with

and demonstrated some "simple flaws" of their data analysis technique. We shall therefore assume for present that there has been a detectable evidence for the cosmological variation of α.

[3]By second relativity postulates (SRP) we mean Eq.(7.5) of the last chapter, which is an outcome of CVL postulate of Einstein in conjunction with the principle of relativity (see Sec.(6.2)

the resolution of the (apparent) violation of the GZK limit observed in the ultra high energy cosmic ray spectrum.

The basic idea is as follows. As already pointed out, some studies on distant quasars indicate deviations in the value of the fine structure constant from its laboratory value. Some authors tend to attribute this variation of dimensionless α parameter to the variation of the speed of light c as a function of red-shift of quasars. According to some popular interpretations, this red-shift dependence of the speed of light ($c = c(z)$) is viewed as the time dependence of the same, since the red-shift as is believed is caused by the expansion of the universe and hence through Hubble's law z measures the distance of a galaxy. Again since light takes finite time to reach us, increasing z implies going back more to the past. Webb et.al's results thus are interpreted to signify the variation of c with respect to time. This chain of arguments can be summarized by saying that light propagation was faster in the past. The present endeavour allows us to look it rather differently. The red-shift of a QSO (or to be precise, that of the absorber system) is directly connected with its recession speed and hence any variation (possible or observed) of the speed of light with red-shift can then be interpreted as a variation in the value for the same as a function of the recession speeds of the frames of reference (QSO's) in which the measurements of the fine structure constant are referred to. This is precisely in consonance with the deformed relativistic kinematics developed in the last chapter in connection with the UHECR paradox. Indeed it appears that a VSL kind of cosmology emerges naturally from the NDSR proposed earlier by us, which was successful in dealing with a problem appearing elsewhere in physics. In the next section we develop this basic idea with a brief recapitulation of the NDSR discussed in connection with the UHECR paradox. The penultimate section will be

of chapter VI for details)

devoted to discussing Webb et.al and other's observations vis-a-vis our proposals. In the last section we will summarize the whole idea where some final remarks will be made in favour of our proposal.

7.2 The New Kinematics

In chapter VI we developed in the context of the UHECR paradox, a novel kinematics based on the notion of the existence of a preferred frame, which is believed to be the one at rest with the cosmic microwave background radiation. The solar system moves through this CMBR frame at a speed of approximately $10^{-3}c$. We have seen how the UHECR paradox can be explained in terms of a non-preferred frame effect of the solar system according to the kinematics proposed therein. In dealing with a preferred frame it has been found advantageous to start from the Dolphin transformations developed by Ghosal et.al[31]. The general form of which has been given by Eqs.(6.3) and (6.4) of the last chapter. We reproduce these equations by replacing a_0, a_{kx}, a_{ky} in it by c_0, c_{kx} and c_{ky} where c_0 is the isotropic speed of light in the preferred frame Σ_0 and c_{kx} and c_{ky} represent the two-way speeds of the same along the x and the y directions respectively.

$$x_k = (c_{kx}/c_{ky})(1 - u_{0k}^2/c_0^2)^{-1/2}(x_0 - u_{0k}t_0), \tag{7.1}$$

$$t_k = (c_0/c_{ky})(1 - u_{0k}^2/c_0^2)^{-1/2}(t_0 - u_{0k}x_0/c_0^2). \tag{7.2}$$

Recall that x_0, t_0 and x_k, t_k refer to space-time coordinates as measured with respect to the CMBR frame Σ_0 and any moving frame Σ_k respectively and u_{0k} denotes the relative velocity of Σ_k with respect to Σ_0. In the previous equations (Eqs.(6.3) and (6.4)) a_0 denotes the isotropic one-way or two-way "acoustic" signal speed whereas a_{kx} and a_{ky} are TWS' of the synchronizing signal in Σ_k parallel and perpendicular to its direction of motion respectively.

Note that up to this point no explicit assumptions regarding the behavior of the speed of light as a function of the speed u_{0k} of the reference frame Σ_k with respect to Σ_0 have been made. In the case of optical signal synchronization, one may thus write generally

$$c_{kx} = c_{kx}(u_{0k}), \tag{7.3}$$

$$c_{ky} = c_{ky}(u_{0k}). \tag{7.4}$$

Note that one recovers from Eqs.(7.1) and (7.2) the familiar relativistic transformations if one explicitly uses the CVL postulate:

$$c_{kx}(u_{0k}) = c_{ky}(u_{0k}) = c_0. \tag{7.5}$$

One may now ask "what happens if the CVL postulate is approximately correct?" We have already discussed that this CVL postulate may be minimally modified by preserving the isotropy ingredient of the relativity theory while assuming the TWS of light to vary only in a minute way. In particular we have assumed

$$c_{kx} = c_{ky} \approx c_0[1 + \eta(u_{0k})], \tag{7.6}$$

where $\eta(u_{0k})$ is assumed to take *very small* values so that the assumption does not contradict SR in the tested domain. The last equation represents our MSRP as explained earlier. The specific form for η has been assumed to be

$$\eta = \alpha u_{0k}^2/2c_0^2. \tag{7.7}$$

We have shown that an assumption of a very small value $\eta \approx 1.71 \times 10^{-22}$ with respect to the solar system ($u_{0k} = 10^{-3}c_0$) can resolve the Fly Eye's detection of 3.2×10^{20} eV particle which apparently crosses the GZK limit. What is crucial here is that the dependence of the speed of light on the speed of the reference frame apparently finds sanction in the context of the resolution of the UHECR paradox.

7.3 MSRP, Quasar Absorption Spectra and VSL

In order to accommodate Webb et.al's results (following their spectral studies of distant quasars) of possible time variation of fine structure constant in the light of VSL, we propose to recast Eq.(7.6) in a new form:

$$c(u_{0k}) = c_0[1 + (u_{0k}/\xi)^{2\mu}]^{1/2}, \tag{7.8}$$

where ξ is a new velocity parameter and μ is a dimensionless constant of the theory. Note that the existence of two velocity parameters (c_0 and ξ) in the theory (instead of one in the usual relativity theory) allows us to call it a DSR theory of a new variant (dubbed as NDSR in the previous section).

We now introduce two dimensionless parameters

$$u_k = u_{0k}/c_0, \tag{7.9}$$

and

$$w = \xi/c_0, \tag{7.10}$$

and rewrite Eq.(7.8) as

$$c(u_k) = c_0[1 + (u_k/w)^{2\mu}]^{1/2}. \tag{7.11}$$

Note that according to Eq.(7.6) of the previous chapter $\mu = 1$, but for now we do not hand put any value for it. In what follows we will be interested in relating observations in three reference frames— the preferred frame Σ_0, the frame of the solar system Σ_s, and that of the distant gas clouds (along the line of sight of background quasars) Σ_q and accordingly we use subscripts "0", "s" and "q" for relevant quantities in these frames respectively. For example, we may write appropriately in Σ_s and Σ_q as

$$c_s = c_0[1 + (u_s/w)^{2\mu}]^{1/2}, \tag{7.12}$$

and

$$c_q = c_0[1 + (u_q/w)^{2\mu}]^{1/2}. \tag{7.13}$$

One may now connect, in a simple minded way, the red-shift of distant quasars with theirs recession speeds using relativistic formula for Doppler effect. If z denotes the red-shift parameter, one can write in a straight forward way for the recession speed u_k of a quasar (or absorber system):

$$u_k = c_0[((1 + z)^2 - 1)/((1 + z)^2 + 1)]. \tag{7.14}$$

Note that the red-shift parameter is defined as

$$z = -\Delta\nu/\nu_0, \tag{7.15}$$

where $\Delta\nu$ denotes the frequency shift while ν_0 represents the proper frequency of the source. In arriving at Eq.(7.14) we have made use of the standard relativistic formula,

$$\nu = \nu_0[(1 - u_k/c_0)/(1 + u_k/c_0)]^{1/2}, \tag{7.16}$$

where ν denotes the observed frequency.

We now wish to write Eq.(7.11) in a different form so that it can directly relate $\Delta c/c_0$ with the speed of concerned reference frame. Writing $c - c_0 = \Delta c$ one thus obtains

$$\Delta c/c_0 \approx \frac{1}{2}(u_k/w)^{2\mu}. \tag{7.17}$$

In writing the above approximate form we have made use of the fact that $\Delta c/c_0 << 1$ in relevant situations. Referring to the two frames Σ_s and Σ_q we can write the above equation as

$$(\Delta c/c_0)_s = \frac{1}{2}(u_s/w)^{2\mu}, \tag{7.18}$$

and

$$(\Delta c/c_0)_q = \frac{1}{2}(u_q/w)^{2\mu}, \tag{7.19}$$

which are nothing but the approximate forms of Eqs.(7.12) and (7.13). Given that the recent measurements and claims for variation of fine structure constants from the studies of spectra of distant quasars can be modeled by a scenario in which only the speed of light varies, one may ask the following questions. Are these observed values for $\Delta\alpha/\alpha = -(\Delta c/c_0)_q$ consistent with the value for η $(= (\Delta c/c_0)_s)$ used earlier in the context of our attempt to resolve the UHECR paradox? In other words we ask keeping Eqs.(7.18) and (7.19) in mind, what values of w and μ are consistent with $\Delta\alpha/\alpha$ values obtained from the recent quasar studies given the constraint $(\Delta c/c_0)_s \approx 1.71 \times 10^{-22}$ imposed by the UHECR data. To answer this question we proceed as follows. We mainly use the most widely quoted non-null results for $\Delta\alpha/\alpha$ obtained by a group and the corresponding red-shifts[16, 20, 21, 25]. For the latter we take the average value of the red-shift range provided by the authors. Below in Table (7.1) we quote some of the representative results reported by the group at different times. Up to the third row the entries correspond to measurements by essentially one group using the Keck/HIRES instrument. Fourth row however refers to the group who claims null result for variation of α following observation with UVES spectrograph on VLT[24]. The entries in row 5 correspond to results following a reanalysis of the data of Ref.[24].

Table 7.1: Values of parameters of the theory from the relative variation of α measurements.

Group	**Red-shift range**	z_{avg}	$\Delta\alpha/\alpha = -\Delta c/c$	u_q	w	2μ
Murphy et.al (2001a) (Keck/HIRES)	0.5-3.5	2	-0.72×10^{-5}	0.800	5.60	5.726
Murphy et.al (2003) (Keck/HIRES)	0.2-3.7	1.95	-0.543×10^{-5}	0.793	5.91	5.96
Murphy et.al (2004) (Keck/HIRES)	0.2-4.2	2.2	-0.573×10^{-5}	0.822	6.11	5.67
Srianand et.al (2004) (VLT/UVES)	0.4-2.3	1.55	-0.06×10^{-5}	0.733	9.06	5.42
Murphy et.al (2007) (VLT/UVES)	0.4-2.3	1.35	-0.64×10^{-5}	0.693	4.78	5.83

The table is self explanatory. The entries in column 1 refer to the names of the groups and their instruments (telescope and the spectrometer systems). In column 3 we give the average red-shift z_{avg} from the given red-shift range (column 2) provided by the groups. The recession speeds u_q have been calculated using the formula given in Eq.(7.14). These have been provided in column 4. The quoted central values $\Delta\alpha/\alpha$ are entered in column 5. From the entries w and 2μ have been calculated using Eqs.(7.18) and (19), and are displayed in column 6 and 7 respectively.

Although it would have been appropriate to use a particular value of $\Delta\alpha/\alpha$ for a given absorber with its corresponding red-shift value and use the data to obtain w and μ , we have opted for the average or mean values of z and $\Delta\alpha/\alpha$ for the fact that since the error margins for individual results are quite high, there is no point in giving any extra credence to a particular obtained value for $\Delta\alpha/\alpha$ against the corresponding red-shift from a given set of observations. However, although the correctness of the theory should demand a unique set of values for w and μ, variations in the values of these parameters corresponding to different set of observations have been made explicit. These variations only reflect the fact that outcome of the measurements of fine structure constants from quasar absorption spectra has not yet reached the desired confidence level.

Assuming any particular set of values for w and μ one may study the nature of variation of the speed of light $\Delta c/c_0 (= \Delta\alpha/\alpha)$ as a function of red-shift by using Eq.(7.14). In Fig.(7.1a) we display the theoretical curves for variations of α against red-shift (for $w = 5.6, 2\mu = 5.726$). Fig.(7.1b) and (7.1c) provides the same for other two values of the paired parameters w and μ (corresponding to rows 2 and 3). No graph has however been drawn for the data entered in row 4 since the claimed constraints therein for $|\Delta\alpha/\alpha|$ is one order less than those of the other entries. Indeed Murphy et.al revised Srianand et.al's null result derived from VLT/UVES quasar absorption spectra and after correcting the "flawed analysis" of the latter, concluded that the same data gives a weighted mean value $\Delta\alpha/\alpha = (-0.640 \pm 0.360) \times 10^{-5}$. The $\Delta\alpha/\alpha - z$ variation following this revised analysis is shown in Fig.(7.1d). The corresponding variation of c have been displayed in Fig.(7.2a-d). (For some data points see Fig.(7.3) which has been reproduced from Refs.[1, 22]).

Pending any rebuttal of Murphy et.al's claim one may assume that variation of fine structure constant with red-shift to be a reality. However if in the ultimate analysis the claim of Ref.[21] proves to be correct, the possibility of α-parameter variation (or variation of the speed of light) may still be a possibility as the authors' findings also allow for a slight relative variation of α (up to 0.6 per million). If however one would still like to believe that Srianand et.al analysis indeed puts stronger constraints (near null) on the α-variation (or c-variation), we can still

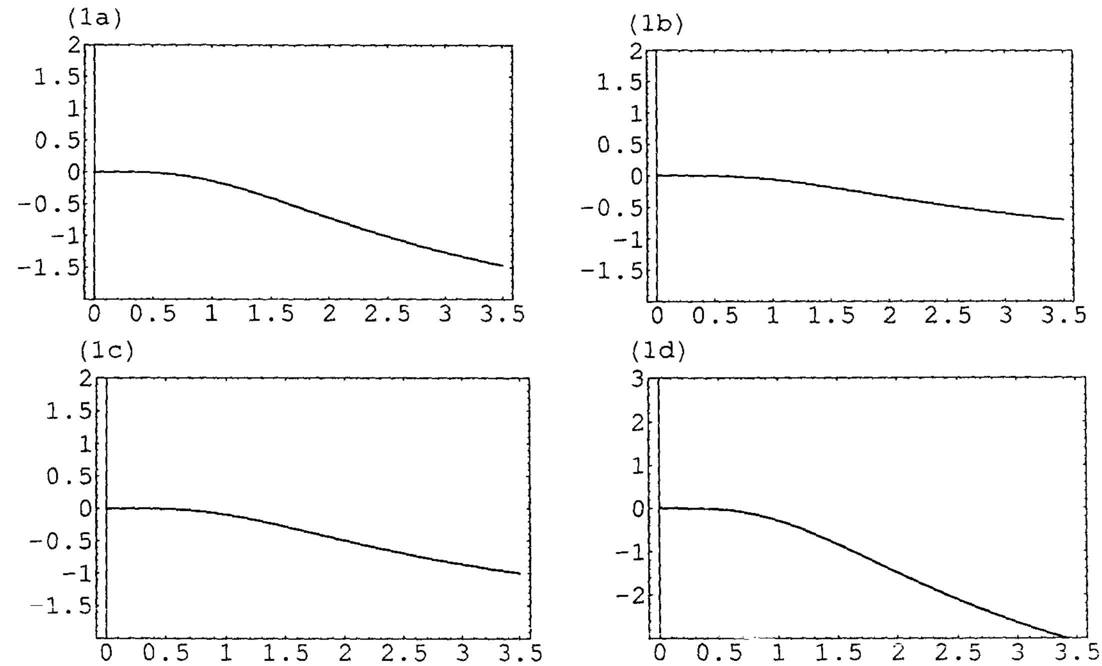

Figure 7.1: Theoretical curve showing the variation of $\Delta\alpha/\alpha \times 10^{-5}$ (vertical axes) with red-shift z (horizontal axes) in the simple minded VSL model for (a) $w = 5.60$ and $2\mu = 5.726$ (obtained from Murphy et.al 2001)[16] (b) $w = 5.91$ and $2\mu = 5.96$ (obtained from Murphy et.al 2003)[20] (c) $w = 6.11$ and $2\mu = 5.67$ (obtained from Murphy et.al 2004)[21] (d) $w = 4.78$ and $2\mu = 5.83$ (obtained from Murphy et.al 2007)[25].

develop our NDSR of the last chapter to accommodate Srianand et.al's claim. The importance of exploring this possibility lies in the fact that the current observational status regarding the α-variation in respect of accuracy, confidence level etc. allows us to doubt any claim or counter claim (with certainty) regarding the issue.

Recall that in the previous chapter we proposed a c-variation kinematics which enabled us to suggest a resolution of the UHECR paradox. The original suggestion required the c-variation in the following form:

$$c_k = c_0(1 + u_k^2/\xi^2)^{1/2}. \tag{7.20}$$

If we identify ξ with w of the present theory, we can see (by comparing Eq.(7.20) with Eq.(7.8)) that the above equation is equivalent to the assumption $\mu = 1$. We can also write the equation (with $\xi = w$) in a more aesthetic form:

$$c_k = c_0/(1 - u_k^2/w^2)^{1/2}, \tag{7.21}$$

since $u_k^2/w^2 << 1$ in the relevant situations.

Note the existence of *two* velocity parameters c_0 and w which allowed us to term the theory (in a lighter vein though) a doubly special relativistic one in a sense different from the current DSR theories. In the present chapter we have called it neo-DSR or NDSR. In the frame of reference of the absorber systems we replace the subscript k by q and write

$$c_q = c_0/(1 - u_q^2/w^2)^{1/2}, \tag{7.22}$$

which gives,

$$\Delta c/c_0 = (c_q - c_0)/c_0 = \frac{1}{2}(u_q/w)^2. \tag{7.23}$$

One thus is able to calculate $\Delta\alpha/\alpha = -\Delta c/c_0$ from the known value of w and u_q. The recession speed of the absorber system can be obtained from its red-shift value while w is already known from the cosmic ray spectrum data. The value of w has already been quoted as

$$w = 5.4 \times 10^7. \tag{7.24}$$

From this value of w and u_q from Srianand et.al's data (see Table (7.1)), which is $0.733c_0$, one obtains

$$\Delta\alpha/\alpha = -1.097 \times 10^{-16}, \tag{7.25}$$

which is surely consistent with Srianand et.al's findings.

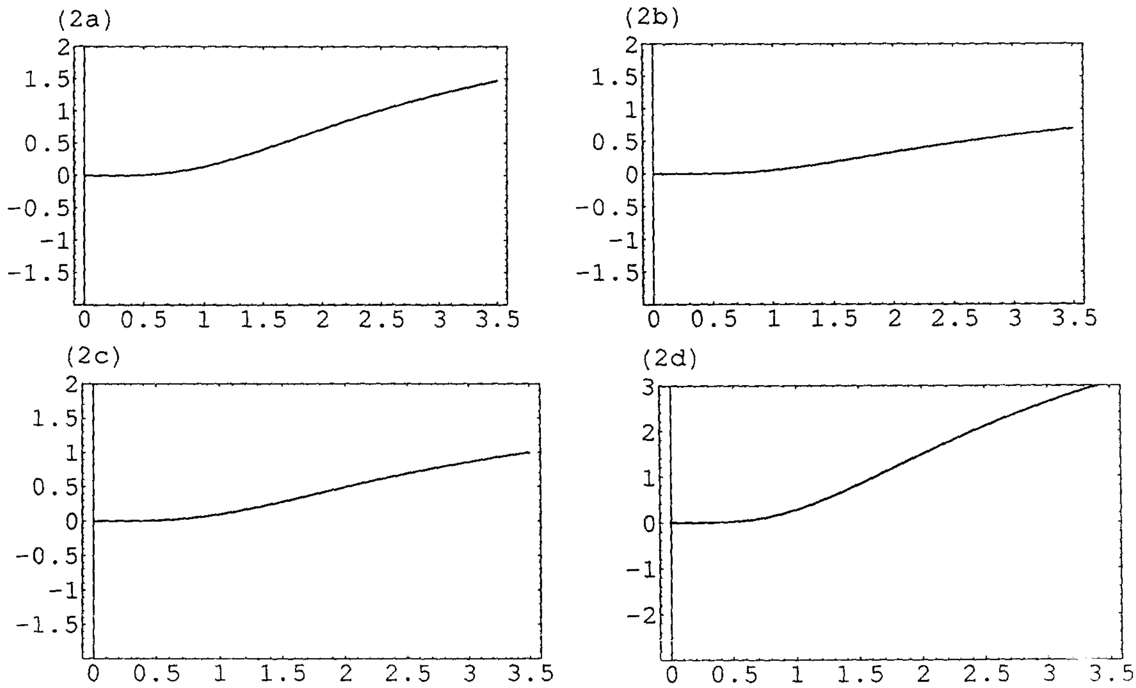

Figure 7.2: Theoretical curve showing the variation of $\Delta c/c_0 \times 10^{-5}$ (vertical axes) with red-shift z (horizontal axes) in the simple minded VSL model for (a) $w = 5.60$ and $2\mu = 5.726$ (obtained from Murphy et.al 2001)[16] (b) $w = 5.91$ and $2\mu = 5.96$ (obtained from Murphy et.al 2003)[20] (c) $w = 6.11$ and $2\mu = 5.67$ (obtained from Murphy et.al 2004)[21] (d) $w = 4.78$ and $2\mu = 5.83$ (obtained from Murphy et.al 2007)[25].

7.4 Summary and Conclusion

Let us now summarize our proposal for a VSL theory. The effort to resolve the UHECR paradox has guided us to assume a c-variation as a function of the velocity of the (inertial) frame of reference relative to the so called cosmological preferred (CMBR) frame, the specific form of function being

$$c_k = c_0[1 + (u_k/w)^{2\mu}]^{1/2}, \tag{7.26}$$

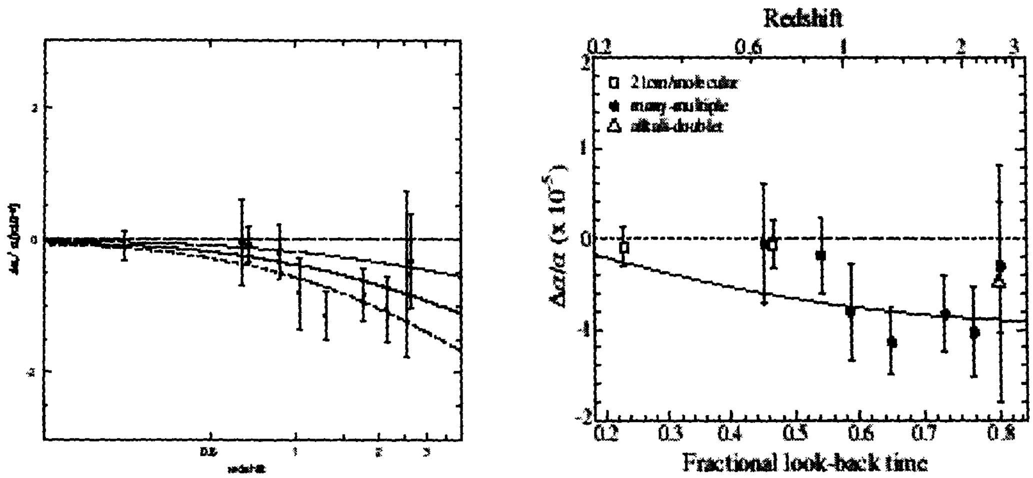

Figure 7.3: The data points are the absorption line in quasar (QSO) results from changing α. The solid line shows theoretical predictions in several varying α model[1,22].

where c_k is the speed of light measured with respect to Σ_k. As the recession velocities of distant galaxies can directly be linked to its age (look back time), one is free to interpret the proposed c-variation (as a function of recession speed) as the time variation of the speed of light. Clearly however the term 'VSL' in the present context has intrinsically a different connotation. This difference in connotation allows us to keep some basic principles of physics intact. For example, the existence of uniformly moving isolated particle is one of the fundamental notion of both classical mechanics and relativity theory; but the usual VSL theory requires that a photon or light pulse should accelerate on its own. In order to obtain a remedy of this unwanted feature Stepanov[32] has to assume a *spatial dependence* in addition to the temporal dependence of c. Clearly the present VSL is devoid of such malady. With respect to any inertial frame a light pulse will remain unaccelerated although its value differs in different inertial frames (with respect to cosmological rest frame its value is given by $c = c_0 = 3 \times 10^8$ m/s, the usual value for the "speed of light" in free space).

From the observational point of view, the idea of VSL seems to be supported by some recent observations of variation of fine structure constant from the study of absorption systems in the spectra of distant quasars, Paul Davies and collaborators[33] have suggested that in principle it is possible to disentangle which of the dimensional constants (the elementary charge e, Planks constant h, and the speed of light c of which the fine structure constant is composed) is responsible for the variation. They have argued that the black hole thermodynamics favours theories in which c decreases with time. Some authors however have disputed this claim[34, 35] not intending to delve into the controversy on the issue we have directly proposed the scenario in which we assume that only the speed of light varies. Originally this is done in the context of the UHECR paradox, but now we find that the proposed theory has the ability to account for the observed variations of fine structure constant. It is however too early to say a final word regarding the values of the parameters w and μ. The range of the values for these has been tabulated. With increased accuracy and more reliable data from quasar spectra in the near future will hopefully narrow down our choices. We do not wish to term this whole exercise of obtaining a theory capable of explaining the UHECR paradox and VSL as merely a phenomenological one. Although Einstein once apparently lamented that "... a physical theory can be satisfactory only when it builds up its structures from *elementary* foundation..."[36, 37] and his own SR failed on that count, the truth is – the apparent weak point of his "principle theory " (as opposed to "constructive theory") to our mind is indeed the strength of one most celebrated physical theories of science i.e relativity. One should not compare the situation with classical thermodynamics (a "principle theory") vis-a-vis the statistical mechanics (a "constructive theory"), in which case the macro (average property of a system) is governed by the properties of the micro structure of a system (working substance of a heat engine for example) and not the other way round. In case of relativity one obtains physics on the basis of the principle of relativity and the whole micro world seems to adjust itself to this principle only. The constructive accounts of relativistic effects (length contraction and time dilation for example) on the other hand still illusive.

What about the present theory? It definitely is not an example of a "constructive theory" which is built on some elementary foundations (a quantum gravity kind of explanation for the specific forms for the c-variation Eqs.(7.11) and (7.22) might enable us to say so). Is it a principle theory? Apparently no. But we would be inclined to say that it is much akin to a "principle theory" in its characteristics. We have already mentioned in the last chapter that one of the operative inputs of the principle of relativity being that the speed of light is the (i) same and (ii) isotropic with respect to all inertial observers.

What we have assumed so far amounts to saying that the principle of relativity holds only approximately which has led to our MSRP. The theory is still in the making. The time has not yet come to enable one to give a *principled* basis for a definite form of the equation

$$c = c(u_k), \tag{7.27}$$

for example, Eqs.(7.11), (7.22) or any other possible forms of MSRP, but that it is approximately a constant function has already given us a lot. However we have a long way to go. Surely there will be more questions prompted by our proposal than it has endeavoured to answer; nevertheless the reader can consider it just to be a humble beginning of a long voyage.

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THE PRINCIPLE OF EQUIVALENCE AND THE TWIN PARADOX

S. K. Ghosal*, **Saroj Nepal, and Debarchana Das**

Department of Physics, North Bengal University
Dist. Darjeeling(WB), Pin: 734 430, India
E-mail: ghosalsk@hotmail.com

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The canonical twin paradox is explained by making a correct use of the principle of equivalence. The role of the principle of equivalence is to provide a physical agent i.e gravity which can supply the required extra aging to the rocket-bound sibling during its acceleration phase through a gravitational time-offset effect. We follow an approach where a novel variation on the twin paradox is used to connect gravity with the desynchronization in the clocks of two spatially distant, identically accelerated observers. It is shown that this approach removes certain drawbacks of an earlier effort which claims to exploit the equivalence principle in explaining the differential aging in the paradox.

Key words: special relativity, general relativity, twin paradox, equivalence principle, gravitational slowing down of clocks, conventionality of simultaneity, Zahar transformation.

1. INTRODUCTION

The principle of equivalence between acceleration and gravity is considered as a *cornerstone* of Einstein's theory of gravitation or that of general relativity (GR). According to Einstein, the principle states that: "A system in a uniform acceleration is equivalent to a system at rest immersed in a uniform gravitational field" [1]. Text books often introduce GR by first demonstrating that the equivalence principle (EP) predicts gravitational red-shift, which Einstein viewed as a test of general relativity. However, we now regard it as a more basic test

*Author to whom all correspondences should be made.

of EP and the existence of curved space-time [2]. The phenomenon of gravitational red-shift, which has been tested by precision experiments by Pound-Rebka and Snider in the sixties [3, 4] is also interpreted as that of gravitational slowing down of clocks (GSDC). The GSDC has now been tested with much accuracy by using a hydrogen maser clock with extraordinary frequency stability flown on a rocket to an altitude of about 10,000 km [2]. In the literature GSDC phenomenon has been found to play an important role in resolving the notorious twin paradox [5].

In the canonical version of the twin paradox, of the two twins initially living on earth (assumed to be an inertial frame), one leaves the earth by a fast rocket to a distant star and then returns to meet her stay-at-home brother to discover that they age differently. This as such is not a paradox since the rocket-bound sibling, on account of her high velocity will suffer relativistic time dilation of her (biological) clock throughout her journey and will therefore return younger with respect to her brother. Indeed, with respect to the inertial frame of the stay-at-home twin, the world lines of the twins in the Minkowski diagram are different (although from the description of the problem, the end points of these lines, i.e the time and the place of departure and that of their reunion, meet) and hence the asymmetry in the aging can be attributed to the fact that proper time is not integrable [6]. The paradox arises if one näively treats the perspectives of the twins symmetrically. For example, if the traveller twin considers herself to remain stationary and relate the motion to her brother, she would (erroneously) expect her brother to stay younger by believing that the Lorentz transformation (LT) predicts reciprocal time dilation of moving clocks. Qualitatively the resolution lies in the observation that one of the twins is in an accelerated (non-inertial) frame of reference and hence the postulates of special relativity (SR) are not applicable to it and therefore the claim of reciprocity of time dilation between the frames of reference of the twins falls through. Indeed, Einstein himself found this sort of argument preferable in dismissing the paradoxical element in the twin problem [7]. However, this suggestion should not be construed as a statement that the resolution of the paradox falls outside the purview of SR . On the contrary much of the expositions found in the literature on the subject deal with the problem in the frame work of SR alone[1], although many tend to believe that the introduction of GR and a gravitational field at the point of acceleration is the right way to understand the asymmetry in the perspectives of the twins. Bohm notes in the context that " two clocks running at places of different gravitational potential will have different rates" [10]. This suggests that EP can directly be used to explain the asymmetry (difference between the experiences of the

[1]Very extensive treatment is available in Special Relativity Theory-Selected Reprints [8], (see also Ref. [9]). For newer expositions see for example Ref. [6] and references therein.

rocket-bound and the stay-at-home twin). However, as pointed out by Debs and Redhead [6] and also others [11], that since in the twin problems one deals with flat space-time, any reference of GR in this context is quite confusing.

Coming back to the issue of acceleration, one finds often that the direct role of acceleration of the rocket-bound twin in causing the differential aging has been much criticized although it is quite clear that in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of the twins have to be compared at the same space-time events) one cannot avoid acceleration.

In an interesting article Gruber and Price [12] dispel the idea of any direct connection between acceleration and asymmetric aging by presenting a variation of the paradox where although one twin is subjected to undergo an arbitrarily large acceleration, no differential aging occurs. That the acceleration per se cannot play a role is also evident from the usual calculation of the age difference from the perspective of the inertial frame of the stay-at-home twin if one notes that the duration of the turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be understood in terms of the usual relativistic time dilation of the traveller twin during essentially the unaccelerated segment of her journey[2].

One is thus caught in an ambivalent situation that, on the one hand the acceleration does not play any role, on the other hand the paradox is not well posed unless there is a turn-around (acceleration) of the traveller twin[3].

In order to get out of this dichotomy it is enough to note that from the point of view of the traveller twin, the acceleration (or the change of reference frame in the abrupt turn-around scenario) is important. The consideration of this acceleration only has the ability to explain that the expectation of symmetrical time dilation of the stationary twin from the point of view of the rocket-bound twin is incorrect.

In an interesting paper A.Harpaz [5] tries to explain the twin paradox by calculating the age difference from the perspective of the traveller twin directly by applying EP i.e by introducing GSDC. From the previous discussions it may seem unnecessary (or even confusing) to invoke gravity in the essentially special relativistic problem. However the fact is, Harpaz's approach apparently provides an alternate

[2] In such a calculation the time dilation is also calculated during the acceleration phase (assuming the clock hypothesis to be true [6]) and is shown to contribute arbitrarily small value in the age offset if the duration of the acceleration phase is assumed to tend to zero.

[3] Here we are considering the standard version of the paradox and the variation where the twins live in a cylindrical universe [13,14] has been kept out of the present scope.

explanation for the differential aging from the traveller's perspective.

The author of the pedagogical article observes that although the special relativistic approach can correctly account for the age difference between the twins, " it does not manifest the 'physical agent' responsible for the creation of such a difference" [5]. It is held that EP provides such an agent and that is gravity. But how does gravity find way into the problem? Gravity enters through EP and its connection with the resolution of the paradox can symbolically be written as

$$\textit{Acceleration} \overset{EP}{\longrightarrow} \textit{Gravity} \longrightarrow \textit{Gravitational red-shift} \longrightarrow \textit{GSDC} \longrightarrow \textit{Extra aging},$$

where the last item of the flow diagram indicates that with respect to the rocket-bound twin, GSDC provides the extra aging of the stay-at-home one, explaining the asymmetrical aging of the problem.

However while there is as such no harm in understanding the twin problem from a different perspective (here, this is in terms of GSDC), Harpaz's approach suffer from two fold conceptual difficulties which we will elaborate in the next section. These difficulties include the fact that the calculations are only approximate. The other difficulty will be seen to be of more fundamental in nature. The aim of the present paper is to remove these difficulties and give an *accurate* account of the asymmetric aging from the perspective of the rocket-bound twin directly in terms of a time-offset between the siblings which is introduced due to the pseudo-gravity experienced by the traveller twin.

2. GSDC AND EXTRA AGING

In the standard version of the twin paradox the differential aging from the perspective of the stay-at-home (inertial) observer A can easily be calculated assuming that for the most parts of the journey of the traveller twin B, the motion remains uniform except that there is a turn-around acceleration of the rocket so that finally the siblings are able to meet and compare their ages. In the Minkowski diagram the whole scenario is characterized primarily by three events: (1) Meeting of the world lines of A and B when the voyage starts taking place, (2) the turn around of B and (3) meeting of the world lines when A and B reunite. For the paradox it is not necessary that at events (1) and (2), the relative velocity between A and B has to be zero, since ages or clocks can be compared at a point even if the observers are in relative motion, therefore the analysis of the problem can be done by considering the acceleration only during the turn-around. The duration of the acceleration phase can be considered to be arbitrarily small compared to the time it takes during its forward and return journeys and hence the age difference occurs due to the usual relativistic time dilation of a clock for its uniform motion. This is clearly given by

$$\text{Age difference} = 2t_A(1-\gamma^{-1}) \approx 2t_A v^2/c^2, \tag{1}$$

where $2t_A$ is the time the rocket takes for its entire journey (up and down) in uniform speed v and $\gamma = (1-\frac{v^2}{c^2})^{-1/2}$ is the usual Lorentz factor.

The paradox is resolved if one can show that B also predicts the same difference in spite of the fact that the time dilation effect is reciprocal. Clearly some new considerations (that were absent in arriving at Eq.(1)) must offset this reciprocal time dilation and also this must provide some extra aging to A from the point of view of B so that the age difference remains independent of the two perspectives. One of these new considerations, as has already been pointed out, is the one of a synchronization gap that B discovers due to her change of inertial frame during her entire voyage. This has been clearly demonstrated by Bondi [15] in the context of Lord Hulsbery's three brother approach [6] to understanding the twin paradox.

The other way of understanding the same thing is the consideration of pseudo-gravity experienced by B because of its turn-around. In order to demonstrate how EP plays the role in the analysis, Harpaz uses the gravitational red-shift formula, which can be obtained heuristically (using the EP) as

$$\Delta\nu = \nu_0(1 + gh/c^2), \tag{2}$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed at a distance h from the source where the frequency of the same light is seen to be ν_0. Interpreting this red-shift effect in terms of GSDC, the formula can be written as

$$t_1 = t_2(1 + \Delta\Phi/c^2), \tag{3}$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\Phi = gh$, is the potential difference between these p oints. It has been shown that with respect to B the acceleration plays a role by providing an extra time difference between B and A, because of the integrated effect of GSDC during the (arbitrarily) short duration of B's acceleration. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount. As pointed out earlier there are two conceptual difficulties in understanding the treatment. First, in an effort to find a "physical agent" responsible for the extra aging, Harpaz relies on some approximate formulae including that of the gravitational red-shift because of his assumption, $v^2/c^2 \ll 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make any such small velocity approximation for the problem. One might of course argue that for the author's stated purpose it would be enough to show that the " physical agent" i.e. gravity is at work when B's point of view is considered. However, it will be shown that such an argument would also not hold good and the reason for it concerns the second difficulty. The explanations based on SR relies on the fact that during the direction reversing acceleration, the travelling twin changes from one reference frame to another and the lack of simultaneity of one reference frame with respect to the other provides the "missing time" which constitutes the reason for the differential aging [6]. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula (see for example [16–18]) which is also followed by the author of Ref. [5], one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $Z = gh/c^2$ is only approximate and is derived by making use of the classical Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator [5]. According to EP an observer within the elevator will " attribute his observations in the elevator, to the existence of a uniform gravitational field in a rest system of reference" [5]. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation! How then is GSDC able to account for an effect, viz. the lack of simultaneity which is essentially a standard relativistic phenomenon?

In the next section we will show that indeed the EP can explain the twin paradox exactly provided the connection of EP and GSDC is obtained using the full machinery of SR.

3. EP AND THE GRAVITATIONAL TIME OFFSET

In an interesting paper Boughn [19] presents a variation of the twin paradox where two twins A and B on board two identical rockets (with equal amount of fuel), initially at rest a distance x_0 apart in an inertial frame S, get identical accelerations for some time in the direction AB (x-direction say), and eventually come to rest (when all their fuel has been expended) with respect to another inertial frame S' moving with velocity v along the positive x-direction with respect to S. From the simple application of Lorentz transformation Boughn obtains a very surprising result that after the acceleration phase is over, the age of A becomes less than that of B.

The result is counter-intuitive by virtue of the fact that the twins throughout have identical local experiences but their presynchronised (biological) clocks go out of synchrony. The amount of this time offset turns out to be

$$\Delta t' = -\gamma v x_0/c^2. \tag{4}$$

The result follows from the simple application of LT which one may write for time as

$$t_k{}' = \gamma(t_k - v x_k/c^2), \tag{5}$$

where t_k and x_k denote the time and space coordinates of the observer k (k stands for A or B) with respect to S and the prime refers to the corresponding coordinates in S'.
From Eq.(5) it follows that

$$t_B{}' - t_A{}' = \gamma[(t_B - t_A) - v(x_B - x_A)/c^2]. \tag{6}$$

Assuming the clocks of the observers A and B are initially synchronized in S, i.e assuming $t_B - t_A = 0$ and also noting that $x_B - x_A = x_0$ remains constant throughout their journeys, the time offset between these clocks is given by the expression (4) provided $\Delta t'$ is substituted for $t_B{}' - t_A{}'$.

The paradox however can be explained by noting that for spatially separated clocks the change of relative synchronization cannot be unequivocally determined. The clocks can only be compared when they are in spatial coincidence. For example, when in S' either of the observers can slowly walk towards the other or both the observers can walk symmetrically (with respect to S') towards the other and compare their clocks (ages) when they meet. However in that case one can show [20] that they do not have identical local experiences— thus providing the resolution of the paradox.

While the paradoxical element of the problem goes away, the fact remains that the result (4) is correct and this time offset remains unchanged even if they slowly walk towards each other and compare their clocks (ages) when they meet [21].

This temporal offset effect of identically accelerated clocks gives an important insight into the behaviour of clocks in a uniform gravitational field, for, according to EP "...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system" [17]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. With this insight we write Eq.(4) as

$$t - t_0 = -\gamma(t) v(t) x_0/c^2 = -f(t), \tag{7}$$

where now t and t_0 are the readings of two clocks at higher and lower potentials respectively and also $f(t)$ stands for the right hand side of Eq.(4) without the minus sign

$$f(t) = \gamma(t) v(t) x_0/c^2. \tag{8}$$

In terms of differentials one may write Eq.(7) as

$$\delta t - \delta t_0 = -\dot{f(t)}\delta t, \tag{9}$$

where the time derivative $\dot{f(t)} = \frac{gx_0}{c^2}$, with $g = \frac{d}{dt}(\gamma v)$ is the *proper acceleration.*
We may now replace δt and δt_0 by n and n_0, where the later quantities corresponds to the number of ticks (second) of the clocks at their two positions. We therefore have

$$(n - n_0)/n_0 = -\dot{f(t)}, \tag{10}$$

or, in terms of frequency of the clocks,

$$-\delta\nu/\nu_0 = \dot{f(t)}, \tag{11}$$

where $\delta\nu$ refers to the frequency shift of an oscillator of frequency ν_0. The slowing down parameter for clocks, $-\delta\nu/\nu_0$ in Eq.(11) is nothing but the so called red-shift parameter Z for which we obtain the well-known formula[4]

$$Z = gx_0/c^2. \tag{12}$$

One thus observes that the time-offset relation (7) of Boughn's paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins.[5] We shall see the importance of the time-offset relation (7) in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin. However before that, in the next section we show that the connection of the time-offset and GSDC is purely relativistic in nature.

4. BOUGHN'S PARADOX IN THE CLASSICAL WORLD

The origin of Boughn's paradox can be traced to the space dependent part in the time transformation of LT. The existence of this term is indeed the cause of relativity of simultaneity in SR.

The notion of relativity of simultaneity however can also be imported to the classical world. By classical or Galilean world we mean a kinematical world endowed with a preferred frame (of ether) S with

[4]In terms of *ordinary* acceleration $\bar{g} = dv/dt$, measured with respect to S the formula comes out to be $Z = (\bar{g}\gamma x_0/c^2)(1 - v^2\gamma^2/c^2)$ which for small velocities can also be written as $Z = \bar{g}x_0/c^2$.

[5]The connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur [22], who derived the approximate formula for the "clock rate difference" mentioned in the previous foot-note.

respect to which the speed of light c is isotropic and moving rods and clocks do not show any length contraction and time dilation effects. However the speed of light measured in any other inertial frame S' moving with velocity v with respect to S will change and will depend on direction. The synchronization of spatially separated clocks is generally not an issue in this world as clocks can be transported freely without having to worry about time dilation, therefore all clocks can be synchronized at one spatial point and then may be transported with arbitrary speed to different locations. (The process is generally forbidden in SR). Clearly one uses the Galilean transformation (GT) to compare events in different inertial frames. Using GT one can show that the two way speed (TWS) of light $\overleftrightarrow{c}$ in S' along any direction θ with respect to the x-axis (direction of relative velocity between S and S') is given by

$$\overleftrightarrow{c}\ (\theta) = c(1-\beta^2)/(1-\beta\sin^2\theta)^{1/2}. \tag{13}$$

According to GT this TWS is not the same as the one-way speed (OWS) of light, for example, along the x-axis it is $c - v$ and $c + v$ in the positive and negative x-directions respectively, while the two way speed, i.e the average round-trip speed of light along the x-direction is given by $c(1 - v^2/c^2)$. However, in a playful spirit one may choose to synchronize the clocks in S' such that the one way speeds, to and fro are, the same as $\overleftrightarrow{c}$. This is similar to Einstein's stipulation in SR which is commonly known as the standard synchrony. In the Galilean world the synchrony is somewhat an awkward one but none can prevent one in adopting such a method. For this synchrony GT changes to the following transformations

$$x' = (x - vt), \quad t' = \gamma^2(t - vx/c^2), \tag{14}$$

which was first obtained by E. Zahar and is therefore known as the Zahar transformation (ZT) [23–26]. The transformations have been successfully used to clarify some recently posed counter-intuitive problems in SR [27, 28]. The presence of the phase term and γ^2 in Eq.(14) distinguishes the ZT from GT. Clearly the appearance of these terms is just an artifact of this synchrony.

One is thus able to recast Boughn's paradox using the above transformations and extending the arguments leading to the Eq.(4), one obtains, for the differential aging,

$$\Delta t' = -\gamma^2 v x_0/c^2. \tag{15}$$

The above expression for the differential aging between two spatially separated twins is also therefore an artifact of the synchrony.

Let us note that ZT has many interesting features which include the existence of apparent time dilation and length contraction effects as observed from an arbitrary reference frame S'. (With respect to the preferred frame however there are no such effects). We have already pointed out that the temporal offset between clocks cannot have any unequivocal meaning unless it corresponds to measurement at one spatial point.

One may therefore define without much ado the reality of the temporal offset effect due to Boughn (hereafter referred to as Boughn-effect), provided the clocks are finally compared when they are brought together. In the relativistic world a clock is slowly transported towards the other in order to minimize the time dilation effect in the process. In this world if one of the pre-synchronized spatially separated clocks is brought to the other in an arbitrarily slow motion, it can be seen that when they are compared at the position of the second clock, they remain synchronized. In other words if two clocks have an initial temporal offset between them (due to Boughn-effect or otherwise) when separated, the value for this offset will remain unchanged when they are brought together for comparison. Boughn-effect is thus a real effect (according to the definition) in the relativistic world. In the classical world the situation is different. Below we calculate the effect of clock transport from ZT.

From ZT between a preferred frame S_0 and an arbitrary frame S, one may write the transformation equation between any inertial frames S_i and S_k as

$$x_i = {\gamma_k}^2(1 - v_i v_k/c^2)x_k - (v_i - v_k)t_k, \tag{16}$$

$$t_i = {\gamma_i}^2[(1 - v_i v_k/c^2)t_k - (\gamma_k^2/c^2)(v_i - v_k)x_k], \tag{17}$$

where the suffixes i and k of coordinates x, t, and v refer to the coordinates in S_i and S_k and velocities of the concerned frames with respect to S_0, respectively; also $\gamma_i = (1 - v_i^2/c^2)^{-1/2}$ and $\gamma_k = (1 - v_k^2/c^2)^{-1/2}$.

Clearly a clock stationary with respect to S_k will suffer a time "dilation" according to

$$\Delta t_i = [(1 - v_i v_k/c^2)/(1 - v_i^2/c^2)]\Delta t_k, \tag{18}$$

where Δt_k refers to the proper time between two events at the same point of S_k and Δt_i is the corresponding time measured by observers in S_i.

Consider now two synchronized clocks are spatially separated by a distance x in S_i and a third clock attached to S_k slowly covers the distance. The time taken by the clock to cover this distance in S_i is given by

$$\Delta t_i = x/w, \tag{19}$$

where w is the relative velocity of S_k with respect to S_i. The corresponding time measured by the third clock (S_k - clock) may be obtained from Eq.(18).

From ZT the relative velocity formula is obtained as

$$w = (1 - v_i^2/c^2)(v_k - v_i)/(1 - v_i v_k/c^2). \tag{20}$$

Using Eqs. (18), (19), and (20), one obtains for the difference of these two times

$$\delta t' = \Delta t_k - \Delta t_i = (v_i x/c^2)\gamma_i^2. \tag{21}$$

This non-vanishing integrated effect of the time dilation in the classical world due to clock transport is independent of the speed (v_k) at which the clock is transported. In contrast, in the relativistic world one finds different values for the effect for different velocities and in particular the value is zero when the speed is vanishingly small.

If now the two stationary (with respect to S_i) clocks refer to two Boughn's observers A and B, they have precisely this amount (Eq.(21)) of temporal offset with a negative sign and hence if the observer A walks towards B no matter whether slow or fast, the result will be the zero time difference between the clocks when compared at one spatial point. This observation demonstrates that although Boughn's paradox can be recast in the Galilean world the time-offset effect is just an artifact and not real according to our definition of "reality" of the effect. Thus GSDC cannot be obtained from this Boughn's effect in the classical world via EP. Conversely Boughn's temporal offset may be regarded as an integrated effect of GSDC while in the classical world if it exists is just an artifact of the synchrony.

5. RESOLUTION

Let us now move on to the details of the arguments leading to Eq.(1). The outward trip of the traveller twin B from the point of view of the earth twin A is composed of two phases. In the first phase, the rocket moves a distance L_A in time t_{A1} with uniform velocity v which is given by

$$t_{A1} = L_A/v, \tag{22}$$

and in the second phase, which corresponds to the deceleration phase of the rocket which finally stops before it takes the turn-around, the time t_{A2} taken by B is given by

$$t_{A2} = \gamma v/g, \tag{23}$$

where the proper acceleration g has been assumed to be uniform with respect to the earth frame. In the present analysis this term does not contribute since we consider the abrupt turn-around scenario where t_{A2} tends to zero as $g \to \infty$; however for the time being we keep it. Therefore the total time elapsed in S for the entire journey is given by

$$T_A = 2L_A/v + 2t_{A2}. \tag{24}$$

Now we compute this time as measured in B's clock by taking the time dilation effect from the point of view of A. For phase 1 this time t_{B1} may be computed as

$$t_{B1} = \gamma^{-1} t_{A1} = \gamma^{-1} L_A / v, \tag{25}$$

where we have applied the simple time dilation formula. For phase 2 however this time dilation formula is differentially true as the speed is not a constant i.e., one may write

$$dt_{B2} = (1 - v^2/c^2)^{1/2} dt_{A2} = (1 - v^2/c^2)^{1/2} d(\gamma v)/g. \tag{26}$$

Hence, after integration, one obtains [29]

$$t_{B2} = \frac{c}{2g} ln \left(\frac{1 + v/c}{1 - v/c} \right). \tag{27}$$

However once again this tends to zero as $g \to \infty$. In any case we shall however not need this expression any more. Therefore the total elapsed time measured in B's clock for the complete journey is given by

$$T_B = 2\gamma^{-1} L_A / v + 2t_{B2}. \tag{28}$$

The differential aging from the point of view of A is thus

$$\delta T_A = T_A - T_B = (2L_A/v)(1 - \gamma^{-1}) + 2(t_{A2} - t_{B2}). \tag{29}$$

From the point of view of B the stay-at-home observer A is moving in the opposite direction and as before one may divide the relative motion of A into two phases, phase I and phase II, where the later corresponds to the acceleration phase. The phase II may be interpreted as turning on of a gravitational field. When this field is switched off (marking the end of the acceleration phase), the phase I starts where the stay-at-home observer A moves with a velocity $-v$ up to a distance L_B which on account of the Lorentz contraction of L_A is given by,

$$L_B = L_A (1 - v^2/c^2)^{\frac{1}{2}}, \tag{30}$$

and the corresponding elapsed time t_{BI} is given by,

$$t_{BI} = \frac{L_B}{v} = \gamma^{-1} L_A / v. \tag{31}$$

This obviously comes out to be the same as t_{B1} since the result is obtained from considerations with respect to the inertial observer A.

Similarly t_{BII} i.e. B-clock's time during phase II should be the same as t_{B2} during which the gravitational field is turned on, i.e.,

$$t_{BII} = t_{B2}, \tag{32}$$

and hence the total time

$$\tau_B = 2t_{BI} + 2t_{BII} = 2\gamma^{-1}L_A/v + 2t_{BII} = T_B. \tag{33}$$

The corresponding time of A's clock by taking into account the time dilation effect is

$$t_{AI} = \gamma^{-1}t_{BI} = \gamma^{-2}L_A/v. \tag{34}$$

Writing A-clock's time during phase II from B's perspective as t_{AII}, one may write for A's clock time for the entire journey as

$$\tau_A = 2t_{AI} + 2t_{AII} = 2\gamma^{-2}L_A/v + 2t_{AII}. \tag{35}$$

The difference of these times of clocks A and B as interpreted by the observer B, is given by

$$\delta T_B = \tau_A - \tau_B = (2\gamma^{-1}L_A/v)(\gamma^{-1} - 1) + 2(t_{AII} - t_{BII}). \tag{36}$$

Note that at the moment we do not know the value of t_{AII}, since it refers to the time measured by A as interpreted by B when it is in its acceleration phase. The paradox is resolved if

$$\delta T_A = \delta T_B. \tag{37}$$

In other words, using Eqs. (29) and (36), one is required to have,

$$t_{AII} = (L_A/v)(1 - \gamma^{-2}) + t_{A2} = L_A v/c^2 + t_{A2}. \tag{38}$$

In the abrupt turn-around scenario, as we have already observed $t_{A2} = 0$, one therefore must have

$$t_{AII} = L_A v/c^2 = \gamma L_B v/c^2. \tag{39}$$

The resolution of the twin paradox therefore lies in accounting for this term. It is interesting to note that the term is independent of the acceleration in phase II. This is possibly the implicit reason why the role of acceleration in the explanation of the twin paradox is often criticized in the literature. However we shall now see how, we can interpret this term as an effect of the direction reversing acceleration (or the pseudo-gravity) experienced by the traveller twin.

Now recall the Boughn-effect of temporal offset between two identically accelerated observers. To be specific, consider an inertial frame of reference S attached to the observer B when it is in the uniform motion phase (phase I). Suppose now there is another observer B' at rest in S at a distance L_B behind B and both of them get identical deceleration and eventually come to rest with respect to A in the frame of reference S', which is moving with velocity $-v$ in the x-direction with respect to S. According to Boughn-effect then the clocks of these two observers get desynchronized and the amount of this desynchronization is given by the expression (4) only with the sign changed, that means

$$desync = \gamma v L_B/c^2, \tag{40}$$

which is nothing but t_{AII}. It has already been pointed out that this Boughn-effect may be interpreted as the effect of pseudo-gravity (in this case as experienced by the observer B) according to EP. In terms of the pseudo acceleration due to gravity the above expression can also be obtained as

$$desync = g\Delta t_B L_B/c^2. \tag{41}$$

Note that $g\Delta t_B$ is finite (equal to γv) even if $g \to \infty$.

The observer B' which is L_B distance away from B is spatially coincident with A, hence, in calculating the clock time of A from B's perspective this time-offset due to Boughn-effect must be taken into account. This effect is ignored when the twin paradox is posed by näively asserting the reciprocal time dilation effect for the stay-at-home and the rocket-bound observers. Clearly the paradox is resolved if the Boughn-effect or the pseudo gravitational effect is taken into consideration.

6. CONCLUDING REMARKS: TEST OF BOUGHN-EFFECT

We have seen that the Boughn-effect can be interpreted as the integrated effect of GSDC. The experimental test of GSDC or the gravitational red-shift is therefore a test of a differential Boughn-effect in a way. On the contrary one may directly measure the integrated effect by the following means:

First two atomic clocks may be compared (synchronized) at the sea level, then one of the clocks may be slowly transported to a hill station of altitude h and then kept there for some time T. In this time these two atomic clocks according to Boughn scenario are perpetually accelerated from a rest frame S to a hypothetical inertial frame S' moving with velocity v, with proper acceleration g so that $\gamma v = gT$. Boughn-effect therefore predicts a temporal offset (see Eqs.(40) and (41)),

$$\Delta t_{\text{offset}} = ghT/c^2. \tag{42}$$

This offset can be checked by bringing the hill station clock down and then comparing its time with the sea level one. Any error introduced in the measurement due to transport of clocks can be made arbitrarily small compared to Δt_{offset} by increasing T. As a realistic example for h =7000ft (altitude of a typical hill station in India), and T =1 year and taking the average g to be about $9.8m/sec^2$, the Boughn-effect comes out to be in the micro-second order:

$$\Delta t_{\text{offset}} = 7.3\mu s, \tag{43}$$

which is easily measurable without requiring sophisticated equipments, such as those used in Pound-Rebka type experiments.

It is interesting to note that from the empirical point of view the effect is not entirely unknown. For example Rindler [16], in seeking to cite an evidence for the GSDC effect, remarks: "Indeed, owing to this effect, the US standard atomic clock kept since 1969 at the National Bureau of standards at Boulder, Colorado, at an altitude of 5400ft. gains about five microseconds each year relative to a similar clock kept at the Royal Greenwich Observatory, England, ..." However one can consciously undertake the project with all seriousness, for the accurate determination of the time-offset (with the error bars and all that), not merely to prove GSDC but to verify the Boughn-effect of SR. It is worth while to note that the empirical verification of this time-offset as a function of T would not only test the Boughn-effect and the integral effect of GSDC but it would also provide empirical support for the relativity of simultaneity[6] of SR. So far no experimental test has been claimed to be the one verifying the relativity of simultaneity. Indeed SR is applicable in the weak gravity condition of the earth so that gravity can be thought of as a field operating in the flat (Minkowskian) background of the spacetime [30]. Clearly because of EP, the earth with its weak gravity has the ability to provide a convenient Laboratory to test some special relativistic effects like the relativity of simultaneity or the Boughn-effect.

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[6]In the light of the C-S thesis, however, "relativity of simultaneity" loses its absolute meaning, since for example if absolute synchrony is used, there is no lack of synchrony between two spatially separated events as observed from different inertial frames, however, the differential aging or the temporal offset will pop up as a time dilation effect in the absolute synchrony set-up when the clocks are brought together by slow transport. The details of this issue is a subject matter of another paper by the authors in preparation.

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