

# Hadron-quark phase transition in dense matter

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**Abstract.** The two-Equation of State (EoS) model is used to describe the hadron-quark phase transition in asymmetric matter formed at high density in heavy-ion collisions. For the quark phase, the three-flavor Nambu–Jona-Lasinio (NJL) effective theory is used to investigate the influence of dynamical quark mass effects on the phase transition. At variance to the MIT-Bag results, with fixed current quark masses, the main important effect of the chiral dynamics is the appearance of an end point for the coexistence zone. We show that a first order hadron-quark phase transition may take place in the region  $T = (50 - 80)$  MeV and  $\rho_B = (2 - 4)\rho_0$ , which is possible to be probed in the new planned facilities, such as FAIR at GSI-Darmstadt and NICA at JINR-Dubna. From isospin properties of the mixed phase some possible signals are suggested. The importance of chiral symmetry and dynamical quark mass on the hadron-quark phase transition is stressed.

## 1. Introduction

The exploration of the phase diagram of strongly interacting matter and the search for signals of the hadron-quark phase transition are challenges both in theory and experiment. Intensive studies on these fields have been performed in the last decades based on Lattice QCD at zero or small chemical potential and effective chiral models.

Most effective models describe the phase transition based on quark degrees of freedom. As a matter of fact, at low temperature and small chemical potential, QCD dynamics are governed by hadrons. Therefore, it is natural to describe the strongly interacting matter with hadronic degrees of freedom at low  $T$  and small  $\mu$  and quarks at high  $T$  and large  $\mu$ . This picture can be realized in the two equation of state (Two-EoS) model in which hadronic and quark phases are connected by Gibbs condition. The Two-EoS model is widely used in describing the phase transition in the interior of compact star in weak equilibrium (e.g., [1, 2, 3, 4, 5, 6, 7, 8, 9]).

We remark that, even in the Two-EoS approach, only a few papers have studied the phase diagram of hadron-quark transitions at high baryon density in connection to the phenomenology of heavy-ion collision in ten A GeV range (intermediate energies) [10, 11, 12, 13, 14]. In these studies, current mass (or massless)  $u$ ,  $d$  quarks are taken for the quark phase, where the MIT-Bag like models are used. In order to obtain more reliable results and predict possible observables

in the planned experiments, in [15] we take the Nambu-Jona Lasinio (NJL) model to describe the quark phase with the interaction between quarks, where the chiral symmetry breaking and restoration are well described. We pay more attention to isospin asymmetric matter, and suggest some observable signals, possibly reached through heavy-ion reactions at new planned facilities, such as FAIR at GSI-Darmstadt and NICA at JINR-Dubna, where heavy ion beams (even unstable, with large isospin asymmetry) will be available with good intensities in the 1-30 A GeV energy region.

## 2. The Two-EoS model

In our Two-EoS approach, the hadron matter and quark matter are described by the non-linear Walecka model and the NJL model, respectively. For the mixed phase between pure hadronic and quark phase, the two phases are connected each other with the Gibbs conditions deduced from thermal, chemical and mechanical equilibriums. If we define  $\chi$  as the fraction of quark matter in the mixed phase, the Gibbs conditions for the mixed phase are

$$\begin{aligned}\mu_B^H(\rho_B, \rho_3, T) &= \mu_B^Q(\rho_B, \rho_3, T) \\ \mu_3^H(\rho_B, \rho_3, T) &= \mu_3^Q(\rho_B, \rho_3, T) \\ P^H(\rho_B, \rho_3, T) &= P^Q(\rho_B, \rho_3, T),\end{aligned}\tag{1}$$

where  $\rho_B = (1 - \chi)\rho_B^H + \chi\rho_B^Q$  is the total baryon density in the mixed phase, and the total isospin density is  $\rho_3 = (1 - \chi)\rho_3^H + \chi\rho_3^Q$ . With the initial condition of asymmetric parameter  $\alpha^H$  in heavy-ion collision, the global asymmetry parameter  $\alpha$  for the mixed phase should be

$$\alpha \equiv -\frac{\rho_3}{\rho_B} = \frac{(1 - \chi)\rho_3^H + \chi\rho_3^Q}{(1 - \chi)\rho_B^H + \chi\rho_B^Q} = \alpha^H \Big|_{\chi=0} = \alpha^Q \Big|_{\chi=1},\tag{2}$$

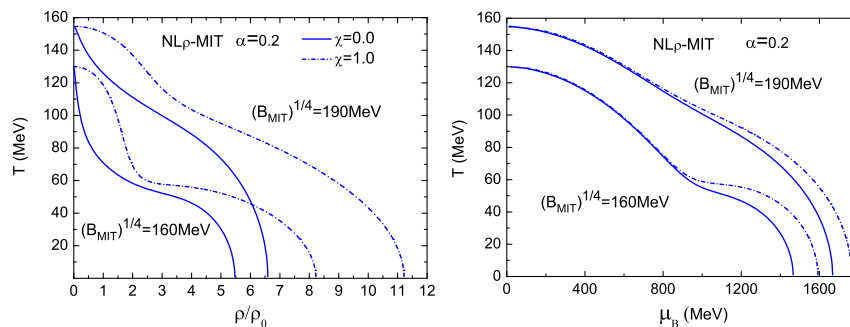
according to the charge conservation. For details, please refer to [15] and refs. therein.

## 3. Numerical results and discussion

We will pay more attention to the phase diagram of asymmetric matter and focus the attention on experiments with neutron-rich beams, where large intensities can be reached. Here the largest accessible isospin asymmetry parameter  $\alpha$  is just above 0.2 (e.g., we have  $\alpha = 0.227$  in  $^{238}\text{U} + ^{238}\text{U}$  collision, see also the Table II in Ref. [13]). Therefore we will consider  $\alpha = 0.2$  in our calculation, as taken in Refs. [11, 12, 13, 14]. Of course the use of more asymmetric unstable beams will enhance the isospin effects described here.

### 3.1. Some results with the MIT-Bag quark EoS

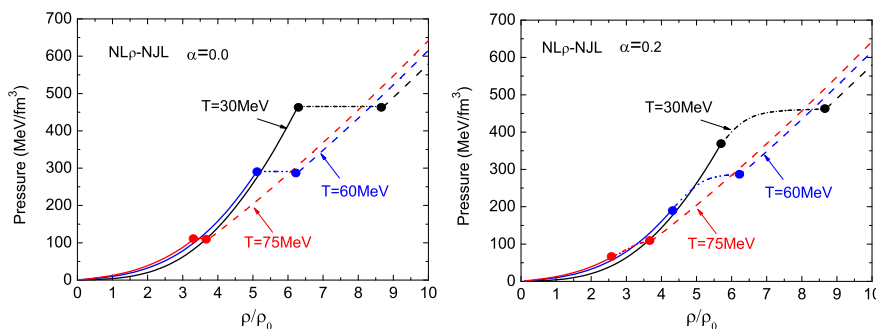
We firstly display some results with the MIT-Bag model in which free fermions are considered with the current mass  $m_u = m_d = 5.5 \text{ MeV}$ . In figure 1 we plot the  $T - \rho_B$  and  $T - \mu_B$  phase diagrams. The solid curves are the phase-transition lines from nuclear matter to quark matter for  $(B_{\text{MIT}})^{1/4} = 160 \text{ MeV}$  (190 MeV), and the corresponding dash-dot curves are the transition lines to pure quark matter.  $NL\rho$  hadron EoS [11] has been used in the calculation. From this figure, it easy to see that the decrease of bag constant reduces the onset density of quark matter, and in general that the phase transition curve highly depends on the value of bag constant, which is one of the motivations of this study to use the NJL model to investigate the phase transition.



**Figure 1.** (Color online) Left(Right): the  $T - \rho_B$  ( $T - \mu_B$ ) plane of asymmetric matter with the isospin ratio  $\alpha = 0.2$  for  $(B_{\text{MIT}})^{1/4} = 160$  MeV and 190 MeV. The region between the solid and dash-dot curve gives the binodal surface of the mixed phase, where the hadron and quark matter coexist.  $NL\rho$  parametrization is used for the hadron phase.

### 3.2. Results with the NJL quark EoS

In figure 2 we present the  $P - \rho_B$  phase diagram, within the  $NL\rho$ -NJL two-EoS scheme [15], respectively for symmetric and asymmetric ( $\alpha = 0.2$ ) matter. Clearly the pressure is a constant in the mixed phase of symmetric matter, just the same as with Maxwell construction. At variance in the asymmetric case, we have a monotonous increase. It is interesting to note that pressure rising is faster in the first part of the mixed phase (more evident for the lower temperatures  $T=30, 60$  MeV). This is due to the isospin distillation effect, i.e., a large  $\rho_d - \rho_u$  asymmetry for reduced quark fractions, see next subsection, which is increasing the quark pressure.



**Figure 2.** (Color online) Left: Pressure of symmetric matter as a function of baryon number density at different temperatures. Right: Pressure of asymmetric matter with  $\alpha = 0.2$ .

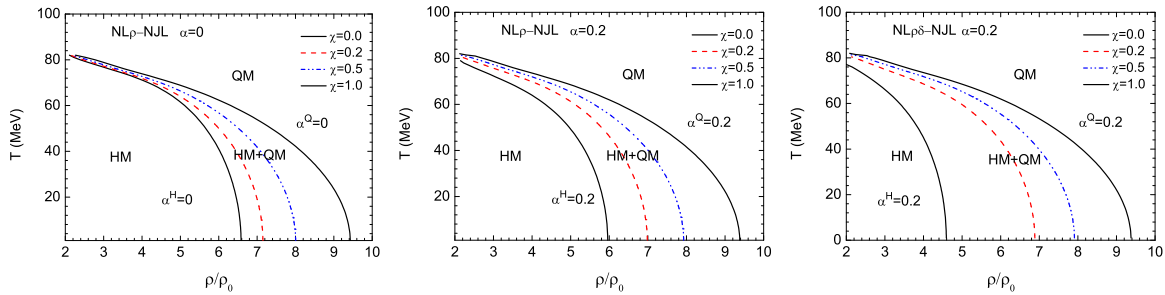
From figure 2, we can also see that the size of the mixed phase shrinks with increasing temperature. Meanwhile the onset density becomes smaller, opening the possibility of probing the coexistence phase in heavy-ion collision at intermediate energies. This effect is further enhanced by the isospin asymmetry as we will discuss in the next subsection.

#### 3.2.1. Symmetry energy effects

In figure 3 we plot the (identical)  $T - \rho_B$  phase diagrams of symmetric and asymmetric matter with different parameters in hadron phase. For different cases, in the mixed phase region, the binodal surface, is rather different. In general the onset density of the mixed phase in asymmetric

matter is smaller than that in symmetric matter, similar to the results obtained by the MIT-Bag model [10, 11, 12, 13, 14].

If the  $NL\rho\delta$  parameter set [11] is used, the onset density will be further reduced, as shown in figure 3. This is nicely due to the fact that at high baryon density the symmetry energy of hadron matter with the parameter set  $NL\rho\delta$  is much larger than in the  $NL\rho$  case, see the following. All that also indicates that isospin effects can be very important for hadron-quark phase transition in heavy-ion collision in order to shed light on nuclear interactions in an hot and dense medium.

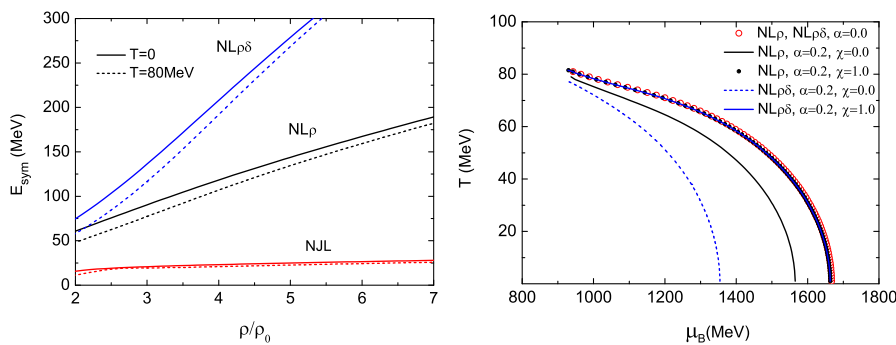


**Figure 3.** (Color online) Left: Phase diagram in the  $T - \rho_B$  plane for symmetric matter with the parameter set  $NL\rho$ . Middle: for asymmetric matter with the parameter set  $NL\rho$ . Right: for asymmetric matter with the parameter set  $NL\rho\delta$ .

To understand the role of the symmetry term on the hadron-quark phase transition, we further investigate the symmetry energy  $E_{sym}$  in the two phases, defined in general by [11, 14]

$$(E/A)_{\alpha_i} = (E/A)_{\alpha_i=0} + E_{sym}\alpha_i^2 \quad (3)$$

where  $\alpha_i = \alpha^H$  for hadron matter,  $\alpha^Q$  for quark matter.



**Figure 4.** (Color online) Left: Symmetry energy of nuclear matter with the parameter sets  $NL\rho$ ,  $NL\rho\delta$  and that of quark matter in the NJL model. Right: Phase diagram in the  $T - \mu_B$  plane for symmetric and asymmetric matter.

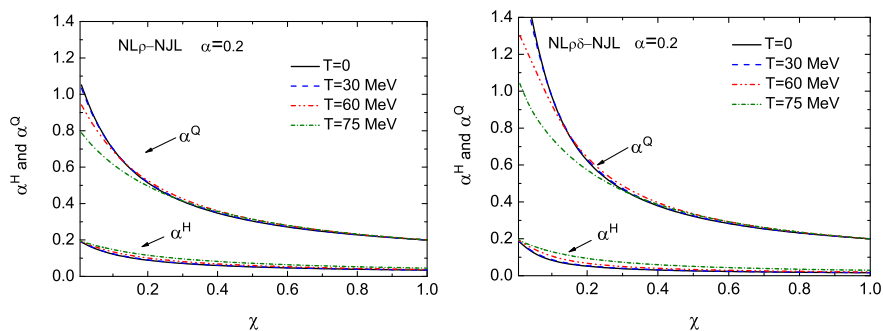
We display the numerical results in the left panel of figure 4. This figure shows that the symmetry energy of hadron matter with the parameter set  $NL\rho\delta$  is larger than that of the  $NL\rho$  case. This leads to a smaller onset density of the transition. It is interesting to note that this is a genuine relativistic effects since the scalar covariant nature of the isovector  $\delta$  meson

contributes to increase the symmetry energy at high densities directly with a larger repulsion and indirectly via a splitting of the neutron/proton effective masses, with  $M_n^* < M_p^*$ , see details in refs. [16, 17].

In the right panel figure 4 we plot the  $T - \mu_B$  phase diagram for symmetric and asymmetric matter with both the parameter sets  $NL\rho$  and  $NL\rho\delta$ . For the symmetric matter at a given temperature,  $\mu_B$  is constant in the mixed phase, i.e., varying the quark matter fraction  $\chi$ , then we have only one transition line (the empty circles). At variance, like in the MIT-Bag calculation of figure 1, in the asymmetric case we have a monotonous  $\mu_B$  increase with increasing  $\chi$ , so there are two curves in  $T - \mu_B$  plane presenting the start and end, respectively, of the transition from nuclear to quark matter. What is important is that the chemical potentials of the onset of the transition ( $\chi = 0$ ) in asymmetric matter are always smaller than the corresponding ones of symmetric matter. This is more evident with the parameter set  $NL\rho\delta$  (dotted curve), which again shows the importance of symmetry energy effects.

### 3.2.2. Isospin Distillation inside the mixed phase

In figure 5, we show the variation of the isospin asymmetry parameters of hadron ( $\alpha^H$ ) and quark ( $\alpha^Q$ ) matter inside the mixed phase at various temperatures, with the global asymmetry  $\alpha = 0.2$ . We clearly see the much larger values of  $\alpha^Q$  when the quark phase starts forming, it reduces to 0.2 when the pure quark phase is reached ( $\chi = 1$ ) with  $\chi = 1$ . This is a nice Isospin Distillation effect ruled by the symmetry energy gap in the two phases, as confirmed by the enhancement in figure 5, where the more repulsive  $NL\rho\delta$  *EoS* is used for the hadron part.



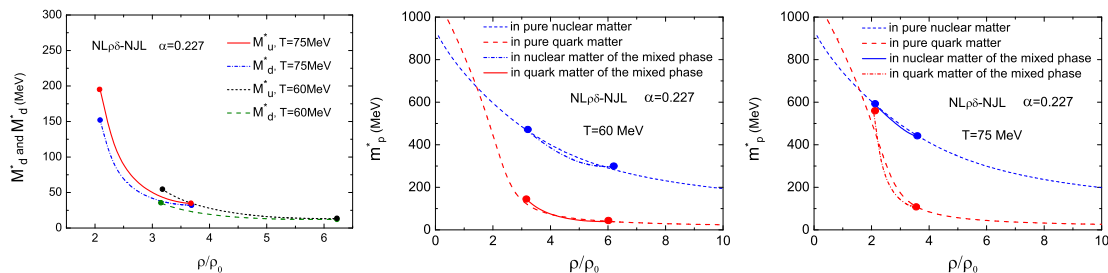
**Figure 5.** (Color online) Asymmetry of quark matter and nuclear matter inside the mixed phase at different temperatures. The parameter set  $NL\rho$  ( $NL\rho\delta$ ) is used for the hadron phase in the left (right) panel.

This behavior of the quark isospin asymmetry inside the mixed phase of the hadron-phase transition will affect the following hadronization in the expansion stage, finally producing some observable signals in heavy-ion experiments. As suggested before in Ref. [11, 12], an inversion in the trend of the emission of neutron rich clusters,  $\pi^-/\pi^+$ ,  $K^0/K^+$  yield ratios in high density regions, and an enhancement of the production of isospin-rich resonances and subsequent decays may be found. Besides, there is a controversial point of view about the enhancement of the yield ratio  $\bar{\Lambda}/\bar{p}$  [18, 19, 20, 21, 22].

### 3.2.3. New effect of the dynamical quark masses: a critical end point?

If we compare MIT-Bag (with fixed current quark masses) and NJL (with chiral restoration mechanism) results for the mixed phase we remark only one main difference. As shown in figure 1, phase diagrams in  $T - \rho_B$  and  $T - \mu_B$  planes are derived, using the MIT-Bag model,

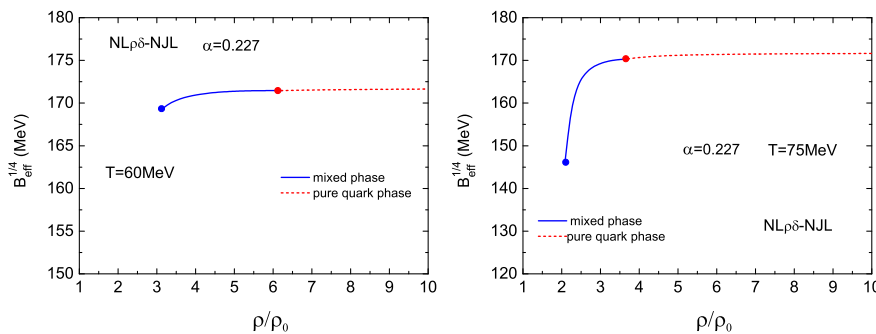
with a critical temperature reached only at zero baryon density. At variance the binodal curves at high temperature and low density cannot be obtained with the NJL model and we see a narrowing of the coexistence region up to a kind of *critical end point*, figure 3. This important result derives from two qualitative new features of the NJL effective theory: i) the quark masses variation due to the chiral restoration, ii) the dependence of the effective bag-constant  $B_{eff}$ , Eq. (17) in Ref. [15], on temperature and baryon density.



**Figure 6.** (Color online) Left: Dynamical masses of  $u$ ,  $d$  quarks inside the mixed phase at different temperatures. Middle (Right): Effective proton masses in the hadron (upper) and quark (lower) matter inside the mixed phase at  $T = 60$  (75) MeV. The solid dots indicate the two density limits of the coexistence region.

- The chiral dynamics largely increases the quark masses at low densities and finite temperatures, see figure 6 where we plot the  $u, d$  effective masses inside the mixed phase at different temperatures, for asymmetric matter. If we reach the limit of a nucleon effective mass in the hadron phase smaller than the corresponding combination of quark effective masses, e.g., for protons  $M_p^* < 2M_u^* + M_d^*$ , we would have unphysical solutions for the Gibbs conditions with  $\rho_B^Q < \rho_B^H$ . This is confirmed by figure 6, where we plot the proton effective mass in the hadron phase (solid line) and in the quark phase (dot-dashed line) inside the coexistence region at  $T = 60$  MeV and  $T = 75$  MeV. We see that at the higher temperature we are close to a crossing, indication of the lack of physical solution at higher temperatures, as already seen in the corresponding  $NL\rho\delta$  results of the previous figure 3.
- Actually we clearly see that in the same  $(T, \rho)$  region we do not have solutions of the Gibbs conditions. This is due to the second qualitative new feature of the  $NJL$  approach, the density and temperature dependence of the “effective bag constant”, also related to the dynamical quark mass variation. As a consequence at low densities and high temperatures, for small values of the bag constant we cannot get mixed phase solutions since the hadron pressure (mostly thermal) cannot equilibrate the quark pressure in the coexistence zone. In figure 7 we show the density dependence of  $(B_{eff})^{1/4}$  at  $T = 60$  and  $T = 75$  MeV (the solid circles give the mixed phase limits). For the  $T = 75$  MeV case we see a sudden drop around the onset of the mixed phase, good indication of a lack of solution for larger temperatures.

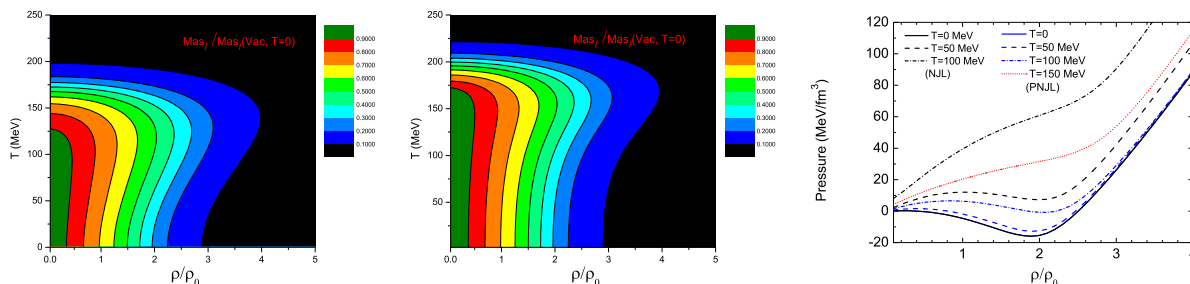
No physical solution existing with the Gibbs conditions at high temperature and low density means that there exists a dynamical competition between hadronic and quark matter. Generally, for the quark phase, the dynamical effects are relevant for both chiral symmetry and (de)confinement. Only chiral dynamics is included in the NJL model in this study. This leads to the pressure of quark phase at low density and high temperature given by the NJL model being always higher than that of hadron phase at the same baryon and isospin chemical potential. So the absence of Gibbs equations solutions in the hadron-NJL model reflects that the consideration of only the chiral dynamics is not enough at low density and high temperature. The confinement



**Figure 7.** (Color online) Left(Right): Baryon density dependence of the NJL “effective bag constant”,  $(B_{eff})^{1/4}$ , at  $T = 60$  ( $75$ ) MeV. The line ending with solid circles indicates coexistence region.

effect has to be included, which will affect the chiral dynamics and the equation of state of quark matter as shown in figure 8.

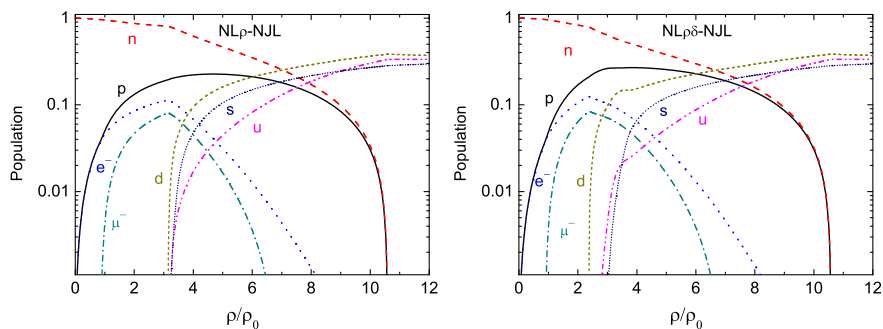
It is expected that the situation at high temperature and low density will be altered when the (de)confinement effect is included. This encourages us to perform a further study, following with the PNJL model for quark phase to get more accurate results about the “critical end point”. One can refer to [23] for the PNJL parameter set as well as the updated result.



**Figure 8.** (Color online) Left (Middle): The contour of  $u, d$  quark mass as functions of temperature and baryon density in the NJL (PNJL) model. Right: Pressure of quark matter in the (P)NJL model. The logarithmic form of Polyakov loop effective potential with rescaled  $T_0 = 210$  MeV [24] is used in the calculation of PNJL model.

### 3.3. Hadron-quark phase transition in neutron star matter

We also give a short discussion about the phase transition in neutron star matter under  $\beta$  equilibrium. As we know that the largest asymmetry parameter in heavy-ion experiment is  $\alpha \simeq 0.227$ . However, for neutron star matter the number density of neutron is much larger than that of proton, so the asymmetry parameter  $\alpha$  can be larger than 0.227. Based on the conclusion obtained in last section, the onset density of hadron-quark phase transition should be much smaller in the interior of neutron star. This judgement is confirmed by the numerical calculation shown in figure 9 for symmetric matter and asymmetric matter. Figure 9 also shows that the onset density of quark matter with  $NL\rho\delta$  is smaller than that with  $NL\rho$ . This is because the symmetry energy with  $NL\rho\delta$  is larger than that of  $NL\rho$ , consistent with above discussion.



**Figure 9.** (Color online) Left (right): Population of neutron star matter under  $\beta$  equilibrium as functions of baryon density with Parameter set of  $NL\rho$  ( $NL\rho\delta$ ). The NJL model is used for quark phase

#### 4. Summary

In this work, we investigate the hadron-quark phase transition in dense matter, particular in isospin asymmetric matter. The novelty of this study is to consider chiral dynamics of the quark masses using the 3-flavor NJL model for interacting quark instead of the MIT-Bag model. We obtain the binodal surface of a first order hadron-quark phase transition in the region of  $\rho_B > 2\rho_0$  and temperature  $T$  less than about 80 MeV. The resulting effect is more relevant just in the region  $T = (50 - 80)$  MeV and  $\rho_B = (2 - 4)\rho_0$ , available with the new planned facilities, where some important observables are suggested and may be found in the new planned facilities, for example, FAIR at GSI-Darmstadt and NICA at JINR-Dubna, with realistic asymmetries for stable/unstable beams.

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