Geometrisation of Electromagnetic Field and Topological Interpretation of Quantum Formalism

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A new concept of geometrization of electromagnetic field is proposed. Instead of the concept of extended field and its point sources, the interacting Maxwellian and Dirac electron–positron fields are considered as a unified closed connected nonmetrized space–time 4-manifold. Within this approach, the Dirac equation proves to be a group-theoretic relation that accounts for the topological and metric properties of this manifold. The Dirac spinors serve as basis functions of its fundamental group representation, while the tensor components of electromagnetic field prove to be the components of a contracted, in a certain way, curvature tensor of the relevant covering space. A basic distinction of the suggested approach from the geometrization of gravitational field in general relativity is that, first, not only the field is geometrized but also are its microscopic sources and, second, the field and its sources are treated not as a metrized Riemannian space–time but as a nonmetrized space– time manifold. A possibility to geometrize weak interaction is also discussed.

Introduction

The possibility of representing physical interaction as a distortion of pseudo-Euclidean space–time metric was first demonstrated in general relativity. However, further attempts at including other interactions and, primarily, electromagnetic interaction into the analogous geometrical approach have failed. It is shown in this work that electromagnetic field and its microscopic sources can be represented as a unified nonmetrized space–time topological manifold. Some of the preliminary results of this work have already been published and delivered at conferences [1]. Let us first use an analogy with the theory of general relativity to outline the main results of this work. It is demonstrated in general relativity that the classical equations of motion in gravitational field can be interpreted as equations for the shortest distances between two points of a curved Riemannian space–time. It is shown in this work that the Maxwell and Dirac equations for the interacting electromagnetic field and its quantum microscopic sources (electron–positron fields) can be interpreted as a group-theoretic relations describing a closed connected nonorientable nonmetrized space–time 4-manifold whose topological invariants are the observable physical characteristics of the system considered. The equations for neutral massive Fermi particles interacting via a weak field generated by them are formulated in terms of the proposed topological approach. This interaction violates the space and time symmetry. The possibility of interpreting these equations as equations describing, in the classical limit, weak interaction of massive neutrinos is discussed.

The interpretation of the microscopic field sources as extended geometrical objects forming, together with field, a unified topological 4-manifold requires a new geometrical interpretation of the mathematical apparatus of quantum mechanics. This new interpretation, above all, should not contradict the traditional interpretation at least in those cases where the latter approach has confirmed its effectiveness experimentally. For this reason, before passing to the main part of this work, it is worthwhile to show that the concept of the quantum object as a special nonmetrized topological manifold is not contradictory to the fundamental positions of the conventional interpretation of quantum mechanics.

1. The de Broglie's original idea of "travelling wave," which is specified for a free particle in the wave–corpuscular duality concept by the function

$$\exp i(\omega t - \mathbf{k}\mathbf{r}) = \exp i(\frac{E}{\hbar}t - \frac{\mathbf{p}}{\mathbf{h}}\mathbf{r}), \tag{1}$$

underlies the traditional interpretation of quantum mechanics. However, Eq. (1) can be recast in the form that allows a different, not wave, interpretation. Let us write Eq. (1) in the "relativistically symmetric" form

$$\exp i(\frac{x_1}{l_1} - \frac{x_2}{l_2} - \frac{x_3}{l_3} - \frac{x_4}{l_4}), \tag{2}$$

where $x_1 = ct, x_2 = x, x_3 = y, x_4 = z$ and $l_1 = 2\pi\hbar c/E, l_2 = 2\pi\hbar/p_x, l_3 = 2\pi\hbar/p_y, l_4 = 2\pi\hbar/p_z$.

In the group theory, function (2) plays the role of a representation of the cyclic Abelian group with four generators $l_1^{-1}, l_2^{-1}, l_3^{-1}$ and l_4^{-1} [2]. Because of this, the object described by this function can be interpreted not only as possessing wave properties but also as a geometrical object possessing a certain symmetry.

2. The notion of a point particle has no physical meaning in the relativistic quantum mechanics. Since a change in momentum during measurement cannot be as large as one likes because of the limiting speed of light, the uncertainty in measuring coordinates cannot be as small as is wished [3]. This does not contradict the treatment of electromagnetic field and its microscopic sources as an extended space–time manifold.

3. The traditional quantum mechanical formalism is based on the statistical probabilistic description. This does not contradict the description of a quantum object as a nonmetrized topological manifold that is "chaotic" by its very topological nature.

4. The suggested geometrical interpretation of the quantum formalism does not involve any "hidden" parameters, whose incorporation in any new possible interpretation of quantum mechanics is forbidden by the relevant theorems [4, 5].

In closing this section, two fundamental distinctions between the suggested topological approach and the geometrization of gravitational field in general relativity should be emphasized.
1. The field sources (masses) in general relativity are considered as nongeometrized point objects. Only the extended field is the object of geometrization. In this work, the microscopic field sources are treated as extended objects which form, together with field, a single unified geometrical object.
2. The gravitational field in general relativity is put in correspondence with a metrized topological manifold, namely, with a curved Riemannian space that has a certain form in every instant of time. In this work, the field and its sources are put in correspondence with an essentially different geometrical object, namely, with a nonmetrized manifold whose metric is undetermined and, hence, which has no definite form and is characterized only by its topological invariants.

Topological derivation of Dirac equation

It is my purpose to prove that the topological characteristics of a certain closed connected 4-manifold are encoded in a certain way in the equations for a classical field interacting with its microscopic sources (electrons and positrons). These equations (Maxwell and Dirac equations) have the form [3]

$$i\gamma_1(\frac{\partial}{\partial x_1} + ieA_1)\psi - \sum_{\alpha=2}^{4} i\gamma_\alpha(\frac{\partial}{\partial x_\alpha} + ieA_\alpha)\psi = m\psi, \tag{3}$$

$$F_{kl} = \frac{\partial A_k}{\partial x_l} - \frac{\partial A_l}{\partial x_k}, \tag{4}$$

$$\sum_{i=1}^{4} \frac{\partial F_{ik}}{\partial x_i} = j_k, \quad j_k = e\psi * \gamma_1\gamma_k\psi. \tag{5}$$

Here $\hbar = c = 1, \quad x_1 = t, \quad x_2 = x, \quad x_3 = y, \quad x_4 = z, \quad F_{kl}$ is the electromagnetic field tensor, A_k is the 4-potential, γ_k are the well-known Dirac matrices, ψ is the Dirac bispinor, and m and e are electron mass and charge, respectively.

To my knowledge, the topological manifolds are not identified by the differential equations. This, in particular, differentiates the suggested approach from general relativity, for which the mathematical apparatus (e.g., equations for geodesics) had well been elaborated by the time the theory emerged. For this reason, I will first demonstrate, by a simple example, how can the differential equations describe, in principle, the topological characteristics of a manifold.

Let us consider the simplest closed connected nonmetrized manifold, namely, a one-dimensional S^1 manifold isomorphous with (equivalent to) ring. It is equivalent in the sense that it can be represented as any of the objects obtained from ring by its deformation without discontinuities. A fundamental group of different classes of closed paths starting and ending at the same point of the manifold is one of the topological invariants of any connected manifold [6]. Free cyclic group isomorphous with the Z group (integer group, a topological invariant of the manifold considered) is the fundamental group of S^1 [7]. In turn, the Z group is isomorphous with the group of parallel translations along a straight line with one generator (the line is called the covering space of our manifold). Denote the generator length by l. Note that the operator T_{lx}

$$T_{lx} = (\frac{il}{2\pi})\frac{d}{dx} \tag{6}$$

can be regarded as a representation of the above group with a basis defined by the function $\varphi_l(x)$

$$\varphi_l(x) = \exp(-2\pi i \frac{x}{l}). \tag{7}$$

Indeed,

$$\varphi_l' = \varphi_l(x + l) = T_{lx}\varphi_l(x). \tag{8}$$

Thus, both the fundamental group (a topological invariant of the manifold considered) and the constraint (metric characteristic of this manifold) on the length of the generator of this group are defined by relationship (8). Consequently, the manifold is fully identified by the differential equation

$$i\frac{d\varphi}{dx} = m\varphi, \qquad m = \frac{2\pi}{l}. \tag{9}$$

which is equivalent to this relationship.

Let us use this approach for the "topological" decoding of differential equation that are more complex than (9), namely, the equation for free Dirac field

$$i\gamma_1\frac{\partial}{\partial x_1}\psi - \sum_{\alpha=2}^{4} i\gamma_\alpha\frac{\partial}{\partial x_\alpha}\psi = m\psi, \tag{10}$$

which is obtained from Eq.(3) at $A_1 = A_2 = A_3 = A_4 = 0$.

To this end, let us again consider, for clearness, one of the simplest and well-studied two-dimensional closed connected nonorientable manifolds, namely, the Klein bottle, and encode its topological and metric properties using the differential equations within the framework of the same approach as was demonstrated by the example of a one-dimensional ring. It will be seen below that the solution of this problem and its generalization to the four-dimensional case allow the topological interpretation of the Dirac equation (10). The Klein bottle is obtained by gluing together two Möbius strips along their edges. Euclidean plane is the covering surface for the Klein bottle, and the sliding symmetry group generated by two parallel translations in the perpendicular directions and two reflections in the directions perpendicular to the translations is its fundamental group [8, 9]. Let us assume that two translation generators l_1 and l_2 satisfy the additional constraint

$$\frac{1}{l_1^2} + \frac{1}{l_2^2} = \frac{1}{l_0^2}, \tag{11}$$

where l_0 is an additional metric characteristic of the manifold. Let us now express the topological characteristic (fundamental group) of the Klein bottle and the additional metric condition (11) in terms of an equation which plays the same role as does Eq. (9) for the ring. Similar to (7), the function

$$\varphi(x,y) = \exp(-2\pi i \frac{x}{l_1} - 2\pi i \frac{y}{l_2}). \tag{12}$$

is chosen as a basis function for the subgroup of two parallel translations along the OX and OY axes. A two-component spinor

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{13}$$

is chosen as a basis for the subgroup of reflections perpendicular to the OX and OY axes. The reason for which the spinor basis is chosen for the reflections is that, as will be seen below, only with such a choice the resulting equation leads to metric condition (11).

As above, the operators of type (6) form the representation of a translation group with basis functions (12). As for the reflections about the planes passing through OX and OY, the Pauli matrices σ_x and σ_y [10]

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{14}$$

form the representation of these reflections for the spinor basis (13). Thus, the functions

$$\psi(x,y) = \chi\varphi(x,y), \tag{15}$$

are the basis functions for the fundamental group of Klein bottle (sliding symmetry), while the operators P_x and P_y

$$P_x = \sigma_x T_{l_1 x}, \qquad P_y = \sigma_y T_{l_2 y}. \tag{16}$$

form the representation of sliding symmetry along the OX and OY axes. It is now straightforward to carry out direct calculation to check on that the equation expressing, similar to Eq. (9), the topological invariant (fundamental group) of the Klein bottle and satisfying metric constraint (10) on the elements of this group has the form

$$(P_x + P_y)\psi = \frac{1}{l_0}\psi. \tag{17}$$

Indeed, one can easily verify that the insertion of Eqs. (12)–(16) into Eq. (17) yields relationship (11). With allowance made for Eq. (16), Eq. (17) can be written in the form of a differential equation

$$i\sigma_x \frac{\partial \psi}{\partial x} + i\sigma_y \frac{\partial \psi}{\partial y} = \mu\psi, \qquad \mu = \frac{1}{l_0}. \tag{18}$$

Let us compare this equation with Eq. (10) for a free Dirac field taking into account that the Dirac matrices γ_i entering Eq. (10) form, in the four-dimensional pseudo-Euclidean space, the representation of reflections about three axes perpendicular to the OX axis, provided that the Dirac bispinors form the basis of this representation [10]. This comparison demonstrates that Dirac equation (10) is the generalization of Eq. (18) for two- dimensional Klein bottle to the four-dimensional pseudo-Euclidean Minkowski space. In this case, the role of metric constraint (10) is played by the energy conservation law

$$E^2 - p_x^2 - p_y^2 - p_z^2 = m^2, \tag{19}$$

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which provides, within the topological interpretation of the free Dirac field as a space–time manifold, the metric relationships between the translation subgroup generators,

$$\frac{1}{l_1^2} - \frac{1}{l_2^2} - \frac{1}{l_3^2} - \frac{1}{l_4^2} = \frac{1}{l_0^2},\tag{20}$$

where $l_0^{-1} = m$, $\quad l_1^{-1} = E$, $\quad l_2^{-1} = p_x$, $\quad l_3^{-1} = p_y$, $\quad l_4^{-1} = p_z$.

So, Dirac equation (10) can be interpreted as a relation encoding the topological and metric properties of a closed nonorientable space–time manifold, whose analogue in the two- dimensional case is provided by the Klein bottle. In this case, the Dirac spinors function as the basis functions of the fundamental group of a manifold, whose covering space is a pseudo-Euclidean Minkowski space. The energy, momentum, and mass of the Dirac field and the energy conservation law are related to the translation subgroup generators by relationships (20). Evidently, the foregoing consideration can be regarded as a "purely" topological derivation of the Dirac equation, i.e., as a derivation based on an assumption that the quantum object represents a certain space–time manifold, without invoking the Lagrangian, Hamiltonian, or any other mechanical formalism.

In closing this section, note that the closeness of a manifold in the pseudo-Euclidean space does not imply any constraints on its extension over the time axis. For example, a circle in the pseudo-Euclidean plane is mapped into an equilateral hyperbola in the usual plane [11].

Geometrisation of electromagnetic interaction

Let us now turn to the fundamental problem of justifying the interpretation of Eqs. (3)–(5) for the interacting electromagnetic field and its sources as group–theoretic relations that encode the topological characteristics and metric constraints of a certain unified nonmetrized space–time 4- manifold. By now, the topology of 4-manifolds is understood to a much lesser extent than the topology of, e.g., two-dimensional manifolds (see, e.g., [12, 13]). The latter are classified in detail, and the parameters of their main topological groups are known [6–9]. For this reason, I will attempt to invoke a possible analogy with similar problem in the two-dimensional topology (much as the Dirac equation was derived topologically in the preceding section), because the successful use of "low- dimensional" analogies is one of the merits of the geometrization of physical and mathematical problems.

Specifically, let us find out what will happen if the different topological properties of the orientable and nonorientable manifolds are combined within a single unified two-dimensional closed manifold. Consider, for example, what is a hybrid of torus and Klein bottle like and how can its topological properties be described. According to the topological classification of two- dimensional manifolds, torus is a "sphere with one arm" (orientable surface of the genus $p = 1$) and the Klein bottle is a "sphere with two holes glued up by Möbius strip" (nonorientable surface of the genus $q = 2$) [6–9]. By the hybrid of torus and Klein bottle can be meant a sphere to which one arm and two Möbius strips are glued simultaneously. Such a surface is nonorientable and belongs to the genus $q' = 2p + q = 2 + 1 = 3$ type [6–9]. For $q > 2$, a hyperbolic plane is a universal covering surface of a manifold and the fundamental group is generated by q sliding symmetries [8, 14]. If one assumes that the analogy with the manifold of the above-mentioned type "operates" in our case, one should expect that Dirac equation (1) can be interpreted as a metric relation for the nonorientable 4-manifold, whose fundamental group is generated by sliding symmetries, while the universal covering space is a four-dimensional pseudo-Euclidean analogue of the hyperbolic plane, namely, a conformally pseudo-Euclidean space [11].

Let us show that the Dirac equation indeed allows the above interpretation. A conformally Euclidean space is said to be a Riemannian space that can be conformally mapped into the Euclidean space. By this is meant that every point $M(x)$ of the conformally Euclidean space can be assigned

a point M_E in the Euclidean space so that the corresponding differentials of arc lengths are related to each other at every point by the relationship [11]

$$ds_E^2 = f(x^0, x^1, x^2, x^3)ds^2, \tag{21}$$

where $f(x)$ is a certain function of coordinates, $ds^2 = g_{ik}dx^i dx^k$ defines the metric of the conformally Euclidean space, and $ds_E^2 = g_{ik}^E dx^i dx^k$ is the squared differential arc length in the Euclidean space (in our case of pseudo-Euclidean space, $g_{00}^E = 1, g_{11}^E = g_{22}^E = g_{33}^E = -1$ and $g_{ik}^E = 0, i \neq k$).

Let us now turn to the left-hand side of Dirac equation (3). Compared to equation (10) for a free electron–positron field, it includes the expression $(\partial/\partial x_l + ieA_l)$ instead of the usual derivative $\partial/\partial x_l$. In electrodynamics, this expression is customarily called "covariant derivative", because it formally resembles the covariant derivative $\bigtriangledown_l$ of a covariant vector field B_m and is written as [6, 11]

$$\nabla_l B_m = \frac{\partial B_m}{\partial x_l} + \Gamma_{ml}^s B_s, \tag{22}$$

where Γ_{ml}^s is the connectivity. The geometrical meaning of a connectivity is, in particular, that the covariant derivative plays the role of a translation group generator for the usual (not spinor) tensor fields on a manifold [6, 11]. (In the Euclidean space, the connectivity is zero and the "usual" derivative $\partial/\partial x_l$ plays the role of a translation group generator). However, for the spin-tensor fields and, in particular, for the four-component first-rank spin tensor entering the Dirac equation, the connectivity with the above mentioned properties does not exist in an arbitrary Riemannian space. This is caused by the fact that spin tensors are Euclidean rather than affine, and the transformation law for their components is specified by the rotation group representation, which cannot be continued to a group of all linear transformations [10, 11]. In other words, the spin tensors can be connected to each other at different points of a curved space only if the orthonormalized frame remains orthonormalized upon the translation in this space.

However, for a particular case of Riemannian spaces, namely, for the conformally Euclidean spaces, a certain vicinity of an arbitrary point M can always be mapped, with retention of metric, into the vicinity of the other arbitrary point M', and this can be done in such a way that the orthonormalized frame specified at point M transforms into an arbitrarily chosen orthonormalized frame at M' [11]. For this reason, the parallel translation of spinor in such a space is determined by metric from the same formulas as occur for usual tensors, with only the Γ_{lp}^p-type connectivity components being nonzero. (Weil was, probably, the first to realize it [15, 16]). Recall that not the physical space of events is assumed to be a conformally Euclidean space in the approach suggested in this work but a universal covering space, which serves only as a mathematical tool for describing the fundamental group of a physical object, namely, of a closed connected 4-manifold of interacting electromagnetic and electron–positron fields.

Let us now assume that the expression ieA_l in Eq. (3) can be regarded as a connectivity Γ_{lp}^p in the conformally Euclidean space. Then the expression $(\partial/\partial x_l + ieA_l)$ entering in Dirac equation (3) can be interpreted as a translation group generator along the universal covering surface of a certain connected 4-manifold. The translation group generator multiplied by the symmetry operator γ_l about the hyperplanes passing through the OX_l axis can be regarded as a generator of a "local" sliding symmetry group in the spinor basis. This leads to the main conclusion of this work: Dirac equation (3) can be interpreted as a group-theoretic relation describing the metric and topological properties of a certain 4-manifold, for which the conformally pseudo-Euclidean space is a universal covering surface, while a group determined by the local sliding symmetry is the fundamental group.

The fact that ieA_l in the above interpretation has a meaning of connectivity on the covering space also allows the geometrical interpretation of the electric and magnetic field components, i.e., of the tensor components F_{ik} of electric and magnetic fields. To this end, let us use the fact that the

curvature tensor $R^q_{lk,i}$ (Riemann–Christoffel tensor defining the deviation from Euclidean geometry in the Riemannian space) is expressed in terms of connectivity as [6, 11]

$$R^q_{lk,i} = \left(\frac{\partial \Gamma^q_{li}}{\partial x_k} - \frac{\partial \Gamma^q_{ki}}{\partial x_l} + \Gamma^q_{kp}\Gamma^p_{li} - \Gamma^q_{lp}\Gamma^p_{ki} \right). \tag{23}$$

(As before, the summation over repeating indices goes from 0 to 3). Let us contract the curvature tensor with respect to its upper and lower right indices (the resulting tensor is denoted by R^0_{lk}),

$$R^0_{lk} = R^q_{lk,q} = \frac{\partial \Gamma^q_{lq}}{\partial x_k} - \frac{\partial \Gamma^q_{kq}}{\partial x_l}. \tag{24}$$

Comparing Eq. (24) with Eq. (4) and using the fact that $\Gamma^q_{mq} = ieA_m$, one obtains

$$ieF_{ik} = R^0_{ik}, \tag{25}$$

i.e., within the geometrical interpretation, the tensor of electric and magnetic fields coincides, except for the factor ie, with certain components of the curvature tensor of a universal covering surface. Therefore, Maxwell Eq. (5) relates the above-mentioned components of curvature tensor to the basis functions of the fundamental group, thereby rendering the system of Eqs. (3)–(5) closed. The curvature tensor for a space with constant curvature K has the form [11]

$$R_{ij,kl} = K(g_{ik}g_{jl} - g_{il}gjk). \tag{26}$$

Comparing Eqs. (26) and (25), one arrives at the conclusion that, within the geometrical interpretation, the electric charge e is proportional to a constant curvature K.

Thus, in the geometrical interpretation, the equations of classical relativistic electrodynamics (3)–(5) have the form

$$i\gamma_l\left(\frac{\partial}{\partial x_l} + \Gamma^p_{lp} \right)\psi = m\psi, \tag{27}$$

$$R^0_{ik} = \frac{\partial \Gamma^p_{ip}}{\partial x_k} - \frac{\partial \Gamma^p_{kp}}{\partial x_i}, \tag{28}$$

$$\sum_{i=1}^{4} \frac{\partial R^0_{ik}}{\partial x_i} = ie^2\psi * \gamma_1\gamma_k\psi. \tag{29}$$

Weak interaction

Let us now show that, in a one-particle approximation adopted in this work (low energies), weak interaction can be represented as a manifestation of the torsion in the covering space of a 4-manifold representing weak field and its sources. In due time, Einstein attempted at including electromagnetic field into a unified geometrical description of physical fields by "adding" torsion to the Riemannian space–time curvature, which reflects the presence of a gravitational field in general relativity [17]. Since the curvature of covering space corresponds within our approach to the electromagnetic field, I will attempt to include weak interaction into the topological approach by including torsion in this space.

The curvature of a covering space (and, hence, the electromagnetic interaction) can be taken into account by replacing usual derivative in Eq. (10) for a free source field by the covariant derivative, which reflects the fact that the field violates the discrete translational symmetry (the solution ceases to be described by a periodic function). It is natural to assume that the inclusion of torsion breaks the mirror symmetry. Let us first consider the case where the electromagnetic field is absent, i.e., the curvature of covering space is zero. A space with torsion but without curvature is called the

space with absolute parallelism [11]. Thus, the challenge is to determine how does free-particle Eq. (10) change if the interparticle interaction is due only to the torsion, which transforms the pseudo-Euclidean covering space into a space with absolute parallelism.

Let us denote the torsion tensor by S_{lm}^k; then the problem can be formulated as follows. It is necessary to "insert" the tensor S_{lm}^k or some of its components into Eq. (10) so that the resulting equation remains invariant about the Lorentz transformations and adequately describes the experimental data (e.g., violation of spatial and time symmetry by weak interaction). Among the spaces with torsion, there are so-called spaces with semisymmetric parallel translation [18]. The torsion tensor S_{lm}^k for such spaces is defined by the antisymmetric part of connectivity and can be represented in the form

$$S_{lm}^k = S_l A_m^k - S_m A_l^k.$$ (30)

Here, S_l is a certain vector and A_l^k is the matrix of rotation group representation. The vector S_l has the property that the infinitesimal parallelogram remains closed upon parallel translation in the hyperplanes perpendicular to this vector.

One may thus assume that in the presence of vector S_l the spatial isotropy breaks in such a way that the isotropy is retained only in the indicated hyperplanes. Assuming that along S_l only the translational symmetry is retained, while the symmetry of Eq. (10) is retained in the hyperplane perpendicular to this vector, one can recast this equation as

$$i(\frac{\partial}{\partial X_1} - S\sigma_z)\varphi - \sum_{k=2}^4 i\sigma_k \frac{\partial \varphi}{\partial X_k} = m\varphi.$$ (31)

Here, the X_1 axis is aligned with the vector S_l, φ is a two-component spiner and σ_α denotes a two-row matrices of rotation group representation in the spinor basis(Pauli matrices). Evidently, Eq. (31) violates both spatial and time symmetry. One can show that the Lorentz invariance of this equation coincides with the local gauge invariance: Lorentz rotation may be here considered as production $U(1)SU(2)$—production of the S-vector rotation and rotation within the 3-dimentional hyperplane perpendicular to this vector. The question of how does Eq. (31) account for the other properties of weak interaction and how does it relate to the results of the standard model of electroweak interaction will be considered in detail elsewhere.

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