NONSTANDARD WEAK BOSONS

J.J. Sakurai*
Max-Planck-Institut für Physik und Astrophysik
Munich, Fed. Rep. Germany

All the low-energy successes of the standard electroweak gauge model are shown to follow in a more phenomenological model based on global SU(2) broken by γ-W^0 mixing. Weinberg's mass predictions need not be valid. Connections with recent composite models of W and Z are also discussed.

* U.S. Senior Scientist Awardee (Alexander von Humboldt-Stiftung) on leave from the University of California, Los Angeles.

Introduction

In the past several years the low-energy predictions of the standard electroweak gauge model of Glashow[1], Salam, Ward[2] and Weinberg[3] have been brilliantly confirmed by various neutral-current experiments[4]. It is yet to be tested whether the model is correct at values of q^2 where the electromagnetic and weak forces are expected to compete in strength. The most central and specefic of the yet-to-be-verified high q^2 predictions are, of course, the fifteen-year-old mass predictions of Weinberg[3] which, when sharpened by the electroweak radiative corrections[5], lead numerically to

$$m_W = (82.0 \pm 2.4)\ \text{GeV},$$
$$m_Z = (93.0 \pm 2.0)\ \text{GeV}. \tag{1}$$

These predictions now (meaning within several months?) being tested at the CERN $\bar{p}$ collider are most remarkable in that the quantitative details of electroweak physics at q^2 of order 8,000 GeV^2 (timelike) are inferred on the basis of our knowledge of weak interaction phenomena at q^2 of order 1-100 GeV^2 (spacelike).

Despite the extraordinary nature of these mass predictions the theoretical physics community appears to be in general agreement with the view that the W and Z bosons will be found at the mass values predicted by (1). The majority opinion - presumably shared also by the Royal Swedish Academy of Sciences - is best summarized by the following remark of Glashow[6] at the LEP Summer Study (September 1978):

> "Since the low-energy limit of the unified theory is so well confirmed, few can doubt the truth of its central prediction: the existence of $W^{\pm}$ at $\sim$ 80 GeV and of Z^o at $\sim$ 90 GeV."

In contrast to this majority view found in countless review papers and even in some text books, I would like to present in this lecture a minority view: The extraordinary successes of the standard electroweak model in accounting for low q^2 phenomena do not immediately imply the correctness of the model at high q^2. Specifically I would like to show that as yet there is no direct experimental evidence for weak-electromagnetic unification in the usual sense, that W and Z need not be gauge bosons, and that the Weinberg mass relations may still fail.

The line of investigation I am reporting here started more than four years ago by Bjorken[7] in a rather unpopular invited talk, "Alternatives to Gauge Theories", presented at the Ben Lee Memorial Conference and was developed further by P.Q. Hung and myself[8]. Subsequently other heretics like Dombey[9] contributed to the propagation of these ideas. Furthermore in the past year or so, our point of view has come to attract an increasing amount of attention in connection with composite models of W and Z, as will be discussed towards the end of this lecture.

γ-W^o Mixing

Let us suppose that the history of neutral-current physics were different. Imagine that the low-energy phenomenology of the neutral-current interactions were first established experimentally, and at that time the theorists were not sophisticated enough to think about the gauge-theory ideology. How might the subject have been developed?

Given the existence of neutral currents as well as charged currents with large parity violation, the most natural thing is to assume (a) pure V-A to ensure maximal parity violation, and (b) weak isospin invariance or global SU(2) to accommodate the charged and neutral currents. So we may begin with

$$L_{eff} = (4G/\sqrt{2})\, \underset{\sim}{J}_\lambda \cdot \underset{\sim}{J}_\lambda \tag{2}$$

where $\underset{\sim}{J}_\lambda$ stands for a triplet of weak isospin currents built up of left-handed fermions. The form (2) is quite natural; indeed, almost immediately after the V-A proposal and long before the birth of the standard electroweak model, it was acutally written down by Bludman[10] and shortly afterwards by Z'eldovich[11].

A closer look at the experimental data reveals that, although (2) is satisfactory for the charged-current interactions, its neutral-current predictions do not quite agree. For one thing the observed neutral currents are known not to be of the pure V-A form. To fit the neutral-current data it is found necessary to modify the interactions of the third current in such a way that the entire Lagrangian may look like

$$L_{eff} = (4G/\sqrt{2}) \sum_{\alpha=1,2} J^\alpha_\lambda J^\alpha_\lambda + (4G/\sqrt{2})(J^3_\lambda - \text{"}\sin^2\theta_W\text{"}\, J^{em}_\lambda)^2 \tag{3}$$

where at this stage "$\sin^2\theta_W$" is some constant experimentally determined to be about 0.23. All the successes of the standard model achieved so far follow just as we let

$$J^3_\lambda \to J^3_\lambda - \text{"}\sin^2\theta_W\text{"} J^{em}_\lambda \tag{4}$$

in the SU(2) symmetric Lagrangian (2). In addition, if we assume that the weak interactions are mediated by a $W^{\pm,o}$ triplet coupled to weak isospin, the modification (4) implies that the W^o couplings have to be altered slightly as we switch on the electromagnetic couplings which are known to break SU(2). We are led to a particular way of breaking global SU(2), namely γ-W^o mixing represented by

$$L_{\gamma W} = -(1/4)\,\lambda\,(F_{\mu\nu} W^3_{\mu\nu} + W^3_{\mu\nu} F_{\mu\nu}) \tag{5}$$

With this the γ-W^o junction goes like q^2, in conformity with electromagnetic gauge invariance.

For definiteness let us now consider simple low-order diagrams for elastic νq scattering. If there were no γ-W^o mixing, then we would have just W^o exchange; on the quark side the W^o is coupled to the third component of weak isospin formed out of u and d. See Fig. 1(a). But because of (5) this is not the whole story. We must also consider Fig. 1(b) where the quark emits a photon that converts itself into a W^o which, in turn, can interact with the neutrino. It is important to keep track of the q^2 dependence here. The $1/q^2$ behavior due to photon exchange just cancels the q^2 behavior coming from the γ-W^o junction. As a result, in the limit where $m_W{}^2$ is much larger than q^2, we have just a current-current coupling of the kind represented by the last term of (3).

The q^2 dependence of the effective $\gamma\bar{\nu}\nu$ vertex that appears in Fig. 1(b) is what we expect from a neutrino with a finite charge radius; because the neutrino is chargeless, its electromagnetic (Dirac) form factor must go like q^2 for small q^2. Historically Bjorken[7] first noted that, as far as low-energy neutrino-induced neutral-current processes are concerned, all the consequences of the standard model follow just by postulating, in addition to global SU(2), a charge radius for the neutrino with magnitude

$$\langle r^2 \rangle = (6G/\sqrt{2}) \sin^2\theta_W/\pi\alpha \quad . \tag{6}$$

However, to fit the electron-deuteron parity violation data at SLAC, the energy dependence of electron-positron annihilation into muon pairs at PETRA, etc., new weak-interaction contributions to the charge radii of e, μ, u and d must also be postulated. Furthermore the amount of the contribution needed in each case is just what we expect if the charge radius arises in a "universal manner" from γ-W^o mixing.

The effective γ-W^o interaction (6) is not a priori weak; to treat its effect quantitatively to all orders, the usual approach based on low-order diagrams is not so convenient. It is much better to use the propagator matrix formalism, as we will see in a moment.

The problem of gauge-invariant mixing between a spin-one object and the photon was discussed extensively in the sixties with reference to vector meson dominance. I even wrote a book on this and related subjects. However, the most transparent (and correct!) treatment of this mixing problem is found in a 1961 paper of Kobzarev, Okun' and Pomeranchuk[13], based on the propagator matrix formalism, whose existence I became aware of only four years ago. I now rewrite their paper in a manner suitable for our situation.

Suppose we have just one particle with no mixing. The propagator in momentum space is

$$D(q^2) = 1/(q^2 + m^2), \tag{7}$$

and the mass is determined by looking at the zero of the inverse propagator

$$q^2 + m^2 = 0. \tag{8}$$

If we have two particles with mixing, the propagator is a 2 x 2 matrix, denoted by $\Delta(q^2)$, and the masses are determined by the roots of the equation

$$\det (\Delta^{-1})\Big|_{q^2=-m_i^2} = 0 \ (i = 1,2). \tag{9}$$

We now specialize to the γ-W^o complex. Our basic Lagrangian is written as

$$L = -\frac{1}{4}\phi^T_{\mu\nu} K \phi_{\mu\nu} - \phi^T_\mu M^2 \phi_\mu + \phi^T_\mu J_\mu \tag{10}$$

where

$$\phi_{\mu\nu} = \begin{pmatrix} F_{\mu\nu} \\ W^o_{\mu\nu} \end{pmatrix}, \qquad \phi_\mu = \begin{pmatrix} A_\mu \\ W^o_\mu \end{pmatrix}, \qquad J_\mu = \begin{pmatrix} eJ^{em}_\mu \\ gJ^3_\mu \end{pmatrix} \tag{11}$$

There is a mixing term present only in the kinetic part, not in the mass part ("current mixing" in the unfortunate terminology of Kroll, Lee and Zumino[14]):

$$K = \begin{pmatrix} 1 & \lambda \\ \lambda & 1 \end{pmatrix}, \qquad M^2 = \begin{pmatrix} 0 & 0 \\ 0 & m_W^2 \end{pmatrix}. \tag{12}$$

Notice that the diagonal elements in the M^2 matrix are 0 and m_W^2; we start with a W^o mass identical to the charged W mass, in agreement with the idea of global SU(2) before mixing.

The Lagrangian (10) can easily be shown to lead to the inverse propagator matrix

$$\Delta^{-1}(q^2) = \begin{pmatrix} q^2 & \lambda q^2 \\ \lambda q^2 & q^2+m_W^2 \end{pmatrix}. \tag{13}$$

The photon and the Z mass are to be obtained by looking for the zeros of $\det(\Delta^{-1})$. First, we see that

$$\det (\Delta^{-1}(q^2))\Big|_{q^2=0} = 0 \tag{14}$$

is automatically satisfied. An initially massless photon remains massless in the presence of the gauge invariant mixing term (5), a hardly surprising result. More interesting is the second root obtained by solving

$$\det\left(\Delta^{-1}(q^2)\right)\Big|_{q^2=-m_Z^2} = 0 \tag{15}$$

with m_Z^2 assumed to be nonvanishing where, after mixing, we have renamed the neutral boson

$$W^o \xrightarrow{\text{mixing}} Z. \tag{16}$$

Specifically (15) leads to

$$\begin{vmatrix} -m_Z^2 & -\lambda m_Z^2 \\ -\lambda m_Z^2 & m_W^2 - m_Z^2 \end{vmatrix} = 0 \tag{17}$$

In this way we obtain a very important mass formula[8)]

$$m_Z^2 = m_W^2/(1 - \lambda^2). \tag{18}$$

Note that, as the W^o changes into Z, its mass is raised by a factor of $1/\sqrt{(1-\lambda^2)}$.

We can also compute the current-current interaction that follows from (10) by writing $(1/2)\, J^T \Delta J$ in terms of the Z boson propagator. A straightforward computation gives[8)]

$$L_{\text{eff}} = \frac{1}{2}\left\{ e^2 J_\lambda^{em} \frac{1}{q^2} J_\lambda^{em} + \frac{g^2}{1-\lambda^2}\left[J_\lambda^3 - \frac{e}{g}\lambda J_\lambda^{em}\right] \frac{1}{q^2+m_Z^2}\left[J_\lambda^3 - \frac{e}{g}\lambda J_\lambda^{em}\right]\right\}. \tag{19}$$

The first term is due to photon exchange, the second due to Z exchange. The low q^2 limit of (19) is of vital interest. The dimensionless constant for the neutral-current interactions, according to (19), is larger than g^2 by a factor of $1/(1-\lambda^2)$, but m_Z^2 is also heavier by the same factor [see (18)]. As a result, in the $q^2 \to 0$ limit, the neutral-current strength is unaltered from its global SU(2) value; it is still characterized by the same G (à la Fermi) as in the charged-current interactions. Explicitly we have for the neutral-current part of (19)

$$L^{NC}(q^2 \cong 0) = (4G/\sqrt{2})\left[J_\lambda^3 - (e\,\lambda/g)J_\lambda^{em}\right]^2 \tag{20}$$

where we have used the charged-current relation

$$G/\sqrt{2} = g^2/8m_W^2. \tag{21}$$

What the experimentalists call " ρ ", the neutral-to-charged current strength ratio, is seen to be unity. The only modification is in the structure of the current, which is precisely of the form (3) provided we identify

$$e\lambda/g = \text{"}\sin^2\theta_W\text{"}. \tag{22}$$

So our very simple considerations based on global SU(2) broken by γ-W^o mixing give rise to exactly the same low q^2 limit - both in form (the J^3 - "$\sin^2\theta_W$"J^{em} combination) and in strength (" ρ" = 1) - as the standard electroweak gauge model[8].

Weak-Boson Mass Formula

Even though our low q^2 predictions are identical to those of the standard model, there can be substantial differences at high q^2. To see this let us go back to the weak-boson mass relation (18) and rewrite it using (22):

$$m_Z^2 = m_W^2/[1 - (g/e)^2\text{"}\sin^4\theta_W\text{"}]. \tag{23}$$

Furthermore we can eliminate g in favor of Fermi's G and m_W^2 [see (21)]. The final result can then be written as[15]

$$m_Z^2 = m_W^2/[1 - (m_W/37.4\ \text{GeV})^2\text{"}\sin^4\theta_W\text{"}]. \tag{24}$$

Notice that we cannot obtain m_W and m_Z separately even if $\sin^2\theta_W$ is known. However, the boson mass formula (24) is nontrivial in that, knowing one of them (say, m_W), we can infer the value of the other (m_Z). See Fig. 2.

In this model what the experimentalists call " ρ " is unity; yet m_Z^2 is, in general, not equal to $m_W^2/\cos^2\theta_W$. From the mere fact that " ρ " is unity experimentally[4] (within a few %), we cannot conclude that the Z mass is equal to $1/\cos\theta_W$ times the W mass. After all, " ρ " = 1 is nothing more than the statement that global SU(2) survives at $q^2 = 0$ as far as the absolute strength of the neutral-current interactions is concerned.

There is another important point that follows from our formalism. We first note that the mixing parameter λ cannot exceed unity in magnitude. Otherwise after diagonalization we would not have the usual particle interpretations for

Z. A practical consequence of this is that the denominator of (24) cannot become non-negative, which leads to an absolute upper bound for the $W^\pm$ mass:

$$m_W < 37.4\ \text{GeV}/\text{"}\sin^2\theta_W\text{"} \cong 170\ \text{GeV for "}\sin^2\theta_W\text{"} \cong 0.22, \qquad (25)$$

a result first obtained by Bjorken[7] from somewhat more general considerations. The inequality (25) is quite remarkable. From the fact that "$\sin^2\theta_W$" is no smaller than 0.22, we have deduced that the range of the charged-current interactions cannot be shorter than

$$(1/170)\ \text{GeV}^{-1} \cong 1.2 \times 10^{-16}\ \text{cm}. \qquad (26)$$

A multi-boson generalization of (25) will appear later.

Unlike the $W^\pm$ mass there is no analogous bound for the Z mass. Indeed, as the W mass approaches its upper bound, the Z mass increases indefinitely; see (24) and Fig. 2. However, there may be other reasons for believing that the Z mass cannot be ridiculously high. For one thing a high Z mass would imply a large dimensionless constant for the neutral-current interactions, which makes it difficult to understand the successes of lowest order calculations for weak interaction processes. As Llewellyn Smith[16] argues, it is consistent to use single Z exchange to describe the low-energy interactions only if m_Z is substantially less than the center-of-mass energy $\sqrt{s}_o$ at which unitarity violation takes place. By requiring m_Z to be less than half of $\sqrt{s}_o$, he predicts

$$m_W < 140\ \text{GeV}, \quad m_Z < 280\ \text{Gev}. \qquad (27)$$

The Unification Condition

Compare our mass formula (24) with Weinberg's results[3]

$$m_W = 37.4\ \text{GeV}/\sin\theta_W,$$
$$m_Z = m_W/\cos\theta_W. \qquad (28)$$

Clearly Weinberg has more predictions. This is not surprising. In our approach based on γ-W^o mixing, in addition to e and G there are two independent parameters e/g and λ (or equivalently e/g and "$\sin\theta_W$"). In contrast in the standard electroweak model there is only one adjustable parameter e/g, which is equal to $\sin\theta_W$.

Weinberg's relations (28), represented by a point in Fig. 2, are seen to be a special case of our formula (24); they are obtainable by imposing a particular relation between the coupling constant ratio e/g and the mixing coefficient λ, viz.

$$e/g = \lambda = \sin\theta_W. \tag{29}$$

We wish to call this the unification condition. Let us write it as

$$e = g\sin\theta_W. \tag{30}$$

This relation, first written down by Glashow[1] in 1961, connects measurable quantities in the electromagnetic and weak interactions. The two quantities, g and $\sin\theta_W$, that appear on the right-hand side of (30) can be deduced, respectively, by measuring the decay width of $W^- \to e^- \bar{\nu}_e$ [or, more practically, by knowing the W mass and using (21)] and by studying the neutral-current structure at low energies, as has been done already. So what we usually regard as purely weak interaction quantities are sufficient to determine Millikan's electronic charge that appears on the left-hand side of (30), hence weak-electromagnetic unification! But this unification condition has not yet been tested experimentally because the W mass has not been determined, and as a consequence g is still an unknown quantity. The fact that eight different ways to determine $\sin^2\theta_W$ agree more or less with each other does not throw light on the question of whether or not the unification condition (29) is correct. From the empirical observation that the neutral currents can be written as a linear combination of J^3_λ and J^{em}_λ we cannot conclude that the weak and electromagnetic forces are unified.

The main message I am trying to convey is the following. The extraordinary successes of the standard electroweak gauge model in accounting for low-energy neutral-current data also follow in a wider framework based on global SU(2) broken by γ-W^0 mixing. These successes by no means guarantee Weinberg's mass relations and are, in fact, logically independent of the unification idea for which the 1979 Nobel Prize was given. So we must still spend \$ 2×10^8 or so to see whether Weinberg is really right.

Let us examine a little more deeply the physical meaning of the unification condition. We go to high q^2 regions and look at the behavior of the effective interaction (19). Let us rewrite it this time using J^3_λ and J^Y_λ (the weak hypercharge current) defined by

$$J^Y_\lambda \equiv J^{em}_\lambda - J^3_\lambda . \tag{31}$$

For $q^2 \gg m_W^2$, m_Z^2, the effective interaction (19) reduces to[8)]

$$L_{eff} \cong \frac{1}{2q^2} \{ \frac{e^2}{1-\lambda^2} J_\lambda^Y J_\lambda^Y + \left(\frac{e^2+g^2-2eg}{1-\lambda^2} \right) J_\lambda^3 J_\lambda^3 + [\frac{2e(e-\lambda g)}{1-\lambda^2}] J_\lambda^Y J_\lambda^3 \} . \tag{32}$$

If there is no constraint between e/g and λ, SU(2) is still violated even at high values of q^2. For SU(2) to become exact we must have

$$L_{eff} \cong \frac{1}{2q^2} (g^2 J_\lambda^3 J_\lambda^3 + \text{const } J_\lambda^Y J_\lambda^Y). \tag{33}$$

Here the coupling coefficient of $J_\lambda^3 J_\lambda^3$ is g^2, the same as the charged-current constant, and the $J_\lambda^Y J_\lambda^3$ term that violates SU(2) is absent. The second term of (33) can be regarded as being due to the exchange of a <u>singlet</u> B boson coupled to weak hypercharge.

Comparing the SU(2) expected behavior (33) with the general behavior (32), we infer that SU(2) becomes exact at high q^2 if e/g is set equal to λ, which is precisely the unification condition (29). So the unification condition ensures a kind of "asymptotic restoration" of SU(2)[7),8)]; the interactions of the $W^{\pm,0}$ triplet again become weak-isospin invariant while the photon, in Bjorken's words, is "transmogrified" into the U(1) B boson coupled to pure weak hypercharge. Needless to say, this restoration idea refers to a short-distance behavior yet to be tested experimentally. It is by no means guaranteed by the extraordinary successes of the standard model in accounting for low q^2 phenomenology.

It has been argued that the imposition of the unification condition is almost sufficient to convert the phenomenological γ-W^0 mixing model into the conventional electroweak gauge model. (I say here "almost" because the Higgs sector, or more generally the weak-boson-mass generation mechanism, is yet to be specified.) There are two points worth mentioning in this connection. First, suppose we compute

$$\nu_e + \bar{\nu}_e \rightarrow W^+ + W^- \tag{34}$$

due to electron exchange (in the t channel) and Z exchange (in the s channel) within the framework of our general γ-W^0 mixing model. When λ is assumed to be unrelated to e/g, the amplitude for the production of longitudinally polarized $W^{\pm}$'s is seen to violate tree unitarity. However, when we impose the unification condition, we obtain a high energy behavior as good as that which follows from the conventional electroweak gauge model[8),17)].

Next, I would like to report on an interesting observation made by Feynman, who, in 1978-79, gave lectures on weak interactions at Caltech. Rather than digesting other people's papers, he wanted to reconstruct the standard electroweak model in his own original way. To this end, instead of the usual $SU(2) \otimes U(1)$ gauge interactions of $W^{\pm,0}$ and B, he considers Yang-Mills type $W^{\pm,0}$ interactions, both among themselves and with quarks and leptons, and the photon (not B) interactions that take into account the fact that the $W^{\pm}$, as well as the quarks and leptons, are electrically charged. When he attempts to impose the Yang-Mills gauge invariance and the electromagnetic gauge invariance, he realizes that something is lacking for the two sets of invariance to be preserved simultaneously. The needed term turns out to be precisely the γ -W^0 mixing interaction with the unification $\lambda = e/g$. Feynman never published this observation; so I published it in my 1979 Hawaii lectures[18)] which interested people may consult for details.

Multiboson Models

We have considered so far a broken SU(2) model based on just one W triplet. We may still keep the idea of global SU(2) before mixing with the photon but allow the possibility that many weak-boson triplets - or even a continuum of weak quanta - may participate in the weak interactions. Suppose there are N triplets altogether all coupled to weak isospin. The strength of the i^{th} triplet to $\underline{J}_\lambda$ is characterized by g_i, and the neutral member of the triplet mixes with the photon with coupling constant λ_i. We can derive the analog of (19), which turns out to be[19),20)]

$$L_{eff} = \frac{1}{2}\left[\frac{e^2}{q^2}(J_\lambda^{em})^2 + \sum_{i=1}^{N} \frac{f_i^2\,(J_\lambda^3 - b_i J_\lambda^{em})^2}{q^2 + m_{Zi}^2}\right] \tag{35}$$

with

$$\sum_{i=1}^{N} (f_i^2/m_{Zi}^2) = \sum_{i=1}^{N} (g_i^2/m_{Wi}^2) = 8G/\sqrt{2}$$

$$\sum_{i=1}^{N} (f_i^2 b_i/m_{Zi}^2) = e\sum_{i=1}^{N} (\lambda_i g_i/m_{Wi}^2) = (8G/\sqrt{2})\text{"}\sin^2\theta_W\text{"}. \tag{36}$$

The low q^2 limit of the neutral-current part of (35) is

$$L^{NC}(q^2 \cong 0) = (4G/\sqrt{2})\,\{(J_\lambda^3 - \text{"}\sin^2\theta_W\text{"}J_\lambda^{em})^2 + C(J_\lambda^{em})^2\} \tag{37}$$

where the coefficient C is given by

$$C = \frac{1}{2} e^2 \sum_{i,j} [\frac{(\lambda_i g_i - \lambda_j g_i)^2}{m_{Wi}^2 m_{Wj}^2}] \Big/ [\sum_k (g_k^2/m_k^2)]^2 \geq 0 \qquad (38)$$

Comparing (37) with (20), we observe that this generalized model does give low q^2 results different from those of the standard model by the presence of an extra $(J_\lambda^{em})^2$ term, usually referred to as the C term[21].

Physically speaking, the coefficient C is a measure of deviations from the single Z hypothesis. Gounaris and Schildknecht[22] have derived a sum rule for this coefficient in terms of weighted cross sections in e^+e^- collisions:

$$C = \frac{1}{16} \frac{\int ds\, \sigma(e^+e^- \to \gamma, Z_1, Z_2, \ldots Z_N \to \text{all})/s - \int ds\, \sigma(e^+e^- \to \gamma, Z \to \text{all})/s}{[1 + (1-4\sin^2\theta_W)^2] \int ds\sigma\, (e^+e^- \to \gamma, Z \to \text{all})/s} \qquad (39)$$

where $\int ds\, \sigma(e^+e^- \to \gamma, Z \to \text{all})/s$ is to be evaluated in a single Z model; the value of this integral can be shown to be independent of the Z mass once "$\sin^2\theta_W$" is given.

At present only e^+e^- data at low energies are available; so we cannot evaluate (39) directly. Furthermore we cannot detect the presence of the C term in neutrino-induced reactions because the neutrino is chargeless, nor in the SLAC parity experiment because $(J_\lambda^{em})^2$ is purely parity-conserving. Fortunately the C term could reveal its presence in

$$e^+ + e^- \to e^+ + e^-, \mu^+ + \mu^- . \qquad (40)$$

The current PETRA limit reported elsewhere at this winter school appears to be around

$$C < 0.025. \quad (95\% \text{ CL}) \qquad (41)$$

The weak-boson mass formula (24) has a multiboson generalization. First, we define the average boson masses by

$$1/\bar{m}_W^2 \equiv \{\sum_i (g_i^2/m_{Wi}^4)\} \Big/ \{\sum_k (g_k^2/m_{Wk}^2)\}$$

$$1/\bar{m}_Z^2 \equiv \{\sum_i (f_i^2/m_{Zi}^4)\} \Big/ \{\sum_k (f_k^2/m_{Zk}^2)\} \quad . \tag{42}$$

Then the multiboson analog of (24) can be derived to be[19]

$$\bar{m}_Z^2 = \frac{\bar{m}_W^2}{1 - (\bar{m}_W/37.4\ \text{GeV})^2 \text{"}\sin^4\theta_W\text{"}} \quad . \tag{43}$$

Again we note the positivity requirement

$$\bar{m}_W < 170\ \text{GeV}\ (\text{"}\sin^2\theta_W\text{"} = 0.22). \tag{44}$$

We see that the main activities in the charged-current channel must take place at energies below 170 GeV even in multiboson models.

In a single Z model we have a sum rule for the colliding beam cross section R,

$$\bar{R} = \int_{Z\ \text{region}} ds R/s \equiv (3/4\pi\alpha^2) \int ds\sigma\ (e^+e^- \to Z \to \text{all}) = (3/8\ \alpha^2)[(1-4\sin^2\theta_W)^2+1] G m_Z^2/\sqrt{2}, \tag{45}$$

which easily follows from integrating the usual one-level resonance formula. If m_Z assumes the standard model value, the right-hand side of (45) is a very big number, ~ 500. Using Schwarz-inequality type arguments, we can derive an analogous expression in the multiboson case, which is now an inequality[20),22)]

$$\bar{R} \equiv (3/4\ \pi\alpha^2) \sum_i \int_{Z_i\ \text{region}} ds\ \sigma(e^+e^- \to Z_i \to \text{all}) \geq (3/8\ \alpha^2)\{(1-4\sin^2\theta_W)^2+1\} G\bar{m}_Z^2/\sqrt{2}. \tag{46}$$

This is great! If we build an e^+e^- colliding beam apparatus that covers $\sqrt{s} \sim \bar{m}_Z$, a large cross section is guaranteed. But the bad news is that $\bar{m}_Z$ need not necessarily lie in the energy range covered by LEP, SLC or some other favorite project of yours. It can, for instance, be at 160 GeV. Clearly it is safer to plan an e^+e^- collider with as high an energy as the funding agencies - or ultimately the taxpayers - allow.

Composite W and Z

In the past few years a number of authors[23),24),25)] have speculated on the possibility that not only the quarks and leptons but also the W and Z bosons are bound states of some more fundamental objects - preons, rishons, haplons, etc. In such composite models the weak interactions we observe - nuclear beta decay, inelastic neutrino scattering, etc. - are phenomenological manifestations of a new kind of superstrong confinement dynamics mediated by hypergluons in much the same way as the hadronic interactions we used to study in the sixties - $\rho \longrightarrow \pi\pi$, Kp scattering, etc. - are now regarded as phenomenological manifestations of QCD, the fundamental dynamics of strong interactions. From this point of view the weak forces we observe are like Vander Waal forces.

A historical analogy may be of some pedagogical value here. In 1960 I thought that the particles later identified as ρ, ω and ϕ were the gauge particles of strong interactions[26)]. (I was trying to construct a gauge theory of strong interactions at a time when the concept of colored quarks was yet to be born!) By 1970 most people came to accept the view that ρ, ω and ϕ are not "elementary" gauge particles but quark-antiquark bound states. In 1980, right after the originators of the standard electroweak gauge model were canonized in Stockholm, the physics community almost unanimously believed that W and Z were the gauge particles of weak interactions. By 1990 most people will perhaps be led to the view that W and Z are composites of some more fundamental objects, just as ρ and ω are. The gauge theory of strong interactions based on ρ, etc. is now considered to be obsolete. Will the gauge theory of weak interactions based on W and Z become obsolete by 1990?

Numerous workers have constructed composite models. At this moment we don't yet know which, if any, of the various composite models proposed so far is likely to survive. Here I mention just two of them, a model proposed by Abbott and Farhi[24)] and another model proposed by Fritzsch and Mandelbaum[25)]. I regard these two models as representative examples of recent unorthodox approaches to electroweak physics rather than as candidates of the "ultimate theory".

The fundamental constituents of the Abbott-Farhi model are a non-Hermitian scalar doublet and a fermion doublet, both confined. In this model a familiar left-handed quark or lepton is a composite object made up of an "elementary" fermion and a scalar boson. The weak bosons W and Z, which are more relevant to my talk here, are supposed to emerge as bound states of scalars

$$W^+_\mu = \phi^+ D_\mu \phi^o - \phi^o D_\mu \phi^+, \text{ etc.}$$

where D_μ stands for the covariant derivative appropriate for the new underlying gauge theory.

The second model I consider is QHD ("Quantum Haplo Dynamics") of Fritzsch and Mandelbaum[25]. (An anonymous Greek physicist pointed out to me that "haplo" means "naive" as well as "simple".) Here the fundamental constituents are a fermion doublet α, β with $Q = -1/2$ and $1/2$, respectively, and two scalar singlets x, y with $Q = -1/6$ and $1/2$, respectively. The leptons and quarks are bound states of spin 1/2 fermions and spinless bosons:

$$\begin{aligned} \nu_e &= (\bar{\alpha}\ \bar{y}), \qquad u = (\bar{\alpha}\ \bar{x}), \\ e^- &= (\bar{\beta}\ \bar{y}). \qquad d = (\bar{\beta}\ \bar{x}). \end{aligned} \tag{47}$$

In contrast to the Abbott-Farhi model, the weak bosons are more like the ρ meson in that they are fermion-antifermion bound states:

$$W^+ = (\bar{\alpha}\ \beta), \quad W^- = (\bar{\beta}\ \alpha), \quad W^o = [(\bar{\alpha}\ \alpha) - (\bar{\beta}\ \beta)]/\sqrt{2} \tag{48}$$

In both models the W interactions with the quarks and leptons are "residual" interactions of the basic confinement dynamics. Fig. 3 shows how the vertex

$$\nu_e \leftrightarrow W^+ + e^- \tag{49}$$

may look in terms of the fundamental constituents of the Fritzsch-Mandelbaum model. In the absence of the electromagnetic couplings the phenomenological W interactions with the left-handed quarks and leptons can be constructed to satisfy global SU(2) à la Bludman[10].

In these models, even though the $W^{\pm,o}$ are composite, the photon, like the gluon and hypergluon, _is_ elementary. From a certain point of view this is highly satisfactory. The "elementary" gauge bosons of the world all remain exactly massless. There is no such thing as a spontaneously broken local gauge symmetry - except in condensed matter physics. The ugly Higgs world is banished away.

The observed neutral-current structure at low q^2 can be obtained by taking advantage of the γ-W^o mixing mechanism discussed earlier. This mechanism now looks completely analogous to γ - ρ^o mixing. See Fig. 4. There is no reason to expect that the constant λ that characterizes the strength of Fig. 4(b) is related to the coupling constant ratio e/g where g characterizes the strength of the phenomenological $\nu_e \leftrightarrow W^+e^-$ vertex appearing in Fig. 3. In models where the photon is elementary but the weak bosons are composite, the "fundamental" electromagnetic forces and the "phenomenological" weak forces are not unified.

Because the unification condition, in general, fails in this class of composite models, the Weinberg mass relations are predicted to be invalid. The weaker mass relation (24) may still hold provided excited weak bosons and/or a weak continuum make negligible contributions to low q^2 physics.

It is, however, possible that the unification condition (29) may be approximately satisfied for dynamical reasons. For example, the underlying confinement dynamics may respect SU(2) at asymptotically high values of q^2, which, as we saw earlier, leads to the unification condition. The unification condition may also be realized if the W boson "imitates" the vector-meson dominance idea of the sixties[12),14),27)]. If the isovector electromagnetic form factor of the neutrino, etc. is completely dominated by a single W^o pole, we can easily show that λ must be equal to e/g[28)].

If the unification condition is a consequence of some dynamical approximation, the Weinberg mass relations are most likely to be satisfied not exactly but only approximately, say, within $\pm$ 20%. In contrast, in the standard electroweak gauge model, the Weinberg mass predictions are, in principle, as accurate as the g-2 prediction for the muon.

<u>Strength of the γ-W^o Junction</u>

I have repeatedly emphasized that the γ-W^o junction is very much like the γ-ρ junction. But empirically the γ-W^o constant λ^2 is much larger than the analogous γ-ρ^o constant. From $\rho^o \to e^+e^-$ and ρ dominance in photoproduction, etc., the γ-ρ^o constant normalized in the same manner can be inferred to be[12),29)]

$$\lambda^2_{\gamma\rho} \cong 1/300, \tag{50}$$

which is of order 1/137. In contrast, for the γ-W^o junction we have from (22) and the empirical value of $\sin^2\theta_W$

$$\lambda^2 = \text{"}\sin^4\theta_W\text{"}/(e/g)^2 \cong 0.23\ (m_W/79\ \text{GeV})^2. \tag{51}$$

How are we going to account for this big difference?

I now show that a value of λ^2 as large as (51) is not at all unreasonable in composite models characterized by a large mass scale. My argument is based on the concept of Q^2 duality[30),31)], which I briefly review.

Consider a series of vector mesons $V_1, V_2, \ldots$ which can be regarded as quark-antiquark bound states ($Q\bar{Q}$). The Q^2 duality hypothesis states that the vector meson peaks we see in the cross section ratio R for electron-positron annihilations into hadrons, when suitably averaged, closely approximate the free quark-pair cross section ratio for

$$e^+ + e^- \to Q + \bar{Q}. \tag{52}$$

Quantitatively it enables us to relate the strength of the $\gamma \to V_i$ junction to the vector meson spacing and the constituent charge. This hypothesis can be justified using a finite-energy (or rather finite-Q^2) sum rule[31] and also has been tested in the theoretical laboratories of nonrelativistic potential models[32] and two-dimensional QCD[33] with remarkable successes.

When applied to our situation, the Q^2 duality hypothesis leads to[34]

$$\lambda^2 = (\alpha/3\pi) N_H e_H^2/m_W^2 \; P(m_W^2). \tag{53}$$

Here e_H stands for the constituent charge, N_H denotes the number of hypercolors in the theory, and $P(m_W^2)$ is the density of vector boson spectrum, i.e., the number of vector bosons per unit mass squared interval. For definiteness we may use the Fritzsch-Mandelbaum model. The constituent charge is given by

$$e_H^2 = \left\{ \frac{1}{\sqrt{2}} \left[\frac{1}{2} - \left(-\frac{1}{2}\right) \right] \right\}^2 = \frac{1}{2}. \tag{54}$$

The hypercolor factor N_H is unspecified but we may take it to be 3 just as in QCD. (There may be an additional factor of 3 if α and β have ordinary colors also.) In the absence of detailed dynamics the hardest thing to guess is $P(m_W^2)$. It is undoubtedly related to the mass scale of QHD. An analogy with QCD may be helpful here. The scale of QCD is set by the famous parameter Λ_C, estimated to be of order 0.1 GeV. With this value of Λ_C we know empirically that $P(m^2)$ in hadron spectroscopy turns out to be of order 1 GeV^{-2}, a typical Regge slope for hadrons; more specifically, $P(m^2)$ for the I=1 vector mesons is computed to be 0.8 GeV^{-2} from ρ (0.78) and ρ'(1.60). In the QHD case the parameter Λ_H, analogous to Λ_C, is conjectured by Fritzsch and Mandelbaum to be of order 0.1 TeV. If QHD is like QCD with 1 GeV replaced by 1 TeV, a reasonable guess for $P(m_W^2)$ will be in the neighborhood of 1 TeV^{-2}.

From (51) and (53) we can deduce $P(m_W^2)$ needed to fit the observed value of $\sin^2\theta_W$. This leads to[34]

$$P(m_W^2) = 0.81 \; (79 \text{ GeV}/m_W)^4 \; \mathrm{TeV}^{-2}, \tag{55}$$

in excellent accord with our conjecture that $P(m_W^2)$ is of the order 1 TeV^{-2}. So the observed large value of λ^2, or of "$\sin^2 \theta_W$", is no mystery in QHD characterized by a large (~ 1 TeV) mass scale.

We see that W and Z are anomalously light on the mass scale of QHD:

$$m_{W,Z}^2 \sim 0.01 \text{ TeV}^2 \ll 1 \text{ TeV}^2. \tag{56}$$

From this point of view W and Z are like the pion of QCD. The importance of having a large mass scale in QHD can also be inferred by estimating the "size" of W and Z from the bound-state wave function needed to explain the observed large value of λ^2 [35).

Because the weak boson spacing is predicted to be of order 1 Tev^2, "excited" W and Z are likely to play insignificant roles in electroweak physics of the eighties and even of the nineties. The mass formula (24) based on the single pole hypothesis is probably an excellent approximation to the more general mass formula (43). Furthermore the C coefficient that appears in multiboson models is probably negligible.

Even though I have estimated the strength of γ-W^o mixing in the particular context of the Fritzsch-Mandelbaum model, I believe that the main conclusion reached is largely independent of the specific model used. To make λ^2 sizable, the important thing is that the mass squared difference between the photon and the W boson (before mixing) is very much smaller than the characteristic squared mass of the dynamical mechanism responsible for the W boson as a bound state. On the scale of QHD the W boson looks nearly massless, almost like the photon. It then follows, as in any quantum-mechanical system with a small energy-level difference, that γ and W^o must be strongly mixed.

Conclusion

I now summarize the main results.

(a) The striking successes of the standard electroweak gauge model in accounting for low q^2 neutral-current data can be reproduced equally well in a more phenomenological model based on global SU(2) broken by γ-W^o mixing.

(b) The boson masses, m_W and m_Z, need not satisfy the Weinberg mass relations. Instead we expect the weaker mass formula (24). There is an upper bound for m_W, in the neighborhood of 170 GeV.

(c) The validity of the Weinberg mass relations or, more generally, the correctness of the weak-electromagnetic unification idea, rests on asymptotic restoration of SU(2), a yet-to-be-tested short-distance behavior.

(d) Multiboson or continuum generalizations also lead to m_W < 170 GeV in the charged-current channel.

(e) Models with composite W and Z (but elementary γ) must rely on the γ-W^o mixing mechanism to reproduce the observed low q^2 behavior of neutral-current phenomenology.

(f) It is not unreasonable to obtain the observed "large" γ-W^o mixing constant in composite model characterized by a TeV mass scale.

If the standard electroweak gauge model fails at high timelike values of q^2, we will undoubtedly enter one of the most exciting eras in the history of particle physics.

References

1) S.L. Glashow, Nucl. Phys. 22, 579 (1961).
2) A. Salam and J.C. Ward, Phys. Lett. 13, 168 (1964); A. Salam, Elementary Particle Theory, ed. N. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 367.
3) S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
4) For a review see e.g. J.E. Kim, P. Langacker, M. Levine and H.H. Williams, Rev. Mod. Phys. 53, 211 (1981); P.Q. Hung and J.J. Sakurai, Ann. Revs. Nucl. Sci. 31, 375 (1981).
5) W.J. Marciano and A. Sirlin, Nucl. Phys. B189, 442 (1981); C.H. Llewellyn Smith and J.F. Wheater, Phys. Lett. 105B, 486 (1981).
6) S.L. Glashow, Proc. LEP Summer Study, CERN Yellow Report 79-01 (1979).
7) J.D. Bjorken, Proc. Ben Lee Memorial International Conference on Parity Nonconservation, Weak Neutral Currents and Gauge Theories, ed. D.B. Cline and F.E. Mills (Hardwood Academic Pub., London, 1978) p. 701; J.D. Bjorken, Proc. XIII Rencontre de Moriond, ed. Tran Than Van (Editions Frontières Dreux, France, 1978) Vol II, p. 491; J.D. Bjorken, Phys. Rev. D19, 335 (1979).
8) P.Q. Hung and J.J. Sakurai, Nucl. Phys. B143, 81 (1978).
9) N. Dombey, Proc. 1981 Banff Summer Institute on Particles and Fields (to be published).
10) S.A. Bludman, Nuov. Cim. 9, 443 (1958).
11) Y.B. Zel'dovich, JETP 9, 682 (1959).
12) J.J. Sakurai, Currents and Mesons (University of Chicago Press, 1969).
13) I. Yu. Kobzarev, L.B. Okun' and I. Ya. Pomeranchuk, JETP 14, 355 (1962).
14) N.M. Kroll, T.D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).
15) The boson mass formula (24) first appeared in this form in Ref. 8. However, it is implicit also in Bjorken's work (Ref. 7).
16) C.H. Llewellyn Smith, Proc. 1981 Banff Summer Institute on Particles and Fields (to be published).
17) This result is anticipated in view of the well-known theorem of C.H. Llewellyn Smith, Phys. Lett. 46B, 233 (1973), and J.M. Cornwall, D. Levin and G. Tiktopoulos, Phys. Rev. D10, 1145 (1974) stating that tree unitarity enforces gauge invariant vertices.
18) J.J. Sakurai, Proc. Eighth Hawaii Topical Conference on Particle Physics (University of Hawaii Press, Honolulu, 1981).
19) E.H. de Groot and D. Schildknecht, Zeit. Phys. C10, 55 (1981).
20) N. Wright, UCLA preprint, UCLA/81/TEP/14.
21) The importance of the C term in broken SU(2) models was first stressed by J.D. Bjorken (Ref. 7).

22) G.J. Gounaris and D. Schildknecht, Zeit. Phys. C12, 57 (1982).
23) H. Terazawa, Y. Chikashige and K. Akama, Phys. Rev. D15, 480 (1977); H. Harari, Phys. Lett. 86B, 83 (1979); M.A. Shupe, Phys. Lett. 86B, 87 (1979); O. Greenberg and J. Sucher, Phys. Lett. 99B, 339 (1981); R. Barbieri, A. Masiero and R. Mohapatra, Phys. Lett. 105B, 369 (1981).
24) L. Abbott and E. Farhi, Phys. Lett. 101B, 69 (1981).
25) H. Fritzsch and G. Mandelbaum, Phys. Lett. 102B, 319 (1981).
26) J.J. Sakurai, Ann. Phys. 11, 1 (1960).
27) M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).
28) This line of thinking has recently been advocated particularly by R. Kögerler and D. Schildknecht, CERN preprint, TH 3231-CERN (1982).
29) In the vector dominance notation of Ref. 12, $\lambda_{\gamma\rho}$ here is to be identified with e/f_ρ.
30) A. Bramón, E. Etim and M. Greco, Phys. Lett. 41B, 609 (1972).
31) J.J. Sakurai, Phys. Lett. 46B, 207 (1973).
32) K. Ishikawa and J.J. Sakurai, Zeit. Phys. C1, 117 (1979); J.S. Bell and R.A. Bertlmann, Zeit. Phys. C4, 11 (1980).
33) J.A. Bradley, C.S. Langensiepen and G. Shaw, University of Manchester preprint, M/C TH 81/09.
34) P. Chen and J.J. Sakurai, Phys. Lett. 110B, 481 (1982).
35) H. Fritzsch and G. Mandelbaum, Phys. Lett. 109B, 224 (1982).

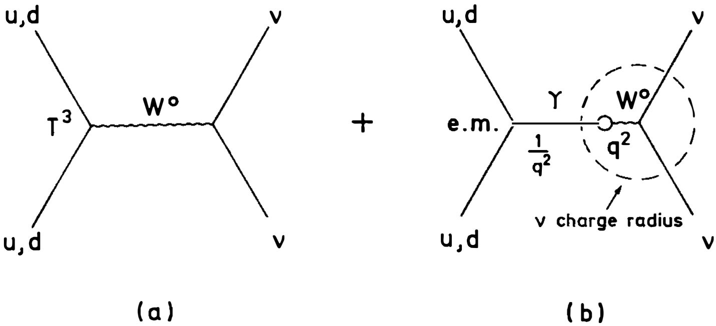

Fig. 1 Simple low-order diagrams for νq scattering.

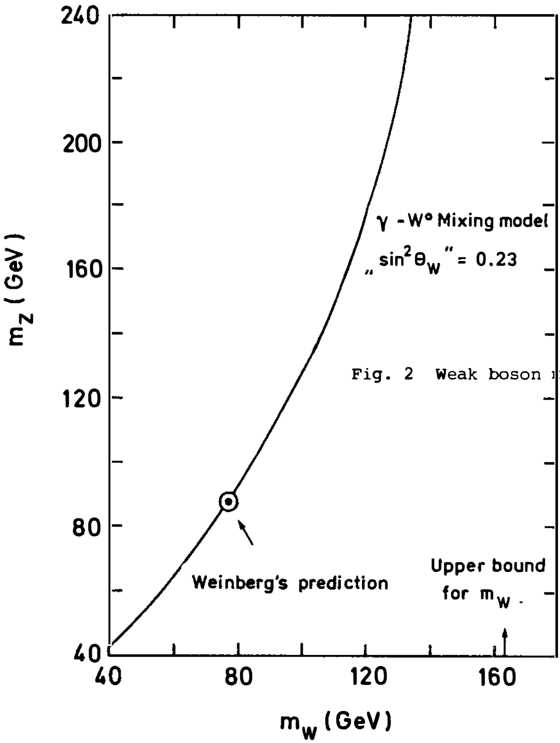

Fig. 2 Weak boson mass relation.

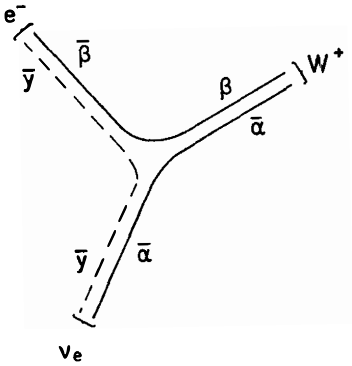

Fig. 3 $\nu_e \leftrightarrow W^+e^-$ vertex

Fig. 4 (a) γ-ρ^o junction; (b) γ-W^o junction.