Refractive index of the hadronic medium in presence of a magnetic field

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Introduction

It is believed that a strong transient magnetic field ($\sim m_\pi^2 \sim 10^{18}$ G) is produced in peripheral ultra-relativistic heavy-ion collisions due to the motion of charged spectator protons. This magnetic field decays with time, and in principle, affects the basic thermodynamic, transport properties, and various susceptibilities of the evolving partonic and hadronic matter. The susceptibilities play a crucial role in describing the QCD phase transition and are related to conserved charge fluctuations of the system through the fluctuation-dissipation theorem. In this work, we estimate the magnetic susceptibility (χ_M^2) and electrical susceptibility (χ_Q^2) considering an interacting hadronic medium having both the attraction and repulsion parameter. The variation of these susceptibilities is studied as a function of temperature in the presence of an external static magnetic field. With the information of χ_M^2 and χ_Q^2, the magnetic permeability (μ_r) and electrical permitivity (ϵ_r) is obtained . Using μ_r and ϵ_r, one can get an idea about the optical properties of the system. We estimate the refractive index (RI) as a function of temperature and magnetic field, which reveals information about the speed of light in the hadronic medium.

Formalism

The partition function for ith particle species in a Grand Canonical Ensemble (GCE) of Ideal Hardon Resonance Gas (IHRG) is given as

$$lnZ_i^{id} = \pm V g_i \int \frac{d^3p}{(2\pi)^3}\, ln\{1 \pm \exp[-(E_i - \mu_i)/T]\}, \tag{1}$$

where, T is the temperature of the system and V represents the volume. The notations g_i, $E_i = \sqrt{p^2 + m_i^2}$, m_i and μ_i are for the degeneracy, energy, mass, and chemical potential of the ith hadron, respectively. Here, id denotes to the ideal. The plus and minus signs ($\pm$) correspond to baryons and mesons, respectively. The ideal pressure in terms of partition function can be expressed as:

$$P^{id} = T\frac{\partial}{\partial V}(lnZ^{id}). \tag{2}$$

In the presence of a magnetic field, the pressure for charged particles is defined as [1]

$$P_{c,i}^{id,z}(T, \mu_i, B) = \pm \frac{T g_i |Q_i| B}{2\pi^2} \sum_k \sum_{sz} \times \int_0^\infty dp_z \; ln\{1 \pm \exp[-(E_{c,i}^z - \mu_i)/T]\} \tag{3}$$

where,

$$E_{c,i}^z(p_z, k, s_z) = \sqrt{p_z^2 + m_i^2 + 2|Q_i|B\left(k + \frac{1}{2} - s_z\right)}, Q_i \neq 0 \tag{4}$$

The pressure for neutral particles is given as:

$$P_i^{id}(T, \mu_i) = \pm T g_i \int \frac{d^3p}{(2\pi)^3}\, ln\{1 \pm \exp[-(E_i - \mu_i)/T]\} \tag{5}$$

The total pressure of the system is the sum of pressure due to the neutral particles and the charged particles.

$$P = P_{c,i}^{id,z}(T, \mu_i, B) + P_i^{id}(T, \mu_i) \tag{6}$$

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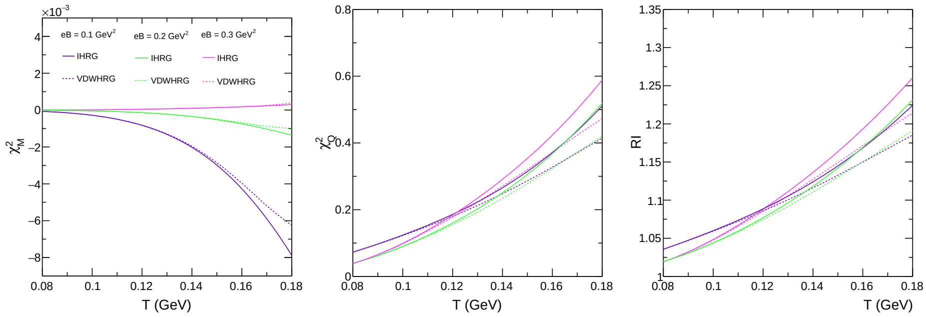

FIG. 1: (Color online) The magnetic susceptibility (left panel), electrical susceptibility (middle panel) and refractive index (right panel) as functions of temperature for $eB = 0.1$, 0.2, 0.3 GeV^2 [1].

The second-order derivative of pressure concerning the magnetic field is called magnetic susceptibility and is given by,

$$\chi_M^2 = \frac{\partial^2 P}{\partial (QB)^2}. \quad (7)$$

The second-order susceptibility corresponding to the electric charge is called electrical susceptibility, and is given by,

$$\chi_Q^2 = \frac{1}{T^2}\frac{\partial^2 P}{\partial \mu_Q^2} \quad (8)$$

Results and Discussion

The left panel of Fig. 1 shows the temperature dependence of magnetic susceptibility for three different values of the magnetic field i.e. eB = 0.1, 0.2, 0.3 GeV^2. One can observe that the magnetic susceptibility is negative for the lower two values of the magnetic field, and its value tends towards positive for a higher magnetic field both for the IHRG and van der Waals HRG (VDWHRG) models. The details regarding the VDWHRG model are described in Ref. [1]. So a clear observation of the diamagnetic to paramagnetic transition happens in both models. With the knowledge of χ_M^2, one can calculate the μ_r of the medium under consideration using the relation $\mu_r = \frac{\mu}{\mu_0} = \frac{1}{1-e^2\chi_M^2}$. The middle panel of Fig. 1 shows the electrical susceptibility increases with an increase in temperature. With a higher magnetic field, the electrical susceptibility is found to be suppressed at lower temperatures, and it starts to increase beyond a certain value of temperature. This limiting temperature was found to decrease with an increase in the magnetic field. This is because the spin-0 particles don't contribute to susceptibility in the presence of the magnetic field. However, as temperature increases, the higher non-zero spin resonance particles start contributing to susceptibility, and hence susceptibility is found to increase with the magnetic field at a higher temperature. Having the information of χ_Q^2, one can obtain the ϵ_r of the medium using the formula $\epsilon_r = 1 + \chi_Q^2$. The knowledge of the μ_r and ϵ_r of the hadronic medium motivates us to calculate one of the most important optical properties, called the refractive index using the standard relation RI $= \sqrt{\epsilon_r \mu_r}$. The right panel of Fig. 1 shows the refractive index variation as a function of temperature; the refractive index of the medium increases as the temperature increases. It is due to the fact that the number density of the system increases with temperature in the HRG model, and a denser medium causes a higher RI. The RI of the medium decreases with the magnetic field at lower temperatures and starts to increase at higher temperatures. The detailed explanation can be found in Ref. [1].

References

[1] B. Sahoo, K. K. Pradhan, D. Sahu and R. Sahoo, [arXiv:2306.03477 [hep-ph]].