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GENERATION OF SAWTOOTH CORRELATION FOR BUNCHING FACTOR ENHANCEMENT

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Abstract

Bunch trains have been considered as a promising means of generating intense, coherent radiation in compact accelerator facilities. However, conventional methods, which impart a sinusoidal modulation on the beam's longitudinal phase space, are inefficient for generating a high bunching factor density modulation. Only a small fraction of a sinusoidal modulation, which has linearity, primarily forms density spikes while other particles under nonlinear correlation have limited contribution to these spikes. One way to improve such bunching efficiency is imparting a saw-tooth correlation, which has piecewise-linearities. This correlation maximizes the peaks of density spikes as more than 90% of particles will contribute to the spikes. While such correlation can be generated by a series of transverse wigglers, a single transverse wiggler with shaped poles to introduce higher harmonics can generate saw-tooth or saw-tooth-like correlations. We present a recent study on this new approach, employing a shaped-pole transverse wiggler.

INTRODUCTION

Most bunch train generation methods impart a sinusoidal modulation on the longitudinal phase space and convert this momentum modulation to a density modulation via appropriate R_{56}. When the microbunch is compressed by the modulation and R_{56}, the particles' longitudinal positions overlap, and they form a density spike. Although this approach generates reasonable density spikes, it has an intrinsic limitation originating from nonlinearity. Only the linear fraction of the sine curve contributes to the density spike while the rest of the particles move close to the peak but do not reach the peak location. The obvious solution to improve performance is to eliminate the nonlinearity from the sinusoidal modulation. Therefore, as shown in Fig. 1 a triangular modulation is a promising candidate which can significantly increase the peak density.

Recently, a method to impart arbitrary correlation on the phase space was proposed [1]. The key equipment of the proposed method is the transverse wiggler, which is a finger-sized wiggler rotated by 90° from its nominal orientation [2]. Since the transverse wiggler imparts sinusoidal modulation on the transverse phase space, it is possible to generate transverse density spikes. This transverse modulation can be converted into a longitudinal bunch train via emittance exchange [3–5]. One of the research directions in this area is improving the equipment to provide better resolution control or minimize the required number of wigglers. As a part of this effort, we considered shaping the pole or geometry of the wiggler to control the harmonic content.

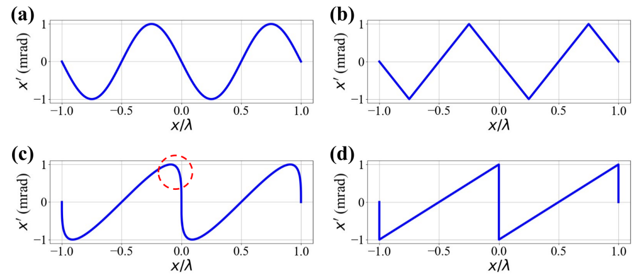

Figure 1: Comparison of bunching by sinusoidal and triangular modulations. Particles in the red circle do not reach the bunching point due to nonlinearity.

It has been known that controlling the shape of the pole tip can strengthen higher-order components and change the magnetic field pattern. We have observed that attaching sharp tips on the Halbach array can convert the sinusoidal magnetic field from a Halbach array to a triangular field. This paper describes recent preliminary simulation results to explore this opportunity. Magnet simulations were carried out using 2D magnetostatic simulation code, FEMM [6]. Particles were tracked numerically without collective effects.

WIGGLER AND POLE TIP GEOMETRY

As a preliminary study, we adjusted the Halbach array. As shown in Fig. 2, we assumed permanent magnets magnetized parallel to the axis and pure Iron poles with triangular tips. The period was fixed at 4 mm, and the pole width, w, and tip height, h, were varied. The gap (g) was fixed at 0.5 mm. Note that the gap changes the magnetic field strength but does not significantly change the field pattern. Therefore, the simulation results regarding the gap control are not included in this paper.

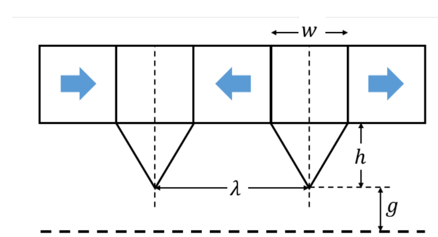

Figure 2: Wiggler geometry.

To estimate the impact of geometry changes, we generated a 4D Gaussian bunch in the x-y plane and numerically applied the modulation using the simulated magnetic field. The modulated bunch drifted to an appropriate distance that

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maximizes the peak density. Due to the flat geometry of the wiggler, we assumed the use of a flat beam and adopted parameters from Ref. [7]. Since the wiggler's magnetic field depends on both the particle's horizontal and vertical positions, we estimated the peak density with a few different vertical beam sizes. The beam parameters are summarized in Table. 1.

Table 1: Beam parameters

Parameter	Value
Energy	44 MeV
σ_x	7 mm
ε_{nx}	175 µm
$S_{x,y}$	$0\,\mathrm{m}^{-1}$
σ_y	0.12, 0.18, 0.24 mm
ε_{ny}	1 µm

Figure 3 shows the numerical test results. It shows peak density achieved with various pole widths and tip heights. Each panel corresponds to different vertical beam sizes: (a) 0.12 mm, (b) 0.18 mm, and (c) 0.24 mm. As expected, it is evident that the peak density increases with increasing tip height. Because a triangular field will provide the maximum peak density, the peak density in the figure converges as the height increases. Additionally, it is apparent that the wider poles are preferred.

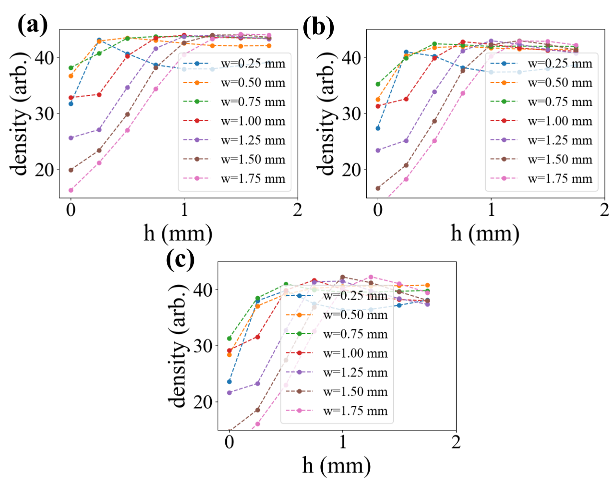

Figure 3: Estimated maximum peak densities for various pole width and tip height combinations.

It is also worth noting that the field pattern can be adjusted by controlling the pole width without the tip. As shown in Fig. 4, a narrow pole reduces the curvature of the field near its maximums while a wide pole increases the curvature. Since the narrow pole without a tip provides a triangular field, the peak density from this geometry is already close to the best case of other geometries. However, for w=0.25 mm (panel (a) in Fig. 4), a kink is already present, which indicates that the field is over-corrected. When the tip is added to this geometry, the magnet passed the correction point and introduces extra nonlinearities. It results in the reduction of the peak density.

The vertical magnetic field generated by the transverse wiggler includes a dependency on the vertical position. As particles are located closer to the magnet surface, they obtain a stronger kick from the magnet. Thus, this vertical dependency introduces spreads of particles around the bunching point, which reduces the peak density. We can observe that the peak density decreases as the initial vertical beam size increases from Fig. 3. Here, the beam started with a fairly large horizontal emittance. Thus, the decrease of the peak density is not as clear as expected because the maximum peak density is strongly limited by the horizontal emittance. This is an important factor to consider.

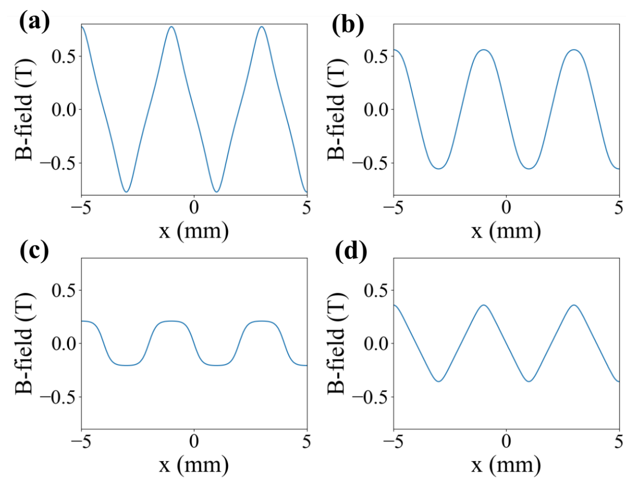

Figure 4: Simulated magnetic fields. Each panel corresponds to a different geometry. (w,h) are (a) (0.25, 0.00) mm, (b) (1.00, 0.00) mm, (c) (1.75, 0.00) mm, and (d) (1.00, 0.75) mm.

IMPACT OF HORIZONTAL EMITTANCE

As mentioned in the previous section, horizontal emittance is an important factor that determines the effectiveness of field shaping by the sharp tip. If the displacement from the ideal spot due to sinusoidal nonlinearity is smaller than the displacement caused by the emittance, the nonlinearity's impact will be overshadowed by the emittance effect. Figure 5 compares the peak density for different geometries and horizontal emittances. The peak density increases as the horizontal emittance decreases. Furthermore, results here show how strongly emittance limited the peak density is in Fig. 3. We can also observe that the difference between different sharpness becomes more prominent when the emittance is low enough. When the sharp-tip pole is wide enough (i.e., dominantly determines the field shape), it generates a factor of 1.5-3 enhancement of the peak density compared to the flat pole. Thus, the advantage of adopting this sharp-tip geometry will depend on the incident beam quality.

WIGGLER'S IMPACT ON VERTICAL EMITTANCE

While the transverse wiggler introduces interesting new opportunities, it also brings significant side effects on the

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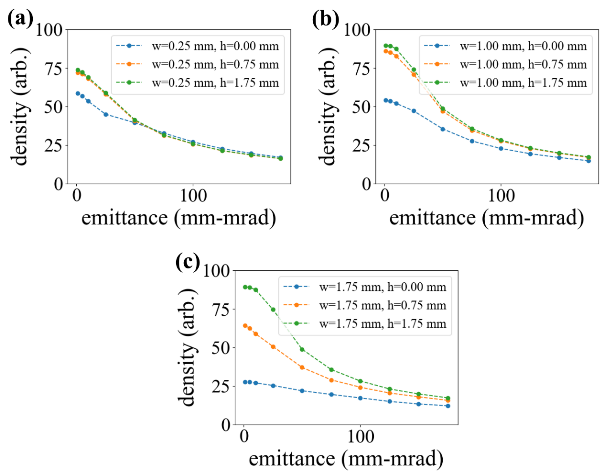

Figure 5: Peak densities with various horizontal emittances.

beam due to $B_x(x)$ field. Because of this horizontal magnetic field, the wiggler focuses the beam in vertical direction and modulates it along the horizontal direction. Such correlation between x- and y-directions leads to emittance growth. However, the bunch train generated by this modulation method includes regions that are lengthened. Particles in these regions are not of interest to us, so we have examined vertical emittance as a function of the cut level. Figure 6 shows the x-y image and vertical phase space of a single transverse beamlet. Here, we only plot the particles whose initial horizontal position was within ± 25% of the wavelength from the zero-crossing points (i.e., only counting particles experiencing bunching). Colors were assigned based on the particle's horizontal position. It is obvious that particle spread in the vertical phase space strongly depends on the horizontal position.

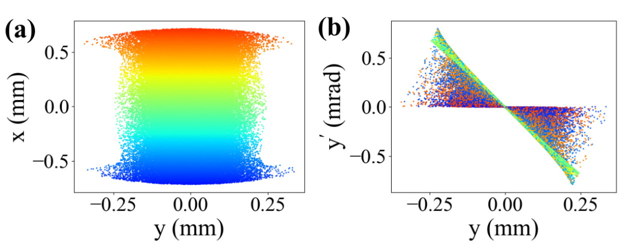

Figure 6: x-y image and vertical phase space of a single transverse beamlet.

As we cut out more particles, the growth of vertical emittance will be suppressed. Figure 7 shows the vertical emittance's response to the cut. Cutting more particles results in a decrease in emittance growth as expected. An interesting point to note is that the emittance decreases quite rapidly within the first 5%. Additionally, the sharp-tip configuration shows a smaller emittance growth compared to the flat-poles. Moreover, the emittance from the sharp-tip decreases much faster than the flat-pole when the width of the pole is not too narrow. This could be another advantage of using a sharp-tip wiggler.

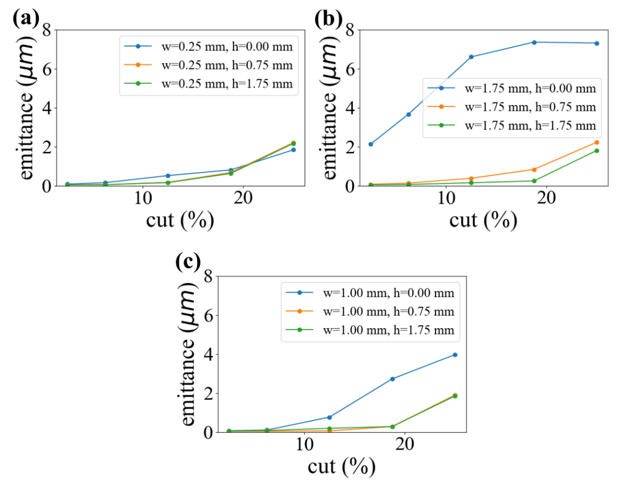

Figure 7: Vertical emittance with various horizontal cut.

SUMMARY

We conducted preliminary simulation studies to explore the opportunity for field shaping with the transverse wiggler. By controlling the width and sharpness of the poles, we were able to adjust the sharpness of the sinusoidal modulation. As expected, this adjustment allowed us to generate a magnetic field closer to a triangular shape, which increased the peak density. This effect was particularly pronounced when the beam's horizontal emittance was sufficiently low. We also observed vertical emittance growth due to the $B_x(x)$ field. The emittance growth was significantly suppressed when we cut the particles outside of ±20% of the wavelength. The sharp-tip configuration showed much smaller growth compared to the flat-pole case. In addition, the emittance growth from the sharp-tip configuration decreased faster than the flat-pole case as the cut range decreased.

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