Nonsense, Conspiracy, and Possible Singular Residues in Regge Pole Theory

The history of what has come to be called Regge pole theory has been a living testimonial against the often quoted maxim that spin is an unessential complication in collision theory. In fact a number of the most interesting aspects of Regge theory arise when one is dealing with particles with spin.

The important concept of "nonsense" points[1] on Regge trajectories is a good example. A nonsense value of the angular momentum, J, can be defined as one of the integer values of J (or $J - \frac{1}{2}$ if one is dealing with a trajectory with baryon number ± 1) which is smaller than the difference of the helicities of two particles coupled to the trajectory. For two spin-zero particles, $J = -1$ is a nonsense point; for two spin-one particles with helicities $+1$ and -1, $J = 1$ is a nonsense point; for a spin-one particle with helicity -1 and a spin-one half-particle with helicity $+\frac{1}{2}$, $J = \frac{1}{2}$ is nonsense. In the process $\pi^- + p \rightarrow \pi^0 + n$ the differential cross section shows a pronounced dip at $t =$ (momentum transfer)$^2 = -0.6$ $(\text{BeV}/c)^2$. This is neatly explained by the fact that the ρ trajectory, which is the most important one, passes through zero at this value of t. It is a feature of the theory that when the trajectory passes through zero (a nonsense point for this process) and, as in this case, the corresponding value of the angular momentum does not correspond to a particle (for the ρ trajectory, $J = 1, 3, 5, \ldots$ are particles) the Regge amplitude vanishes. There are several examples of this general principle[2]; whenever a trajectory passes through a nonsense point of the wrong signature one expects a dip in the differential cross section.

Another striking example of the role of spin in scattering theory and in Regge analysis is the following. Consider a general process $a + b \rightarrow c + d$ which we refer to as the s channel. The crossed, or t-channel reaction is taken to be $D + b \rightarrow c + A$ where A (D) is the antiparticle of a (d); it is the Regge poles in the t channel which dictate the large s behavior. Let us restrict attention to processes in which $m_a = m_c$ and $m_b = m_d$ so that $t = 0$ corresponds to scattering through zero degrees in the s channel (θ_s, the angle between a and c, is zero). Then if we use the letters a b c d to denote helicities, only the helicity amplitudes $f_{cd;ab}$ for which $\lambda = a - b$

is equal to $\mu = c - d$ are different from zero. This is because at $\theta_s = 0$, λ and μ are initial and final angular momenta *along* the direction of motion and must therefore be conserved. The theory even predicts the minimum power of t with which the vanishing amplitudes must go to zero as $t \to 0$. Now since the s-channel helicity amplitudes are related by analytic continuation to the t-channel ones, and their numbers are the same, if certain s-channel amplitudes vanish at $t = 0$, there must be an appropriate set of linear relations among the t-channel amplitudes to make the numbers balance.

The first time such conditions were noted was in connection with nucleon-nucleon scattering in pre-Regge days.[3] The relevance to Regge theory was pointed out by Volkov and Gribov[4] in a widely ignored paper. Here one notes that the five usual independent helicity amplitudes degenerate to three at $t = 0$. There must be then two relations among the t-channel amplitudes. It turns out that one of these is trivial, saying only that a certain partial wave t-channel amplitude $f^J(t)$ must vanish near $t = 0$ like $t^{1/2}$ for all J. The second condition is much more involved and when translated into Regge theory states that there must be a distinct relation between Regge trajectories (and residues) with quite distinct quantum numbers at $t = 0$, i.e., a conspiracy among trajectories. If the trajectories are classified according to the nucleon-antinucleon states which are allowed for physical J, one finds that

$$\alpha[{}^1J_J,\, t = 0] = \alpha[{}^3(J + 1)_J,\, t = 0] = \alpha[{}^3J_J,\, t = 0] \pm 1$$

and a relation among residues. The only way to avoid the conspiracy is to have the pole residues vanish like t near $t = 0$. This cowardly way out has profound effects on the spin dependence of the differential cross section near $t = 0$.

Another interesting example[5] which also has relevance for the last question we shall discuss is the case of the scattering of massive spin-one particles by spin-zero particles (ρ–π scattering). Suppressing the spin-zero particle helicity labels, we have in general four s-channel amplitudes $\mathfrak{M}_{1,1}$, $\mathfrak{M}_{0,0}$, $\mathfrak{M}_{1,0}$, and $\mathfrak{M}_{1,-1}$. At $t = 0$, the last two vanish ($\mathfrak{M}_{1,0} \sim t^{1/2}$, $\mathfrak{M}_{1,-1} \sim t$ near $t = 0$). Designating the t-channel amplitudes by M_{11}, M_{00}, M_{10}, and M_{1-1}, the condition $\mathfrak{M}_{1,-1} \sim t$ becomes, using the crossing relations of Trueman and Wick,[6] $M_{11} - M_{00} + M_{1-1} \sim t$, which we call the conspiracy condition. Since the ρ mesons (neutral in our example) are identical, we are concerned only with one type of Regge trajectory, namely those with $C = +1$, even signature. It turns out that to satisfy the conspiracy condition, if there is a Regge trajectory which passes through some value J_0 near $t = 0$, there *must* be a consort which passes through $J_0 - 2$. Furthermore if the residue at either of these poles for $M_{11} - M_{00}$ is called a and that of M_{1-1} called b, we must have $a(t) - J_0(t)[J_0(t) - 1]b(t) \sim t$

near $t = 0$. Now suppose $J_0(t) = 1 + \alpha'(0)t + \ldots$; *if* $b(t)$ is singular near $t = 0$, i.e. $b(t) \sim 1/t$, $a(0) \neq 0$. This possibility has the following interesting consequence: The s-channel amplitudes are, at $t = 0$, $\mathfrak{M}_{1,1} = M_{11} + M_{1-1}$, $\mathfrak{M}_{0,0} = M_{11} - M_{1-1}$. The Regge-pole contribution to M_{1-1} is proportional to $bJ_0(J_0 - 1)$; if b is singular near $t = 0$ when $J_0 \to 1$, which is, of course, what the Pomeranchuk trajectory is supposed to do, then as shown above, it will give a nonzero contribution to $\mathfrak{M}_{1,1}$ and $\mathfrak{M}_{0,0}$. Thus, contrary to what is often said, the Pomeranchuk trajectory *would* lead to spin dependence at high energies, $\mathfrak{M}_{1,1} \neq \mathfrak{M}_{0,0}$. Whether Nature chooses this option remains to be seen.

The question of singular residues is very important for processes involving real photons. Consider as an example Compton scattering on pions. Assume that the Pomeranchuk trajectory couples to the two photons; the s-channel amplitude $\mathfrak{M}_{1,1}$ is related to the t-channel amplitude M_{1-1}, for all t, by $\mathfrak{M}_{1,1} = M_{1-1}$. A Regge pole with trajectory $\alpha(t)$ contributes something of the form

$$\mathfrak{M}_{1,1} \sim \frac{\beta(t)\alpha(t)[\alpha(t) - 1][1 + e^{-i\pi\alpha(t)}]s^{\alpha(t)}}{\sin \pi\alpha(t)}.$$

If $\alpha(t) = 1 + \alpha'(0)t$ near $t = 0$, *unless* $\beta(t) \sim 1/t$, this Pomeranchuk trajectory will not contribute to $\mathfrak{M}_{1,1}$ at $t = 0$ and thus the Compton cross section would not approach a constant at high energy.[7] This seems very strange since one expects in this "classical" electromagnetic problem that the characteristic diffraction behavior should obtain. We have been able to show[5] that singular residues (which in this problem at least may also be viewed in a certain way as indicating a fixed singularity at $J = 1$ as well as a true Pomeranchuk trajectory) are not in conflict with the factorization constraints on Regge residues in the processes $\gamma + \gamma \to \gamma + \gamma$ and $\pi + \pi \to \pi + \pi$. Very similar conditions and results hold for Compton scattering on spin-$\frac{1}{2}$ particles.

It should be clear, hopefully, that if spin is not essential, it is surely very interesting!

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