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# Primordial Magnetogenesis from Killing Vector Fields

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**Abstract**

Papapetrou showed that the covariant derivative of a Killing vector field satisfies Maxwell's equations in vacuum. Papapetrou's result is extended, in this article, and it is shown that the covariant derivative of a Killing vector field satisfies Maxwell's equations in non-vacuum backgrounds as well if one allows electromagnetic currents of purely geometric origin. It is then postulated that every Killing vector field gives rise to a physical electromagnetic field and, in a non-vacuum background, a physical electromagnetic current—hereafter called *Killing electromagnetic field* and *Killing electromagnetic current*, respectively. It is shown that the Killing electromagnetic field of the flat FLRW (Friedmann–Lemaître–Robertson–Walker) universe comprises a Killing magnetic field and a rotational Killing electric field; an upper bound on the Killing magnetic field is derived, and it is found that the upper bound is consistent with the current observational bounds on the cosmic magnetic field. Next, the time-like Killing vector of the Schwarzschild spacetime is shown to give rise to a radial Killing electric field. It is also shown that in the weak field regime—and far from the matter distribution—the back reaction of the radial Killing electric field changes the Schwarzschild metric to the Reissner–Nordström metric, establishing a partial converse of Wald's result. Drawing upon Rainich's work on Rainich–Riemann manifolds, the etiological question of how a physical electromagnetic field can arise out of geometry is discussed; it is also argued that detection of the Killing electric field of flat FLRW spacetime may be within the current experimental reach. Finally, this article discusses the relevance of Killing electromagnetic currents and the aforementioned transmutation of Schwarzschild spacetime to Reissner–Nordstrom spacetime, to Misner and Wheeler's program of realizing "charge without charge".

**Keywords:** magnetogenesis; Killing vectors; cosmic magnetic field; Schwarzschild spacetime; Reissner–Nordström spacetime

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## 1. Introduction

The origin of the cosmic magnetic field—a weak magnetic field that pervades the intergalactic region—remains a mystery [1–5]. The current observational bounds on the strength of the cosmic magnetic field (CMF)—also called the intergalactic or extragalactic magnetic field—and the sourcing mechanisms that have been proposed to explain its origin are summarized in Section 2.1. Given the vastness of the intergalactic region and the relatively sparse distribution of the galaxies that punctuate it [6], it is believed that the ubiquitous CMF likely did not originate from any post-recombination astrophysical process(es) [4,5] but, instead, is likely of primordial origin and was generated by a mechanism that was operative over the entire vastness of intergalactic region [1,4,5,7].

A second, possibly related, enigma can be stated, using Gamow's words [8], as the following question: Why is it that "…*all successive degrees of accumulation of matter, such* *as planets, stars and galaxies, are found in the state of more or less rapid axial rotation...*"? In Section 2.2, a related conjecture of Gamow's is discussed; Gamow's conjecture does not resolve but rather amplifies the enigma. Section 2.2 also presents a summary of the follow-up work on Gamow's conjecture, the current constraints on vorticity in the universe, and previous attempts at explaining the origin of axial rotations of galaxies.

Against the backdrop of the above two enigmas, this article revisits an intriguing observation that Papapetrou made in 1966 [9]. Specifically, Papapetrou showed that the covariant derivative of a Killing vector field of a spacetime resembles the electromagnetic field strength tensor and satisfies Maxwell's equations in vacuum [9]. Wald used Papapetrou's observation to derive the electromagnetic field around a Kerr black hole placed in a uniform magnetic field that is parallel to the black hole's axis of rotation [10]. Wald's construction has been used in the study of Kerr black holes by a number of authors [11–15]. Most notably, Blandford and Znajek used Wald's construction to propose the so-called BZ mechanism, which suggests that the astrophysical jets of a Kerr black hole are powered by its spin energy [16].

In this article, Papapetrou's result is extended to show that the covariant derivative of a Killing vector field satisfies Maxwell's equations, even in non-vacuum backgrounds, if one allows electromagnetic currents of purely geometric origin; see Appendix A.

Guided by the results mentioned in the previous paragraphs, this article proposes the following two postulates:

Postulate 1: Every Killing vector field gives rise to a *physical* electromagnetic field whose field strength tensor is proportional to the covariant derivative of the Killing vector.

Postulate 2: In a non-vacuum background, every Killing vector field gives rise to a *physical* electromagnetic current whose density is proportional to the contraction of the Killing vector with the Ricci tensor.

The electromagnetic field and the electromagnetic current—mentioned in Postulates 1 and 2—will be called the *Killing electromagnetic field* (KEF) and *Killing electromagnetic current* (KEC), respectively; the KEF and the KEC density arising from a Killing vector field are defined in (A1) and (A9) in Appendix A. Hereafter, Postulates 1 and 2 will be referred to as the KEF and KEC postulates, respectively.

At the outset, a natural criticism of the postulates is that the postulated emergence of physical electromagnetic field and current out of geometry (Killing vectors) seems far-fetched. *What is the mechanism—the manner of causation—by which Killing symmetries, which are geometric structures, give rise to physical, measurable electromagnetic fields and currents?*

Less than a decade after the formulation of general relativity, Rainich [17] published a seminal calculation that effectively dissolved the distinction between source-free electromagnetism and the geometry of spacetime. Summarized in Section 5.4, Rainich's calculation—which also served as the foundation for Misner and Wheeler's work [18]—shows how source-free electromagnetism 'emerges' from geometry. Rainich's calculation suggests a conjectural explanation of how KEF and KEC can 'emerge' from Killing vectors and why the KEF and KEC postulates may not be far-fetched. Admittedly, the postulates are speculative conjectures until they are experimentally verified or falsified, and in Section 5.3, it is argued that falsifying the KEF postulate may be within the current experimental reach.

Predicated on the validity of the KEF postulate, Sections 3 and 4 explore its consequences in two spacetimes—the flat FLRW spacetime, which is believed to describe the large-scale structure of our universe [19], and the Schwarzschild spacetime.

It is shown that three of the Killing vectors of the flat FLRW spacetime generate a Killing magnetic field that pervades the spacetime. An upper bound on the strength of the Killing magnetic field is derived using the self-consistency constraint. It is found that the derived upper bound is consistent with the known observational bounds on the strength of the CMF. Three other Killing vectors of the flat FLRW spacetime generate a Killing electric field, which has a nonzero 'curl'; the rotational Killing electric field raises the speculative question of whether *the Killing electric field may have seeded rotation of primordial, charged matter* and whether *it continues to power the "axial rotation" of charged matter today.* The answer to the preceding question hinges on the strength of the Killing electromagnetic fields in flat FLRW spacetime; the article discusses bounds on the strengths of Killing electromagnetic fields in flat FLRW and Schwarzschild spacetimes in Sections 3 and 4.

Finally, the consequences of the KEF postulate in the Schwarzschild spacetime are explored. The timelike Killing vector of the Schwarzschild spacetime generates a radial Killing electric field. The back reaction of the radial Killing electric field on the Schwarzschild spacetime in the weak field regime is derived, and it is found that, far from the distribution of matter, the back reaction changes the Schwarzschild metric to the Reissner–Nordström metric. Thus, if the KEF postulate is valid, then an observer far from the spherical distribution of electrically uncharged matter concludes that the spacetime is described by the Reissner–Nordström metric; Section 5.5 discusses the implications of the above transmutation of the metric by the Killing electric field.

The paper is organized as follows. Section 2.1 summarizes the known bounds on the CMF and the mechanisms that have been proposed to explain its origin. Section 2.2 summarizes the highlights of previous work on the puzzle of ubiquitous vorticity in accumulations of matter. Appendix A extends Papapetrou's derivation to non-vacuum backgrounds and to conformal Killing vectors. Section 3 derives the Killing magnetic field and the Killing electric field generated by the Killing vectors of the flat FLRW spacetime and discusses their conjectured cosmological implications. In Section 4, the radial Killing electric field generated by the time-like Killing vector of Schwarzschild spacetime is derived, and it is shown that, in the weak field regime and far from the distribution of matter, the Killing electric field changes the Schwarzschild metric to a Reissner–Nordström metric. In Section 5.1, the gauge invariance of the KEF is discussed. In Section 5.2, the plausibility of the KEF and KEC postulates is discussed. In Section 5.3, it is argued that falsifying the KEF postulate is possibly within the current experimental reach. Section 5.4 draws upon Rainich's results to provide a conjectural description of how KEF and KEC could 'emerge' from Killing vectors. Section 5.5, discusses the transmutation of the Schwarzschild metric, by its Killing electric field, into the Reissner–Nordström metric, as well as the postulated KEC, in the context of Misner and Wheeler's results on "charge without charge". In Section 5.6, the main features of KEF and KEC that distinguish magnetogenesis sourced by Killing vectors from previous proposals for primordial magnetogenesis are summarized. In Section 6, some directions for future work are discussed.

## 2. Background

In the first subsection below, the main bounds on the CMF and the mechanisms that have been proposed to explain its origin are summarized. In the second subsection, the previous work on understanding the "axial rotation" observed in all accumulations of matter up to galactic length scales is summarized.

### *2.1. Cosmic Magnetic Field*

A homogeneous CMF affects the isotropy of the cosmic microwave background (CMB) radiation. Hence, the known bounds on CMB anisotropy can be used to derive an upper bound on the strength of a homogeneous CMF. An upper bound of $3.4 \times 10^{-9}(\Omega_0 h_{50}^2)^{\frac{1}{2}}$ G on the current strength of any primordial homogeneous magnetic was derived by Barrow et al. based on an analysis of CMB anisotropy data [4]; $\Omega_0$ is the cosmological density parameter, and $h_{50}$ is the current value of the Hubble constant in units of 50 km·s$^{-1}$·Mpc$^{-1}$. A homogeneous magnetic field Faraday-rotates polarized light from distant sources, and hence, the Faraday rotation can be used to estimate the strength of the CMF. Using rotation measure data from 309 galaxies and quasars, Vallee [20] derived an upper bound of $10^{-9}(\Omega_{IG}h_{100}/0.01)^{-1}$ G on the strength of the CMF; $\Omega_{IG}$ is the ionized gas density, as a fraction of the critical density, and $h_{100}$ is the Hubble constant in units of 100 km·s$^{-1}$·Mpc$^{-1}$. The recently discovered evidence of ultrahigh-energy cosmic ray events emanating from the Perseus–Pisces supercluster was used by Neronov et al. to derive an upper bound of about $10^{-10}$ G on the CMF [21]; this upper bound is arguably the strongest to date. Finally, known constraints on big bang nucleosynthesis (BBN) also impose an upper bound on the strength of the CMF, although the upper bound derived from BBN is weaker than that derived from CMB anisotropy. Therefore, the upper bound derived from BBN [5] is not discussed further, and instead the following paragraph presents the more interesting discussion of the lower bound on the strength of the CMF.

The electromagnetic cascade that is triggered when the TeV photons from blazars interact with the extragalactic photon background gives rise to secondary GeV photons. The charged sector of the cascade couples to the CMF. The non-detection of secondary GeV photons by the Fermi/LAT telescope was used by Neronov and Vovk [22] and Ackerman et al. [23] to establish a lower bound of $B_{CMF} \geq 3 \times 10^{-16}$ G on the CMF. The order of magnitude of Neronov and Vovk's lower bound was reiterated subsequently by Tavecchio et al. [24] and Dolag et al. [1]. Dermer et al. [25] argue that the lower bound would be lower and that $B_{CMF} \gtrsim 10^{-18}$ G if the TeV activity of the blazars is assumed to last for less time than in the above reports. Regardless, the GeV photon flux data from blazars provides a nonzero lower bound on the strength of the CMF.

The remainder of the subsection discusses the previously proposed sourcing mechanisms for the CMF. Thorne [26] conjectured that the CMF must have come into existence at the big bang. Specifically, Thorne [26] wrote: "*All attempts to understand how galactic magnetic fields could have arisen since the big-bang creation of the universe (assuming the big-bang theory to be correct) face difficulties, which seem to be insurmountable (see, e.g., Hoyle 1958). The only way in which the existence of magnetic fields can be understood today is by assuming that they, like the matter in the universe, were created in the big bang. Although, this suggestion seems distasteful and ad hoc to many astrophysicists, the absence of any other explanation suggests that it be considered seriously (Zel'dovich 1964a).*" It should be noted that the Killing magnetic field described in this paper is generated at the birth of the flat FLRW spacetime and supports Thorne's conjecture in that the emergence of a magnetic field at the big bang does not have to postulated; it can be derived as an inescapable consequence of spacetime's Killing symmetries.[1]

The vorticity in primordial plasma has been investigated as a possible source of the magnetic field. Harrison [27] has suggested that if one assumes the existence of vorticity in the pre-recombination era, then the differences in the Thomson cross-sections of the electrons and protons can give rise to a seed magnetic field. However, If vorticity was not present soon after the big bang, and if the assumptions of the Helmholtz–Kelvin circulation theorem hold, significant vorticity cannot emerge during subsequent evolution [5]. Rebhan [28] has studied conditions under which the Helmholtz–Kelvin theorem can be circumvented. The constraints on vorticity in the early universe come from CMB anisotropy limits, through which Rebhan has also derived the upper bound on the strength of the current magnetic field. It has also been argued that the non-linear evolution of primordial density perturbations can generate a vorticity of ions and electrons at the second order of perturbation [29–32]. Siegel and Fry [33] have argued that cosmological perturbations in the pre-recombination era produced charge separations and currents that produced seed magnetic fields.

Vilenkin and Leahy [34] have argued that parity violation in the electroweak theory gives rise to parity-violating currents, which can seed a primordial magnetic field of sufficient strength. Brizard et al. [35] have argued that in the presence of vorticity, plasma, and neutrino–antineutrino asymmetry, the neutrino–plasma interactions can generate magnetic fields.

Inflation provides a kinematic means for exponentially increasing the coherence lengths of the magnetic fields that are produced either before or during inflation. However, if the universe is conformally flat—as FLRW spacetimes are—then, due to the conformal invariance of electromagnetism, the magnetic field decays too fast (as $1/a^2$) with expansion to allow meaningfully large magnetic fields to exist at later times. Therefore, previous researchers have explored mechanisms to break the conformal invariance of electromagnetism.

Turner and Widrow [36] considered different models in which conformal invariance is broken; specifically, they did this by coupling the electromagnetic field to gravity, or a scalar field that is not conformally invariant, or to axions. Since the scale factor grows exponentially during inflation, the strengths of the generated magnetic fields are sensitive to the model parameters [7]. For example, Ratra [37] proposed a model in which the electromagnetic field is coupled to the inflaton (scalar) field; the parameters in the model enable the production of magnetic fields from $\lesssim 10^{-65}$ G to $\gtrsim 10^{-10}$ G. Dolgov [38] has considered the breaking of conformal invariance due to the trace anomaly in curved spacetime.

As discussed in [7,39–42], inflationary magnetogenesis has to address two problems: the back reaction problem and the strong coupling problem. If the energy density of the produced electromagnetic field is too high—as would be the case for some values of parameters—then the electromagnetic field has a nontrivial back reaction on the background. Demozzi et al. [42] argue that if the back reaction is assumed to be not strong enough to subvert inflation, then the produced magnetic field would be too weak. For more information, see [37,38,43–47].

The second problem in inflationary magnetogenesis—called the strong coupling problem—is that, in order to obtain sufficient magnetic energy density at the present time, the renormalized charge in the early stages of inflation may need to be too large, putting the theory in the strong coupling regime, where perturbative analysis of electromagnetism does not hold. Models that seek to circumvent the strong coupling problem have been proposed by several authors [48–50], including a proposal to use a higher-dimensional scale factor [51].

Campanelli [52] has shown that the expected value of the 2-point function of the magnetic field in de Sitter spacetime is scale-invariant, without explicitly breaking conformal invariance. Also, see [53,54] for discussion of the physical relevance of Campanelli's result. Gasperini et al. [47] have considered primordial magnetogenesis in string cosmology due to the coupling of the electromagnetic field to a dilaton background.

In addition to models that generate non-helical magnetic fields, models that produce helical magnetic fields have also been proposed. For example, coupling the $F_{\mu\nu}\tilde{F}^{\mu\nu}$ term to a pseudo-scalar field, such as an axion, produces a helical magnetic field [55]. Despite the possible amplification of helical magnetic fields by an inverse cascade [56], Durrer et al. [55] concluded that the helical magnetic fields cannot produce the observed field strength over the observed correlation lengths [57].

In addition to inflationary magnetogenesis and the generation of magnetic fields by primordial vorticity, the phase transitions in the early universe—particularly the electroweak phase transition (EWPT) and the QCD phase transition (QCDPT)—have also been investigated as possible settings for magnetogenesis. Although experimental constraints have not definitively ruled out the possibility, it is believed that the EWPT and QCDPT were not first-order transitions [58]. If the transitions were of first order, then bubbles of new phases formed and grew in a sea of old phases. Baym et al. [59] argue that magnetic fields of sufficient strength can be generated in a first-order EWPT by shock waves caused by the expansion of bubbles of the new phase. Brandenburg and Subramanian [60] argue that the turbulence caused by the collision of bubbles could amplify magnetic fields due to dynamo action. Vachaspati [61] proposed that the gradients in the Higgs field—arising from bubbles of the new phase having different vacuum expectation values—could give rise to magnetic fields with strengths of the order of $10^{23}$ G. The mechanism proposed by Vachaspati has been further analyzed by Davidson [62] and by Grasso and Riotto [63]. The conclusions of Vachaspati [61] and Grasso and Riotto [63] were numerically confirmed by Diaz-Gil et al. [64,65]. Numerical simulations of the mechanism suggested by Vachaspati have also been reported in [66–68]. For suggested links between magnetogenesis and baryogenesis during the EWPT, as well as numerical tests of the suggestion, see [66,69].

Similarly, if the QCDPT was of first order, the expanding bubbles of hadronic phase would have released latent heat in the form of shock waves into the quark–gluon plasma [70]. The difference in the responses of the positive and negative charges to the shock waves, Quashnock et al. [70] argued, could produce magnetic field of the order of 5 G. Cheng and Olinto [71] argue that the currents produced at the interface between the hadronic phase and the quark–gluon plasma can produce magnetic fields of up to $10^8$ G.

Boyarsky et al. [72] have suggested that the parity violation in the weak force and the chiral anomaly generate a long-range magnetic field in the ground state of the Standard Model. Boyarsky et al. [73] have argued that the coherence scale of the abovementioned magnetic field could increase due to the so-called $\alpha$-effect. For other work on helicity generation stemming from parity violation in the weak force, see [74–77].

Finally, it has been proposed that the CMF could also be seeded in the post-recombination era by outflows from magnetized galaxies into the intergalactic void [2,78–81]. For reviews of previous work on primordial magnetogenesis, see [5,7,58,82–85].

### 2.2. Rotation of Observable Matter

In 1946, Gamow called the “axial rotation” of “all successive degrees of accumulation of matter”, “One of the most mysterious results of the astronomical studies of the universe” [8]. He then sought to expand the scope of the above “mystery” by asking if, in addition to the observed rotations up to the scale of galaxies, “whether it is not possible to assume that *all matter in the visible universe is in a state of general rotation around some center located far beyond the reach of our telescopes?*” Gödel, who discovered the rotating universe solution to Einstein’s equations a few years later [86], referred to Gamow’s observation in his lecture in 1949 [87].

Much work has been done both to follow up on Gamow’s hypothesis *that all matter is rotating about some cosmic center*, as well as on vorticity in the universe in general. For example, Li observed that Gamow’s hypothesis, if valid, can explain galactic rotations as the effect of Coriolis force on radially collapsing matter in the course of galaxy formation [88]. Li showed that Gamow’s hypothesis can also explain the empirically observed relation [89–92] between the angular momentum $J$ and mass $M$ of galaxies

$$J \propto M^{5/3}.$$

Whereas Gamow hypothesized that all visible matter may be in a state of rotation, Hawking [93] and Hawking and Collins [94] considered smaller scales of about 100 Mpc and derived limits on vorticity from the CMB data in various spatially homogeneous models of the universe. Other upper bounds on vorticity have been derived in [95–101].

Gamow's hypothesis and the follow-up work on it focus on the length scales up to which visible matter may be rotating. On the other hand, the aforementioned bounds on vorticity focus on the rate of rotation of visible matter. Neither line of inquiry, however, addresses the "mystery" of *why* all visible matter is in a state of axial rotation. In other words, a fundamental question is the following: *What, possibly primordial, mechanism is responsible for injecting nonzero angular momentum into all accumulations of matter—at least up to the length scales of galaxies?*

As mentioned above, Li [88] argues that Gamow's global rotation would explain the origin of the rotation of galaxies. One concern with Li's explanation is that it would imply anisotropy in the axes of rotation of galaxies; the existence and extent of such anisotropy are still being debated [102–104]. Hoyle [105] suggested that the tidal torques from neighboring proto-accumulations of matter could be responsible for seeding rotations in proto-galaxies. Hoyle's suggestion was investigated further by Peebles [106], von Weizsacker [107], and Gamow [6], who, on the other hand, argued that the rotations of galaxies could be seeded by primordial turbulence, although Gamow notes that "...it is, however, difficult to see how such a (turbulent) motion could originate in a uniformly expanding homogeneous material." Finally, Chernin [108] and Doroshkevich [109] have suggested that vortices in primordial matter—arising from shock waves generated by gravitational instability—could be a possible explanation for the origin of the rotations in galaxies.

## 3. Killing Electromagnetic Field of Flat FLRW Spacetime

Observational evidence strongly suggests that the universe is described by the flat FLRW metric [19]. The metric of flat FLRW spacetime can be written as

$$ds^2 \;=\; c^2dt^2 - a(t)^2(dx^2 + dy^2 + dz^2) \tag{1}$$

where $a(t)$ is the scale parameter, and $x, y, z$ are the comoving spatial coordinates. The homogeneity of a spatial hypersurface allows one to choose the spatial origin to be at an arbitrary point on the hypersurface.

It is easily verified that

$$\left(K^{(1)}\right)^\mu \;=\; \left(\frac{1}{l_f}\right)(0,0,-z,y),\; \left(K^{(2)}\right)^\mu \;=\; \left(\frac{1}{l_f}\right)(0,z,0,-x),\; \left(K^{(3)}\right)^\mu \;=\; \left(\frac{1}{l_f}\right)(0,-y,x,0), \tag{2}$$

are contravariant Killing vectors of the flat FLRW metric (1). The constant[2] $l_f$ has the dimension of length and makes the covariant Killing vectors $(K^{(i)})^\mu$ dimensionless.

The *electromagnetic field strength tensor* (EFST) corresponding to the vector $K^{(i)}, i = 1, 2, 3$ is defined as

$$F_{\mu\nu}\left[K^{(i)}\right] \;:=\; \zeta\left(K^{(i)}_{\nu;\mu} - K^{(i)}_{\mu;\nu}\right), \qquad \zeta := B_f l_f \tag{3}$$

where $B_f$, a constant[3], has the dimension of magnetic field strength to ensure that the EFST $F_{\mu\nu}$ has the required dimension of magnetic field strength as well. Since $K^{(i)}$ is a Killing vector field, $K^{(i)}_{\mu;\nu} = -K^{(i)}_{\nu;\mu}$, and thus,

$$F_{\mu\nu}\left[K^{(i)}\right] \;=\; 2\,\zeta\,K^{(i)}_{\nu;\mu} \tag{4}$$

Papapetrou [9] showed that if $K$ is a Killing vector field, then $F_{\mu\nu}[K]$ satisfies the Maxwell's equations in a vacuum. See Appendix A.

Since every linear combination of $K^{(1)}, K^{(2)}$, and $K^{(3)}$, shown in (2), is also a Killing vector, this article considers the symmetric linear combination of $K^{(1)}, K^{(2)}$, and $K^{(3)}$, namely,

$$K(\beta) \;=\; \left(\frac{\beta}{l_f}\right)(K^{(1)} + K^{(2)} + K^{(3)}) \;=\; \left(\frac{\beta}{l_f}\right)(0, z-y, x-z, y-x)$$

where $\beta$ is a free parameter that will be constrained later. A straightforward calculation, using the nonzero Christoffel symbols of the flat FLRW metric[4] shows that the nonzero components of the EFST $F_{\mu\nu}[\,K(\beta)\,]$ are

$$\begin{aligned} F_{0i}[\,K(\beta)\,] &= -F_{i0}[\,K(\beta)\,] = -\left(\frac{2\zeta\, a^2\, H}{c}\right) K^i(\beta), \quad i = 1,2,3, \quad H := \frac{\dot{a}}{a} \\ F_{12} &= -F_{21} = F_{23} = -F_{32} = F_{31} = -F_{13} = -2B_f\,\beta\, a^2 \end{aligned} \tag{6}$$

The standard observer,[5] in flat FLRW spacetime, has 4-velocity $u^\mu = (1,0,0,0)$ in comoving coordinates. The magnetic field 4-vector seen by the observer is given by [7,110,111]

$$B^\mu_{(K)} \;=\; \frac{1}{2}\frac{\mathcal{A}^{\mu\nu\alpha\beta}}{\sqrt{-g}} u_\nu F_{\alpha\beta} \;=\; \frac{2\,\beta\, B_f}{a(t)}\,(0,1,1,1) \tag{7}$$

where $\mathcal{A}^{\mu\nu\alpha\beta}$ is the totally antisymmetric symbol, with $\mathcal{A}^{0123} = 1$. The subscript $_{(K)}$ denotes that the field arises out of the Killing symmetry of the spacetime. Since $B^\mu_{(K)}$, a 4-vector, is nonzero in the comoving coordinates, it is nonzero for every observer, and hence, every observer sees a nonzero magnetic field arising out of the Killing vectors in the flat FLRW spacetime.

The projection tensor

$$h_{\mu\nu} \;:=\; g_{\mu\nu} - u_\mu u_\nu \tag{8}$$

can be used to project any tensor onto the spatial hypersurface $S$, whose tangent plane, at every point on $S$, is orthogonal to a given unit time-like (congruence) vector field $u^\mu$.

Projecting $B^\mu_{(K)}$ onto $\mathcal{S}$, the spatial hypersurface orthogonal to the 4-velocity of the standard observer, one obtains $B^\mu_{(p)}$.

$$B^\mu_{(p)} \;=\; h^\mu{}_\nu B^\nu_{(K)} \;=\; (\delta^\mu{}_\nu - u^\mu u_\nu) B^\nu_{(K)} \;=\; B^\mu_{(K)}$$

confirming the obvious fact that $B^\mu_{(K)}$ lies in $\mathcal{S}$. Thus, in comoving coordinates, the standard observer sees a homogeneous magnetic field

$$\vec{B}_{(K)} \;=\; \frac{2\,\beta\, B_f}{a(t)}\,(1,1,1) \tag{9}$$

that is confined to the spatial hypersurface orthogonal to his world line. The magnetic field in (9) decreases with $a(t)$ in comoving coordinates. Assuming that the observations of Pomakov et al. [112] (Table 6) were made in comoving coordinates, (9) is consistent with their results, which show an increase in the strength of the magnetic field with redshift in the primordial magnetogenesis model. The invariant scalar of the magnetic field in (9) is $B^\mu_{(K)}(B_{(K)})_\mu = -12\beta^2 B_f^2$, which is nonzero if $\beta \neq 0$.

The magnetic field $B^\mu_{(K)}$—generated by Killing symmetries—is of primordial origin, coming into existence at the birth of the flat FLRW spacetime itself. As mentioned in Section 2.1, Thorne conjectured that the CMF ought to have emerged at the big bang, but did not provide a mechanism by which a magnetic field arises at the big bang. The above discussion strengthens Thorne's conjecture by showing that the birth of the CMF at the big bang does not have to be postulated but can be derived as a consequence of the spacetime's Killing symmetries (see Note 1).

The electric field 4-vector seen by the standard observer is given by [7,110,111]

$$E^{\mu}_{(K)} \;=\; F^{\mu\nu}u_{\nu} \;=\; -\frac{2B_f \beta H}{c}(0, z-y, x-z, y-x) \tag{10}$$

Projecting $E^{\mu}_{(K)}$ to $\mathcal{S}$, one obtains $E^{\mu}_{(p)}$.

$$E^{\mu}_{(p)} \;=\; h^{\mu}{}_{\nu}E^{\nu}_{(K)} \;=\; (\delta^{\mu}{}_{\nu} - u^{\mu}u_{\nu})E^{\nu} \;=\; E^{\mu}_{(K)} \tag{11}$$

again confirming the obvious fact that $E^{\mu}_{(K)}$ lies in $\mathcal{S}$. Similarly, the projection of the covariant derivative of the electric field $\nabla_{\mu}\left(E_{(K)}\right)_{\nu}$ onto $\mathcal{S}$ yields

$$\nabla^{(p)}_{\mu}\left(E_{(K)}\right)_{\nu} \;=\; h_{\mu}{}^{\alpha}h_{\nu}{}^{\beta}\nabla_{\alpha}\left(E_{(K)}\right)_{\beta}$$

Since the electric field resides in the spatial hypersurface orthogonal to the 4-velocity of the standard observer, one can covariantly construct the 'curl'[6] of the electric field in $\mathcal{S}$, denoted $\omega^{\mu}$, as [113]

$$\omega^{\mu} \;:=\; \left(\frac{\mathcal{A}^{\mu\nu\alpha\beta}}{\sqrt{-g}}\right)u_{\nu}\nabla^{(p)}_{\alpha}\left(E_{(K)}\right)_{\beta} \;=\; \left(\frac{\mathcal{A}^{\mu\nu\alpha\beta}}{\sqrt{-g}}\right)u_{\nu}h_{\alpha}{}^{\rho}h_{\beta}{}^{\sigma}\nabla_{\rho}\left(E_{(K)}\right)_{\sigma} \tag{12}$$

where $A^{\mu\nu\alpha\beta}$ is the totally antisymmetric symbol, with $\mathcal{A}^{0123} = 1$. It is easy to see that in a locally inertial frame, $\omega^{\mu} = (0, \nabla \times \vec{E}_{(K)})$; that is, $\omega$ effectively reduces to the three-dimensional curl.

Using (10) and (12), one can calculate $\omega^{\mu}$ to obtain

$$\omega^{\mu} \;=\; -\left(\frac{4B_f \beta}{c}\right)\left[\frac{H}{a(t)}\right](0, 1, 1, 1) \tag{13}$$

Since the 4-vector $\omega^{\mu} \neq 0$ is in comoving coordinates, every observer sees a nonzero $\omega^{\mu}$. In particular, the standard observer sees a homogeneous nonzero curl of the electric field (10) at every point on the spatial hypersurface $\mathcal{S}$.

Like the Killing magnetic field $B^{\mu}_{(K)}$, the rotational electric field $E_{(K)}$—generated by Killing symmetries—is also of primordial origin, coming into existence at the birth of the flat FLRW spacetime. The nonzero curl of the primordial $E_{(K)}$ would have induced pervasive vorticity in the charged matter that existed in the pre-recombination early universe, raising the following speculative question: *Does the vorticity that the rotational electric field $E_{(K)}$ induces in charged matter in the early universe provide a viable mechanism for seeding the "axial rotation" that one observes today in "all successive degrees of accumulation of matter, such as planets, stars, and galaxies"* [8]? See Section 2.2 for a discussion of the background for the above question.

Secondly, $\omega^{\mu}$, the curl of the $E_{(K)}$ is still nonzero today. Hence, it is reasonable to ask the following question: *Can the Killing electric field $E_{(K)}$ be detected in the laboratory today?* In Section 5.3, it is argued that, if $E_{(K)}$ indeed exists, then its detection may be within the current experimental reach. The detectability of $E_{(K)}$ depends on the magnitude of the parameter $\beta$, for which an upper bound is derived in the following discussion.

The homogeneous magnetic field $B_{(K)}$, seen by the standard observer, is at odds with the underlying assumption of isotropy of the flat FLRW spacetime. Therefore, the above derivations would be self-consistent only if the Killing electromagnetic field has a negligible back reaction on the flat FLRW metric. The self-consistency constraint enables us to derive an upper bound on the strength of the Killing magnetic field, or in other words, an upper bound on $\beta$.

The Hilbert stress–energy tensor for the electromagnetic field arising from the Killing field $K(\beta)$, shown in (6), is, in comoving coordinates $(t, x, y, z)$,

$$\left(T_{(K)}\right)_{\mu\nu}(t,0,0,0) = \frac{1}{2\mu_0}\left[-F_{\mu\alpha}F_{\nu}{}^{\alpha} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right]\Big|_{(t,0,0,0)} = \frac{\beta^2 B_f^2}{\mu_0}\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & a^2 & -2a^2 & -2a^2 \\ 0 & -2a^2 & a^2 & -2a^2 \\ 0 & -2a^2 & -2a^2 & a^2 \end{bmatrix} \tag{14}$$

On the other hand, the nonzero components of the Einstein tensor for the flat FLRW metric, in comoving coordinates $(t, x, y, z)$, are

$$G_{00} = \frac{3H^2}{c^2}, \quad G_{11} = G_{22} = G_{33} = \frac{2a\ddot{a} + \dot{a}^2}{c^2} \tag{15}$$

Assuming that the universe is populated by matter whose energy–momentum tensor, $T^{\mu\nu}_{(m)}$, is proportional to the Einstein tensor, shown in (15), and that Einstein's equation is satisfied in the absence of the KEF, one concludes that

$$T^{\mu\nu}_{(m)} = \left(\frac{c^4}{8\pi G}\right)G^{\mu\nu} \tag{16}$$

The $T^{\mu\nu}_{(K)}$ arising from the KEF is regarded as a perturbation of $T^{\mu\nu}_{(m)}$. Then,

$$T^{\mu\nu}_{(total)} = T^{\mu\nu}_{(m)} + T^{\mu\nu}_{(K)}$$

$T^{\mu\nu}_{(K)}$ would be negligibly smaller than $T^{\mu\nu}_{(m)}$, and hence, $T^{\mu\nu}_{(total)} \approx T^{\mu\nu}_{(m)}$ if

$$\beta^2 \ll \left[\frac{\mu_0 c^2}{8\pi G B_f^2}\right](\ddot{a}/a + H^2/2)$$

The value of the deceleration parameter today is [114]

$$q_0 = -\frac{\ddot{a}}{aH^2} = -0.55$$

Hence, the bound on $\beta$, derived from the self-consistency requirement, is

$$\beta \ll \sqrt{\frac{\mu_0 c^2 H^2}{8\pi G B_f^2}} := \beta_0 = 4.98 \times 10^{-25} \tag{17}$$

The upper bound on the right in (17) is denoted as $\beta_0$ for later convenience. Taking the current value of $a = 1$, the magnetic field arising out of the Killing vector $K(\beta)$, given by (9), is thus constrained by the self-consistency condition to satisfy[7]

$$|\vec{B}| = 2\sqrt{3}\,\beta\,B_f \ll 2\sqrt{3}\,\beta_0\,B_f = 2\sqrt{3}\left[\frac{\mu_0\,c^2\,H^2}{8\pi G}\right]^{1/2} \sim 6.21 \times 10^{-4}\,G \tag{18}$$

which is consistent with the bounds on the CMF mentioned in Section 2.1. In particular, if the Killing magnetic field, shown in (9), is the CMF, denoted as $B_{CMF}$, then the recent upper bound of $\sim 10^{-10}$ G on $B_{CMF}$, derived from the ultrahigh-energy cosmic ray hotspot in the Perseus–Pisces region [21], yields the following upper bound on $\beta$

$$2\sqrt{3}\,\beta\,B_f \sim |B_{CMF}| \quad \lesssim \quad 10^{-10}\,G \quad \Longrightarrow \quad \beta \lesssim 8\times 10^{-32} \sim 10^{-7}\,\beta_0 \tag{19}$$

which is not inconsistent with the self-consistency condition $\beta \ll \beta_0$ shown in (17).

Using the lower bounds on the CMF, deduced by Neronov and Vovk [22], Tavecchio et al. [24], and Dolag et al. [1], one can also obtain a lower bound on $\beta$ under the assumption that the Killing magnetic field of the flat FLRW spacetime is the CMF that is observed today.

$$\left|2\sqrt{3}\,\beta B_f\right| \sim |B_{CMF}| \geq 3\times 10^{-16}\,G \quad \text{or} \quad \beta \quad \gtrsim \quad 2.4\times 10^{-37} \tag{20}$$

From (19) and (20), under the assumption that the Killing magnetic field of the flat FLRW spacetime is the CMF, one can conclude that

$$-36.61 \quad \lesssim \quad \log_{10}(\beta) \quad \lesssim \quad -31.10 \tag{21}$$

The back reaction of the KEF on the flat FLRW metric was not considered in the above analysis, in which, for self-consistency, the KEF was constrained to make a negligibly small contribution to the overall stress–energy tensor. A more rigorous derivation would involve extremizing the following action:

$$S = \int d^4x\sqrt{-g}\left[\left(\tfrac{c^3}{16\pi G}\right)\mathcal{R} - \frac{1}{4\mu_0}\left\{F^{\mu\nu}[\sum_a \beta^{(a)}K^{(a)}(g_{\mu\nu},g_{\mu\nu,\alpha})]F_{\mu\nu}[\sum_a \beta^{(a)}K^{(a)}(g_{\mu\nu},g_{\mu\nu,\alpha})]\right\} + \mathcal{L}_m\right] \tag{22}$$

The metric $g_{\mu\nu}$ that appears above depends on both the electromagnetic fields arising from the Killing symmetries of $g_{\mu\nu}$ and the distribution of other matter in the universe described by $\mathcal{L}_m$. The Killing vectors, in turn, are determined by the metric and its derivatives. The superscripts on the Killing vectors $K^{(a)}$ label the different Killing vectors determined by the metric $g_{\mu\nu}$ and its derivatives $g_{\mu\nu,\alpha}$. Each Killing vector $K^{(a)}$ is scaled by a dimensionless constant $\beta^{(a)}$; the values of the $\beta^{(a)}$s are determined by minimizing the action $\mathcal{S}$. Since $\beta^{(a)}$s determine the strengths of the KEFs, the metric $g_{\mu\nu}$ depends on $\beta^{(a)}$s as well. Minimizing the above action, to simultaneously derive both the metric of spacetime as well as the KEFs of that metric, is a formidable problem.

If the KEF postulate is indeed valid, then the emergence of a magnetic field and an electric field with nonzero curl—each of which introduces a preferred direction—shows that a purely flat FLRW spacetime is untenable. The CMF with large correlation length that has been observed in the universe demonstrates a weak deviation from the isotropic flat FLRW spacetime, at least at the correlation length scale of the CMF.

The degeneracy in the choice of the basis vectors in the space-like hypersurface $\mathcal{S}$ implies degeneracy of the Killing magnetic fields that can arise in the flat FLRW spacetime, with the degeneracy possibly broken either by metric fluctuations or by the stochastic deviations from isotropy of the other electrically charged matter and dipoles in the universe.

## 4. Killing Electromagnetic Field of Schwarzschild Spacetime

As is well known [10], the KEF, $F_{\mu\nu}[K]$, generated by the time-like Killing vector, $K$, of the Kerr metric is proportional to the electromagnetic field strength tensor $F^{(KN)}_{\mu\nu}$ of the Kerr–Newman metric. If the angular momentum of the rotating mass $J \to 0$, then the Kerr (resp. Kerr–Newman) metric reduces to the Schwarzschild (resp. Reissner–Nordström) metric. Thus, it is also well known that the Killing electromagnetic field, generated by the time-like Killing vector of the Schwarzschild metric, is proportional to the electromagnetic field of the Reissner–Nordström metric.

The discussion below considers the following converse question about uniqueness: *Does the back reaction of the KEF of the time-like Killing vector of the Schwarzschild spacetime change the Schwarzschild spacetime to a Reissner–Nordström spacetime?* It will be shown that, far away from the Schwarzschild mass distribution, the back reaction of a sufficiently weak KEF does indeed change the Schwarzschild metric to the Reissner–Nordström metric.

Since the Schwarzschild metric of a mass distribution with mass $M$, defined as

$$ds^2 \;=\; c^2dt^2\chi(r) - \frac{dr^2}{\chi(r)} - r^2\Big(d\theta^2 + \sin^2(\theta)d\varphi^2\Big), \qquad \chi(r) := \left(1 - \frac{2GM}{c^2 r}\right), \tag{23}$$

does not depend on $t$, $(K^{(t)})^\mu(\beta) \;=\; \beta(1,0,0,0)$ is a Killing vector of the spacetime, where $\beta$ is a dimensionless parameter. The EFST of $K^{(t)}(\beta)$ is

$$F_{\mu\nu}\Big[K^{(t)}(\beta)\Big] \;=\; 2\,\zeta\, K^{(t)}_{\nu;\mu}, \qquad \zeta := \frac{ec}{4\pi\epsilon_0 GM}, \tag{24}$$

For brevity, in the rest of the section, the dependence on $K^{(t)}$ and $\beta$ is omitted, and the EFST is written as just $F_{\mu\nu}$ or $F^{\mu\nu}$. The nonzero components of the EFST are

$$F_{01} \;=\; -F_{10} \;=\; \beta\left(\frac{\kappa}{r^2}\right), \quad \kappa := -\left[\frac{2\,\zeta\, G\, M}{c^2}\right] \tag{25}$$

The nonzero components of $F^{\mu\nu} \;=\; g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}$ are

$$F^{01} \;=\; -F^{10} \;=\; -\beta\left(\frac{\kappa}{r^2}\right) \tag{26}$$

$F^{\mu\nu}$, shown in (26), represents a radial electric field and does not contain a magnetic field. Both the Schwarzschild metric as well as the KEF (26) are *static* and *spherically symmetric*, suggesting the following ansatz for the modified metric:

$$ds^2 \;=\; e^{p(r)}c^2dt^2 - e^{q(r)}dr^2 - r^2(d\theta^2 + \sin^2(\theta)d\varphi^2) \tag{27}$$

with the constraint[8] that as $\beta \to 0$, one recovers the Schwarzschild metric. That is,

$$\lim_{\beta\to 0} e^{p(r)} \;=\; \chi(r), \qquad \lim_{\beta\to 0} e^{q(r)} \;=\; \chi^{-1}(r) \tag{28}$$

Secondly, the EFST, shown in (25), vanishes at spatial infinity. Therefore, when the KEF is turned on in Schwarzschild spacetime, the modified metric—as a result of the back reaction due to the KEF—will remain asymptotically flat and regular at spatial infinity, even for nonzero $\beta$. Thus, outside the Schwarzschild radius, $e^{p(r)}$ can be expanded as

$$e^{p(r)} \;=\; \chi(r) + \beta\left[\frac{a_k}{r^k} + \frac{a_{k+1}}{r^{k+1}} + \dots\right] \qquad k \geq 1 \tag{29}$$

where $a_k$ is the first nonzero coefficient in the series[9].

Since the metric in (27) is also independent of $t$, $Q^\mu = \beta(1,0,0,0)$ is a Killing vector of the metric. The KEF associated with $Q^\mu$ is

$$F_{\mu\nu}[Q] \;=\; 2\,\zeta\, Q_{\nu;\mu} \tag{30}$$

The nonzero components of $F_{\mu\nu}[Q]$ are

$$F_{01} \;=\; -F_{10} \;=\; -\beta\,\zeta\, e^{p(r)}\, p'(r) \tag{31}$$

Differentiating (29), one obtains

$$e^{p(r)}\,p'(r) \;=\; \left(\frac{2GM}{c^2}\right)\frac{1}{r^2} - \frac{\beta}{r^{k+1}}\left[k\,a_k + (k+1)\left(\frac{a_{k+1}}{r}\right) + \dots\right]$$
$$\;=\; \left(\frac{2GM}{c^2}\right)\frac{1}{r^2} - \frac{\beta}{r^{k+1}}\left[k\,a_k + h(r)\right], \qquad k \geq 1 \tag{32}$$

where

$$h(r) \;:=\; (k+1)\left(\frac{a_{k+1}}{r}\right) + \dots$$

Since $h(r) \to 0$ as $r \to \infty$, there exists an $r_0$ such that for $r > r_0$, $|h(r)| < |a_k|$. Then, for $r > r_0$,

$$|k\,a_k + h(r)| \;\leq\; k|a_k| + |h(r)| \;\leq\; (k+1)|a_k|$$

For $r > r_0$, and in the weak-field regime, defined as,

$$\beta \;\ll\; \frac{2GM}{c^2(k+1)|a_k|}r_0^{k-1}, \qquad k \geq 1 \tag{33}$$

one can conclude that

$$\left|\frac{\beta}{r^{k+1}}(k\,a_k + h(r))\right| \;\leq\; \frac{\beta}{r^2\,r_0^{k-1}}(k+1)|a_k| \;\ll\; \left(\frac{2GM}{c^2}\right)\frac{1}{r^2}$$

That is, in the weak-field regime, and for $r > r_0$, the $O(\beta)$ term in (32) is negligibly smaller than the leading term and hence can be omitted. The nonzero components of the EFST (30) are then

$$F_{01} \;=\; -F_{10} \;=\; -\beta\left(\frac{A}{r^2}\right), \qquad A := r_s\zeta, \quad r_s := \frac{2GM}{c^2} \tag{34}$$

Two of the nonzero components of the Ricci tensor of the modified metric (27) are

$$R_{00} \;=\; \frac{e^{p-q}}{2}\left[p'' + \frac{(p')^2}{2} + 2\left(\frac{p'}{r}\right) - \frac{p'q'}{2}\right], \quad R_{11} \;=\; -\frac{p''}{2} - \frac{(p')^2}{4} + \frac{p'q'}{4} + \frac{q'}{r} \tag{35}$$

where $p'$ and $q'$ denote derivatives of $p$ and $q$ with respect to $r$.

The corresponding components of the stress–energy tensor of the EFST (30)

$$T_{\mu\nu} \;=\; \frac{1}{2\mu_0}\left[-F_{\mu\alpha}F_{\nu}{}^{\alpha} + \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right] \tag{36}$$

are

$$T_{00} \;=\; \eta e^{2p-q}, \quad T_{11} \;=\; -\eta e^{p}, \qquad \eta := \frac{(\beta\zeta p')^2}{4\mu_0} \tag{37}$$

From (37), it follows that

$$\frac{T_{00}}{e^{p(r)}} + \frac{T_{11}}{e^{q(r)}} \;=\; 0 \tag{38}$$

Outside the compact spherical mass $M$ that gives rise to the Schwarzschild metric (23), Einstein's equation is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \;=\; \left( \frac{8\pi G}{c^4} \right) T_{\mu\nu} \tag{39}$$

where $R$ is the Ricci scalar, and $T_{\mu\nu}$ is the stress–energy tensor of the KEF shown in (36). Since $T_{\mu\nu}$ is traceless[10], the Ricci scalar vanishes[11], and Einstein's equation becomes

$$R_{\mu\nu} \;=\; \left( \frac{8\pi G}{c^4} \right) T_{\mu\nu} \tag{40}$$

From (38) and (40), one concludes that

$$\frac{R_{00}}{e^{p(r)}} + \frac{R_{11}}{e^{q(r)}} \;=\; 0$$

or using (35), one obtains the following equation at all finite $r$ greater than the radius of the spherical mass $M$

$$p'(r) + q'(r)) \;=\; 0$$

That is to say, $p(r) + q(r)$ is independent of $r$. The objective is to compute the modified metric when the EFST, shown in (25), is turned on in Schwarzschild spacetime. Since the EFST vanishes as $1/r^2$ as $r \to \infty$, the spacetime inhabited by the KEF and the compact spherical mass $M$ is asymptotically flat or equivalently

$$\lim_{r\to\infty} p(r) \;=\; \lim_{r\to\infty} q(r) \;=\; 0$$

from which it follows that

$$p(r) + q(r) \equiv 0 \implies p(r) \;=\; -q(r) \tag{41}$$

Using (41), the remaining nonzero components of the Ricci tensor are

$$R_{22} \;=\; 1 - \left( r\, e^{p(r)} \right)' \qquad R_{33} \;=\; \sin^2(\theta) \left[ 1 - \left( r\, e^{p(r)} \right)' \right] \tag{42}$$

In the weak-field regime and for $r > r_0$, $T_{22}$ is given by

$$T_{22} \;=\; \frac{\kappa}{r^2} \qquad \kappa := \beta^2 \left( \frac{\zeta^2\, r_s^2}{4\, \mu_0} \right) \tag{43}$$

Using (42) and (43), the Einstein equation $R_{22} \;=\; \left( \frac{8\pi G}{c^4} \right) T_{22}$ becomes

$$1 - \left( r\, e^{p(r)} \right)' \;=\; \frac{\xi}{r^2}, \qquad \xi := \beta^2 \left( \frac{8\pi G}{c^4} \right) \frac{(\zeta r_s)^2}{4\mu_0} \tag{44}$$

The solution of (44) is

$$e^{p(r)} \;=\; 1 + \frac{C}{r} + \frac{\xi}{r^2}$$

where $C$ is an arbitrary constant. The constraint (28) implies that

$$C \;=\; -\frac{2GM}{c^2}$$

Therefore, in the weak-field regime and for $r > r_0$, the modified metric (27) has the form

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} + \frac{\xi}{r^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r} + \frac{\xi}{r^2}\right)} - r^2\left(d\theta^2 + r^2\sin^2(\theta)d\varphi^2\right) \tag{45}$$

which coincides with the Reissner–Nordström metric, with mass $M$ and charge $Q$,

$$ds^2_{RN} = \left(1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{4\pi\epsilon_0 c^4 r^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{4\pi\epsilon_0 c^4 r^2}\right)} - r^2\left(d\theta^2 + r^2\sin^2(\theta)d\varphi^2\right) \tag{46}$$

if

$$Q = \beta\left[\frac{2\sqrt{2}\,\pi\,\zeta\,r_s}{\mu_0 c}\right] \tag{47}$$

In particular, in the weak-field regime an observer, far from a Schwarzschild black hole ($r > r_0$), would conclude that it is actually a Reissner–Nordström black hole containing an electric charge given by (47). Clearly, the above derivation remains valid for any spherical distribution of stationary matter. Thus, every spherical distribution of stationary matter—regardless of the amount of matter—appears electrically charged to a distant observer in the weak-field regime. Finally, it should be noted that, in the above arguments, attention was restricted to the weak-field regime only to make the calculation more tractable. It is worth exploring if the same conclusion can be obtained even when the weak-field assumption is dropped.

Penrose's *weak cosmic censorship hypothesis* yields an upper bound on $\beta$, even if it is a rather weak bound. In the Reissner–Nordström metric (46), if

$$\frac{Q}{M} > \sqrt{4\pi\epsilon_0 G} \tag{48}$$

then $g_{00} > 0$, and the spacetime has no horizon. And yet, $r = 0$ is a singularity—the so-called *naked singularity*, since it would not be hidden inside a horizon when (48) holds. Assuming the validity of Penrose's weak cosmic censorship hypothesis [115], which posits that naked singularities do not exist, one obtains the condition

$$\frac{Q}{M} \leq \sqrt{4\pi\epsilon_0\,G} \tag{49}$$

From (47) and (49), one obtains the following upper bound on $\beta$:

$$\beta \leq \sqrt{4\pi\epsilon_0 G}\left[\frac{\mu_0\,c\,M}{2\sqrt{2}\,\pi\,\zeta\,r_s}\right] \tag{50}$$

## 5. Discussion

The gauge invariance of the KEF is verified in Section 5.1, below. Subsequently, the plausibility of the KEF and KEC postulates is discussed in Section 5.2. It is also argued, in Section 5.3, that falsifying the KEF postulate may be within the current experimental reach. In Section 5.4, drawing upon Rainich's result [17], the etiological question associated with the KEF and KEC postulates is discussed. Section 5.5 discusses the relevance of the aforementioned transmutation of the Schwarzschild metric and the KEC postulate to Misner and Wheeler's program of realizing "charge without charge" [18]. Finally, in Section 5.6, the features of the KEF and KEC that distinguish magnetogenesis sourced by Killing vectors from previous proposals for primordial magnetogenesis are discussed.

### 5.1. Gauge Invariance

Under a gauge transformation of the Killing vector, $K^\mu \to \tilde{K}^\mu = K^\mu + \partial^\mu \Lambda$, $F_{\mu\nu} \to \tilde{F}_{\mu\nu}$, and

$$\tilde{F}_{\mu\nu} = \zeta\left[\tilde{K}_{\nu;\mu} - \tilde{K}_{\mu;\nu}\right] = F_{\mu\nu} + \zeta\left\{\left[\partial_\mu\partial_\nu\Lambda - \Gamma^\lambda{}_{\mu\nu}\partial_\lambda\Lambda\right] - \mu \leftrightarrow \nu\right\} = F_{\mu\nu} \tag{51}$$

Thus, the EFST $F_{\mu\nu}$ is invariant under a general gauge transformation.

One can easily verify that $\tilde{F}_{\mu\nu}$ also satisfies Maxwell's equations. Specifically,

$$\tilde{F}_{\alpha\beta;\mu} + \tilde{F}_{\mu\alpha;\beta} + \tilde{F}_{\beta\mu;\alpha} = 0 \tag{52}$$

$$\tilde{F}^{\mu\nu}{}_{;\nu} = \frac{J^\mu_{(K)}}{\epsilon_0\, c}, \qquad J^\mu_{(K)} = 2\,\epsilon_0\, c\,\zeta\, R^{\mu\nu} K_\nu \tag{53}$$

Equation (52) follows immediately from (51) and the proof of (A3), presented in Appendix A.

$$\tilde{F}_{\alpha\beta;\mu} + \tilde{F}_{\mu\alpha;\beta} + \tilde{F}_{\beta\mu;\alpha} = F_{\alpha\beta;\mu} + F_{\mu\alpha;\beta} + F_{\beta\mu;\alpha} = 0 \tag{54}$$

Equation (53) also follows immediately from (51) and the proof of (A2) presented in Appendix A.

$$\tilde{F}^{\mu\nu}{}_{;\nu} = F^{\mu\nu}{}_{;\nu} = \frac{J^\mu_{(K)}}{\epsilon_0\, c} = 2\,\zeta\, R^{\mu\nu} K_\nu \tag{55}$$

It should be noted that, in (55), the source term, $2\,\zeta\, R^{\mu\nu}\, K_\nu$, is written in terms of the Killing vector $K_\nu$ and not the transformed vector $\tilde{K}_\nu$, although the left-hand side $\tilde{F}^{\mu\nu}{}_{;\nu}$ is written in terms of the transformed tensor. Equation (55) is elaborated further, in the following discussion.

Let $\mathcal{S}$ denote the set of all twice continuously differentiable scalar fields on the spacetime manifold. Define

$$\mathcal{S}_0 := \left\{\Lambda \in \mathcal{S} \mid \nabla_\mu \nabla_\nu \Lambda = 0,\ \mu,\nu = 0,1,2,3\right\} \tag{56}$$

The set of all gauge transforms of a Killing vector $K$ is

$$\mathcal{G}(K) := \left\{\tilde{K}^\mu \mid \tilde{K}^\mu = K^\mu + \partial^\mu\Lambda,\ \Lambda \in \mathcal{S}\right\}$$

Also, define

$$\mathcal{G}_0(K) := \left\{\tilde{K}^\mu \mid \tilde{K}^\mu = K^\mu + \partial^\mu\Lambda,\ \Lambda \in \mathcal{S}_0\right\}, \qquad \mathcal{G}_1(K) := \mathcal{G}(K) \setminus \mathcal{G}_0(K) \tag{57}$$

First, note that for a gauge transform $\tilde{K}^\mu = K^\mu + \partial^\mu\Lambda \in \mathcal{G}(K)$,

$$\tilde{K}_{\nu;\mu} + \tilde{K}_{\mu;\nu} = \left[K_{\nu;\mu} + K_{\mu;\nu}\right] + 2\,\nabla_\mu\nabla_\nu\Lambda = 2\,\nabla_\mu\nabla_\nu\Lambda \tag{58}$$

From (58) and (56), it follows that every vector field in $\mathcal{G}_0(K)$ is a Killing vector field. Conversely, none of the vector fields in $\mathcal{G}_1(K)$ is a Killing vector field. *In general, the gauge transform of a Killing vector is not necessarily a Killing vector.*

A gauge transform $\tilde{K}^\mu \in \mathcal{G}_0(K)$ satisfies the following important property:

$$R^{\mu\nu}\tilde{K}_\nu = R^{\mu\nu} K_\nu \tag{59}$$

where $R^{\mu\nu}$ is the Ricci tensor.

To prove the above property, using (51) and recalling that $\tilde{K}^\mu$ is a Killing vector, one observes that

$$\tilde{F}_{\mu\nu} \;=\; \zeta\,(\tilde{K}_{\nu;\mu} - \tilde{K}_{\mu;\nu}) \;=\; 2\,\zeta\,\tilde{K}_{\nu;\mu} \;=\; F_{\mu\nu} \;=\; 2\,\zeta\,K_{\nu;\mu}, \;\Longrightarrow\; \tilde{K}_{\nu;\mu} \;=\; K_{\nu;\mu} \tag{60}$$

For an arbitrary Killing vector $W^\mu$, using the symmetries of the Riemann tensor, one concludes that

$$R^{\mu\nu}W_\nu \;=\; = g^{\mu\alpha}R^{\lambda}{}_{\nu\lambda\alpha}W^\nu \;=\; g^{\mu\alpha}\left(W^{\lambda}{}_{;\alpha;\lambda} - W^{\lambda}{}_{;\lambda;\alpha}\right) \;=\; g^{\mu\alpha}g^{\lambda\rho}\left(W_{\rho;\alpha;\lambda} + W_{\lambda;\rho;\alpha}\right) \tag{61}$$

Using (54) and (60), it follows that

$$\tilde{K}_{\rho;\alpha;\lambda} + \tilde{K}_{\lambda;\rho;\alpha} \;=\; -\tilde{K}_{\alpha;\lambda;\rho}, \qquad K_{\rho;\alpha;\lambda} + K_{\lambda;\rho;\alpha} \;=\; -K_{\alpha;\lambda;\rho} \tag{62}$$

From (61), (62), and (60), one concludes that

$$R^{\mu\nu}\tilde{K}_\nu \;=\; -g^{\mu\alpha}g^{\lambda\rho}\tilde{K}_{\alpha;\lambda;\rho} \;=\; -g^{\mu\alpha}g^{\lambda\rho}K_{\alpha;\lambda;\rho} \;=\; R^{\mu\nu}K_\nu$$

Finally, consider (55), which states that the gauge transform $\tilde{F}_{\mu\nu}$ obeys Maxwell's equation, in which the current density (right-hand side) is expressed in terms of the untransformed field $K^\mu$ and not the transformed field $\tilde{K}^\mu$. Given a gauge transform $\tilde{K}^\mu$, one needs to specifiy how the source term $2\,\zeta\,R^{\mu\nu}\,K_\nu$ in (55) can be constructed. Note that $\tilde{K}^\mu \;=\; K^\mu + \partial^\mu\Lambda$ for some $\Lambda \in \mathcal{S}$. Let $\Lambda'$ be a solution of the following equation:

$$\nabla_\mu\nabla_\nu\Lambda \;=\; \frac{1}{2}\left[\,\tilde{K}_{\mu;\nu} + \tilde{K}_{\nu;\mu}\,\right] \tag{63}$$

If $\tilde{K}^\mu \in \mathcal{G}_0(K)$, then $\Lambda' \;=\; 0$ is a possible solution of (63). For $\tilde{K}^\mu \in \mathcal{G}_1(K)$, $\Lambda' \neq 0$. Consider

$$Q^\mu \;:=\; \tilde{K}^\mu - \partial^\mu\Lambda' \;=\; K^\mu + \partial^\mu\Lambda'', \qquad \Lambda'' \;=\; \Lambda - \Lambda', \tag{64}$$

Thus, $Q^\mu$ is a gauge transform of $K^\mu$, and from the definition of $\Lambda'$, it follows that $Q^\mu$ is a Killing vector. Therefore,

$$Q_{\nu;\mu} + Q_{\mu;\nu} \;=\; 2\nabla_\mu\nabla_\nu\Lambda'' \;=\; 0$$

showing that $Q^\mu \in \mathcal{G}_0(K)$. Therefore, from (59), one can conclude that $R^{\mu\nu}Q_\nu \;=\; R^{\mu\nu}K_\nu$. In summary, the source term $2\,\zeta\,R^{\mu\nu}K_\nu$ in (55), is constructed, using $\tilde{K}^\mu$, as follows. First, solve (63), and then, using the solution $\Lambda'$, construct $Q^\mu$ as in (64). $R^{\mu\nu}Q_\nu$ yields the required $R^{\mu\nu}K_\nu$.

### *5.2. Plausibility*

The main plausibility argument for the postulates is that they can, if valid, potentially explain two enigmas—the CMF and the axial rotation of all matter. As mentioned in Section 2, Thorne writes [26],"All attempts to understand how galactic magnetic fields could have arisen...face difficulties, which seem...insurmountable. The only way in which the existence of the magnetic fields can be understood...is by assuming that they...were created in the big bang." However, Thorne does not explain *why* magnetic fields should be created in the big bang. Assuming that the flat FLRW spacetime emerged at—or shortly after—the big bang, the KEF postulate, if valid, could potentially explain why Thorne's conjecture could be true. The second argument that makes KEF an appealing candidate for the CMF is that the primordial KEF does not depend on any model of particle physics in the early universe. Finally, the CMF is ubiquitous; a natural source for the CMF should pervade the entire intergalactic region, as the KEF does.

The sourcing mechanism for the pervasive vorticity—observed at all length scales—remains unknown. If the KEF postulate is valid, and the early universe was a flat FLRW spacetime, then the rotational KEF in flat FLRW spacetime could potentially provide a natural and ubiquitous sourcing mechanism for vorticity.

A curious plausibility argument for the KEC, $J^{\mu}_{(K)}$, stems from the observation that all known electrically charged particles have nonzero rest mass. For some—as yet unknown—reason, the existence of massless electrically charged particles seems to be forbidden. $J^{\mu}_{(K)}$ is nonzero—and gives rise to a nonzero charge density—only where the Ricci tensor is nonzero, and hence, the stress–energy tensor is nonzero[12]. Thus, the KEC $J^{\mu}_{(K)}$ stipulates that the presence of nonzero charge density at a location is necessarily accompanied by a non-vanishing gravitational source at the location; such a stipulation is consistent with the observed nonzero rest mass of all electrically charged particles.

### *5.3. Falsifiability*

If the Killing electric field of the flat FLRW spacetime is indeed real, then it accelerates a point charge, such as an electron[13]. Using (10), and taking $\beta \sim 10^{-32}$, the acceleration $a$ of an electron, induced by the Killing electric field, is

$$a \quad \sim \quad \frac{e\,\beta B_f H}{m_e c} \sim 10^{-31}\,\mathrm{m}\cdot\mathrm{s}^{-2}$$

Over a year's time, the acceleration induces a displacement $d$ that is approximately

$$d \quad \sim \quad 10^{-16}\,\mathrm{m}. \tag{65}$$

The existing state-of-the-art laser interferometers [116] are able to detect displacements as small as $10^{-19}$ m (see note[14]), making it plausible that, with a suitably designed experiment, the measurement of the displacement $d$ is within the current experimental reach.

While it is possible to shield an electron from electromagnetic fields that arise from external charges and currents, an electron cannot be shielded from the Killing electric field, which arises out of spacetime itself. Hence, the effect of the KEF can likely be isolated. Assuming the correctness of the bounds in [1,21,22,24], the non-detection of the predicted KEF-induced displacement, corresponding to the range (21), would show that the CMF is not the KEF and would effectively falsify the KEF postulate.

The KEC $J^{\mu}_{(K)} \propto R^{\mu\nu}K_{\nu}$ is a weak current. Even in extremal objects such as neutron stars, which have matter density of the order of $10^{18}$ kg/m$^3$ [117], $R^{\mu\nu} \sim 10^{-8}$ m$^{-2}$; the nontrivial magnetic field and plasma surrounding neutron stars make it challenging to isolate the effect of a weak KEC. The other arena in which the stress–energy tensor is significant is the early universe. Possible approaches to discern the signature of $J^{\mu}_{(K)}$, from the available data about the early universe, are currently being investigated.

### *5.4. Maxwell Tensor from Ricci Tensor*

In 1925, Rainich [17] derived an important result, which provides insight into the deep connection between electromagnetism and geometry. Specifically, he showed that a spacetime inhabited only by a source-free electromagnetic field is a *Rainich–Riemann manifold*—a Riemannian manifold in which the Ricci tensor is non-null[15] and satisfies the Rainich conditions[16]—and further that the source-free electromagnetic field is uniquely determined by the Ricci tensor and its derivative up to an overall global phase (constant of integration)[17]. A non-null Ricci tensor that satisfies the Rainich conditions will be called a *Rainich–Ricci tensor* in the following discussion.

Rainich's result, hailed as an "already unified theory" by Misner and Wheeler [18], rigorously showed that source-free electromagnetism ($F_{\mu\nu}$) can be expressed entirely in terms of geometry (in terms of $R_{\mu\nu}$ and its derivative). The result has a profound implication: *What we physically perceive as—what we interpret as—a source-free electromagnetic field is, at a more fundamental level, merely a geometric structure constructed from the Ricci tensor and its derivative in a special Riemannian manifold—the Rainich–Riemann manifold.* Thus, source-free electromagnetism does not arise out of—and is not separate from—geometry. *Source-free electromagnetism is geometry.*

Rainich's result provides a possible answer to the following etiological question that confronts the KEF postulate: *What is the mechanism—the manner of causation—by which a physical electromagnetic field (KEF) arises out of geometry (Killing vector field)?* Rainich's result suggests the following conjecture as an answer: *The Killing electromagnetic field is the way we physically perceive—the way we interpret—the special geometric structure constructed from the covariant derivative of a Killing vector.* The KEF does not arise out of geometry. *The KEF is geometry*. Similarly, it is conjectured that the KEC does not arise out of geometry. *The KEC is the way we perceive the special geometric structure constructed from the second covariant derivative of a Killing vector*.

### 5.5. "Charge Without Charge"

Rainich's calculations were followed up by Misner and Wheeler [18], who speculated that the universe has nontrivial topology and, in particular, "wormholes." In a Rainich universe, inhabited only by a source-free electromagnetic field, the electric lines of force do not terminate at charges; however, an observer without the ability to probe spacetime with sufficient resolution would interpret the radial lines of force emerging from (or going into) the mouth of a sufficiently small wormhole as lines of force emerging from (or going into) a point charge. Such an observer—Misner and Wheeler argued—would perceive the mouth of a wormhole as a point "charge," "without" the actual existence of point "charge."

The postulated KEC, $J^{\mu}_{(K)}$, provides a different mechanism for the emergence of charge density $J^{0}_{(K)}$—from geometry—without requiring the universe to have nontrivial topology (and wormholes). In other words, the KEC realizes Misner and Wheeler's vision of a "charge without charge," even in a universe with a trivial topology.

If the KEF postulate is valid, then (26) is an actual, physical electric field, which is denoted $E_S$. $E_S$ exerts electric force on a charged particle, making the particle's motion deviate from a geodesic trajectory, while an uncharged particle moves along a geodesic trajectory—exactly as one would expect in a Reissner–Nordstrom spacetime. Thus, the 'charge' (47) not only formally resembles the Reissner–Nordstrom charge but also physically behaves like a real Reissner–Nordstrom charge and, hence, can be regarded as a 'charge without charge'. Lacking a good estimate of $\beta$, however, one cannot, at present, estimate the magnitude of the charge.

### 5.6. Magnetogenesis from Killing Vectors

Assuming that the two (KEF and KEC) postulates are indeed valid, the KEF has two features that make it an appealing candidate for the CMF. First, the KEF does not depend on any particle physics model of the early universe. In fact, it does not even rely on the big bang model. The only underlying assumption is that the spacetime is described by the flat FLRW metric. Secondly, since the KEF arises everywhere in spacetime, it can provide a natural explanation for the large observed correlation length of the CMF.

## 6. Future Work

This section describes some ongoing work, as well as a few problems that warrant further investigation. As was argued in Section 5.4, the detection of the KEF in flat FLRW spacetime may be within the current experimental reach. An important follow-up task would be to design a suitable experiment for detection of the KEF. Currently, the possible approaches to discern the signature of the KEC from the available data about the early universe are being investigated, alongside efforts to establish good upper and lower bounds on the strength of the Killing electric field in Schwarzschild spacetime. Assuming that the CMF is a KEF, the current observational bounds on the CMF were used to derive upper and lower bounds for the KEF in flat FLRW spacetime (see (21)); however, it would be of greater interest to obtain an *ab initio* estimate of the strength of the KEF by extremizing the action (22).

The primordial rotational electric field, described in Section 3, generates vorticity. It is worth investigating if the conjectured primordial vorticity generated by Killing vectors can provide the initial conditions needed in Harrison's magnetogenesis model. Furthermore, the possible role of the conjectured vorticity, generated by Killing vectors, in structure formation also warrants further investigation.

Finally, it would be of interest to study the KEFs and KECs arising from the Killing vectors that have not been considered—in both the Schwarzschild and other spacetimes. As remarked in Appendix A, it would also be of interest to extend the KEF and KEC postulates to conformal Killing vectors and study the implications of the extended postulates.

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## Appendix A. Killing Vectors and Maxwell's Equations

The *Killing electromagnetic field* strength tensor $F_{\mu\nu}$ derived from a Killing vector $K^{\mu}$ is defined as

$$F_{\mu\nu} \quad := \quad \zeta(K_{\nu;\mu} - K_{\mu;\nu}) \;=\; 2\,\zeta\, K_{\nu;\mu}, \tag{A1}$$

where $\zeta$ is a dimensionful constant that endows $F_{\mu\nu}$ with the correct dimension. The second equality follows from the Killing equation $K_{\mu;\nu} + K_{\nu;\mu} \;=\; 0$. In the following discussion, it is shown that $F^{\mu\nu}$ satisfies the Maxwell's equations

$$F^{\mu\nu}{}_{;\nu} \quad = \quad \frac{J^{\mu}}{\epsilon_0 c} \tag{A2}$$

$$F_{\alpha\beta;\mu} + F_{\mu\alpha;\beta} + F_{\beta\mu;\alpha} \quad = \quad 0 \tag{A3}$$

First, consider (A3).

$$\left(\frac{1}{\zeta}\right)\left[\, F_{\alpha\beta;\mu} + F_{\mu\alpha;\beta} + F_{\beta\mu;\alpha}\,\right] \quad = \quad 2K_{\beta;\alpha;\mu} + 2K_{\alpha;\mu;\beta} + 2K_{\mu;\beta;\alpha} \tag{A4}$$

Using the Killing equation $K_{\beta;\mu} \;=\; -K_{\mu;\beta}$ and noting that

$$K_{\mu;\alpha;\beta} - K_{\mu;\beta;\alpha} \quad = \quad R^{\lambda}{}_{\mu\alpha\beta}K_{\lambda} \tag{A5}$$

where $R^{\lambda}{}_{\mu\alpha\beta}$ is the Riemann tensor, one can conclude that

$$\left(\frac{1}{\zeta}\right)\left[\,F_{\alpha\beta;\mu} + F_{\mu\alpha;\beta} + F_{\beta\mu;\alpha}\,\right] \;=\; (R^{\lambda}{}_{\beta\alpha\mu} + R^{\lambda}{}_{\alpha\mu\beta} + R^{\lambda}{}_{\mu\beta\alpha})K_{\lambda} \;=\; 0$$

The last equality follows from the identity

$$R^{\lambda}{}_{\beta\alpha\mu} + R^{\lambda}{}_{\alpha\mu\beta} + R^{\lambda}{}_{\mu\beta\alpha} \;=\; 0 \tag{A6}$$

Next, consider (A2). Since $g^{\alpha\beta}{}_{;\mu} \;=\; 0$,

$$F^{\mu\nu}{}_{;\nu} \;=\; \left(g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}\right)_{;\nu} \;=\; g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta;\nu} \tag{A7}$$

Recalling that $F_{\alpha\beta} \;=\; 2\,\zeta\,K_{\beta;\alpha}$, and using (A3) and (A4), one obtains

$$g^{\nu\beta}F_{\alpha\beta;\nu} \;=\; 2\zeta R^{\lambda}{}_{\alpha}K_{\lambda} \tag{A8}$$

where $R_{\lambda\alpha}$ is the Ricci tensor. Therefore, using (A7) and (A8), in addition to the fact that the Ricci tensor is symmetric, one can conclude that

$$F^{\mu\nu}{}_{;\nu} \;=\; \frac{J^{\mu}{}_{(K)}}{\epsilon_0 c}, \qquad \text{where} \qquad J^{\mu}{}_{(K)} := 2\epsilon_0 c\,\zeta\,R^{\mu\nu}K_{\nu} \tag{A9}$$

If the Ricci tensor vanishes, as in vacuum, then one obtains the Maxwell's equations in vacuum. On the other hand, non-vanishing Ricci tensor gives rise to the *Killing Electromagnetic Current* (KEC) density 4-vector $J^{\mu}{}_{(K)}$, which is shown in (A9). The subscript $K$ in $J^{\mu}{}_{(K)}$ indicates that the current density emerges from a Killing vector field.

An EFST

$$\mathcal{F}_{\mu\nu} \;=\; \zeta(\xi_{\nu;\mu} - \xi_{\mu;\nu})$$

derived from a conformal Killing vector $\xi$ also satisfies Maxwell's equations; as before, $\zeta$ is a suitable dimensionful constant.

Noting that a conformal Killing vector $\xi$ satisfies

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} \;=\; \frac{2}{d}(\xi^{\alpha}{}_{;\alpha})g_{\mu\nu}$$

where $d$ is the dimension of spacetime, the argument, presented earlier—for the EFST derived from Killing vectors—can be easily adapted to show that $\mathcal{F}_{\mu\nu}$ satisfies the homogeneous Maxwell's equation

$$\mathcal{F}_{\alpha\beta;\mu} + \mathcal{F}_{\mu\alpha;\beta} + \mathcal{F}_{\beta\mu;\alpha} \;=\; 0$$

and also the Maxwell's equation involving the source term

$$\mathcal{F}^{\mu\nu}{}_{;\nu} \;=\; \frac{\mathcal{J}^{\mu}}{\epsilon_0 c}, \qquad \mathcal{J}^{\mu} \;=\; \epsilon_0 c\zeta\left[2R^{\mu\lambda}\xi_{\lambda} + \frac{2(d-1)}{d}(\xi^{\beta}{}_{;\beta})^{;\mu}\right] \tag{A10}$$

If $\xi$ is a Killing vector, then $\xi^{\beta}{}_{;\beta} \;=\; 0$, and (A10) reduces to (A9). It would be of interest to explore the consequences of extending the KEF and KEC postulates to conformal Killing vectors.

## Notes

1 The assertion is predicated on the assumption that the flat FLRW spacetime emerged at the big bang.

2 $l_f \;=\; \dfrac{h}{m_e c} \sim 2.4 \times 10^{-12} m$

3 Also called the critical magnetic field strength in the literature, $B_f \;=\; \sqrt{\dfrac{2m_e^4\, c^5}{\epsilon_0\, h^3}} \approx 3.6 \times 10^{16}\, T$ in SI units.

4
$$\Gamma^0{}_{ii} \;=\; a\,\dot{a}/c, \quad \Gamma^i{}_{0i} \;=\; \Gamma^i{}_{i0} \;=\; H/c, \quad i \;=\; 1,2,3, \quad H := \frac{\dot{a}}{a} \tag{5}$$

5 Also called the comoving observer, a standard observer is actually a family of subobservers—one observer at each triple of comoving spatial coordinates—with the comoving spatial coordinates of each subobserver being independent of the "bookkeeper" coordinate $t$.

6 Strictly speaking, the curl of a 4-dimensional vector field is not a vector, but a bivector. However, both the electric field $E_{(K)}$ and $\omega^\mu$, defined in (12), lie in the three-dimensional hypersurface $\mathcal{S}$, in the sense that the projection of $E_{(K)}$ (resp. $\omega^\mu$) to $\mathcal{S}$ coincides with $E_{(K)}$ (resp. $\omega^\mu$). Therefore, with some abuse of notation, the 4-vector $\omega^\mu$ will be called a'curl', with the understanding that the zeroth component of $\omega^\mu$ will be ignored.

7 In accordance with the standard terminology, the symbol $G$ has been used to denote both the gravitational constant as well as Gauss, the unit of magnetic field strength. The discerning reader will determine what the symbol stands for from the context.

8 Note that $p(r)$ and $q(r)$ depend on $\beta$, although the dependence is not explicitly shown.

9 The terms of $O(\beta^2)$ and higher need not be included in this expansion, since they also have the same form as the $O(\beta)$ term and hence can be merged with the $O(\beta)$ term, the effect of which would be to replace the $\beta$ in the $O(\beta)$ term with some function $f(\beta)$, which can then be redefined as the new small parameter $\beta' = f(\beta)$.

10 $T^\mu{}_\mu \;=\; -F_{\mu\nu}F^{\mu\nu} + \dfrac{1}{4}\delta^\mu{}_\mu F_{\alpha\beta}F^{\alpha\beta} \;=\; 0.$

11 Taking the trace on both sides of (39) and noting that $T^\mu{}_\mu \;=\; 0$, one concludes that $-3R \;=\; \left(\frac{8\pi G}{c^4}\right) T^\mu{}_\mu \;=\; 0.$

12 In four-dimensional spacetime, if the Einstein tensor $G_{\mu\nu}$ vanishes, then so does the Ricci tensor $R_{\mu\nu}$.

$$G_{\mu\nu} \;=\; 0 \implies R_{\mu\nu} \;=\; \frac{1}{2}g_{\mu\nu}R \implies R \;=\; 2R \implies R \;=\; 0 \implies G_{\mu\nu} \;=\; R_{\mu\nu} \implies R_{\mu\nu} \;=\; 0$$

13 The energy loss due to radiation from the accelerating electron is $\sim 10^{-116}$ W/s and will be ignored.

14 For dynamic measurements, such as in LIGO, the strain sensitivity around 100 Hz is less than $10^{-23}/\sqrt{Hz}$, which, for an arm length of 4 km, yields a displacement sensitivity of $4 \times 10^{-20}$ m.

15 $R_{\alpha\beta}R^{\alpha\beta} \neq 0.$

16 Rainich conditions on the Ricci tensor are

$$R^\mu{}_\mu \;=\; 0, \quad R_{\mu\alpha}R^{\nu\alpha} \;=\; \delta_\mu{}^\nu\left(\frac{1}{4}R_{\rho\sigma}R^{\rho\sigma}\right), \quad R_{00} \geq 0 \tag{66}$$

$$\chi_{\rho;\sigma} - \chi_{\sigma;\rho} \;=\; 0, \qquad \chi_\beta \;=\; \frac{\sqrt{-g}\,\mathcal{A}_{\beta\lambda\mu\nu}\,R^{\lambda\gamma;\mu}\,R_\gamma{}^\nu}{R_{\rho\sigma}R^{\rho\sigma}} \tag{67}$$

Note that the algebraic conditions (66), as well as the differential condition (67), are purely geometric constraints that involve only the Ricci tensor. As before, $\mathcal{A}_{\beta\lambda\mu\nu}$ is the antisymmetric symbol.

17 Specifically, the Maxwell tensor (electromagnetic field strength tensor) can be written in terms of the Ricci tensor as follows:

$$F_{\mu\nu} \;=\; \xi_{\mu\nu}\cos\alpha + *\xi_{\mu\nu}\sin\alpha$$

where $\xi_{\mu\nu}$ is the extremal "Maxwell square root" of the Ricci tensor obtained as the solution of the equation

$$\xi_{\alpha\beta}\xi_{\mu\nu} \;=\; -\frac{1}{2}E_{\alpha\beta\mu\nu} - \frac{1}{2}\frac{E_{\alpha\beta\gamma\delta}E_{\mu\nu}{}^{\gamma\delta}}{(R_{ab}R^{ab})^{\frac{1}{2}}}, \qquad \text{where} \quad E_{\tau\sigma}{}^{\mu\nu} := \frac{1}{2}\left(-\delta_\tau{}^\mu R_\sigma{}^\nu + \delta_\sigma{}^\mu R_\tau{}^\nu - \delta_\sigma{}^\nu R_\tau{}^\mu + \delta_r{}^\nu R_\sigma{}^\mu\right)$$

and $*\xi_{\mu\nu} = \frac{1}{2}\sqrt{-g}\mathcal{A}_{\mu\nu\alpha\beta}\xi^{\alpha\beta}$ is the dual of $\xi_{\mu\nu}$. $\alpha$ is also determined by $R_{\mu\nu}$, up to a constant of integration, and is the solution of the equation

$$\frac{\partial\alpha}{\partial x^{\beta}} = \frac{\sqrt{-g}\,\mathcal{A}_{\beta\lambda\mu\nu}R^{\lambda\gamma;\mu}R_{\gamma}{}^{\nu}}{R_{\rho\sigma}R^{\rho\sigma}}$$

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