

# Linearized metric solutions in ghost-free nonlocal gravity

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**Abstract.** In this manuscript we review some aspects of linearized metric solutions in ghost-free nonlocal gravity, in which the action is made up of non-polynomial differential operators containing covariant derivatives of infinite order. By working with the simplest model of such a wide class of infinite derivative theories of gravity, we will first compute the spacetime metric generated by a static point-like source and show that all curvature invariants are nonsingular at the origin. Secondly, a similar computation is performed for an electrically charged source and also in this case the regularizing feature of nonlocality plays a crucial role. As a third case, we consider the spacetime metric generated by a Dirac delta distribution on a ring and show that, at least in the linear regime, Kerr-like singularities can be avoided in ghost-free non-local gravity.

## 1. Introduction

Einstein's general relativity (GR) has been the most successful theory of gravity so far, indeed its predictions have been tested to very high precision in the infrared (IR) regime, i.e. at large distances and late times [1]. For instance, we can think of the recent observation of gravitational wave emission [2], which can represent one of the greatest triumphs of theoretical physics. However, despite its great achievements, there are still unsolved problems which suggest that Einstein's GR can be only seen as an effective field theory of gravitational interaction, which works very well at low energy but breaks down in the ultraviolet (UV) regime. In fact, at the classical level the Einstein-Hilbert Lagrangian,  $\sqrt{-g}\mathcal{R}$ , suffers from the presence of blackhole and cosmological singularities (short-distance regime), while at the quantum level it turns out to be non-renormalizable from a perturbative point of view (high-energy regime).

It is worthwhile emphasizing that our knowledge of the gravitational interaction at short distances is really limited. It suffices to think that, from a pure experimental point of view, Newton's  $1/r$  law has been tested only up to a distance of  $5.6 \times 10^{-5}$  meters, which in terms of energies means 0.01eV [3]. Therefore, there exists a very wide desert of roughly thirty orders of magnitude, from 0.01eV to the Planck scale  $M_p \sim 10^{19}\text{GeV}$ <sup>1</sup>, in which we do not know anything about gravity. This is the place where UV extensions of GR can play a crucial role.

<sup>1</sup> In this paper we work with Natural Units in which  $c = 1 = \hbar$  and the Coulomb constant is  $k_e = 1$ . Moreover, we adopt the mostly positive convention for the metric signature,  $\eta = \text{diag}(-1, +1, +1, +1)$ .



One way to extend GR is to add terms quadratic in the curvatures in the gravitational action, like for example  $\mathcal{R}^2$  and  $\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}$ . This kind of action was shown to be power counting renormalizable in Ref. [4, 5], but still non-physical because of the presence of a massive spin-2 ghost degree of freedom which causes Hamiltonian instabilities, classically, and breaking of the unitarity condition of the S-matrix, quantum mechanically<sup>2</sup>. See Refs. [10, 11, 12, 13, 14, 15, 16, 17, 18, 19] for other relevant works on applications involving gravitational actions quadratic in the curvature invariants.

The emergence of ghost modes is related to the presence of higher order time derivatives in the field equations [22]. In the last four decades, it was realized that unhealthy degrees of freedom can be still avoided in higher derivative theories if the order of the derivatives is not finite but *infinite*. Indeed, by introducing certain non-polynomial differential operators in the action, like for example  $e^\square$ , one can prevent the appearance of extra poles in the physical spectrum, which was already noticed in the last century by [23, 24, 25, 26]. The presence of non-polynomial derivatives makes the action *nonlocal*, and this kind of nonlocal models were already studied in the early fifties in relation with the improved UV behavior of loop integrals, see Refs. [27]. This possibility turned out to be very promising and has motivated a deeper investigation of this unexplored sector of *nonlocal (or infinite derivative) field theories*.

First relevant applications of nonlocal field theory in a gravitational context were made in Refs. [28, 29, 30, 31] where the authors explicitly show the possibility to construct a quadratic curvature theory of gravity which is classically stable and unitary at the quantum level. It was also noticed that the presence of nonlocality can regularize infinities and many efforts have been made in order to resolve blackhole [29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47] and cosmological [28, 48, 49, 50] singularities. Further applications appear in the context of inflation [51] and thermal field theory [52, 53, 54].

This class of nonlocal field theories shows an improved UV behavior [55, 58, 57, 58] but a rigorous proof of renormalizability is only available for some peculiar non-polynomial operators [57]. See also Refs. [60, 61, 62, 63, 64] for recent progresses on infinite derivative field theories, and in particular Refs. [65, 66, 67, 68] where the authors prove perturbative unitarity.

It is worthwhile emphasizing that similar infinite derivative operators also appear in string field theory and p-adic string [69, 70, 71, 72, 73].

Our aim is to review some classical aspects of ghost-free infinite derivative gravity (IDG); in particular, we will find linearized metric solutions for several sources and discuss the physical implications of nonlocality. The manuscript is organized as follows. In Section 2, we introduce the gravitational action and, we will find the propagator and the linearized field equations around Minkowski background. In Section 3, we will find the linearized metric solution for a static neutral point-like source. In Section 4, we will do the same for an electrically charged point-like source. Moreover, in Section 5 we will find the spacetime metric for a Delta Dirac distribution on a ring. Finally, in Section 6 we will draw our conclusions and discuss the outlook.

## 2. Nonlocal gravitational action

The class of ghost-free nonlocal gravitational actions is very wide but we will only consider one particular model for our analysis, which will capture all the main classical features due to nonlocality. Let us consider the following infinite derivative action [29, 30, 32]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ \mathcal{R} + G_{\mu\nu} \frac{e^{-\square/M_s^2} - 1}{\square} \mathcal{R}^{\mu\nu} \right\}, \quad (1)$$

<sup>2</sup> See Refs. [6, 7, 8, 9] for recent interesting works, in which the authors introduce a new quantization prescription through which the ghost is converted into a fake degree of freedom (*fakeon*), in such a way that the optical theorem is still satisfied and so perturbative unitarity is maintained.

where  $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - g_{\mu\nu}\mathcal{R}/2$  is the Einstein tensor,  $\kappa^2 \equiv 8\pi G$  with  $G = 1/M_p^2$  being the Newton constant, while  $M_s$  is a new fundamental energy scale at which nonlocal effects should manifest and is mathematically needed to make the exponent of the exponential dimensionless.

Note that the non-polynomial form-factor in Eq. (1) is

$$\mathcal{F}(\square) \equiv \frac{e^{-\square/M_s^2} - 1}{\square} = - \sum_{n=1}^{\infty} \frac{1}{n!} \frac{(-\square)^{n-1}}{M_s^{2n}}, \quad (2)$$

which is analytic and modify the short-distance (UV) behavior of Einstein's GR as only positive power of  $\square$  appear. It is worthwhile mentioning that in the literature there are also examples of non-analytic form-factors which are responsible for IR modifications; see for instance Refs. [74, 75, 76, 77, 78, 79].

### 2.1. Graviton propagator and linearized field equations

Let us now consider linear perturbations around the Minkowski background,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad (3)$$

where  $h_{\mu\nu}$  is the graviton perturbation, so that by expanding the action up to order  $\mathcal{O}(h_{\mu\nu}^2)$  [30] we obtain:

$$S^{(2)} = \frac{1}{4} \int d^4x h_{\mu\nu} e^{-\square/M_s^2} \mathcal{O}^{\mu\nu\rho\sigma} h_{\rho\sigma}, \quad (4)$$

where

$$\mathcal{O}^{\mu\nu\rho\sigma} := \frac{1}{4} (\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) \square - \frac{1}{2} \eta^{\mu\nu}\eta^{\rho\sigma} \square + \frac{1}{2} (\eta^{\mu\nu}\partial^\rho\partial^\sigma + \eta^{\rho\sigma}\partial^\mu\partial^\nu - \eta^{\mu\rho}\partial^\nu\partial^\sigma - \eta^{\mu\sigma}\partial^\nu\partial^\rho) \quad (5)$$

is a four-rank operator  $\mathcal{O}^{\mu\nu\rho\sigma}$  which is totally symmetric in all its indices.

By inverting the kinetic operator  $e^{-\square/M_s^2} \mathcal{O}^{\mu\nu\rho\sigma}$  one obtains the graviton propagator around Minkowski, and its gauge independent and saturated part is given by [23, 26, 29, 30]

$$\Pi_{\mu\nu\rho\sigma}(k) = e^{-k^2/M_s^2} \left( \frac{\mathcal{P}_{\mu\nu\rho\sigma}^2}{k^2} - \frac{\mathcal{P}_{s,\mu\nu\rho\sigma}^0}{2k^2} \right) \equiv e^{-k^2/M_s^2} \Pi_{GR,\mu\nu\rho\sigma}(k), \quad (6)$$

where  $\Pi_{GR} = \mathcal{P}^2/k^2 - \mathcal{P}_s^0/2k^2$  is the graviton propagator in Einstein's GR, while  $\mathcal{P}^2$  and  $\mathcal{P}_s^0$  are the spin-2 and spin-0 projection operators; see Refs. [80, 81] for more details.

Very interestingly, the nonlocal modification in the propagator turns out to be a simple extra factor  $e^{-k^2/M_s^2}$ , which does not introduce any new pole in the theory other than the standard massless spin-2,  $k^2 = 0$ .

The linearized field equations corresponding the the action in Eq. (4) are given by [30]

$$e^{-\square/M_s^2} \mathcal{O}_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} = -16\pi G T_{\mu\nu}, \quad (7)$$

where

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}} \quad (8)$$

is the stress-energy tensor of the matter sector.

In the following sections we will solve the field equations in Eq. (7) for three different choices of  $T_{\mu\nu}$ .

### 3. Spacetime metric for a static point-like source

In this Section we wish to find a spherically symmetric static metric solution in presence of a Delta Dirac distribution placed at  $r = 0$ . By working in the conformal Newtonian gauge, we can write the perturbed metric in Eq. (3) in isotropic coordinates as follows:

$$ds^2 = -(1 + 2\Phi(r))dt^2 + (1 - 2\Psi(r))(dr^2 + r^2 d\Omega^2), \quad (9)$$

where  $r = \sqrt{x^2 + y^2 + z^2}$  is the isotropic radial coordinate, so that the field equations in Eq. (7) become

$$\begin{aligned} e^{-\nabla^2/M_s^2} \nabla^2 \Phi &= 4\pi G(T + 2T_{00}), \\ e^{-\nabla^2/M_s^2} \nabla^2 \Psi &= 4\pi G T_{00}. \end{aligned} \quad (10)$$

where  $T \equiv \eta^{\mu\nu} T_{\mu\nu}$ , and we have used  $\kappa h_{00} = -2\Phi$ ,  $\kappa h_{ij} = -2\Psi\delta_{ij}$ ,  $\kappa h = 2(\Phi - 3\Psi)$  and  $\square \simeq \nabla^2$ .

In the case of a neutral static point-like source, the stress energy tensor is given by

$$T_{\mu\nu} = m\delta_\mu^0 \delta_\nu^0 \delta^{(3)}(\vec{r}), \quad (11)$$

so that the modified Poisson equations in Eq. (10) reduce to

$$e^{-\nabla^2/M_s^2} \nabla^2 \Phi = e^{-\nabla^2/M_s^2} \nabla^2 \Psi = 4\pi m \delta^{(3)}(\vec{r}), \quad (12)$$

implying that the two metric potentials are equal,  $\Phi = \Psi$ . Note that we have to deal with a differential equation of infinite order due to the presence of the exponential operator  $e^{-\nabla^2/M_s^2}$ ; however it can be easily solved by going to Fourier space and then anti-transforming back to coordinate space. Indeed, by doing so we obtain the following solution:

$$\Phi(r) = -4\pi Gm \int d^3k \frac{e^{-k^2/M_s^2}}{k^2} e^{i\vec{k}\cdot\vec{r}} = -\frac{2Gm}{\pi} \frac{1}{r} \int_0^\infty dk \frac{\sin(kr)}{k} e^{-k^2/M_s^2} = -\frac{Gm}{r} \text{Erf}\left(\frac{M_s r}{2}\right), \quad (13)$$

where

$$\text{Erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad (14)$$

is the so called error-function.

Very interestingly, note that the gravitational potential in Eq. (13) is non-singular at  $r = 0$ , indeed it tends to the finite constant value  $\Phi(0) = 2GmM_s/\sqrt{\pi}$ ; while for large distances we recover the  $1/r$  behavior of the Newtonian potential, as expected. Note that the linearized regime is valid as long as the inequality  $2GmM_s/\sqrt{\pi} < 1$  holds true.

From a physical point of view, nonlocality is able to regularize the singularity, indeed the point-like source at  $r = 0$  is smeared out on a region of size  $1/M_s$  due to the presence of infinite order derivative. Not only the linearized metric potentials but also all the linearized curvature invariants turn out to be non-singular. Furthermore, at  $r = 0$  the metric becomes conformally-flat, since all the components of the Weyl tensor vanish at the origin [38].

### 4. Spacetime metric for an electrically charged point-like source

Let us now consider the case of a static electric charge as a gravitational source and find the corresponding metric potentials for the spherically symmetric metric in Eq. (9). The stress-energy tensor in this case is given by electro-magnetic one which reads

$$T_{\mu\nu} = \frac{1}{4\pi} \left( \eta_{\rho\nu} F_{\mu\sigma} F^{\rho\sigma} - \frac{1}{4} \eta_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right), \quad (15)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the electro-magnetic field strength defined in terms of the potential-vector  $A_\mu$ . In the simplest case of an electric charge, only the components related to the electric field are non-vanishing:

$$F_{10} = -F_{01} = E_r \quad \text{and} \quad E_r = \frac{Q}{r^2}, \quad (16)$$

with  $E_r$  being the radial component of the electric field. Given the stress-energy tensor in Eq. (15) with the components in Eq. (16), the infinite derivative differential equations in Eq. (10) become [41]:

$$\begin{aligned} e^{-\nabla^2/M_s^2} \nabla^2 \Phi &= \frac{GQ^2}{r^4}, \\ e^{-\nabla^2/M_s^2} \nabla^2 \Psi &= \frac{GQ^2}{2r^4}. \end{aligned} \quad (17)$$

Also in this case we can solve the modified Poisson equations by using the Fourier transform method. By making the field redefinitions  $\tilde{\Phi} := e^{-\nabla^2/M_s^2} \Phi$ ,  $\tilde{\Psi} := e^{-\nabla^2/M_s^2} \Psi$ , we obtain the following two solutions from Eq. (17):

$$\begin{aligned} \tilde{\Phi}(r) &= -\frac{C_1}{r} + \frac{GQ^2}{2r^2} + C_2 \\ \tilde{\Psi}(r) &= -\frac{C_1}{r} + \frac{GQ^2}{4r^2} + C_2, \end{aligned} \quad (18)$$

where we fix  $C_1 = Gm$  by requiring that we want to recover the static neutral case in Eq. (13) when  $Q = 0$ , and  $C_2 = 0$  since we want asymptotic flatness (for  $r \rightarrow \infty$ ). We can now go back to the fields  $\Phi$  and  $\Psi$ , which are given by

$$\begin{aligned} \Phi(r) &= -Gme^{\nabla^2/M_s^2} \left( \frac{1}{r} \right) + \frac{GQ^2}{2} e^{\nabla^2/M_s^2} \left( \frac{1}{r^2} \right), \\ \Psi(r) &= -Gme^{\nabla^2/M_s^2} \left( \frac{1}{r} \right) + \frac{GQ^2}{4} e^{\nabla^2/M_s^2} \left( \frac{1}{r^2} \right). \end{aligned} \quad (19)$$

By using the fact that  $4\pi/k^2$  is the Fourier transform of  $1/r$ , we can write

$$e^{\nabla^2/M_s^2} \left( \frac{1}{r} \right) = \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{k^2} e^{-k^2/M_s^2} e^{i\vec{k}\cdot\vec{r}} = \frac{2}{\pi} \int_0^\infty dk \frac{\sin(kr)}{kr} e^{-k^2/M_s^2} = \frac{1}{r} \text{Erf} \left( \frac{M_s r}{2} \right), \quad (20)$$

which gives the neutral part of the potentials; then, by using the Fourier transform  $\frac{2\pi^2}{k} \text{sign}(k)$  of  $1/r^2$ , we can write

$$e^{\nabla^2/M_s^2} \left( \frac{1}{r^2} \right) = \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k} \text{sign}(k) e^{-k^2/M_s^2} e^{i\vec{k}\cdot\vec{r}} = \int_0^\infty dk \frac{\sin(kr)}{r} e^{-k^2/M_s^2} = \frac{M_s}{r} \text{F} \left( \frac{M_s r}{2} \right), \quad (21)$$

where

$$\text{F}(x) := e^{-x^2} \int_0^x e^{t^2} dt \quad (22)$$

is the so called Dawson function. Therefore, the two metric potentials in Eq. (19) read [41]

$$\begin{aligned} \Phi(r) &= -\frac{Gm}{r} \text{Erf} \left( \frac{M_s r}{2} \right) + \frac{GQ^2 M_s}{2r} \text{F} \left( \frac{M_s r}{2} \right), \\ \Psi(r) &= -\frac{Gm}{r} \text{Erf} \left( \frac{M_s r}{2} \right) + \frac{GQ^2 M_s}{4r} \text{F} \left( \frac{M_s r}{2} \right). \end{aligned} \quad (23)$$

As expected, for large distances  $M_s r \gg 1$ , we recover the metric potentials of the linearized Reissner-Nordström metric of GR [41], while in the limit  $r \rightarrow 0$  the two metric potentials tend to finite values,  $\Phi(0) = -GmM_s/\sqrt{\pi} + GQ^2M_s^2/4$  and  $\Psi(0) = -GmM_s/\sqrt{\pi} + GQ^2M_s^2/8$ , as can be easily checked. The same regularized behavior can be shown for all linearized curvature invariants; in particular, the Weyl tensor vanishes implying that the metric is conformally-flat at the origin [41], as it also happens for the case of a neutral source studied in Section 3. Therefore, also for a point-like electric charge nonlocality is able to regularize the singularity at  $r = 0$ .

The linearized regime holds all the way from  $r = \infty$  up to  $r = 0$ , as long as the inequalities  $2|\Phi(0)|, 2|\Psi(0)| < 1$  are satisfied, which means  $mM_s < M_p^2$  and  $|Q|M_s < M_p$ , where we have neglected constant factors of order one [41].

### 5. Spacetime metric for a Dirac delta distribution on a ring

In this Section, we wish to determine the spacetime metric generated by a Dirac Delta distribution on a ring and show how nonlocality can regularize Kerr-like ring singularities. It is well known that the Kerr metric suffers from the presence of a ring singularity which in Boyer-Lindquist coordinates is described by the equation  $r^2 + a^2 \cos^2 \vartheta = 0$ , or in Cartesian coordinates by  $z = 0, x^2 + y^2 = a^2$ , where  $a$  is the radius of the ring [82].

To mimic such a ring distribution, we consider a ring of radius  $a$  rotating with constant angular velocity  $\omega$ , described by following stress-energy tensor:

$$T_{00} = m\delta(z) \frac{\delta(x^2 + y^2 - a^2)}{\pi}, \quad T_{0i} = T_{00}v_i, \quad (24)$$

where  $v_i$  is the tangential velocity and its magnitude is related to the angular velocity through the relation  $v = \omega a$ ; moreover, by assuming that the rotation happens around the  $z$ -axis, we can write  $v_x = -y\omega$ ,  $v_y = x\omega$  and  $v_z = 0$ . The stress-energy tensor in Eq. (24) will source the following linearized non-diagonal metric:

$$ds^2 = -(1 + 2\Phi)dt^2 + 2\vec{h} \cdot d\vec{x}dt + (1 - 2\Psi)d\vec{x}^2, \quad (25)$$

where  $h_i \equiv h_{0i}$  and the coordinate  $r$  is the isotropic radius which should not be confused with the Boyer-Lindquist radial coordinate used above. The metric components in Eq. (25) can be found by solving a set of decoupled infinite order differential equations [42]:

$$\begin{aligned} e^{-\nabla^2/M_s^2} \nabla^2 \Phi(\vec{r}) &= e^{-\nabla^2/M_s^2} \nabla^2 \Psi(\vec{r}) = 4Gm \delta(z) \delta(x^2 + y^2 - a^2), \\ e^{-\nabla^2/M_s^2} \nabla^2 h_{0x}(\vec{r}) &= -16Gm \omega y \delta(z) \delta(x^2 + y^2 - a^2), \\ e^{-\nabla^2/M_s^2} \nabla^2 h_{0y}(\vec{r}) &= 16Gm \omega x \delta(z) \delta(x^2 + y^2 - a^2). \end{aligned} \quad (26)$$

Also in this case we can solve the modified Poisson equations by using the Fourier transform method. Note that, by going to cylindrical coordinates, we can Fourier transform the stress-energy tensor components as follows [42]:

$$\begin{aligned} \mathcal{F}[\delta(z) \delta(x^2 + y^2 - a^2)] &= \int_{-\infty}^{\infty} dz \delta(z) e^{ik_z z} \int_0^{\infty} d\rho \rho \delta(\rho^2 - a^2) \int_0^{2\pi} d\varphi e^{ik_x \rho \cos \varphi} e^{ik_y \rho \sin \varphi} \\ &= \pi \int_0^{\infty} d(\rho^2) \delta(\rho^2 - a^2) I_0 \left( i\rho \sqrt{k_x^2 + k_y^2} \right) \\ &= \pi I_0 \left( ia \sqrt{k_x^2 + k_y^2} \right), \end{aligned} \quad (27)$$

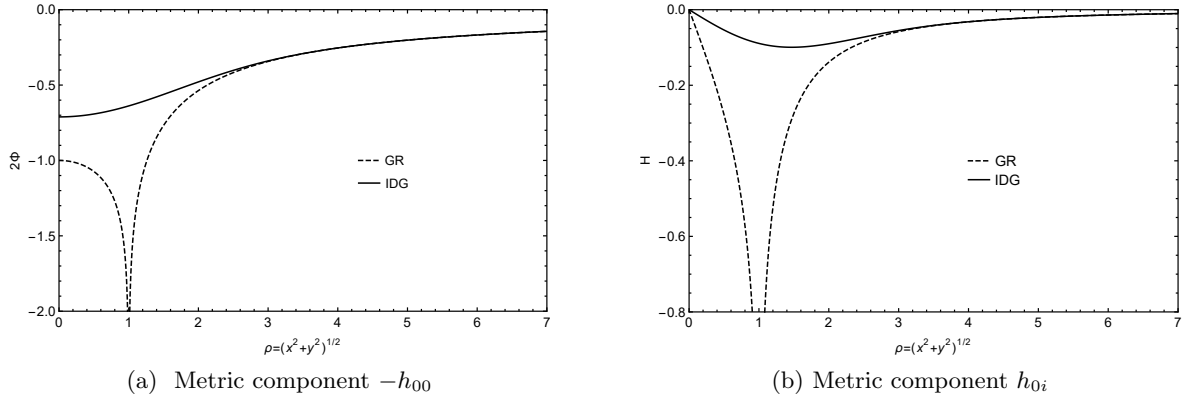


Figure 1: In this plot we have shown the behavior of the components  $-h_{00}$  and  $h_{0i} \sim H$  as functions of the cylindrical radius  $\rho$ , in both cases of IDG (orange line) and IDG (blue line). For convenience, we have set  $M_s = 1.5$ ,  $G = 1$ ,  $m = 0.5$  and  $a = 1$ . It is clear that, while the metric potentials in GR have a singularity for  $\rho = a$ , they turn out to be regular in IDG.

$$\begin{aligned}
 \mathcal{F}[\delta(z)\delta(x^2 + y^2 - a^2)] &= \int_{-\infty}^{\infty} dz \delta(z) e^{ik_z z} \int_0^{\infty} d\rho \rho^2 \delta(\rho^2 - a^2) \int_0^{2\pi} d\varphi e^{ik_x \rho \cos\varphi} e^{ik_y \rho \sin\varphi} \cos\varphi \\
 &= \pi \frac{k_x}{\sqrt{k_x^2 + k_y^2}} \int_0^{\infty} d(\rho^2) \rho \delta(\rho^2 - a^2) I_1 \left( i\rho \sqrt{k_x^2 + k_y^2} \right) \\
 &= \pi a \frac{k_x}{\sqrt{k_x^2 + k_y^2}} I_1 \left( ia \sqrt{k_x^2 + k_y^2} \right),
 \end{aligned} \tag{28}$$

and

$$\mathcal{F}[y\delta(z)\delta(x^2 + y^2 - a^2)] = \pi a \frac{k_y}{\sqrt{k_x^2 + k_y^2}} I_1 \left( ia \sqrt{k_x^2 + k_y^2} \right), \tag{29}$$

where  $\mathcal{F}[\dots]$  stands for the Fourier transform operation,  $I_0$  and  $I_1$  are two Modified Bessel functions. For simplicity, let us only consider the plane  $z = 0$  and work with the cylindrical radial coordinate  $\rho = \sqrt{x^2 + y^2}$ , as we know that in GR the ring singularity lies in the  $x$ - $y$  plane. Therefore, by anti-transforming to coordinate space, the metric potentials will be given by [42]

$$\Phi(\rho) = \Psi(\rho) = -Gm \int_0^{\infty} d\zeta I_0(ia\zeta) I_0(i\zeta\rho) \text{Erfc}\left(\frac{\zeta}{M_s}\right), \tag{30}$$

$$h_{0x}(x, y) = 4Gm\omega a \frac{y}{\rho} H(\rho), \quad h_{0y}(x, y) = -4Gm\omega a \frac{x}{\rho} H(\rho), \tag{31}$$

with

$$H(\rho) := \int_0^{\infty} d\zeta I_1(ia\zeta) I_1(i\zeta\rho) \text{Erfc}\left(\frac{\zeta}{M_s}\right), \tag{32}$$

which in the limit  $M_s \rightarrow \infty$  all reduce to GR case.

The above three integrals cannot be solved analytically but we can do it numerically as shown in Fig. 1. We can explicitly see that in GR the metric potentials suffers from the presence of

a ring singularity at  $\rho = a$ , while in infinite derivative gravity such a singularity is regularized. Also in this case, one can show that all the curvature invariants are non-singular in the entire spacetime and that the Weyl tensor vanishes at  $r = 0$ .

Note that the linearized regime holds from  $r = \infty$  all the way to  $r = 0$  as long as the inequalities  $2|\Phi| < 1$  and  $|h_{0i}| < 1$  are satisfied for any  $r$ , which, by neglecting constant factors of order one, also means  $GmM_s < 1$  and  $GmM_s^2\omega a^2 < 1$ , respectively [42].

## 6. Conclusions and outlook

In this manuscript, we have studied linearized metric solutions for one model of infinite derivative theories of gravity, in which the choice of nonlocal form-factor is the one in Eq. (2). We have considered three different configurations, neutral static point-like source, static electrically charged point-like source and Dirac delta distribution on a ring rotating with constant angular velocity. Our aim was to show how the presence of nonlocality through infinite order derivatives can regularize both point-like and ring singularities, at least at the linearized level.

The next important step is to explore the non-linear regime and understand whether singularities can be also avoided when dealing with the full set of non-linear field equations. Some progress has been made in Ref. [40], where physical arguments have been presented in support of singularity avoidance. However, a rigorous treatment is still lacking and no full non-linear spherically symmetric solution is known in the literature. Such a program is challenging but at the same time very promising, and it will subject of future works.

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