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# Searching optimum equations of state using internal magnetic fields with various shapes

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**Abstract.** Neutron stars (NSs) are highly magnetized rotating compact stars. Recently, three NSs around twice the solar mass have been found. Such massive NSs give strong constraints on equations of state (EoSs) of NS matter. In this study, we calculate masses and radii of NSs and compare them with the observed masses and radii of two NSs and the radius of a NS with 1.4 $M_\odot$ for various EoSs with internal magnetic fields. In our calculation, to investigate the optimum EoS for the neutron matter with a strong magnetic field, we calculate the total masses and radii of NSs by changing the internal magnetic fields formulated by four free parameters. The predictions by several EoSs come into the range determined by observational constraints if suitable values of these free parameters are chosen.

## 1. Introduction

The neutron star (NS) is an interesting and important subject in nuclear physics as a unique object of dense hadronic matter. The nuclear density of NS is getting high toward its core, which can be expected to reach several times larger than the saturated one. Equation of state (EoS) of the nuclear matter is often utilized to discuss various properties of NSs, such as masses, radii, and mass-radius (MR) relations.

In 2010, a NS named PSR J1614-2230 was observed to have a mass of twice the solar mass ($1.97 \pm 0.04\ M_\odot$) [1]. In 2013, a NS named PSR J0348+0432 with a mass of $2.01 \pm 0.04\ M_\odot$ was also observed [2]. Furthermore, mass of the millisecond-pulsar (MSP) J0740+6620 was measured to be $2.14^{+0.10}_{-0.09}\ M_\odot$ in 2019 [3] with additional observation by XMM-Newton. Later a more precise value of $2.072^{+0.067}_{-0.066}\ M_\odot$ was measured in 2021 [4]. Such massive NSs give strong constraints on EoSs of NS matter. Another constraint on the NS radius comes from the observation of the gravitational wave event GW170817 [5], which claims that the NS radius $R$ should have the upper limit of $R \leq 13.76$ km at NS mass of $1.4\,M_\odot$.

Magnetar is a type of the NS with strong magnetic fields ($\geq 10^{15}$G) on the surface. Recently, up to 30 magnetars have been observed [6, 7]. At most, about $2 \times 10^{15}$ G of the magnetic field strength on the surface has been found by the observation [6], although we do not yet know generation mechanism where and how they come from. In our previous study [8, 9, 10], we calculated MR-relations using various EoSs in the presence of magnetic fields together with rotation.

In this study, we investigate masses and radii of NSs for various EoSs with hyperons by changing the shapes of the internal magnetic field characterized by four free parameters, while we compare NSs to those with some measured observational constraints for radii and masses; radii at 1.4 $M_\odot$, and those in the error range of radii and masses at 1.44 $M_\odot$ of a standard pulsar, and 2.072 $M_\odot$ of the observed pulsar.

## 2. Formulations

### *2.1. Equations of State*

In this study we adopt the relativistic mean field (RMF) theory based on the nonlinear Walecka model. We assume that the NS matter is static and uniform in the high density region. The Lagrangians and nuclear properties at each saturation number density $\rho_0$ of employed EoSs are given in Ref. [8, 9, 10]. Here we consider seven EoSs (GM1, GM3, TM1-a, TM2$\omega\rho$-a, NL3-a, NL3$\omega\rho$-a and DDME2-a) with different nuclear properties and other five EoSs (TM1-b, TM2$\omega\rho$-b, NL3-b, NL3$\omega\rho$-b and DDME2-b) where different ratios of hyperons to nucleons (from $R_{\omega\Lambda} = 2/3$ to $R_{\omega\Lambda} = 1$) are employed; see Ref. [11] in details.

### *2.2. Magnetic fields*

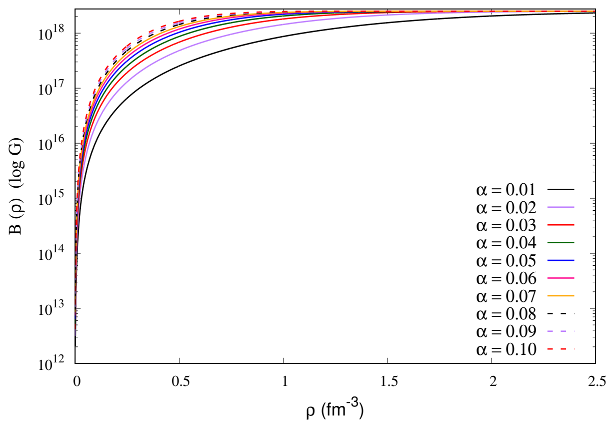


**Figure 1.** Magnetic field strengths $B(\rho)$ as a function of $\rho$ by changing $\alpha$. ($B_0 = 2.5\times10^{18}$ G, $B_s = 10^{12}$ G, and $\gamma = 2$)

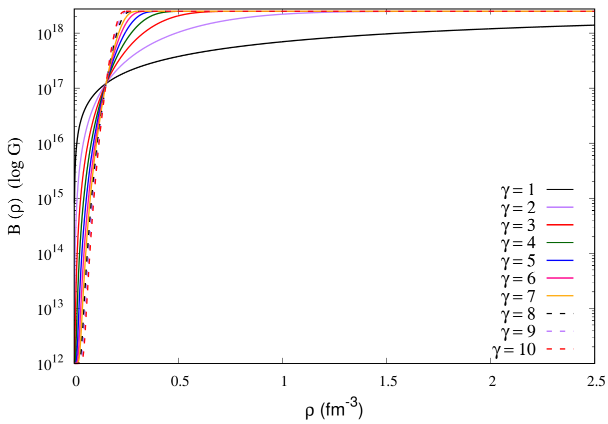


**Figure 2.** Magnetic field strengths $B(\rho)$ as a function of $\rho$ by changing $\gamma$. ($B_0 = 2.5\times10^{18}$ G, $B_s = 10^{12}$ G, and $\alpha = 0.05$)

In this study we adopt a density ($\rho$) -dependent magnetic field whose strength is given by [13, 14],

$$B(\rho) = B_s + B_0\left[1 - \exp\left\{-\alpha\left(\frac{\rho}{\rho_0}\right)^\gamma\right\}\right], \tag{1}$$

where $B_0$ indicates the strength in a denser region than that of the saturation number density $\rho_0$ (0.153 fm$^{-3}$) and $B_s$ determines one on the surface to be fixed constant as $B_s = 10^{12}$ G for typical pulsar and as $B_s = 10^{15}$ G for a magnetar.

Only the strengths of the magnetic fields on the surface are known in observation. Even if one assumes the form of Eq. (1), one still has free parameters $\alpha$, $\gamma$, $B_s$, and $B_0$. The parameters $\alpha$ and $\gamma$ control how fast the central magnetic field strength $B_0$ approaches to the asymptotic value at the surface strength $B_s$. Figure 1 shows the shape of the magnetic field as a function of $\rho$ for various $\alpha$'s with a constant $\gamma$, while Fig. 2 shows the shape of the magnetic field strength as a function of $\rho$ for various $\gamma$'s with a constant $\alpha$. Up to now, no work except our own work [9]

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is reported concerning the change of $\alpha$ and $\gamma$ parameters. In this study we arbitrarily change $\alpha$ and $\gamma$ to see their effects on the radii and masses of NSs to search for optimum EoSs with including hyperons.

The energy density and the pressure of hadronic matter in the presence of the spherically symmetric magnetic fields are given as $\varepsilon = \varepsilon_m + \boldsymbol{B}^2/2,\ p = p_m + \boldsymbol{B}^2/2$ , respectively, where $\varepsilon_m$ and $p_m$ are the energy density and the pressure of hadronic matters with the contribution $\boldsymbol{B}^2/2$ from the magnetic fields being neglected. All the details are given in Appendix A of Ref. [9].

## 3. Results

Figures 3 and 4 show the MR relations for various EoSs without and with the magnetic field, respectively. The unstable region in each EoS is not shown. The orange and light green shades show the MR areas expected from the measurements of pulsars PSR J0740+6620 and PSR J0030+0451 (68% credibility), respectively. The arrow indicates the upper limit of radius for $1.4M_\odot$ from GW170817. The colored lines represent MR-relations by EoSs. The adopted parameters of the magnetic field strength are $B_0 = 2.5 \times 10^{18}$ G, $B_s = 10^{12}$ G, $\alpha = 0.05$ and $\gamma = 2$ in Fig. 4. Comparing Fig. 3 with Fig. 4, only those MR-relations predicted by two EoSs (GM1 and TM2$\omega\rho$-b) satisfy observational constraints. However, the predictions by a numerous number of EoSs come into the range determined by observational constraints if suitable choices of free parameters ($\alpha$ and $\gamma$) are taken.

The strengths of surface magnetic fields $B_s = 10^{12}$ G and $B_s = 10^{15}$ G do not cause any substantial numerical differences for all EoSs. It is because this difference of the strengths of magnetic fields is compensated by changing $\alpha$ and $\gamma$ values. Big changes are seen in masses and radii when parameters $\alpha$ and $\gamma$ are increased. Table 1 shows candidates of suitable EoSs with proper values of $\alpha$ and $\gamma$ for fixed $B_s = 10^{12}$ G and $B_0 = 2.5 \times 10^{18}$ G. Further details are given in Ref. [10].

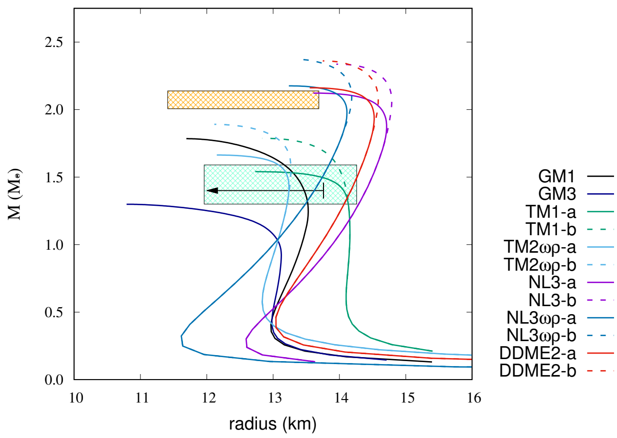


**Figure 3.** MR relations with no magnetic fields for various EoSs.

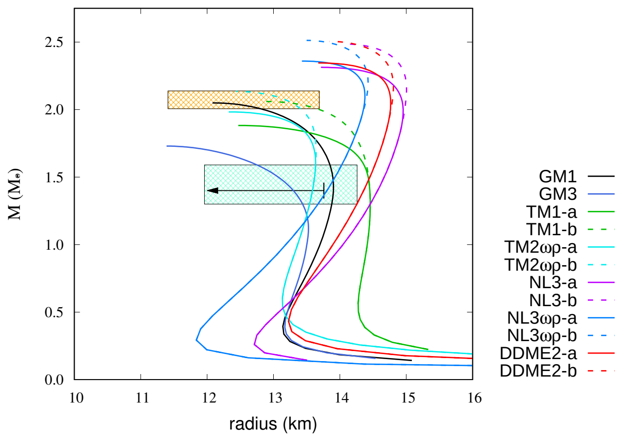


**Figure 4.** MR relations with magnetic fields for various EoSs.

In this work, we considered twelve EoSs to search for the possibility of various slope parameters $L$. Note that we list only discrete values of $\alpha$'s and $\gamma$'s, but they continuously change in certain ranges of values. The results (Table 1) show that two EoSs (GM1 ($L$=93.9 MeV), GM3 ($L$=89.7 MeV)) and two types of TM2$\omega\rho$ ($L$=54.8 MeV) satisfies the conditions of observations. This suggests that high symmetry energy slope $L$ does not necessary saftisfy the conditions of observations. Concerning the compressibility $K$ for two EoSs (GM1 ($K$=300 MeV), GM3 ($K$=240 MeV)) and two types of TM2$\omega\rho$ ($K$=281.7 MeV), even if the value of $K$ is

**Table 1.** The parameter sets ($\alpha$ and $\gamma$) which satisfy observational constraints for various EoSs.

| | GM1 | GM3 | TM2$\omega\rho$-a | TM2$\omega\rho$-b |
|---|---|---|---|---|
| $\gamma$ | | | $\alpha$ | |
| 2 | | | 0.06 | 0.03, 0.04, 0.05, 0.06 |
| 3 | 0.02 | 0.04, 0.05 | 0.02, 0.03, 0.04 | 0.01, 0.02, 0.03 |
| 4 | 0.01 | 0.01, 0.02 | 0.01 | 0.01 |
| 5 | 0.01 | 0.01 | | |

higher than that in experimental measurements, we could suggest their possibilities of satisfing obserbational constraint from the results of the present work.

## 4. Summary

We searched for optimum EoSs with hyperons by calculating MR relations by changing shapes of magnetic field strengths as a function of baryon number density. We studied how the MR relations are changed by $\alpha$ and $\gamma$ parameters in the magnetic field strength. When $\alpha$ is increased, the internal magnetic field is increased gradually, and mass and radius are increased accordingly. When $\gamma$ is increased, the mass and radius are suddenly changed after a certain point. By changing the parameters $\alpha$ and $\gamma$, four EoSs meet the requirements determined by two conditions of observations and come into the allowed range of measured pulsars and GW170817.

Finally, we point out the following: Even if only unique EoS is microscopically determined by the constituents of particles, a NS has its own macroscopically individual character, such as internal magnetic fields and rotation. Therefore, its mass and radius are not determined solely by microscopically calculated EoSs. Thus, MR-relations should be considered by taking care of also the macroscopical characters of individual NSs.

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