Femtoscopy of the J/ψ-nucleon interaction

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Abstract. I discuss the prospects of using femtoscopy in high-energy proton-proton and heavy-ion collisions to learn about the low-energy J/ψ-nucleon interaction. Femtoscopy is a technique that makes it possible to obtain spatiotemporal information on particle production sources at the femtometer scale through measurements of two-hadron momentum correlation functions. These correlation functions also provide information on low-energy hadron-hadron forces as final-state effects. In particular, such correlation functions give access to the forward scattering amplitude. One can express the forward amplitude as the product of the J/ψ chromopolarizability and the nucleon's average chromoelectric gluon distribution, the latter being relevant to the problem of the origin of the nucleon mass. I will present the results of a recent study using the information on the J/ψ-nucleon interaction from lattice QCD simulations to compute J/ψ-nucleon correlation functions. The calculated correlation functions show clear sensitivity to the final-state interaction. I conclude discussing open issues regarding the use of the effective range expansion formula to fit experimental data for small scattering lengths and large effective range parameters.

1 Introduction and Motivation

The interaction of a heavy quarkonium, like the J/ψ, with a nucleon at low energies has a unique place in the wealth of low-energy hadron-hadron interactions. It is unique because the quarkonium-nucleon interaction cannot happen via a one-light-meson exchange, which is the dominant component in one of the most studied hadron-hadron interactions, the nucleon-nucleon interaction. In a one-light-meson exchange at least one valence light quark is involved, but a heavy quarkonium and the nucleon do not share light valence quarks, so it cannot happen. The interaction has to involve, in one way or another, multigluon exchange. Indeed, the low-energy quarkonium-nucleon forward scattering amplitude can be written as a product of the quarkonium-gluon interaction and a matrix element of gluon fields in the nucleon [1–9]. This matrix element comes from the trace of the QCD energy-momentum tensor [10–12]; the trace results from a quantum anomaly and relates to the emergence of hadron masses and the phenomenon of dynamical chiral symmetry breaking [13, 14]. Further interest in the quarkonium-nucleon interaction, particularly in the J/ψ-nucleon (J/ψ-N), is motivated by planned experiments in different laboratories, for example, JLab, FAIR, and NICA, to study J/ψ propagation in a nuclear medium. Such experiments will address questions related to the formation of exotic nuclear-bound states [15–22] and cold matter versus quark-gluon plasma effects in relativistic heavy-ion collisions [23].

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Quarkonium production in electro-and photo-production on a proton provides a means of accessing the J/ψ-N scattering amplitude [24]. Two very recent analyses [25, 26] of a 2019 Glue-X photoproduction experiment at JLab [27] extracted, using the vector meson dominance (VMD) model, a value of the order of 10^{-3} fm for the J/ψ-N scattering length. This value is considerably smaller than most of the theoretical predictions. However, as the authors of Refs. [25, 26] pointed out, their extraction has significant uncertainties. There are uncertainties related to the extrapolation of the data to the forward direction, as the kinematics of the production process forbids direct access to the forward amplitude. There is also an issue related to the fact that the J/ψ might not have had enough time to form in the process, i.e., the VMD process might be incomplete in that experiment. A recent study [28] has shown that VMD, actually, fails for heavy vector mesons like the J/ψ. An explanation, not based on the VMD model, of the Glue-X data was provided in Ref. [29], in that the J/ψ production occurs via $\Lambda_c\overline{D}^*$ intermediate states.

A promising alternative [30] to J/ψ electro-and photo-production is femtoscopy. Femtoscopy is a technique used in ultrarelativistic proton-proton and heavy-ion collisions to obtain spatio-temporal information on particle production sources at the femtometer scale through measurements of two-hadron momentum correlation functions [31, 32]. Remarkably, these correlation functions also provide information on low-energy hadron-hadron forces as final-state effects [33, 34]. In particular, femtoscopy gives direct access to the quarkonium-nucleon forward scattering amplitude, a feature relevant to the gluon distribution in the nucleon; in femtoscopy, there are no kinematic constraints forbidding forward scattering. In addition, the J/ψ-N correlation functions are not affected by the Coulomb force and quantum statistics, effects that can overwhelm the strong interaction between two hadrons. These and other aspects are the subjects of the present communication. Specifically, I discuss the results of a recent study [30] using the information on the J/ψ-nucleon interaction from lattice QCD simulations to compute J/ψ-nucleon correlation functions. In particular, I will show how measurements of two-particle momentum correlation functions give access to the matrix element of gluon fields in the nucleon. I will also discuss a possible problem in the use of the effective range expansion formula to fit experimental data. Ref. [35] is a recent publication that discusses the capabilities of femtoscopy to extract hadron-hadron interactions in ultrarelativistic proton-proton collisions at LHC.

2 J/ψ-N femtoscopic correlation function

In femtoscopy, the observable of interest is a two-hadron correlation function $C(\boldsymbol{p}_1, \boldsymbol{p}_2)$ of measured hadron momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ [31, 32]. The extraction of the experimental correlation function involves computing the ratio of two yields, the coincidence yield, formed by pairs with a given relative momentum coming from a single collision event, and an uncorrelated yield formed by pairs with the same relative momentum but collected from different collision events. If the ratio is equal to unity, there is no correlation between the two hadrons. This ratio is usually expressed in terms of the two-hadron relative momentum in their center-of-mass frame, $\boldsymbol{k} = \boldsymbol{p}_1 = -\boldsymbol{p}_2$. For the theoretical interpretation of the experimental correlation function one uses the Koonin-Pratt (KP) formula [33, 36]:

$$C(k) = \int d^3r \, S_{12}(\boldsymbol{r}) \, |\psi(\boldsymbol{k}, \boldsymbol{r})|^2, \tag{1}$$

where $k = |\boldsymbol{k}|$, $\psi(\boldsymbol{k}, \boldsymbol{r})$ the two-hadron relative wave function, and $S_{12}(\boldsymbol{r})$ the two-hadron relative distance distribution in the pair's frame (a.k.a. the emission source). This is an approximate formula; in-depth discussions of the assumptions and approximations behind this formula can be found in Refs. [31, 32, 37, 38].

To make predictions for $C(k)$, one needs a model for computing $\psi(\boldsymbol{k},\boldsymbol{r})$. Lattice QCD studies [39–47] revealed that the J/ψ-N interaction is attractive and not very strong. This allows us to assume that $S-$waves dominate the interaction. Therefore, one can separate from $\psi(\boldsymbol{k},\boldsymbol{r})$ the $l = 0$ component, which contains the effects of the strong interaction, and write $\psi(\boldsymbol{k},\boldsymbol{r})$ as:

$$\psi(\boldsymbol{k},\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \psi_0(k,r) - j_0(kr), \tag{2}$$

where $j_0(kr)$ is the $S-$wave component of the non-interacting wave function, a spherical Bessel function. Moreover, it is a common practice in the experimental extraction of $C(k)$ to take a one-parameter Gaussian form for the emission source [35]: $S_{12}(r) = 1/(4\pi R^2)^{3/2}\,\exp(-r^2/4R^2)$. Under these assumptions, the KP formula can be written as:

$$C(k) = 1 + \frac{4\pi}{(4\pi R^2)^{3/2}} \int_0^\infty dr\, r^2\, e^{-r^2/4R^2} \left[|\psi_0(k,r)|^2 - |j_0(kr)|^2\right]. \tag{3}$$

When most of the emitted hadron pairs are within the interaction range, which is the case of sources with small radii R, one needs the pair wave function $\psi_0(k,r)$ in the entire range $0 \le r \le \infty$ of integration in Eq. (3). Otherwise, when most of the emitted pairs are not under the influence of the interaction, then one can replace $\psi_0(k,r)$ with its asymptotic form:

$$\psi_0^{asy}(k,r) = \frac{\sin(kr+\delta_0)}{kr} = e^{-i\delta_0}\left[j_0(kr) + f_0(k)\frac{e^{ikr}}{r}\right] \quad \text{with} \quad f_0(k) = \frac{e^{i\delta_0}\sin\delta_0}{k}, \tag{4}$$

where $f_0(k)$ is the scattering amplitude and δ_0 the phase shift. Replacing $\psi_0(k,r)$ by $\psi_0^{asy}(k,r)$ in Eq. (3), one obtains for $C(k)$ [34]:

$$C(k) \;=\; 1 + \frac{|f_0(k)|^2}{2R^2}\left(1 - \frac{r_0}{2\sqrt{\pi}R}\right) + \frac{2\mathrm{Re}f_0(k)}{\sqrt{\pi}R}F_1(2kR) - \frac{\mathrm{Im}f_0(k)}{R}F_2(2kR), \tag{5}$$

where

$$F_1(x) = \frac{1}{x}\int_0^x dt\, e^{t-x}, \qquad F_2(x) = \frac{1}{x}\left(1 - e^{-x^2}\right). \tag{6}$$

The term $1 - r_0/2\sqrt{\pi}R$ in Eq. (5) is a correction term accounting for the error one makes when replacing $\psi_0(k,r)$ by $\psi_0^{asy}(k,r)$, where r_0 the effective range parameter of the effective range expansion (ERE) formula for $f_0(k)$:

$$f_0(k) = \frac{1}{k\cot\delta_0 - ik} \xrightarrow{k\approx 0} \frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0\,k^2 - ik}, \tag{7}$$

and a_0 is the scattering length. In the derivation of this correction term, one assumes that the effective range parameter r_0 is of the order of the interaction range, which is not always the case. I will come back to this point shortly ahead. The expression in Eq. (5) is known as the Lednicky-Lyuboshits (LL) model.

In summary, when one replaces $\psi_0(k,r)$ by $\psi_0^{asy}(k,r)$ and uses the ERE for $f_0(k)$, Eq. (5) depends only on three numbers: a_0 and r_0, related to the hadron-hadron interaction, and R, related to the emission source. It is a universal formula, in the sense that no further knowledge is required to compute the correlation function.

3 $C(k)$ and the gluon distribution in the nucleon

When one expresses $C(k)$ is terms of the scattering amplitude $f_0(k)$ as in Eq. (5), one can relate $C(k)$ at small k to the matrix element of the average chromoelectric gluon distribution in the nucleon, $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$, where $\boldsymbol{E}^a, a = 1, \cdots, 8$, is the chromoelectric gluon field, g the strong coupling constant, and $|N\rangle$ the on-shell nucleon state. The matrix element $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$ is related to to the trace of the QCD energy-momentum tensor and the nucleon mass through the inequality [48]:

$$\langle N| \left[(g\boldsymbol{E}^a)^2 - (g\boldsymbol{B}^a)^2\right] |N\rangle = -\frac{1}{2}\langle N|g^2 G^a_{\mu\nu} G^{a\mu\nu}|N\rangle = \frac{16\pi^2}{9} m_N \leq \langle N|(g\boldsymbol{E}^a)^2|N\rangle.0 \tag{8}$$

where, in the chiral limit [10, 11, 49–51]

$$T^\mu_\mu(x) = -\frac{9}{32\pi^2}\, g^2 G^a_{\mu\nu}(x) G^{a\mu\nu}(x), \tag{9}$$

The normalization of the nucleon state is such that the expectation value of $\langle N|T^{00}|N\rangle$ is the nucleon energy [21]. For finite current-quark quark masses, Eq. (9) contains the contribution of the σ-term, which is of the order of 60 MeV [21].

If the J/ψ-N interaction is weak and $S-$wave dominated [39–47], the forward amplitude at small values of k is real and determined by the scattering length, $f_0(k) \simeq -a_0$. On the other hand, using the QCD multipole expansion, one can express the forward scattering amplitude in terms of $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$ and write [3, 8, 21]:

$$a_0 = \frac{\mu}{4\pi}\, \alpha_\psi\, \langle N|(g\boldsymbol{E}^a)^2|N\rangle, \tag{10}$$

where μ is the J/ψ-N reduced mass and α_ψ the J/ψ chromopolarizability. Given these results, one can relate $C(k)$ to $\langle N|(g\boldsymbol{E})^2|N\rangle$. A particularly enlightening expression for $C(k)$ at small values for k can be obtained when $a_0/R \ll 1$, namely:

$$C(k^2) \underset{k\to 0}{\simeq} 1 - \frac{1}{2\pi^{3/2}} \left(1 - \frac{8}{3} k^2 R^2\right) \frac{\mu}{R}\, \alpha_\psi\, \langle N|(g\boldsymbol{E}^a)^2|N\rangle, \tag{11}$$

in which $F_1(x) \simeq 1 - 2/3\, x^2$ and $F_2(x) \simeq x$ was used. It is important to stress that this result for $C(k^2)$ was obtained under several assumptions, the most important ones being the validity of the multipole expansion and of the LL model, and $a_0/R \ll 1$.

The main result is: a measurement of the J/ψ-N correlation function at small values of k allows us to obtain the product $\alpha_\psi\, \langle N|(g\boldsymbol{E}^a)^2|N\rangle$. If α_ψ is known from an independent source, one obtains $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$. The argument can be reversed, one can obtain an approximate value for α_ψ if one uses Eq. (8), i.e. $\langle N|(g\boldsymbol{E}^a)^2|N\rangle = 9/(16\pi^2 m_N)$. Away from the LL model, the link between $C(k)$ and $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$ is less direct. One would still have access to information on the interaction, e.g. on scattering parameters, but the theoretical interpretation of the data would be more subtle.

4 Predictions for $C(k)$

We compute $C(k)$ in Eq. (5) using results from lattice QCD simulations. Specifically, we use the results of Refs. [39–41], which provide values for the $S-$wave scattering length a_0 and effective range r_0 parameters, and of Ref. [42] which gives, in addition, an $S-$wave J/ψ-N potential extracted with the HALQCD method. These lattice results were obtained either with quenched gluon configurations [39, 41] or large pion masses [40, 42]. Therefore, they need to be extrapolated to the physical pion mass. We employ the extrapolated results obtained with the quarkonium-nucleon effective field theory (QNEFT) of Ref. [52]. The QNEFT obtained expressions for a_0 and r_0 at leading order (LO) and next-to-leading order (NLO) in the pion mass. The QNEFT extrapolations considered spin-1/2 and spin-3/2 degeneracy, an approximation that requires further scrutiny. The QNEFT potential contains contact terms and a long-range, model-independent van der Waals type of potential of range $1/2m_\pi \simeq 0.7$ fm, with a strength controlled by the J/ψ chromopolarizability, namely:

$$V_{\rm vdW}(r) = \frac{3g_A^2}{128\pi^2 F^2}\left\{c_{di}\left[6 + m_\pi r(2 + m_\pi r)(6 + m_\pi r(2 + m_\pi r))\right] + c_m m_\pi^2 r^2 (1 + m_\pi r)^2\right\} \frac{e^{-2m_\pi r}}{r^6}, \tag{12}$$

where $g_A = 1.27$ is the nucleon axial charge, $F = 93$ MeV the pion decay constant, and c_{di} and c_m are low-energy constants that can be determined by using the QCD trace anomaly [9]. The explicit expressions for the couplings c_{di} and c_m are given in Eq. (5) of Ref. [52]; they depend on α_ψ, which is the only free parameter in Eq. (12). Ref. [52] extracted the value $\alpha_\psi = 0.24$ GeV^{-3} by fitting $V_{\rm vdW}(r)$ to the $J/\psi\, N$ HALQCD potential [42]. Using the lattice values for a_0 and r_0 from Refs. [39–41], the corresponding QNEFT-extrapolated values are -0.71 fm $\leq a_0 \leq -0.35$ fm, and 1.29 fm $\leq r_0 \leq 1.35$ fm.

We note that since the range of the potential in Eq. (12) is $1/2m_\pi \simeq 0.7$ fm and its strength at $r = 1/2m_\pi$ is rather small, 3 MeV, and in addition taking into account the extrapolated values of a_0 and r_0, one can use of Eq. (5) to compute $C(k)$ for $R = 1$. This value of the source radius is a typical radius used by the Alice Collaboration for pp collisions; see for example the recent measurement of ϕ-N correlation function reported in Ref. [53]. Moreover, since the value of the effective range r_0 does not vary much within the uncertainties of the QNEFT extrapolation, results will be shown for $r_0 = 1.3$ fm only.

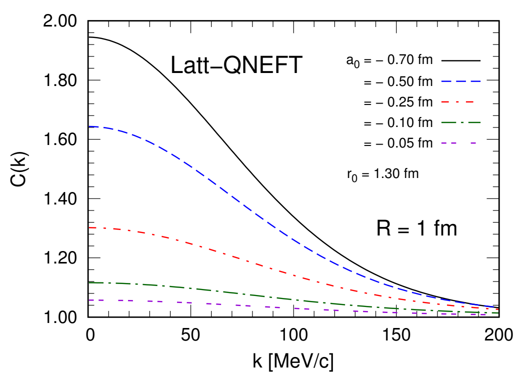

Figure 1. J/ψ-N correlation function computed with Eq. (5) for a source radius of $R = 1$ fm. The a_0 and r_0 values are from the lattice QCD simulations of Refs. [39–41], extrapolated to the physical pion mass with the QNEFT of Ref. [52].

Figure 1 displays results for $C(k)$. The figure reveals the expected trend about correlation strength as a function of the scattering length a_0: the smaller the value of a_0, the weaker the correlation. Moreover, the correlation strengths are comparable to those extracted for the ϕ-N system in Ref. [53] for similar values of the scattering length. Further results for $C(k)$ are shown in Ref. [30].

5 $C(k)$ and the effective range expansion

There are situations that the scattering length a_0 is not much larger than the physical range $R_{\rm pot}$ of the potential. In such situations, the effective-range expansion (ERE) is not useful for assessing the physical range of the potential, as the effective range r_0 can be very different from $r_{\rm pot}$. The ERE provides a useful low-energy expansion of the scattering amplitude only when $a_0/r_{\rm pot} \gg 1$—for a recent discussion and early references on this issue, see Ref. [54].

To exemplify with a concrete and simple example, I consider the spherical finite-well model of Ref. [55]. This model addresses the J/ψ-N interaction within the hadro-charmonium picture of Ref. [56], in that the J/ψ interacts as a compact object within the volume of a light hadron. This model was used in Ref. [30] to compute the J/ψ correlation function with the full wave function. The potential is given by:

$$V(r) = \begin{cases} -\frac{2\pi}{3}\left(\frac{\alpha_\psi}{R_N^3}\right) m_N & \text{for} \quad r < R_N \\ 0 & \text{for} \quad r > R_N \end{cases} . \tag{13}$$

Table 1 shows the values of a_0 and r_0 for this potential for different values of the J/ψ polarizability α_ψ and range of the potential equal to $R_N = 1$ fm. One sees that the range of the potential $R_{\rm pot} = R_N$ and r_0 can be very different. Moreover, one sees that the difference between $R_{\rm pot}$ and r_0 increases a lot when a_0 gets much smaller than $R_{\rm pot}$.

Table 1. ERE parameters for a spherical well with $R_N = 1$ fm for different α_ψ.

α_ψ [GeV^{-3}]	0.10	0.20	0.5	1.0
a_0 [fm]	−0.02	−0.04	−0.1	−0.2
r_0 [fm]	22.0	11.0	4.8	2.7
$\tilde{r}_0$ [fm]	1.0	1.0	1.0	0.9

Ref. [54] shows that, instead of the traditional ERE expansion, Eq. (7), one obtains a more useful expansion of the scattering amplitude by expanding the function $\tan\delta_0/k$ for small k:

$$\frac{1}{k}\tan\delta_0 \sim -a + \frac{1}{6}\tilde{r}_0^3 k^2 + \cdots , \tag{14}$$

in which $\tilde{r}_0$ is a “new effective range´´ parameter. For the finite-well potential in Eq. (13), $\tilde{r}_0$ is given by [54]:

$$\tilde{r}_0^3 = R_N^3 - 3a_0^2 R_N. \tag{15}$$

In the limit of $a_0 \to 0$, $\tilde{r}_0$ becomes the actual range $R_{\rm pot} = R_N$ of the potential and $\tilde{r}_0$ remains close to $R_{\rm pot}$ for $a_0 \ll R_{\rm pot}$. As a_0 increases, $\tilde{r}_0$ changes sign. In Table 1 one can see this trend of $\tilde{r}_0$ being closer to the physical range of the potential when a_0 is small.

Work is underway to assess the implication for the femtoscopic correlation function $C(k)$ when using this alternative low-energy expansion of the scattering amplitude.

6 Conclusions and perspectives

I presented results from a prospective study [30] of using femtoscopy in high-energy proton-proton and heavy-ion collisions for learning about the low-momentum J/ψ-nucleon (J/ψ-N) interaction. The motivation for such a study stems from the fact that femtoscopic correlation measurements offer the opportunity to access information on low-energy hadron-hadron forces inaccessible by other means. Within the QCD multipole expansion framework, the forward J/ψ-nucleon scattering amplitude is given in terms of the J/ψ chromopolarizability and a matrix element involving the square of the chromoelectric field in the nucleon. This matrix element comes from the QCD trace anomaly and is linked to the origin of hadron masses. Given the present knowledge about the J/ψ-N interaction coming from lattice QCD simulations, this study revealed that sizable correlations can be expected. The strength of the correlation for low values of the relative J/ψ-N momentum is similar to that recently extracted by the ALICE Collaboration for the ϕ-N system, which in some respects is a system similar to the J/ψ-N.

I have not addressed experimental issues that can impact the extraction of a low-momentum J/ψ-nucleon correlation function. Nontrivial issues include source form and size, momentum resolution, and non-femtoscopic correlations. Notwithstanding these issues, I hope that the positive prospects of this theoretical study motivate an experimental study as well. In this respect, the issue regarding the use of the effective range expansion formula to fit experimental data for small scattering lengths and large effective range parameters is under study.

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