Background Dependence of Pole Position in Kaon Photoproduction

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Abstract. Pole position has become a new important parameter to describe the properties of nucleon resonance. Compared to the Breit-Wigner parameters, the pole position is less model dependent. The shape of resonance based on our previous formalism is strongly affected by the hadronic form factors. We study the effect of different background parameters on the extracted pole position. We have also used different hadronic form factors to see the effect on the background dependence of pole position. We obtained the necessary parameters by fitting to about 9000 experimental data points. We expect to find appropriate hadronic form factors that lead to the background independence. Therefore, this study will shed a light on the choice of the appropriate hadronic form factors.

1. Introduction

In recent years, the Particle Data Group (PDG) has been prioritizing the pole position over the Breit-Wigner parameters. It is shown in the particle listing of PDG that the pole position of a resonance is written before the Breit-Wigner parameter. Theoretically, pole position is less model dependent compared to the Breit-Wigner paramaterization [1, 2]. Therefore, it provides a good benchmark in many theoretical model that gives resonances position. However, in practice, it could depend on the background by various reasons. Furthermore, there has been a study showing that both properties are needed to describe the resonances [3]. On the other hand, the formalism of consistent interaction using the Lagrangian effective theory to evaluate the scattering amplitude for high spin resonances in kaon photoproduction has been succsesfully constructed and proven to be better than other models [4, 5]. However, there are some problems rising due to the strong momentum dependence in the scattering amplitude. Two of these problems are the false position of resonances and the divergences of amplitude at higher energies. This defect does not occur in our recent study because it is concealed by the background and it is suppressed by the hadronic form factor. Vrancx *et al.* [6] tried to attack this defect by using the suitable hadronic form factor that leads to the correct resonances position and convergence at high energies. This is the reason to study the background independence of pole position of the nucleon resonances in the $\gamma p \to K^+\Lambda$ reaction by using various hadronic form factors.

We use our recent model in kaon photoproduction [5, 7] as the benchmark model to investigate the effect of diferent form factors and variation of the background. The corresponding Feynman diagrams used in this study are shown in Figure 1. This model involves nucleon resonances with spins up to 9/2 and the spin-1/2 and -3/2 hyperon resonances as the background. The background of course varies by the involvement of spin-3/2 hyperons. Furthermore, we use three

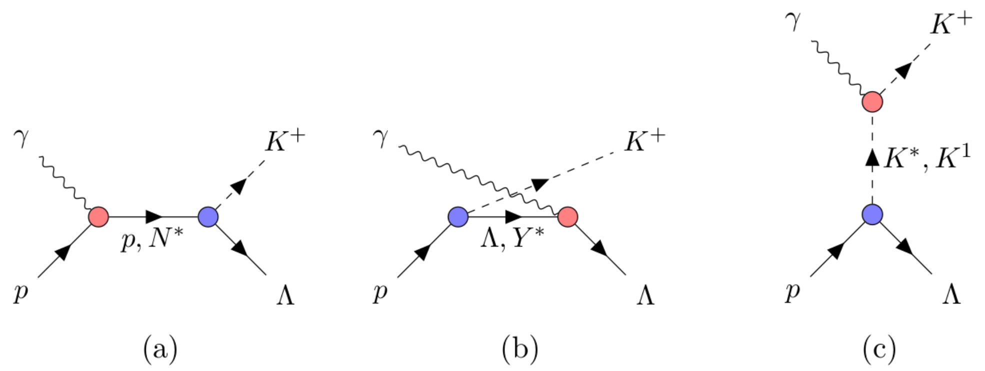

Figure 1. Feynman diagrams for kaon photoproduction $\gamma + p \to K^+ + \Lambda$. Each diagram describes the (a) s-channel, (b) u-channel and (c) t-channel.

different form factors. Each model is fitted to around 9000 experimental data. The extracted Breit-Wigner parameter is used to evaluate the pole position. The effect of certain resonances will be presented by the plots of total cross section.

2. The Pole Position

In principle, the pole position is evaluated at the zero value of denominator in the scattering amplitude. In this way, the scattering amplitude of a resonance will be very large near the pole position. As a consequence, the background amplitude will not affect the property of resonances evaluated at the pole position. Thus, the projection of experimental data to the resonance region must have certain value of pole position that is not affected by the background.

The pole position is defined by

$$\sqrt{s_R} = W_R = M_{\rm pole} - i\Gamma_{\rm pole}/2 \ , \tag{1}$$

which can be easily obtained by imposing the denominator of scattering amplitude to vanish, i.e.,

$$s_R - m_R^2 + im_R\Gamma(s_R) = 0 \ . \tag{2}$$

In this work we use the energy dependent width [8] that is directly proportional to the total width Γ_R. As a consequence, we cannot solve this equation analitically. Only the numerical calculation works in this case. The initial values of m_R and Γ_R are obtained from the particle listing of the Particle Data Book. These values are actually the Breit-Wigner parameters that were used as free parameters in the fitting process, where the upper and lower limits are bounded to the PDG values. The obtained parameters are used to calculate the pole positions of resonances.

3. The Hadronic Form Factor

The problem of divergent scattering amplitude of the resonances in high energy region is an old issue. The well known solution is the introduction of hadronic form factors to suppress this divergence. The various form factors were constructed mainly by specific purpose and sometimes the formula becomes trivial. The most common form factor widely used is the dipole form factor. In our previous study, we used the dipole hadronic form factor for all resonances that reads

$$F_{\rm dip}(s,\Lambda) = \frac{\Lambda^4}{(s-m_R^2)^2 + \Lambda^4} \ , \tag{3}$$

Table 1. Properties of the spin-1/2 and -3/2 hyperon resonances obtained from Particle Data Book [1]. Here J^P is the spin-parity of the resonance and the units of mass and width are expressed in MeV.

Y^*	J^P	Mass	Width
$\Lambda(1405)$	$1/2^-$	$1405^{+1.3}_{-1.0}$	50.5 ± 2
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0
$\Lambda(1600)$	$1/2^+$	1600^{+100}_{-40}	150 ± 100
$\Lambda(1670)$	$1/2^-$	1670 ± 10	35 ± 15
$\Lambda(1690)$	$3/2^-$	1690 ± 5	60 ± 10
$\Lambda(1800)$	$1/2^-$	1800^{+50}_{-80}	300 ± 100
$\Lambda(1810)$	$1/2^+$	1810^{+40}_{-60}	150 ± 100
$\Lambda(1890)$	$3/2^+$	1890^{+20}_{-40}	100^{+100}_{-40}

Y^*	J^P	Mass	Width
$\Sigma(1385)$	$3/2^+$	1383.7 ± 1.0	36 ± 5
$\Sigma(1660)$	$1/2^+$	1660 ± 30	100^{+100}_{-60}
$\Sigma(1670)$	$3/2^-$	1670^{+15}_{-5}	60 ± 20
$\Sigma(1750)$	$1/2^-$	1750^{+50}_{-20}	90^{+70}_{-30}
$\Sigma(1880)$	$1/2^+$	1821 ± 17	300 ± 59
$\Sigma(1940)$	$3/2^-$	1940^{+10}_{-40}	220^{+80}_{-70}
$\Sigma(2080)$	$3/2^+$	2091 ± 7	186 ± 48

where Λ is the cutoff parameter and s is the square of total c.m. energy. Unfortunately in our recent formalism this form factor is too weak to suppress the strong momentum dependence of high spin resonances amplitude. However, as mentioned before, this problem can be also suppressed by the background. However, suppresing the divergence in this way could lead to the background dependence of pole position, since the resonances-like structure is obtained with the help of the background.

Our formalism of high spin resonances interaction is based on the Pascalutsa [9] and Vrancx *et al.* [6] works. In their paper, Vrancx *et al.* discuss the suitable form factor for this formalism. They claim that it can be preserve the position of resonances in terms of their mass and width. The form factor is called multidipole-Gauss as it is made by combining the multidipole and Gaussian form factors. It will be seen that it is actually not the multidipole, but rather the modified form factor that preserves the interpretation of decay width as the FWHM of the resonance. The form factors reads

$$F_{\rm mmG}(s,\Lambda) = \left\{ \frac{m_R^2 \tilde{\Gamma}_R^2}{(s-m_R^2)^2 + m_R^2 \tilde{\Gamma}_R^2} \right\}^{J-1/2} \exp\left\{ -\frac{(s-m_R^2)^2}{\Lambda^4} \right\} \tag{4}$$

where J is the spin of resonance and $\tilde{\Gamma}_R$ is the modified width that reads as

$$\tilde{\Gamma}_R = \frac{\Gamma_R}{\sqrt{2^{1/(2J)}-1}} \; . \tag{5}$$

This form factor can be modified by subtituting $m_R^2 \tilde{\Gamma}_R^2$ with the cutoff parameter Λ to obtain the actual multidipole-Gauss one. Therefore, along with those two kind of form factors we try to test this form factor in our recent model of kaon photoproduction with two variations of the background.

4. Results and discussion

The hyperon resonances (Y^*) used in the background can be seen in Table 1. There is two variations of the background used in this work, i.e., the model without the spin-3/2 hyperon resonances and that with the spin-3/2 hyperon resonances. Along with this, we fitted 3 kinds of hadronic form factors to test the background independence of resonances pole position. The obtained pole positions compared to the data from PDG are shown in Table 2. From this Table we can clearly see that in all models the resonances $N(1520)$, $N(1675)$, $N(1680)$ and $N(1720)$ are consistent with the PDG. There are a number of resonances which have large discrepancies with the PDG values, but consistent in all models, i.e., the $N(1880)$, $N(1990)$, $N(2000)$, $N(2060)$ and $N(2220)$. The result can be used to evaluate the actual pole position of the resonances or for further study of the resonance role in our model.

In this work our concern is not to compare our result with the PDG, but instead we want to see the background dependence of pole position. Thus, we try to calculate the absolute value of of pole position difference of the two different backgrounds, in each of hadronic form factors, i.e.,

$$|\Delta W_R/W_R| = \sqrt{\frac{[m_R(1)-m_R(2)]^2+[\Gamma_R(1)-\Gamma_R(2)]^2/4}{m_R^2(1)+\Gamma_R^2(1)/4}}\ , \tag{6}$$

where the notations (1) and (2) refer to the models without and with spin-3/2 hyperon resonances, respectively. Table 3 presents the result of numerical calculation. In general, we found that models using multidipole-Gauss form factor are more independent than those using other two form factors. The use of modified multidipole-Gauss form factors leads to the worst result. By looking at Table 3 one can say that in general the result is consistent for all models because the largest discrepancy is only 4.4%. However, this is not completely true, since we have just involved the spin-3/2 hyperon resonances. Furthermore, the inclusion one nuleon resonance might become a background for another nucleon resonance. Thus, we trivially set the accepted difference to be 1%. As a consequence, based on the Table 3, in the model with modified multidipole-Gauss form factor more nucleon resonances are independent to the change of background compared to dipole one. Interestingly, the $N(1535)$ resonance has the difference value larger than 1% for all form factor models. This indicates that the $N(1535)$ is probably

Table 2. Comparison of the $M_{\rm pole}$ and $\Gamma_{\rm pole}$ obtained from Particle Data Book [1] and those obtained from calculation of the extracted mass and width of 6 different models with 3 different form factors and 2 variations of background. The notations (1) and (2) in the column title refer to the model without and with spin-3/2 hyperon resonances, respectively.

Resonans	PDG		$F_{\rm dip}$ (1)		$F_{\rm dip}$ (2)		$F_{\rm mmG}$ (1)		$F_{\rm mmG}$ (2)		$F_{\rm mG}$ (1)		$F_{\rm mG}$ (2)	
	$M_{\rm pole}$	$\Gamma_{\rm pole}$	$M_{\rm pole}$	$\Gamma_{\rm pole}$	$M_{\rm pole}$	$\Gamma_{\rm pole}$	$M_{\rm pole}$	$\Gamma_{\rm pole}$	$M_{\rm pole}$	$\Gamma_{\rm pole}$	$M_{\rm pole}$	$\Gamma_{\rm pole}$	$M_{\rm pole}$	$\Gamma_{\rm pole}$
N(1440)	1369	189	1305	174	1305	174	1382	177	1344	160	1305	174	1305	174
N(1520)	1510	110	1495	102	1487	115	1477	113	1485	100	1487	115	1487	115
N(1535)	1510	170	1508	169	1475	163	1430	209	1474	225	1474	225	1475	163
N(1650)	1655	135	1604	261	1613	232	1653	175	1639	183	1627	171	1627	171
N(1675)	1660	135	1640	148	1640	148	1627	166	1627	166	1640	148	1640	148
N(1680)	1675	120	1651	123	1652	123	1645	132	1645	132	1635	130	1642	122
N(1700)	1700	200	1670	129	1692	152	1687	153	1693	153	1598	154	1584	169
N(1710)	1720	230	1658	175	1658	175	1689	64	1689	65	1710	75	1710	75
N(1720)	1675	250	1643	204	1652	188	1639	208	1639	207	1654	186	1658	188
N(1860)	1830	250	1856	221	1862	217	1851	225	1844	229	1793	218	1787	228
N(1875)	1875	200	1714	244	1787	277	1779	252	1771	269	1740	255	1724	248
N(1880)	1870	220	1786	298	1786	298	1770	285	1766	282	1786	298	1786	298
N(1895)	1907	100	1879	208	1876	225	1873	245	1872	251	1875	230	1875	233
N(1900)	1920	215	1867	253	1862	261	1824	307	1820	311	1867	253	1867	253
N(1990)	2030	240	1866	241	1875	237	1908	279	1841	252	1900	220	1900	220
N(2000)	2030	380	1933	274	1933	274	1859	265	1902	283	1904	284	1904	284
N(2060)	2030	400	1857	341	1857	341	1874	335	1858	325	1836	329	1836	329
N(2120)	2115	345	1925	349	1885	355	1911	341	1911	341	1946	361	1951	363
N(2190)	2075	450	2046	252	2027	245	2028	249	2045	252	1965	227	1970	233
N(2220)	2170	480	2020	228	2020	228	2031	226	2048	221	2048	221	2048	221
N(2250)	2200	450	2060	274	2085	266	2087	299	2089	298	2138	285	2120	291

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Table 3. Comparison of the absolute value of the nucleon resonance pole position diference between two different background models for each of the hadronic form factors ($|\Delta W_R/W_R|$). The result is given in percentage (%).

Resonans	J^P	$F_{\rm dip}$	$F_{\rm mmG}$	$F_{\rm mG}$
N(1440)	$1/2^+$	0	2.8	0
N(1520)	$3/2^-$	0.7	0.7	0
N(1535)	$1/2^-$	2.2	3.1	2.1
N(1650)	$1/2^-$	1.1	0.9	0
N(1675)	$5/2^-$	0	0	0
N(1680)	$5/2^+$	0.1	0	0.5
N(1700)	$3/2^-$	1.5	0.4	1
N(1710)	$1/2^+$	0	0	0
N(1720)	$3/2^+$	0.7	0	0.2
N(1860)	$5/2^+$	0.3	0.4	0.4
N(1875)	$3/2^-$	4.4	0.7	0.9

Resonans	J^P	$F_{\rm dip}$	$F_{\rm mmG}$	$F_{\rm mG}$
N(1880)	$1/2^+$	0	0.2	0
N(1895)	$1/2^-$	0.5	0.2	0.1
N(1900)	$3/2^+$	0.3	0.2	0
N(1990)	$7/2^+$	0.5	3.6	0
N(2000)	$5/2^+$	0	2.4	0
N(2060)	$5/2^-$	0	0.9	0
N(2120)	$3/2^-$	2.1	0	0.3
N(2190)	$7/2^-$	0.9	0.8	0.3
N(2220)	$9/2^+$	0	0.8	0
N(2250)	$9/2^-$	1.2	0.1	0.9

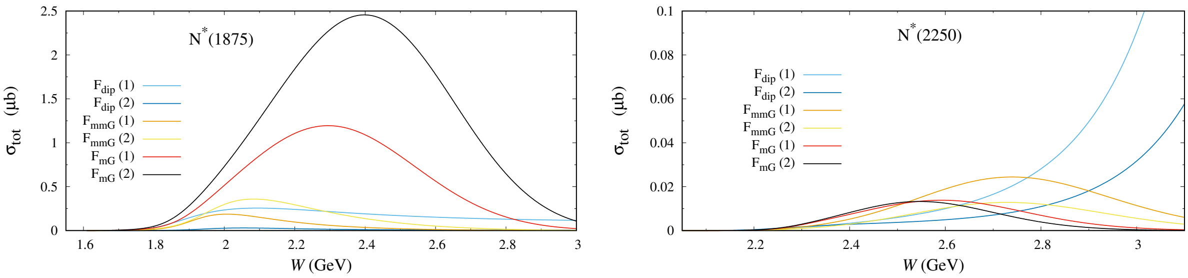

Figure 2. Comparison of the total cross section of $N(1875)$ (left) and $N(2250)$ (right) resonances for models with three different hadronic form factors and two variations of background.

not so important in this reaction.

Figure 2 shows the total cross section for nucleon resonances $N(1875)$ and $N(2250)$, whereas Table 3 for these resonances the modified multidipole-Gauss form factor has the lowest difference. Figure 2 confirms this. For the two resonances the model using this form factor has similar plots, but different in strength. On the other hand, the cross section of $N(1875)$ with multidipole-Gauss form factor has a similar pattern with the modified one. The worst result come from the dipole form factors. For the $N(2250)$ resonance we observe that a divergence in high energy region, but addition of hyperon resonances could suppress it. The similar case is also found for the $N(1875)$ resonance, but the divergence is relatively soft due to the proportionality of momentum dependence to the spin. Furthermore, the result obtained after the inclusion of spin-3/2 hyperon resonances leads to a better resonance-like structure, but it shifts the pole position from its original one, as we can see in Table 3.

5. Summary and conclusion

We have studied the background dependence of pole position of resonances in kaon photoproduction by using 3 different hadronic form factors. The result shows that the multidipole-Gauss form factor is the best for achieving the background independent pole position. However, there is still the lack of knowledge of the source of this problem. Therefore,

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further study to elaborate the source of this problem and to build a suitable form factor, that can improve the model, is strongly advised.

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