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Emittance Growth due to a Small Low-frequency Perturbation

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Emittance Growth due to a Small Low-frequency Perturbation

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Abstract

With a small low-frequency perturbation, the transverse emittance of a storage-ring beam will grow. The growth mechanism due to betatron tune dependence on amplitude is discussed in the Hamiltonian approach. Applications are given when the perturbation is due to Jostlein's scheme[1] of bunch centering as well as ground wave.

1. Introduction

Particle beam in a circular storage ring will experience transverse emittance growth whenever there is a perturbing modulation. The growth is caused mainly by energy spread plus chromaticity and by nonlinear lattice characteristics. The source of the perturbation can be ground waves, Jostlein's bunch centering scheme,[1] rf noise, power noise, etc. The effects of energy plus chromaticity have been analyzed in a previous paper.[2] In this paper, we concentrate only on the nonlinear lattice characteristics or betatron tune dependence on amplitude. The detuning is assumed to be small so that no nonlinear resonances will be encountered. The problem is dealt with in the Hamiltonian approach. The results are applied to the Jostlein's beam-centering scheme and ground motion.

2. The Model

Let X represent the horizontal or vertical position offset of a particle. The equation of motion governing X along longitudinal path length s is

$$\frac{d^2X}{ds^2} + K(s)X = \int d\nu_m a_m \sin\frac{\nu_m s}{R}\sum_{n=0}^{\infty}\delta(s-2\pi nR)\;, \tag{2.1}$$

where $K(s)$ describes the focussing mechanism of the lattice which can be nonlinear, R is the average radius of the storage ring, and ν_m is the perturbing tune (frequency divided by revolution frequency) with a_m the amplitude per unit tune. The perturbation at the j-th turn produces an angular kick

$$\Delta X' = \int d\nu_m a_m \sin 2\pi j\nu_m\;. \tag{2.2}$$

We first define the Floquet variables

$$\left\{\begin{array}{rcl} x &=& \dfrac{X}{\sqrt{\beta}}\;, \\[2mm] d\theta &=& \dfrac{ds}{\nu_0\beta}\;, \end{array}\right. \tag{2.3}$$

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where β is the beta-function at a location and ν_0 is the nominal betatron tune. Equation (2.1) becomes

$$\frac{d^2x}{d\theta^2} + \nu_0^2 x = \int d\nu_m a_m \sqrt{\beta_0}\nu_0 \sin\nu_m\theta \sum_{n=0}^{\infty}\delta(\theta - 2\pi n) , \tag{2.4}$$

with β_0 the beta-function at the location of the kick. The corresponding Hamiltonian is

$$H = \frac{1}{2}p^2 + \frac{1}{2}\nu_0^2 x^2 - xF(\theta) , \tag{2.5}$$

where the perturbing force is

$$F(\theta) = \int d\nu_m a_m \sqrt{\beta_0}\nu_0 \sin\nu_m\theta \sum_{n=0}^{\infty}\delta(\theta - 2\pi n) . \tag{2.6}$$

We next perform a canonical transformation to the action-angle variables (J, ϕ) with the aid of the generating function

$$F_1(x,\phi) = -\frac{1}{2}\nu_0 x^2 \tan\phi . \tag{2.7}$$

The transformation gives

$$\begin{cases} x &= \sqrt{\dfrac{2J}{\nu_0}}\cos\phi , \\ \dfrac{p}{\nu_0} &= -\sqrt{\dfrac{2J}{\nu_0}}\sin\phi . \end{cases} \tag{2.8}$$

For the transformed Hamiltonian, we write specifically

$$H = \nu_0 J - \alpha\frac{4J^2}{\nu_0} - \sqrt{\frac{2J}{\nu_0}}\cos\phi F(\theta) , \tag{2.9}$$

so as to introduce tune dependence on amplitude. This Hamiltonian will be the starting point of our discussion, In the absence of the perturbing force $F(\theta)$, the betatron tune is

$$\nu_\beta = \left.\frac{\partial H}{\partial J}\right|_{F(\theta)=0} = \nu_0 - \alpha A^2 , \tag{2.10}$$

where the betatron amplitude is given by $A = \sqrt{2J/\nu_0}$ and α is the detuning.

In the absence of the perturbing force $F(\theta)$, the Hamiltonian is an invariant, implying that the particle stays on invariant curves in the x-p phase space. These curves are, in fact, circles of constant J. With the perturbing force having an amplitude very much less than the betatron amplitude, or

$$\int d\nu_m a_m \sqrt{\beta_0} \ll \sqrt{\frac{2J}{\nu_0}} , \tag{2.11}$$

we assume that the system remains integrable, at least approximately. The invariant curves will deviate from circles.

3. New Invariant Curves

We try to solve the Hamiltonian in Eq. (2.9) for each perturbing frequency ν_m. For this, we define the *integrated* perturbing amplitude for this particular frequency as

$$\hat{a}_m \equiv a_m \sqrt{\beta} d\nu_m . \tag{3.1}$$

The equation of motion for ϕ is

$$\frac{d\phi}{d\theta} = \frac{\partial H}{\partial J} = \nu_0 + \mathcal{O}\left(\Delta\nu_\beta, \frac{\hat{a}_m}{A}\right) , \tag{3.2}$$

where $\Delta\nu_\beta$ is the nonlinear tune spread due to detuning and is assumed to be small compared with the nominal tune ν_0. These small terms are dropped for the time being. Thus, approximately, we obtain

$$\phi = \phi_0 + \nu_0\theta \ . \tag{3.3}$$

The equation of motion for J is

$$\frac{dJ}{d\theta} = -\frac{\partial H}{\partial\phi} = -\sqrt{\frac{2J}{\nu_0}}\sin\phi F(\theta) \ . \tag{3.4}$$

Substituting Eq. (3.3) for $\phi(\theta)$, Eq. (3.4) can be integrated easily to give

$$\begin{aligned}\sqrt{\frac{2J_N}{\nu_0}} - \sqrt{\frac{2J_0}{\nu_0}} &= -\sum_{n=0}^{N}\bar{a}_m\sin(\phi_0+2\pi N\nu_0)\sin 2\pi N\nu_m \\ &= \frac{\bar{a}_m}{2}\sum_{n=0}^{N}[\cos(\phi_0+2\pi n\nu_+)-\cos(\phi_0+2\pi n\nu_-)] \ ,\end{aligned} \tag{3.5}$$

where J_0 and J_N are, respectively, the actions of the particle after the 0-th and N-th turn, and we have defined

$$\nu_\pm = \nu_0 \pm \nu_m \ . \tag{3.6}$$

To find the invariant curves, we should look at the position of the particle every perturbation period starting from the n_0-th turn. In other words, there is a set of invariant curves for every n_0. Therefore, we let

$$N = n_0 + \frac{\bar{n}}{\nu_m} \qquad \bar{n} \text{ an integer} \ . \tag{3.7}$$

The cosine series can then be summed neatly to give

$$\sum_{n=0}^{N}\cos(\phi_0+2\pi n\nu_\pm) = \frac{\cos(\phi_0+\pi N\nu_0 \pm \pi n_0\nu_m)\sin(\pi N\nu_0 \pm \pi n_0\nu_m + \pi\nu_\pm)}{\sin\pi\nu_\pm} \ , \tag{3.8}$$

and Eq. (3.5) becomes

$$\sqrt{\frac{2J_N}{\nu_0}} - \sqrt{\frac{2J_0}{\nu_0}} = \frac{\bar{a}_m}{4\sin\pi\nu_+\sin\pi\nu_-}\left\{\cdots\right\} \ , \tag{3.9}$$

where

$$\begin{aligned}\left\{\cdots\right\} &= \sin 2\pi n_0\nu_m\left[\sin(\phi_0+2\pi N\nu_0)(\cos 2\pi\nu_0 - \cos 2\pi\nu_m) + \cos(\phi_0+2\pi N\nu_0)\sin 2\pi\nu_0\right] \\ &\quad - \cos 2\pi n_0\nu_m\sin 2\pi\nu_m\sin(\phi_0+2\pi N\nu_0) + \sin\phi_0\sin 2\pi\nu_m \ .\end{aligned} \tag{3.10}$$

If we look at the invariant curves every $1/\nu_m$ turns starting from turn zero ($n_0 = 0$), Eq. (3.9) simplifies to

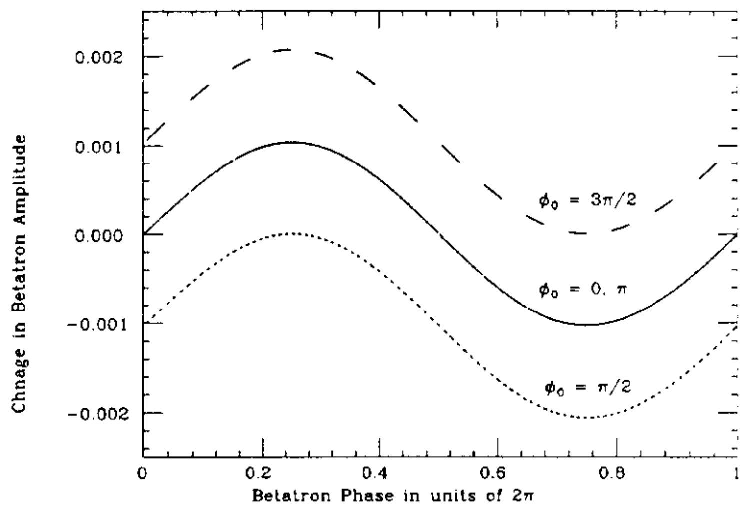
$$\sqrt{\frac{2J_N}{\nu_0}} - \sqrt{\frac{2J_0}{\nu_0}} = -\frac{\bar{a}_m\sin 2\pi\nu_m}{4\sin\pi\nu_+\sin\pi_-}\left[\sin\phi - \sin\phi_0\right] \ , \tag{3.11}$$

where we have substituted $\phi \approx \phi_0 + 2\pi N\nu_0$ according to Eq. (3.3).

To check the existence of invariant curves when the last small term of Eq. (3.2) is included, we perform a turn-by-turn simulation. Initially and after N turns, the positions of the particle are denoted, respectively, by $A_0e^{-i\phi_0}$ and $A_Ne^{-i\phi}$ where the amplitude $A_j = \sqrt{2J_j/\nu_0}$ for $j = 0$ or N. The kick at the j-th turn is $i\bar{a}_m\sin 2\pi j\nu_m$. Then, following the particle turn by turn, we obtain

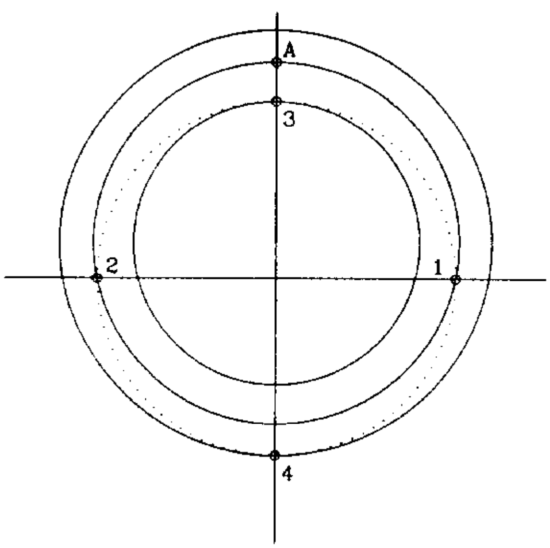
$$A_Ne^{-i\phi} = A_0e^{-i[\phi_0+\sum_{k=1}^{N}\nu_k]} + \sum_{j=1}^{N}i\bar{a}_m\sin 2\pi j\nu_m e^{-i2\pi\sum_{k=j+1}^{N}\nu_k} \ , \tag{3.12}$$

where ν_k is the average betatron tune in the k-th turn and is dependent on the betatron amplitude as given by Eq. (2.10). In the above, the complex notation $(x, p/\nu_0)$, with x and p/ν_0 given by Eq. (2.8), has been used. We followed four particles which were placed initially on the unit circle in the phase plane ($A_0 = 1$ unit), at phases $\phi_0 = 0$, $\pi/2$, π, and $3\pi/2$. The nominal tune was $\nu_0 = 0.4$. The perturbing frequency was $f_m = 20$ Hz, or $\nu_m = 1/172$ for the SSC collider whose revolution frequency is $f_0 = 3.440$ kHz. The kick amplitude was taken as $\bar{a}_m = 0.1$ unit and the detuning varied from 0.00 to 0.01 unit. The amplitude-phase plot is displayed in Fig. 1a after tracking for 5×10^4 turns. The plots demonstrate the existence of the invariance curves for the four particles even when the last term of Eq. (3.2) is included. Also the results are as expected from Eq. (3.11). These invariant curves are plotted in the phase plane in Fig. 1b.



(a)

Tune 0.400, detuning 0.01000, kick 0.1000, f_m 20.00 Hz, kn 0



(b)

Figure 1: Invariant curves for particles 1, 2, 3, and 4 are plotted amplitude-versus-phase in (a) and in the x-p phase plane in (b). The particles are marked on the dotted unit circles in (b) representing the initial invariant curve for all the 4 particles.

Of course, the existence of new invariant curves does not necessarily imply the increase in emittance. It is the spread in betatron tune that leads to the emittance increase. The eventual emittance will be given by the area of the closed invariant curve of particle 4. When particle 1 leads particle 4 by slightly more than $\pi/2$ arriving at point A with particle 4 remaining at the original position, the fractional increase in area will reach roughly half the maximum. The number of turns $N_{\frac{1}{2}}$ required is given by

$$\Delta\nu_\beta = \frac{d\nu_\beta}{dA}\Delta A_{\frac{1}{2}} N_{\frac{1}{2}} \sim \frac{1}{4} \;, \tag{3.13}$$

where $\Delta A_{\frac{1}{2}}$ is the amplitude difference between particles 1 and 4. With the aid of Eq. (2.10),

$$N_{\frac{1}{2}} \sim \frac{1}{8\alpha A_0 \Delta A_{\frac{1}{2}}} \;. \tag{3.14}$$

The maximum fractional increase in emittance (or area) can be obtained from Eq. (3.11), or Eq. (3.9) for all n_0, by integrating ΔA^2 over $\frac{1}{2}d\phi$ and divided by πA_0^2:

$$\frac{\Delta\epsilon}{\epsilon} \approx \frac{\bar{a}_m \sin 2\pi\nu_m}{2A_0 \sin^2 \pi\nu_0} \;, \tag{3.15}$$

where $\nu_\pm$ in the denominator has been replaced by ν_0. A simulation was performed by placing initially 201 particles evenly on a unit circle in the phase plane and tracked for 10^7 turns. The maximum spreads in x and p/ν_0 were found for each turn. The area (or emittance) was defined by multiplying these two maximum spreads together and compared with 4. The emittance computed in this way fluctuated from turn to turn, but we only recorded the emittance which was larger than that of the previous turn. The simulation was performed with betatron tune $\nu_0 = 0.4$, perturbing tune $\nu_m = 1/172$, detuning $\alpha = 0.001$, kick amplitude $\bar{a}_m = 0.2$. The results are plotted in Fig. 2. As expected from Eq. (3.15), the fractional increase in emittance reaches 0.004. We therefore put $\Delta A_{\frac{1}{2}} = 0.001$ in Eq. (3.14), giving $N_{\frac{1}{2}} \sim 1.2 \times 10^5$ turns to reach half maximum. This estimation is in rough agreement with the simulation shown in Fig. 2.

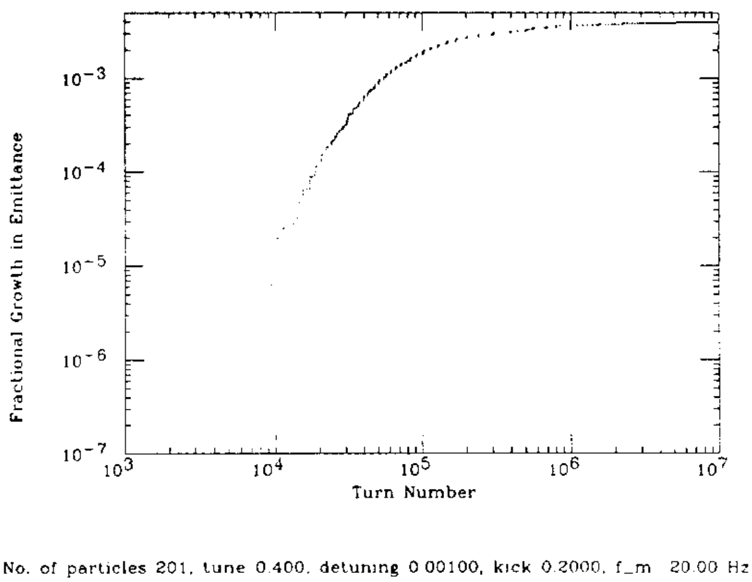


Figure 2: Fraction growth in emittance plotted against turn number, showing the saturation of the growth as predicted.

We want to point out that the fractional growth in emittance, as depicted in Eq. (3.15), is in fact proportional to the incremental kick per turn $\bar{a}_m \nu_m$, and the denominator exhibits resonances whenever $\nu_0 = \pm\nu_m$, results to be expected physically. When the whole spectrum of perturbation frequencies is included, Eq. (3.15) becomes

$$\frac{\Delta\epsilon}{\epsilon} \approx \int d\nu_m \frac{a_m \sin 2\pi\nu_m}{2A_0 \sin^2 \pi\nu_0} \ . \tag{3.16}$$

The invariant curves for different values of n_0 are given by Eq. (3.11). Take for example the situation of largest kick, $n_0 = 1/4\nu_m$. The shift in amplitude becomes

$$\sqrt{\frac{2J_N}{\nu_0}} - \sqrt{\frac{2J_0}{\nu_0}} = \frac{\bar{a}_m \sin 2\pi\nu_m}{4 \sin \pi\nu_+ \sin \pi\nu_-} \left[\sin\phi(\cos 2\pi\nu_0 - \cos 2\pi\nu_m) + \cos\phi \sin 2\pi\nu_0 + \sin\phi_0 \sin 2\pi\nu_m\right], \tag{3.17}$$

where we have replaced $\phi_0 + 2\pi N\nu_0$ by ϕ. The shift in amplitude in Eq. (3.11) when $n_0 = 0$ is of $\mathcal{O}(\bar{a}_m\nu_m)$. However, when $n_0 = 1/4\nu_m$, the shift is of $\mathcal{O}(\bar{a}_m)$ which is $1/\nu_m$ times bigger. In fact, the separation of invariant curves for the four particles with $\phi_0 = 0,\ \pi/2,\ \pi$, and $3\pi/2$ are still of $\mathcal{O}(\bar{a}_m\nu_m)$ just like Fig. 1b. The only difference is that all the invariant curves have just been shifted upward by $\mathcal{O}(\bar{a}_m)$ due to the perturbation amplitude. As a result, we can neglect ν_m in Eq. (3.17) and obtain

$$\sqrt{\frac{2J_N}{\nu_0}} - \sqrt{\frac{2J_0}{\nu_0}} = \frac{\bar{a}_m \cos(\phi + \pi\nu_0)}{2 \sin \pi\nu_0} \ . \tag{3.18}$$

A simulation with betatron tune $\nu_0 = 0.4$, perturbing tune $\nu_m = 1/172$, detuning $\alpha = 0.01$, kick amplitude $\bar{a}_m = 0.1$ is plotted in Fig. 3. The fussiness of the plot is due to the close spacings of the invariant curves of the four particles stated above.

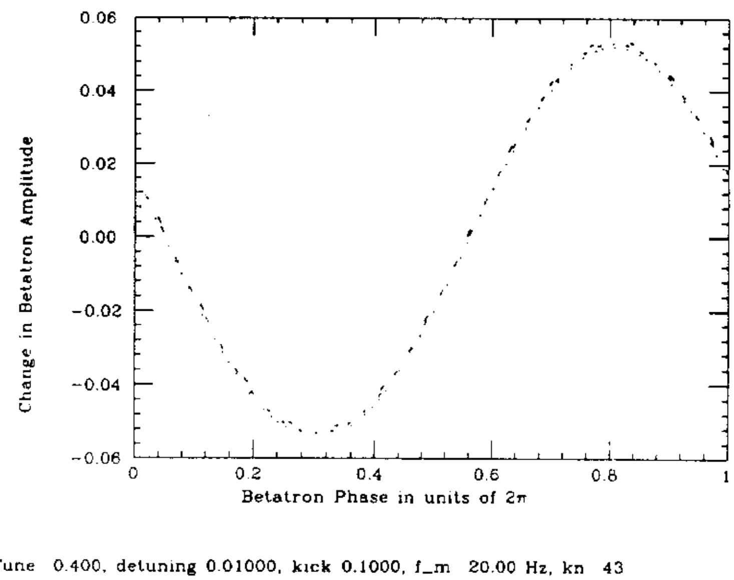


Figure 3: Invariant curves for the same 4 particles as in Fig. 1b when they are viewed starting from turn $n_0 = 1/\nu_m$ when the modulation amplitude is at a maximum. Although the variations are much bigger than those in Fig. 1a, the spacings of the curves are the same.

4. Jostlein's Beam-centering Scheme

In the Jostlein's beam-centering scheme,[1] one beam is rotated about the other at an interaction point, and the resulting variation in luminosity serves to measure the amount and direction by which the two beam centers miss each other. In this situation, each beam acts on the other to first order like a moving quadrupole of focal length $f_Q = \beta_0/4\pi\Delta\nu_{bb}$, where $\Delta\nu_{bb}$ is the head-on beam-beam tune shift. On the j-th turn, each beam receives a kick

$$\bar{a}_m \sin 2\pi j\nu_m = \frac{b_m\sqrt{\beta_0}}{f_Q} \sin 2\pi j\nu_m \ , \tag{4.1}$$

where b_m is the amplitude of the sinusoidal beam modulation. Note that in this consideration only the linear beam-beam effect has been taken into account. With the initial amplitude $A_0 = x_{\max}/\sqrt{\beta_0}$ where $x_{\max}$ is the transverse maximum radius of the bunch, the fractional growth in emittance from Eq. (3.11) becomes

$$\frac{\Delta\epsilon}{\epsilon} = \frac{b_m}{x_{\max}} \frac{\beta_0}{2f_Q} \frac{\sin 2\pi\nu_m}{\sin^2 \pi\nu_0} \ . \tag{4.2}$$

Take the SSC as an example. For linear beam-beam tune shift $\Delta\nu_{bb} = 0.004$, $\beta_0 = 0.5$ m at the interaction point, modulation tune $\nu_m = 1/172$ ($f_m = 20$ Hz), and betatron tune $\nu_0 = 0.4$, the growth in emittance is found to be

$$\frac{\Delta\epsilon}{\epsilon} = 1.0 \times 10^{-3} \frac{b_m}{x_{\max}} \ , \tag{4.3}$$

which is indeed very small since we must choose $b_m \ll x_{\max}$ in practice.

We can estimate the growth time. For the SSC, a typical value for nonlinear detuning is $\mu = 48.0$ m^{-2} found by Yan[3] in simulations using a full spectrum of random errors. If we use $\beta = 390$ m, a value at the F quad, this translates into our detuning $\alpha = \mu\beta = 1.87 \times 10^4$ m^{-1}. At 20 TeV, the rms bunch size is about 0.12 mm at the F quad, thus $A_0 = 6.07 \times 10^{-6}m^{\frac{1}{2}}$. From Eq. (3.14), the time to reach half maximum growth is $7.24 \times 10^8 \times (x_{\max}/b_m)$ turns or $58 \times (x_{\max}/b_m)$ hours.

If the Jostlein's modulation is switched off abruptly, say, at turn number corresponding to $n_0 = 1/4\nu_m$ when the modulation amplitude is largest, the beam will eventually smear out due to nonlinear tune spread. The growth in emittance will therefore be derived from Eq. (3.18) instead, giving

$$\frac{\Delta\epsilon}{\epsilon} = \frac{b_m}{x_{\max}} \frac{\beta_0}{f_Q} \frac{1}{\sin \pi\nu_0} = 0.026 \frac{b_m}{x_{\max}} \ , \tag{4.4}$$

which is 26 times larger. However, this smearing time is extremely long. It takes roughly $(\alpha A_0 \Delta A)^{-1} = 1.1 \times 10^8 \times (x_{\max}/b_m)$ turns or $9.0 \times (x_{\max}/b_m)$ hours. Therefore, the offset bunch can always be kicked back easily to the ideal closed orbit by an active kicker and no emittance growth due to nonlinear tune spread will occur.

5.0 Perturbation due to Ground Motion

5.1 Quarry Blast

There is a quarry blast about 9 miles away from the SSC rings, which may be set off several times in a week. Tunnel site measurement[4] shows that the spectrum is peaked at 1 Hz and 3 Hz with integrated vertical ground displacements[5] $\delta y = 1.43$ and 1.08 microns, respectively. The SSC collider ring consists of 90° cell of length $L = 228.5$ m. The focal lengths of the quadrupoles are therefore $f_q = L/4\sin 45° = 80.79$ m and $\beta = 390.1$ m at the F quad. The beam modulation amplitudes are

$$a_m = \frac{\delta y\sqrt{\beta_0}}{f_q} = \begin{cases} 3.50 \times 10^{-7} \text{ m}^{\frac{1}{2}} \\ 2.64 \times 10^{-7} \text{ m}^{\frac{1}{2}} \end{cases} \tag{5.1}$$

According to Eq. (3.14) or Eq. (3.16), the fractional growth in emittance is 0.00600, where a factor of $\sqrt{1000}$ has been included to account for the $\sim$1000 quadrupoles in the collider ring.

The time required to reach half maximum, estimated from Eq. (3.14), gives 1.2×10^8 turns or 9.7 hours. Both the ground-wave peaks at 1 Hz and 3 Hz have a full width of about 1 Hz, corresponding to a correlation time of $\tau \sim 2$ sec, for which the growth is extremely tiny. The quarry blast usually lasts for only 30 sec. The total growth is still negligibly small.

However, at the end of a correlated wave, the beam can be kicked off-center, resulting in emittance growth due to nonlinear tune spread. If we average over the n_0 in Eq. (3.7), the average amount of off-center shift after the abrupt end of a correlated wave is

$$\langle \Delta A \rangle = \sum \frac{\bar{a}_m}{\pi \sin \pi \nu_0} \times \sqrt{1000} = 6.50 \times 10^{-6} \text{ m}^{\frac{1}{2}} \ , \tag{5.2}$$

which is of the same order of magnitude as A_0, the original size of the bunch. The smearing time is found to be 4.8×10^5 turns or 140 sec. Thus, an active damper can always be used to kick the beam back to its ideal orbit avoiding any nonlinear smearing.

5.2 Crossing Train

The Midlothian train crosses the collider ring at a point where the tunnel depth is only 20 m. Site measurement[4] shows a spectrum having 2 peaks at 3 Hz and 7 Hz with integrated vertical displacements[5] $\delta y = 0.55$ and 0.58 micron, respectively. The beam modulation amplitudes are $\bar{a}_m = 1.34 \times 10^{-7}$ and 1.42×10^{-7} $\text{m}^{\frac{1}{2}}$. The fractional growth in emittance is 0.00232, where a factor of 10 has been included to represent the assumption that 10 nearby quadrupoles are affected by the train and they contribute equally. It takes 3.12×10^8 turns or about 25 hours to reach half maximum. A one-mile train traveling at 30 mph will take about 120 sec to cross the ring. As a result, the growth should be negligibly small.

The peak at 1 Hz has a full width of 1 Hz and the one at 7 Hz has a full width of 12 Hz. The correlation time for the two frequencies are therefore 2 and 0.17 sec, respectively. Again abrupt stopping of a correlated wave will throw the beam off-center. But because of the small nonlinear tune spread, smearing can be avoided by an active damper.

5.3 Ambient Ground Noise

The ambient ground noise measured at several tunnel positions at different times varied over two orders of magnitudes.[4] The integrated average vertical displacement was found to be[5] 0.015 micron having a typical frequency of 3 Hz. For 1000 quadrupoles, the fractional growth in emittance is 5.78×10^{-5} and the time to reach half maximum is 1.25×10^{10} turns or 1012 hours. Therefore, the growth will be negligible even for a duration of a whole day. Again the beam offset due to abrupt stopping of a correlated wave can be restored to the ideal position through an active damper without introducing any nonlinear smearing.

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