Energy loss of open strings with massive endpoints in AdS/CFT

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Abstract. We propose a novel AdS/CFT construction for a heavy quark in an $\mathcal{N} = 4$ SYM theory. Using our new AdS/CFT heavy quark Lagrangian, in which we couple massive endpoints to the usual open string, we compute the stopping distance and differential energy loss for heavy quarks traversing a strongly-coupled $\mathcal{N} = 4$ SYM plasma. We discuss the implications of our new holographic results for heavy-ion physics.

1. Introduction

We wish in heavy ion physics to characterize theoretically and confirm experimentally the non-trivial, emergent, many-body properties of quantum chromodynamics (QCD) at energy scales above the expected deconfinement temperature, at which the relevant degrees of freedom for the theory change from the usual hadronic matter we experience in everyday life to something new, and at nearly zero baryon chemical potential. Experimentally, the field uses colliders to smash nuclear matter together at enormous speeds, the Relativistic Heavy Ion Collider (RHIC) located at Brookhaven National Laboratory in the USA and at the Large Hadron Collider (LHC) located at CERN in Switzerland. The particle species used at these colliders tend to be protons and large nuclei such as gold and lead. On the theoretical side, perturbative quantum chromodynamics (pQCD), lattice QCD (lQCD), and the AdS/CFT conjecture are the three main tools used to derive predictions to be compared with data.

That both pQCD and AdS/CFT are concurrently used reflects the current uncertainty in some of the basic properties of the quark-gluon plasma (QGP) produced in these heavy ion collisions. In particular, it is not clear experimentally or theoretically what, if any, small parameter exists for the description of the QGP at temperatures not far above $\Lambda_{QCD} \sim 180$ MeV$\sim 10^{12}$ K. An enormous amount of progress has been made for high momentum observables using both the assumption that the usual coupling in the QCD Lagrangian, evaluated at the temperature of the plasma, is small: $g(T) \ll 1$ [1–17]. On the other hand, a large swath of low momentum data is better described under the naive assumption that the 't Hooft coupling is large: $\lambda \equiv g^2 N_c \gg 1$ [18–31].

Since in science we seek a consistent picture of the processes we observe in nature, we'd like to resolve the current, seeming paradoxical picture of heavy ion collisions into a unified understanding. There are essentially two paths to this unification: 1) a successful application of weak coupling techniques to low momentum observables or 2) a successful application of strong coupling techniques to high momentum observables. Fascinating recent work has shown promise for the first direction of research [32–37]. We will, however, follow the latter path.

The main phenomenological obstacle faced when comparing high momentum observable predictions from AdS/CFT to data is all research so far has shown that in a strong coupling theory the particles are stopped by the plasma very rapidly [38–41]. Perhaps, it is possible that this very short stopping distance (and thus large energy loss) is due to using an AdS/CFT analog for QCD objects that is not accurate. For example, there are puzzling aspects to using the usual AdS/CFT construction for heavy quarks [42]; e.g., in the presence of a plasma, the mass of the heavy quark *decreases*. Perhaps a more natural construction for a heavy quark in AdS/CFT exists which will simultaneously solve the puzzling aspects of heavy flavor and also yield energy loss results more similar to data.

2. Weighing the string endpoints

We propose to extend the finite endpoint momentum work [43] to allow string endpoints to have both finite momentum and finite mass. We therefore hope to provide an additional connection between heavy ion physics and AdS/CFT; our finite mass endpoint construction should naturally apply to the open heavy flavor physics measured at RHIC and LHC.

The simplest Poincaré invariant action that includes finite momentum and finite mass at the string endpoints may be written as

$$S = -\frac{1}{4\pi\alpha'}\int_M d\tau d\sigma \sqrt{-h}h^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \frac{1}{2}\int_{\partial M} d\xi \left(\frac{1}{\eta}\dot{X}^\mu \dot{X}^\nu G_{\mu\nu} - \eta m^2\right), \tag{1}$$

where ∂M is the boundary of the worldsheet M and $G^{\mu\nu}$ is the metric of the target spacetime. Here h refers to the determinant of auxiliary worldsheet metrics h_{ab} and η is auxiliary field defined at ∂M. The dots denote differentiation with respect to ξ, which is an independent parametrization of the worldsheet boundary. The first term in our action Eq. 1 is the usual Polyakov action describing a bosonic string. The second term is nothing but the action for a relativistic point particle of mass m.

We may find the equations of motion due to Eq. 1 by varying the action with respect to the coordinates X^μ:

$$\begin{aligned}\delta S = &\int_M d\tau d\sigma\ \partial_a(P^a_{\ \mu}\delta X^\mu) - \int_M d\tau d\sigma\ (\partial_a P^a_{\ \mu})\delta X^\mu + \int_M d\tau d\sigma\ P^a_{\ \lambda}\partial_a X^\mu \Gamma^\lambda_{\rho\nu}(\delta X^\rho) \\ &+ \int_{\partial M} d\xi\ \frac{d}{d\xi}(p_\mu \delta X^\mu) - \int_{\partial M} d\xi\ (\dot{p}_\mu)\delta X^\mu + \int_{\partial M} d\xi\ p_\lambda \dot{X}^\mu \Gamma^\lambda_{\rho\mu}\delta X^\rho, \end{aligned}\tag{2}$$

where we defined the worldsheet and endpoint momenta as

$$P^a_{\ \mu} := -\frac{1}{2\pi\alpha'}\sqrt{-h}h^{ab}\partial_b X^\nu G_{\mu\nu}; \qquad p_\mu := \frac{1}{\eta}\dot{X}^\nu G_{\mu\nu}. \tag{3}$$

Applying the two dimensional version of Stokes' theorem over the surface term on the string worldsheet in Eq. 2 gives

$$\int_M d\tau d\sigma\ \partial_a(P^a_{\ \mu}\delta X^\mu) = \int_{\partial M} dn^a \epsilon_{ab} P^b_{\ \mu}\delta X^\mu = \int_{\partial M} d\xi\ \dot{n}^a \epsilon_{ab} P^b_{\ \mu}\delta X^\mu,$$

where $n^a(\xi)$ parametrizes the normal to the worldsheet boundaries and ϵ_{ab} is an antisymmetric symbol with $\epsilon_{\tau\sigma} = 1$. Then by extremizing the string action and after relabelling some dummy indices, the bulk and boundary equations of motion read, respectively as

$$\partial_a P^a_{\ \mu} - \Gamma^\lambda_{\mu\nu}\partial_a X^\nu P^a_{\ \lambda} = 0; \qquad \dot{p}_\mu - \Gamma^\lambda_{\mu\nu}\dot{X}^\nu p_\lambda = \dot{n}^a \epsilon_{ab} P^b_{\ \mu}. \tag{4}$$

Now let us spend a few moments to discuss on the dynamics of the endpoints. A key result in the study of [43] shows that the free string endpoint motion is no longer purely transverse when the endpoints have non-vanishing momentum. Thus the endpoints are driven in such a way that their velocity is pointing at least partially in the same direction as the tangent to the string at the worldsheet boundary. Our massive endpoints move at velocity $v < 1$.

Furthermore we have the following identity

$$\dot{n}^a \epsilon_{ab} P^b_{\ \mu} \pm \frac{\eta}{2\pi\alpha'} p_\mu = 0, \tag{5}$$

which shows that we can separate the motion of the endpoints from that of the interior of string. This separation of motion is actually a direct consequence of the finite endpoint momenta.

Next, by substituting Eq. 5 into Eq. 4, it is straightforward to show – by means of a change of variable – that the endpoints' equation of motion describes a timelike geodesic on the spacetime boundary. Finally, in the rest of this work we shall assume a configuration of strings that prevents a snap-back as the 4-point momentum p^μ vanishes. Indeed we will argue that the endpoint momenta are going to vanish as the string crosses the black hole horizon.

3. Falling strings on spacetime geodesics

The metric of the asymptotic AdS_5 part of the 10-dimensional dual geometry can be written as

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + \frac{1}{f(z)}dz^2 + d\vec{x}^2\right], \qquad f(z) = 1 - \left(\frac{z}{z_h}\right)^4. \tag{6}$$

Here L is the AdS curvature radius, $(t, \vec{x})$ corresponds to 4-dimensional Minkowski coordinates and z denotes the inverse radial coordinate. Thus the spacetime boundary is located at $z = 0$, while the black hole horizon is at $z = z_h$. The temperature of the field theory equals the Hawking temperature of the black hole in the interior of the asymptotic AdS_5 [44]. The AdS/CFT dictionary provides us with a relation between the background temperature and the horizon radius such that $T = 1/(\pi z_h)$.

In the gravity dual, adding a fundamental representation quark in $\mathcal{N} = 4$ SYM is tantamount to adding a D7 brane [42] that fills a transverse S^3 and extends along the radial direction from the boundary of the asymptotic AdS_5 up to some depth z_*. The dual description of probe quark moving through the plasma involves classical open IIB strings whose endpoints are attached to the D7 brane. One can think the string as an holographic representation of the colour field between a quark-antiquark pair.

Again we restrict our attention to the motion of the string in the x-z plane of the asymptotic AdS_5 geometry. We are also interested in a string configuration with an initial pointlike state and no component of the initial velocity along the radial direction. Let us assume that the string is created at some coordinate $z = z_*$, then extends while the endpoints follow timelike geodesics until it ultimately falls into the black hole.

The proper time τ at the worldsheet boundary can be chosen to parametrize the endpoints path. Geodesic equations with respect to the t and x coordinate system in the asymptotic AdS_5 background read

$$\frac{d^2t}{d\tau^2} + \frac{z^2}{L^2 f(z)}\frac{d}{dz}\left(\frac{L^2}{z^2}f(z)\right)\frac{dz}{d\tau}\frac{dt}{d\tau} = 0, \qquad \frac{d^2x}{d\tau^2} + \frac{z^2}{L^2}\frac{d}{dz}\left(\frac{L^2}{z^2}\right)\frac{dz}{d\tau}\frac{dx}{d\tau} = 0, \tag{7}$$

respectively, with the constraint

$$G_{\mu\nu}\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau} = -f(z)\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{f(z)}\left(\frac{dz}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2 = \zeta. \tag{8}$$

Here $\zeta = 0, -1$ for null or timelike geodesics corresponding to massless and massive endpoints respectively. However, we would like a smooth approach to find the limiting case of a lightlike geodesic, that is by setting $m = 0$ everywhere in our results. Notice that since any monotonic function $\tau'(\tau)$ is equivalently a good parameter for the geodesic, then, physically it does not make any difference to parametrize paths by τ or τ'. In order to exhibit the mass term, let us parametrise the geodesics by $\tau' = \tau/m$. Working out the geodesic equations we obtain

$$\frac{dx}{dt} = \frac{h_2}{h_1} f(z), \qquad \frac{dz}{dt} = f(z)\left[1 - \frac{m^2 + (h_2 z/L)^2}{(h_1 z/L)^2} f(z)\right]^{1/2}, \tag{9}$$

where h_1, h_2 are constants of integration. It follows that

$$\frac{dx}{dz} = \frac{v}{f(z_*)}\left[1 - \frac{m^2 + (h_2 z/L)^2}{(h_1 z/L)^2} f(z)\right]^{-1/2}. \tag{10}$$

where v is the initial velocity of the endpoints along the x direction and $h_2/h_1 = v/f(z_*)$. Notice that the results of [43] for massless string endpoints are correctly recovered by setting $m = 0$. Furthermore we can solve for the h_1, h_2 constants from the initial pointlike conditions that yields

$$\frac{dx}{dz} = \frac{v}{f(z_*)}\left[1 - \frac{-v^2 + f(z_*) + (vz/z_*)^2}{(f(z_*)z/z_*)^2} f(z)\right]^{-1/2}. \tag{11}$$

Eq. 11 does not explicitly depend on the endpoint mass. In fact, this parameter is encoded in the initial conditions of our string. To recover the results for massless endpoints we just use the appropriate initial conditions for the massless case, namely $v = \sqrt{f(z_*)}$, which corresponds to the speed of light along the x-direction and at constant depth z_* in the Schwarzchild-AdS$_5$ geometry. Since the endpoints are now massive and follow timelike geodesics, they move with initial velocity $v < f(z_*)$.

4. Maximal stopping distance

Recall first that the velocity of the endpoints vanishes when they cross the horizon. In principle, we will integrate Eq. 11 from $z = z_*$ to $z = z_h$ to find the distance travelled by the string endpoints, but this expression is difficult to find analytically. However, we can estimate the stopping distance by means of a reasonable approximation. The idea consists of assuming the endpoints move at an approximately constant depth $z = z_*$ and constant velocity $v \approx 1$ for a long time compared to z, before plunging rapidly into the horizon. Integrating Eq. 11 in the limit $z_* \ll z_h$ under this assumption yields

$$\Delta x \approx v \frac{z_h^2}{z_*} \frac{\sqrt{\pi}\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})}. \tag{12}$$

One may easily recover the massless limit in [43] by taking $v = 1$. Since $v < 1$, then the string with massless endpoints goes *further* than the massive one provided that both endpoints start at the same depth $z = z_*$.

In particular, we find that

$$\Delta x = v\Delta x_0 = v\left(\frac{2}{\pi^2}\right)^{1/3} \frac{\Gamma(\frac{5}{4})\Gamma(\frac{1}{4})^{1/3}}{\Gamma(\frac{3}{4})^{4/3}} \left[\frac{1}{\sqrt{\lambda}} \frac{E_0}{T^4}\right]^{1/3}, \tag{13}$$

where in the last equality we used the equation $T = 1/(\pi z_h)$ furnished by the AdS/CFT dictionary, and Δx_0 is the result for massless string endpoints [43], λ is the ‘t Hooft coupling, and E_0 is the initial energy of the quark.

5. Heavy-quark energy differential loss in AdS/CFT

We want to compute how quickly the endpoints' energy decreases as the string approaches the horizon. If we assume again that the string endpoints quickly swoop down to the black hole after evolving for a relatively long time at radius $z \approx z_*$ with constant velocity v, we find

$$\frac{dE}{dz} \approx -\frac{\sqrt{\lambda}}{2\pi}\frac{1}{z^2\sqrt{1-\frac{f(z)}{f(z_*)}}}. \tag{14}$$

By comparing Eq. 14 with [43], the massive string endpoints lose approximately the same energy as the massless endpoints, with respect to z. Notice however that we also did an overestimation here.

To read off the energy loss by the quark in field theory, we need to compute the energy loss of the endpoints as they move ahead in the x direction. This is done by means of a chain rule in Eq. 14 and using the geodesic equation, which gives

$$\frac{dE}{dx} \approx -\frac{1}{v}\left(\frac{\sqrt{\lambda}}{2\pi}\frac{f(z_*)}{z^2}\right) \approx -\frac{1}{v}\left(\frac{\sqrt{\lambda}}{2\pi}\frac{1}{z^2}\right), \tag{15}$$

where in the last step we assumed $z_* \ll z_h$. By taking $v \to 1$, we recover one of the main results from [43]. Notice that since the additional factor $v < 1$ the string endpoints lose more energy along a timelike geodesic than a lightlike one.

6. Discussion

In this proceedings we generalized the work of Ficnar and Gubser [43] to allow strings with *both* finite endpoint mass and momentum. Our results are completely consistent with those of Ficnar and Gubser: we recover all their results when we take our endpoint masses to 0. Additionally, our results are quite sensible: 1) the endpoint stopping distance Δx for a massive quark is reduced by a factor of $v < 1$ when the heavy quark starts at the same z depth as the massless quark and 2) the differential energy loss dE/dx is similarly enhanced by a factor of $1/v > 1$ in the massive case compared to the massless case.

From the work of this proceedings, one may improve the modelling of strong-coupling open heavy flavor energy loss calculations used in heavy ion phenomenology.

Naïvely one expects a massive particle *of the same momentum* as a massless particle to propagate further before stopping. We leave checking such an expectation for future work.

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References

[1] Gyulassy M, Levai P and Vitev I 2002 *Phys. Lett.* **B538** 282–288 (*Preprint* nucl-th/0112071)

[2] Vitev I and Gyulassy M 2002 *Phys. Rev. Lett.* **89** 252301 (*Preprint* hep-ph/0209161)

[3] Wang E and Wang X N 2002 *Phys. Rev. Lett.* **89** 162301 (*Preprint* hep-ph/0202105)

[4] Majumder A, Wang E and Wang X N 2007 *Phys. Rev. Lett.* **99** 152301 (*Preprint* nucl-th/0412061)

[5] Dainese A, Loizides C and Paic G 2005 *Eur. Phys. J.* **C38** 461–474 (*Preprint* hep-ph/0406201)

[6] Armesto N, Cacciari M, Dainese A, Salgado C A and Wiedemann U A 2006 *Phys. Lett.* **B637** 362–366 (*Preprint* hep-ph/0511257)

[7] Wicks S, Horowitz W, Djordjevic M and Gyulassy M 2007 *Nucl. Phys.* **A784** 426–442 (*Preprint* nucl-th/0512076)

[8] Majumder A, Nonaka C and Bass S A 2007 *Phys. Rev.* **C76** 041902 (*Preprint* nucl-th/0703019)

[9] Zhang H, Owens J F, Wang E and Wang X N 2009 *Phys. Rev. Lett.* **103** 032302 (*Preprint* 0902.4000)

[10] Vitev I and Zhang B W 2010 *Phys. Rev. Lett.* **104** 132001 (*Preprint* 0910.1090)
[11] Schenke B, Gale C and Jeon S 2009 *Phys. Rev.* **C80** 054913 (*Preprint* 0909.2037)
[12] Young C, Schenke B, Jeon S and Gale C 2011 *Phys. Rev.* **C84** 024907 (*Preprint* 1103.5769)
[13] Majumder A and Shen C 2012 *Phys. Rev. Lett.* **109** 202301 (*Preprint* 1103.0809)
[14] Horowitz W A and Gyulassy M 2011 *Nucl. Phys.* **A872** 265–285 (*Preprint* 1104.4958)
[15] Buzzatti A and Gyulassy M 2012 *Phys. Rev. Lett.* **108** 022301 (*Preprint* 1106.3061)
[16] Horowitz W A 2013 *Nucl. Phys.* **A904-905** 186c–193c (*Preprint* 1210.8330)
[17] Djordjevic M and Djordjevic M 2014 *Phys. Lett.* **B734** 286–289 (*Preprint* 1307.4098)
[18] Teaney D, Lauret J and Shuryak E V 2001 *Phys. Rev. Lett.* **86** 4783–4786 (*Preprint* nucl-th/0011058)
[19] Kolb P F, Heinz U W, Huovinen P, Eskola K J and Tuominen K 2001 *Nucl. Phys.* **A696** 197–215 (*Preprint* hep-ph/0103234)
[20] Hirano T and Tsuda K 2002 *Phys. Rev.* **C66** 054905 (*Preprint* nucl-th/0205043)
[21] Kolb P F and Heinz U W 2003 (*Preprint* nucl-th/0305084)
[22] Teaney D 2003 *Phys. Rev.* **C68** 034913 (*Preprint* nucl-th/0301099)
[23] Hirano T and Gyulassy M 2006 *Nucl. Phys.* **A769** 71–94 (*Preprint* nucl-th/0506049)
[24] Hirano T, Heinz U W, Kharzeev D, Lacey R and Nara Y 2006 *Phys. Lett.* **B636** 299–304 (*Preprint* nucl-th/0511046)
[25] Romatschke P and Romatschke U 2007 *Phys. Rev. Lett.* **99** 172301 (*Preprint* 0706.1522)
[26] Song H and Heinz U W 2008 *Phys. Rev.* **C77** 064901 (*Preprint* 0712.3715)
[27] Schenke B, Jeon S and Gale C 2011 *Phys. Rev. Lett.* **106** 042301 (*Preprint* 1009.3244)
[28] Shen C, Heinz U, Huovinen P and Song H 2011 *Phys. Rev.* **C84** 044903 (*Preprint* 1105.3226)
[29] Gale C, Jeon S, Schenke B, Tribedy P and Venugopalan R 2013 *Phys. Rev. Lett.* **110** 012302 (*Preprint* 1209.6330)
[30] Bzdak A, Schenke B, Tribedy P and Venugopalan R 2013 *Phys. Rev.* **C87** 064906 (*Preprint* 1304.3403)
[31] Casalderrey-Solana J, Liu H, Mateos D, Rajagopal K and Wiedemann U A 2011 (*Preprint* 1101.0618)
[32] Molnar D and Gyulassy M 2002 *Nucl. Phys.* **A697** 495–520 [Erratum: Nucl. Phys.A703,893(2002)] (*Preprint* nucl-th/0104073)
[33] Lin Z w and Ko C M 2002 *Phys. Rev.* **C65** 034904 (*Preprint* nucl-th/0108039)
[34] Bzdak A and Ma G L 2014 *Phys. Rev. Lett.* **113** 252301 (*Preprint* 1406.2804)
[35] Orjuela Koop J D, Adare A, McGlinchey D and Nagle J L 2015 *Phys. Rev.* **C92** 054903 (*Preprint* 1501.06880)
[36] He L, Edmonds T, Lin Z W, Liu F, Molnar D and Wang F 2016 *Phys. Lett.* **B753** 506–510 (*Preprint* 1502.05572)
[37] Lin Z W, He L, Edmonds T, Liu F, Molnar D and Wang F 2015 Elliptic Anisotropy v_2 May Be Dominated by Particle Escape instead of Hydrodynamic Flow (*Preprint* 1512.06465) URL https://inspirehep.net/record/1410941/files/arXiv:1512.06465.pdf
[38] Morad R and Horowitz W A 2014 *JHEP* **11** 017 (*Preprint* 1409.7545)
[39] Casalderrey-Solana J, Gulhan D C, Milhano J G, Pablos D and Rajagopal K 2014 *JHEP* **10** 19 [Erratum: JHEP09,175(2015)] (*Preprint* 1405.3864)
[40] Horowitz W A 2015 *Phys. Rev.* **D91** 085019 (*Preprint* 1501.04693)
[41] Moerman R W and Horowitz W A 2016 (*Preprint* 1605.09285)
[42] Karch A and Katz E 2002 *JHEP* **06** 043 (*Preprint* hep-th/0205236)
[43] Ficnar A and Gubser S S 2014 *Phys. Rev.* **D89** 026002 (*Preprint* 1306.6648)
[44] Witten E 1998 *Adv. Theor. Math. Phys.* **2** 505–532 (*Preprint* hep-th/9803131)