7th International Workshop on Astronomy and
Relativistic Astrophysics (IWARA 2016)
International Journal of Modern Physics: Conference Series
Vol. 45 (2017) 1760060 (4 pages)

DOI: 10.1142/S2010194517600606

Neutral Pion Pole Mass Calculation in a Strong Magnetic Field: Lattice QCD Versus NJL Model

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Published 15 August 2017

Within the framework of cold magnetized SU(2) Nambu-Jona-Lasinio model we evaluate the π_0 and σ pole mass, as well as, f_{π_0}, $g_{\pi_0 qq}$ at zero baryon density. We employ a magnetic field dependent coupling, $G(eB)$, fitted to reproduce lattice QCD results for the quark condensates. In particular, we find that the π_0 meson mass systematically decreases when the magnetic field increases, in good agreement with recent lattice calculations.

Keywords: Neutral Meson Mass, RPA Approximation, NJL Model in a Magnetic Field.

PACS numbers: 14.40.Aq, 12.39.-x, 12.38.-t, 13.40-f, 12.40.-y, 12.38.Gc

1. Introduction

In this work the π_0 neutral meson pole mass in a strong magnetic field is calculated within the random phase approximation (RPA) [1] at zero temperature and baryon density. The model coupling constant, $G(eB)$ [2,3], is taken to be magnetic field dependent and it has been fitted in order to reproduce lattice QCD results for the quark condensates [4]. We employ the magnetic field independent regularization scheme (MFIR) [5].

2. Meson Properties Under Strong Magnetic Field

The SU(2) NJL model lagrangian, in the presence of a magnetic field, is described by

$$\mathcal{L} = \bar{\psi}_f \left(i\not{D} - \tilde{m}\right) \psi_f + G \left[(\bar{\psi}_f \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \vec{\tau} \psi_f)^2\right] - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \ , \tag{1}$$

where a sum over repeated f is implied. The electromagnetic gauge field is represented by A^μ, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\vec{\tau}$ is the isospin matrix, the coupling constant by G while Q=diag(q_u= $2e/3$, q_d=-$e/3$) represents the charge matrix, $D^\mu = (i\partial^\mu - QA^\mu)$ is the covariant derivative, ψ^{T}=(ψ_u, ψ_d) is the quark fermion field and $\tilde{m} = \mathrm{diag}(m_u, m_d)$ represents the bare quark mass matrix. It has been shown by Avancini et al.[1] that within the RPA approximation the π_0 mass can be written as:

$$m^2_{\pi_0}(B) = -\frac{m}{M(B)} \frac{1}{4iGN_cN_f I(m^2_{\pi_0}, B)} \ , \tag{2}$$

where

$$I(m^2_{\pi_0}, B) = \frac{1}{4(2\pi)^3} \sum_{f=u,d} \beta_f \sum_{n=0}^{\infty} g_n I_{f,n}(k^2_\parallel = m^2_{\pi_0}) \ , \tag{3}$$

where $M(B)$ is the effective mass, β_f=$|q_f|B$, $f = (u, d)$, $N_c = 3$, $g_n = 2 - \delta_{n0}$, $p_\parallel = p_0 - p_3$, and

$$I_{f,n}(k^2_\parallel) = \int d^2 p_\parallel \frac{1}{[p^2_\parallel - M^2 - 2\beta_f n][(p+k)^2_\parallel - M^2 - 2\beta_f n]}. \tag{4}$$

It can be shown[1] that the σ-meson mass is given by the relation $m^2_\sigma(B) = 4M^2(B) + m^2_{\pi_0}(B)$ and that the Gell-Mann-Oakes-Renner (GOR) relation in a magnetic medium is recovered[1]. The detailed calculation of f_{π_0} pion decay constant and $g_{\pi_0 qq}$ can be found elsewhere [1]. We use the four different sets of parameters displayed in Table 1.

Table 1. Parameter sets for the NJL model at $T = B = 0$. The correct $eB \to 0$ limit of our ansatz requires that $G_{II} = G(eB = 0)$.

	m_{π_0} (MeV)	m_0 (MeV)	G (GeV^{-2})	Λ (MeV)
Set I	135.62	5.0	$G_I = 4.67$	664.3
Set II	143.31	5.5	$G_{II} = 4.50$	650.0
Set III	417	48.41	$G_{III} = G_I$	664.3
Set IV	417	50.16	$G_{IV} = G_{II}$	650.0

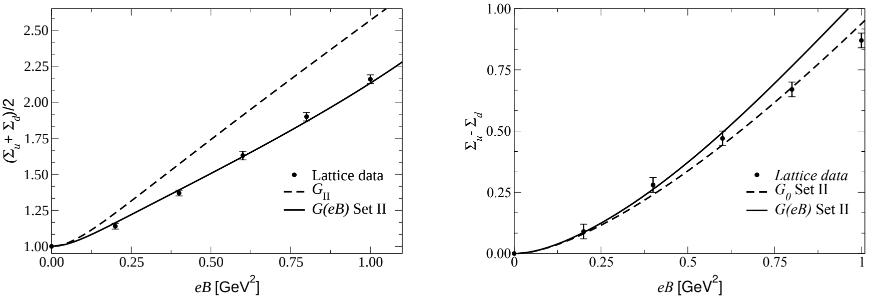

Fig. 1. Condensates average and difference as functions of eB for the NJL model with G_{II}, $G(eB)$ compared to lattice QCD calculations [4].

3. Field Dependent Coupling $G(eB)$

Next, we present our ansatz for $G(eB)$ which reproduces the lattice results of Bali et al. [4]. Lattice simulations for the average $(\Sigma_u + \Sigma_d)/2$ can be obtained by using

$$G(eB) = \alpha \ + \ \beta \ e^{-\gamma \ (eB)^2} \quad , \tag{5}$$

where $\alpha = 1.44373$ GeV^{-2}, $\beta = 3.06$ GeV^{-2} and $\gamma = 1.31$ GeV^{-4}. In Fig.1 we show NJL results for the condensate average and difference using $G(eB)$ and G, and compare with lattice results.

4. Numerical Results and Conclusions

The properties of magnetized neutral mesons have been investigated using a fixed and a B-dependent coupling constant so that model predictions and LQCD results related to inverse magnetic catalysis agree [2,3]. In Fig. 2 our results for the m_{π_0} and $g_{\pi_0 qq}$ are presented for both fixed G and $G(eB)$. We have found that the use

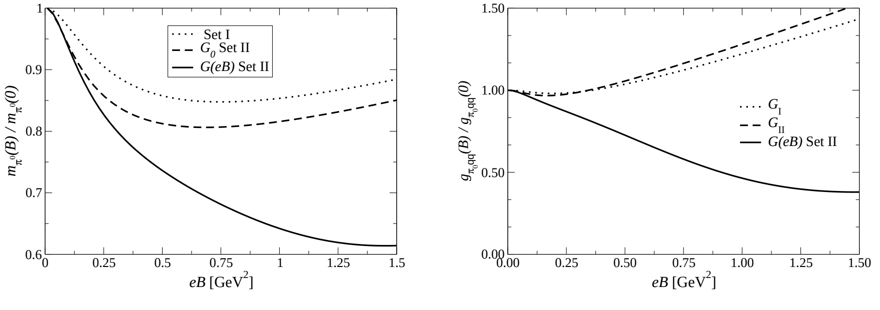

Fig. 2. Normalized π_0 meson mass and meson-quark coupling as a function of eB in the NJL model with different coupling schemes.

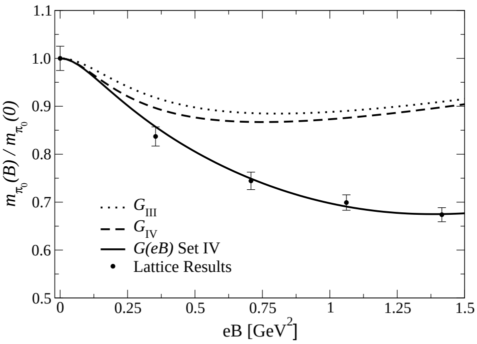

Fig. 3. Normalized neutral pion mass $m_{\pi_0}(eB)/m_{\pi_0}(0)$ in the NJL model with different coupling schemes and a large current quark mass compared to recent lattice results [6].

of $G(eB)$ and G_I (and G_{II}) indicate an opposite behavior for $g_{\pi_0 qq}$ with B. As a major result, we show that the π_0 remains a soft mode even at rather high field strengths ($\approx 1.5\,\mathrm{GeV}^2$) since its mass decreases by about 30%. Our results in Fig.7 show a remarkable agreement with recent LQCD predictions [7].

Acknowledgements

This work was supported by CNPq, FAPESP and CAPES.

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