Theory of rare hadronic decays

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Abstract. In this proceeding we will review the current theoretical status of rare hadronic decays, with a particular focus on decays of the B meson. These decays are indeed excellent indirect probes while searching for New Physics. Given the current experimental situation at colliders, where no new particles have been directly produced, rare decays provide a fundamental and alternative approach in the search for Physics beyond the Standard Model. In particular, the following classes of decays will be here reviewed: $B_q \to \tau\nu$, $B_q \to \mu\mu$, $B \to K^{(*)}\nu\bar{\nu}$, $B \to K^{(*)}\ell\ell$, $B_s \to \phi\ell\ell$ and $b \to s\gamma$. We will provide the most updated Standard Model predictions, highlight which are the main sources of uncertainty, and give the current status of New Physics searches in these channels when confronting the theory predictions to current experimental results.

1 Introduction

Rare B decays are excellent probes for New Physics (NP) searches. Given the current lack of direct production for NP states at present experimental facilities, alternative avenues must be explored to investigate potential extensions of the Standard Model (SM). One promising approach involves the in depth study of rare processes: indeed, even if NP effects would arise only as intermediate, virtual effects, these could still become evident due to the SM contribution being already suppressed.

In these context, most of the rare hadronic decays are mediated by Flavour Changing Neutral Currents (FCNC), which are forbidden in the SM at tree-level. Therefore, these processes occur at the loop-level and are hence very rare, being generally both GIM- and CKM-suppressed. This proceeding aim to review the current theoretical status of the most promising rare B decays. Precise experimental measurements are being confronted with accurate theoretical predictions in (and beyond) the SM, contributing to the ongoing search for NP effects.

2 The Status of Flavour Physics

Before delving in the details of the decays here reviewed, it is useful to briefly review the current status of Flavour Physics, particularly in the context of internal consistency and NP reach.

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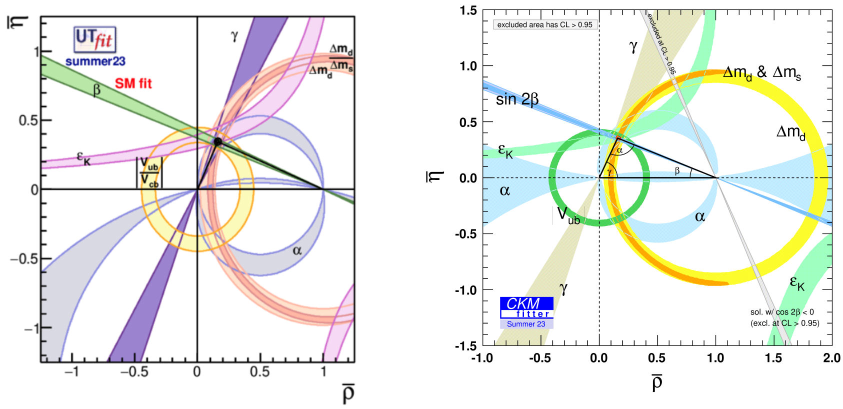

Figure 1. The latest results on the Unitarity Triangle Analysis performed by the UTfit and CKMfitter collaborations. Figures taken from Refs. [1, 2].

Starting from the internal consistency of Flavour measurements, this can be probed by analyzing the several measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Indeed, this unitarity matrix can be parameterized by means of 4 parameters (λ, A, $\bar{\rho}$ and $\bar{\eta}$), with all decays involving a quark flavour change being dependent on (a combination) of (some) of them. This means that the extraction of these 4 quantities from data come from a super-determined system, given the large amount of decays depending on such quantities. A common way to visualize the agreement of these parameters comes from exploiting the unitarity of the CKM matrix and perform Unitarity Triangle Analysis (UTA), as the ones shown in Fig. 1 and performed by the UTfit [1] and CKMfitter [2] collaborations, respectively. As can be inferred from both plots, no evidence of deviation in the UTA has been observed so far, with the current determination for the CKM parameters reading

$$\begin{aligned} A &= 0.827 \pm 0.010\,, & \lambda &= 0.2251 \pm 0.0008\,, \\ \bar{\rho} &= 0.160 \pm 0.009\,, & \bar{\eta} &= 0.346 \pm 0.009\,. \end{aligned} \tag{1}$$

The fact that no deviation has been observed in the UTA does not imply the fact that no NP can be hidden within Flavour processes. Indeed, we are merely in a situation where effects stemming from heavy NP is particularly suppressed - but might still be visible in the rare decays object of this review. The usual way to study those effects at the low scale is by mean of an Effective Field Theory (EFT), where the heavy degrees of freedom characterizing the NP effects have been integrated out and their effects encoded in couplings to effective, local operators. A typical EFT Hamiltonian can be therefore written as

$$H^{\text{eff}}(x) = \sum \frac{c_O}{\Lambda^{dim_O - 4}} O(x)\,, \tag{2}$$

where the c_O are couplings parameterizing the low-scale footprints of heavy degrees of freedom and usually called Wilson coefficients (WC), O is a series of local operator built as monomials in low-energies fields and derivatives, and Λ is the high scale of the heavy degrees of freedom, being also the cutoff of the effective theory. Extending the SM by means of these local operators it is possible to set bounds on the ratios c_O/Λ^{dim_O-4}, therefore effectively obtaining a bound on the high NP scale by requiring perturbativity for the WCs.

3 $B_q \to \tau\nu$

We start by reviewing the leptonic $B_q \to \tau\nu$ decays. While these processes are not mediated by an FCNC, nevertheless these ca be considered rare decays due to helicity suppression. The SM prediction for their Branching Ratio (BR) reads:

$$\mathcal{B}(B_q^+ \to \tau^+\nu_\tau)^{\rm SM} = \tau_{B_q^+} \frac{G_F^2 |V_{qb}|^2 f_{B_q^+}^2 m_{B_q^+} m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_{B_q^+}^2}\right)^2 , \qquad q = u, c \tag{3}$$

where$\tau_{B_q^+}$ and $m_{B_q^+}$ denote the B_q^+ meson lifetime and mass, respectively, G_F is the Fermi constant, and m_τ is the mass of the τ^+ lepton. The main sources of theoretical uncertainty come from the B_q^+ meson decay constants $f_{B_q^+}$ and the CKM elements $|V_{qb}|$. Concerning the former, the most precise measurements to date come from Lattice QCD (LQCD) and read $f_{B_c^+} = 427(6)$ MeV and $f_{B^+} = 190.0(1.3)$ MeV [3]. Regarding the latter, a long-standing discrepancy is currently present among inclusive and exclusive determinations of both $|V_{cb}|$ and $|V_{ub}|$ [4]; however, indirect extractions of these elements can be obtained via global fits feeding in the UTA. The latest predictions performed by the UTfit collaboration [1] for these elements read $|V_{ub}| = 3.70(11) \times 10^{-3}$ and $|V_{cb}| = 42.22(51) \times 10^{-3}$. Using these input, the latest predictions for the two BRs read in the SM:

$$\mathcal{B}(B_c^+ \to \tau^+\nu_\tau)^{\rm SM} = 2.29(9) \times 10^{-2} , \qquad \mathcal{B}(B^+ \to \tau^+\nu_\tau)^{\rm SM} = 0.87(5) \times 10^{-4} . \tag{4}$$

Looking for physics beyond the SM, additional contribution from both vector and scalar currents can affect this class of decays, stemming from the NP operators

$$O_{V_{L(R)}} = (\bar{q}_{L(R)}\gamma_\mu b_{L(R)})(\bar{\tau}_L\gamma_\mu\nu_L) , \qquad O_{S_{L(R)}} = (\bar{q}_{R(L)} b_{L(R)})(\bar{\tau}_R\nu_L) . \tag{5}$$

We will denote with C_i the respective WCs. NP effects coming from the scalar operators $O_{S_{L(R)}}$ can be excellently probed from these decays, due to the induced lift of the helicity suppression:

$$\mathcal{B}(B_q^+ \to \tau^+\nu_\tau) = \mathcal{B}(B_q^+ \to \tau^+\nu_\tau)^{\rm SM} \times \left| 1 - \left(C_{V_R}^q - C_{V_L}^q\right) + \left(C_{S_R}^q - C_{S_L}^q\right) \frac{m_{B_q}^2}{m_\tau(m_b + m_q)} \right|^2 , \tag{6}$$

Models involving a scalar Leptoquark (LQ) and/or an additional Higgs doublet typically produce such contributions (for a study on current and future bounds see, e.g., Ref. [5]).

4 $B_q \to \mu\mu$

A second class of helicity suppressed decays, this time actually mediated by FCNC, consists of $B_q \to \mu\mu$ decays. The BR for these transitions is described in the SM by

$$\mathcal{B}(B_q^0 \to \mu^+\mu^-)^{\rm SM} = \tau_{B_q^0} \frac{G_F^4 |V_{tb}^* V_{tq}|^2 f_{B_q}^2 m_W^4 m_{B_q^0} m_\mu^2}{2\pi^5} \sqrt{1 - \frac{4m_\mu^2}{m_{B_q^0}^2}} \left|C_{10}^{\rm q,SM}\right|^2 , \qquad q = d, s \tag{7}$$

where m_W is the W boson mass and $C_{10}^{\rm q,SM}$ is the SM WC mediating the axial current $Q_{10}^q = \frac{\alpha_e}{4\pi}(\bar{q}_L\gamma_\mu b_L)(\bar{\mu}\gamma^\mu\gamma_5\mu)$. Similarly to the previous case, also for this class of channels the main sources of uncertainty stem from the B_q meson decay constants f_{B_q} and the CKM elements $|V_{tq}|$, whose most precise determinations come from LQCD and UTA analyses, respectively. Concerning the decay constants, we have $f_{B_d} = 190.5(1.3)$ MeV and $f_{B_s} = 230.1(1.2)$

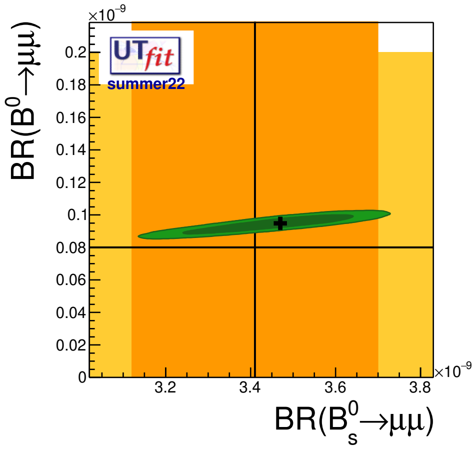

Figure 2. Allowed region in the $\mathcal{B}(B_d \to \mu^+\mu^-) - \mathcal{B}(B_s \to \mu^+\mu^-)$ plane. The vertical (orange) and horizontal (yellow) bands correspond to the present experimental results (1 σ regions). Figure taken from Ref. [1].

MeV [3]; for the CKM elements, the latest determinations read $|V_{ts}| = 41.28(46) \times 10^{-3}$ and $|V_{td}| = 8.59(11) \times 10^{-3}$ [1]. Using these input, we can give the most updated and precise SM predictions for the two BRs:

$$\mathcal{B}(B_d \to \mu^+\mu^-)^{\rm SM} = 9.48(36) \times 10^{-11}\,, \qquad \mathcal{B}(B_s \to \mu^+\mu^-)^{\rm SM} = 3.47(14) \times 10^{-9}\,. \quad (8)$$

If one allows for NP effects, beyond potential modifications to the SM axial WC, this class of decays is sensitive to further contributions also to the additional (pseudo)scalar operators,

$$Q_S = \frac{\alpha_e}{4\pi}\frac{m_b}{m_W}(\bar{q}_L b_R)(\bar{\ell}\ell)\,, \qquad Q_P = \frac{\alpha_e}{4\pi}\frac{m_b}{m_W}(\bar{q}_L b_R)(\bar{\ell}\gamma_5\ell)\,, \quad (9)$$

and to primed operators, obtained replacing $P_{R(L)}$ with $P_{L(R)}$. The modified expression for the BR, where (pseudo)scalar contributions are capable again to lift the helicity suppression, reads

$$\mathcal{B} = \mathcal{B}^{\rm SM} \times \left(\left| \frac{C_{10}^{\rm q,NP} - C_{10}^{\prime\rm q,NP}}{C_{10}^{\rm q,SM}} + \frac{m_{B_q}^2}{2m_\mu m_b}\frac{C_P^{\rm q,NP} - C_P^{\prime\rm q,NP}}{C_{10}^{\rm q,SM}} \right|^2 + \left| \sqrt{1 - \frac{4m_\mu^2}{m_{B_q}^2}}\,\frac{m_{B_q}^2}{2m_\mu m_b}\frac{C_S^{\rm q,NP} - C_S^{\prime\rm q,NP}}{C_{10}^{\rm q,SM}} \right|^2 \right). \quad (10)$$

The current agreement between the experimental measurements and the SM predictions in these channels, as can be observed from Fig. 2, allows us to put stringent bounds on the involved NP couplings. We will focus on the most phenomenologically interesting ones in Sec. 6, in the context of global fits to $b \to s\ell\ell$ data.

5 $B \to K^{(*)}\nu\bar{\nu}$

The FCNC semi-leptonic decays $B \to K^{(*)}\nu\bar{\nu}$ are reviewed in this section. On top of the same kind of theoretical uncertainties presented for $B_s \to \mu\mu$ decays in Sec. 4, they suffer from additional ones due to the presence of form factors, needed to describe the hadronic transitions present in these channels. Indeed, it is possible to parameterize the hadronic matrix elements present in the two channels as

$$\langle \bar{K}(k)|\bar{s}\gamma^\mu b|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^\mu f_0(q^2)\,, \tag{11}$$

$$\begin{aligned}\langle \bar{K}^*(k)|\bar{s}\gamma^\mu(1-\gamma_5)b|\bar{B}(p)\rangle = {}& \epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p^\rho k^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \\ & - i\varepsilon^*_\mu (m_B + m_{K^*}) A_1(q^2) + i(p+k)_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_{K^*}} \\ & + i q_\mu (\varepsilon^* q) \frac{2m_{K^*}}{q^2}\left[\frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2) - A_0(q^2)\right].\end{aligned} \tag{12}$$

The form factors f_0 and f_+ involved in $B \to K$ transitions have been estimated via LQCD [6, 7], with a combination given in Refs. [8, 9]. Concerning the form factors A_0, A_1, A_2 and V mediating $B \to K^*$ transitions, an estimate by means of Light-Cone Sum Rules (LCSR) has been in given in Ref. [10], whose results are also combined with the ones obtained by LQCD in Ref. [11]. We have now all the necessary elements to write down the expression for the differential BRs, which in the SM read as [12]:

$$\frac{d\mathcal{B}}{dq^2}(B \to K\nu\bar{\nu}) = \tau_B \frac{G_F^2 \alpha_{\rm em}^2}{256\pi^5} \frac{\lambda_K^{3/2}}{m_B^3} |C_L^{\rm SM}|^2 |V_{tb}^* V_{ts}|^2 [f_+(q^2)]^2\,, \tag{13}$$

$$\frac{d\mathcal{B}}{dq^2}(B \to K^*\nu\bar{\nu}) = \tau_B \frac{G_F^2 \alpha_{\rm em}^2}{128\pi^5} \frac{\lambda_{K^*}^{1/2} q^2}{m_B^3} (m_B + m_{K^*})^2 |C_L^{\rm SM}|^2 |V_{tb}^* V_{ts}|^2 \Big([A_1(q^2)]^2\,, \tag{14}$$

$$+ \frac{32 m_{K^*}^2 m_B^2}{q^2 (m_B + m_{K^*})^2} [A_{12}(q^2)]^2 + \frac{\lambda_{K^*}}{(m_B + m_{K^*})^4} [V(q^2)]^2\Big), \tag{15}$$

where A_{12} is a linear combination of the form factors A_1 and A_2 [11], and $\lambda_M \equiv \lambda(q^2, m_B^2, m_M^2)$ with $M = K, K^*$, is the Källén-function defined as $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. Furthermore, we have introduced the SM WCs $C_L^{\rm SM} = -6.32(7)$ [12] defined as the flavour-diagonal, universal part of the WC C_L^{ij} associated to the operator $O_L^{ij} = \frac{\alpha_e}{4\pi}(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_j)$. The SM predictions for the integrated BRs of these decays read [9, 13]:

$$\mathcal{B}(B^\pm \to K^\pm\nu\bar{\nu}) = (4.44 \pm 0.30) \times 10^{-6}\,, \qquad \mathcal{B}(B^\pm \to K^{*\pm}\nu\bar{\nu}) = (9.8 \pm 1.4) \times 10^{-6}\,. \tag{16}$$

Due to the impossibility to flavour-tag neutrinos in experiments, it is customary to express NP contributions to these channels in the following way:

$$\begin{aligned} R_{K^{(*)}}^{\nu\bar{\nu}} = \frac{\mathcal{B}(B \to K^{(*)}\nu\bar{\nu})}{\mathcal{B}^{\rm SM}(B \to K^{(*)}\nu\bar{\nu})} = {}& 1 + \sum_i \frac{2{\rm Re}[C_L^{\rm SM}(C_L^{ii} + C_R^{ii})]}{3|C_L^{\rm SM}|^2}\,, \\ & + \sum_{i,j}\left(\frac{|C_L^{ij} + C_R^{ij}|^2}{3|C_L^{\rm SM}|^2} - \eta_{K^{(*)}} \frac{{\rm Re}[C_R^{ij}(C_L^{\rm SM}\delta^{ij} + C_L^{ij})]}{3|C_L^{\rm SM}|^2}\right), \end{aligned} \tag{17}$$

where C_R^{ij} is the WC of the operator $O_R^{ij} = \frac{\alpha_e}{4\pi}(\bar{s}_R\gamma_\mu b_R)(\bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_j)$, and we have introduced $\eta_K = 0$, $\eta_{K^*} = 3.33(7)$. The Belle II collaboration recently measured for the first time

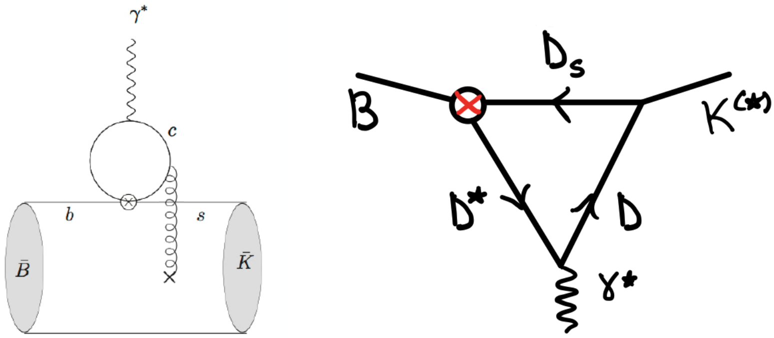

Figure 3. Examples of diagrams mediated by the non-local matrix elements $h_\lambda(q^2)$. Figures taken from Refs. [17, 18].

$R_K^{\nu\bar{\nu}} = 5.4 \pm 1.5$ [14], obtaining a result 2.9σ larger than its SM prediction; when combined with previous upper limits, this result yields $R_K^{\nu\bar{\nu}} = 2.8 \pm 0.8$. On the other hand, up to now we have for the $B^\pm \to K^{*\pm}\nu\bar{\nu}$ decay only upper limits, with the best being set by the Belle collaboration at $R_{K^*}^{\nu\bar{\nu}} < 2.7$ with 90% C.L. [15].

The interpretation of the potential excess observed by Belle II in the $K^\pm$ channel in terms of NP effects is not trivial, once confronted with current limits present in the $K^{*\pm}$ one [9, 13]. Indeed, an explanation in terms of a flavour-universal NP shift to C_L would be clearly viable only after a decrease of the discrepancy in the $K^\pm$ channel. Introducing non-universal contributions is however strongly constrained by current data on ratios of muon to electron BRs in $b \to s$ transitions, as we will see below in Sec. 6; a non-universal component would have to be therefore predominantly connected to ν_τ. Nevertheless, e.g. in a LQ scenario, such components would yield to additional contributions in $b \to c\tau\nu$ transitions, hence needing to confront with data and anomalies in that sector as well [16].

6 $B \to K^{(*)}\ell\ell$, $B_s \to \phi\ell\ell$

The next class of decays includes rare semi-leptonic B decays involving charged leptons in the final states, namely $B \to K^{(*)}\ell\ell$ and $B_s \to \phi\ell\ell$, with $\ell = e, \mu$. In analogy to $B \to K^{(*)}\nu\bar{\nu}$, one of the main sources of uncertainty in these class of channels stem from the form factors, whose number grows, due to the inclusion of tensor matrix elements as well, to three in the case of $B \to K$ transitions and to seven in the cases of $B \to K^*$ and $B_s \to \phi$ ones:

$$\langle \bar{K}(k)|\bar{c}\sigma_{\mu\nu}b|\bar{B}(p)\rangle = i\left(p_\mu k_\nu - p_\nu k_\mu\right)\frac{2f_T(q^2)}{m_B + m_K}\,, \tag{18}$$

$$\begin{aligned}\langle \bar{K}^*(k)|\bar{c}\sigma_{\mu\nu}b|\bar{B}(p)\rangle = i\epsilon_{\mu\nu\alpha\beta}\Bigg[&-\varepsilon^{*\alpha}(p+k)^\beta T_1(q^2) + \varepsilon^{*\alpha}q^\beta\frac{m_B^2 - m_{K^*}^2}{q^2}[T_1(q^2) - T_2(q^2)]\\ &+(\varepsilon^* q)p^\alpha k^\beta\frac{2}{q^2}\left(T_1(q^2) - T_2(q^2) - \frac{q^2}{m_B^2 - m_{K^*}^2}T_3(q^2)\right)\Bigg]\,. \end{aligned} \tag{19}$$

Analogously to the previous class of decays, these additional form factors have been estimated by LQCD for the former case [6, 7], and by a combination of LQCD and LCSR for the latter ones [10, 11].

Differently from the previous case, a second source of uncertainty is introduced here due to the presence of non-local matrix elements involving the four-quark operator Q_2^c =

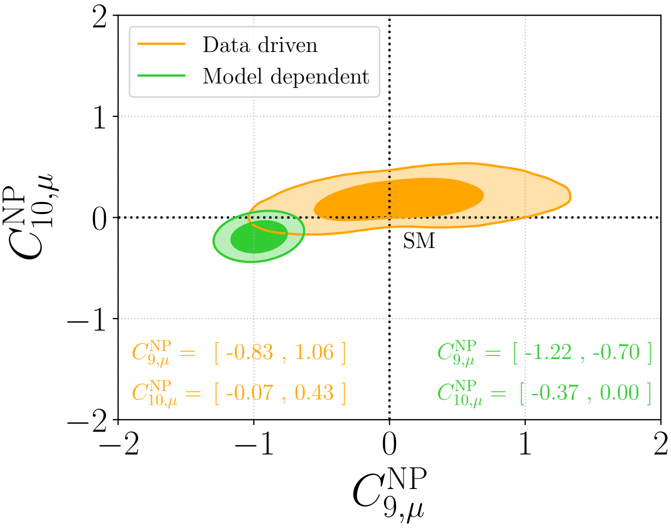

Figure 4. Joint posterior distributions for NP contributions to muon C_9 and C_10. 68% and 95% probability regions are shown. While the result given in green is based on the h_λ estimates from Refs. [17, 20–22] and is not capable to reproduce the LFUV data, the orange one is based indeed on a direct fit for this hadronic contributions to data. Figure taken from Ref. [18].

$(\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$, particularly in proximity of the $c\bar{c}$ threshold. This additional term yields non-factorizable power corrections from the time-ordered product

$$h_\lambda(q^2) = \frac{\epsilon^*_\mu(\lambda)}{m_B^2} \int d^4x\, e^{iqx} \langle \bar{K}^* | \mathcal{T}\{ j^\mu_{\mathrm{em}}(x) Q_2^c(0)\} | \bar{B} \rangle\,, \tag{20}$$

with $\lambda = \{0, +, -\}$ representing the helicity and $j^\mu_{\mathrm{em}}(x)$ being an electromagnetic current. While considerable progress has been made in estimating (at least part of) these amplitudes using LCSR [17, 19] and analyticity supplemented with perturbative QCD in the Euclidean q^2 region [20–22], fully calculating these hadronic contributions remains an open problem. Examples of diagrams stemming from this kind of contributions are showed in Fig. 3. Moreover, $h_\lambda(q^2)$ can mimic the presence of NP effects in these channels as it enters in the amplitudes as a Lepton Flavour Universal (LFU) shift to the coefficient C_9 mediating the operator $Q_9 = \frac{\alpha_e}{4\pi}(\bar{s}_L\gamma_\mu b_L)(\bar{\mu}\gamma^\mu\mu)$ [23], hence polluting the cleanness of the plethora of angular observables that can be defined for these three- and four-body decays beyond the usual BRs [24, 25]. To this end, Lepton Flavour Universality Violating (LFUV) ratios has been defined as $R_{K^{(*)}} = \mathcal{B}(B \to K^{(*)}\mu\mu)/\mathcal{B}(B \to K^{(*)}ee)$ [26], where the uncertainties introduced by these non-local matrix elements largely cancel out yielding to the clean theoretical prediction of $R_{K^{(*)}} = 1.00(1)$ [27].

In the last decades an increasing number of so-called *anomalies* has been measured by the LHCb collaboration, both in angular analyses of decays with vector mesons in the final state and in the BRs of $B \to K^{(*)}\mu\mu$ and $B_s \to \phi\mu\mu$ (see, e.g., Refs. [28–32] for the most updated results). While all these measurements are potentially plagued by the presence of non-hadronic matrix elements, a claim for NP effects coupled to the muon current was advocated due to observation by the LHCb collaboration of under-abundance of muon productions in the measurements of LFUV ratios R_K and R_{K^*} [33, 34]. However, after a recent re-analysis of LHCb data concerning these ratios [35], there is no longer evidence for lepton-flavour violating NP effects in these channels, as shown in Fig. 4 [18].

Nevertheless, the presence of LFU NP effects in this channel cannot be excluded yet, particularly in the context of global fits where all decays involving $b \to s\ell\ell$ transitions are taken into account. Indeed, remembering the constraints on C_{10} coming from current data on $B_s \to \mu\mu$ mentioned in Sec. 4, the presence of LFU NP effects in C_9 is still not excluded, albeit with different significance according to the treatment of non-local hadronic uncertainties [18, 36, 37].

7 $b \to s\gamma$

We conclude our review with the study of the radiative decays mediated by the $b \to s\gamma$ transition. Starting from the inclusive decay $B \to X_s\gamma$, the SM prediction for its BR is based on the equation

$$\mathcal{B}_{s\gamma} \equiv \mathcal{B}(B \to X_s\gamma)_{E_\gamma > E_0} = \mathcal{B}(B \to X_c\ell\nu) \left|\frac{V_{tb}^* V_{ts}}{V_{cb}}\right|^2 \frac{6e^2}{4\pi^2 C}[P(E_0) + N(E_0)]\,, \tag{21}$$

where C is the so-called semi-leptonic phase-space factor, $E_0 = 1.6$ GeV, and $P(E_0)$ and $N(E_0)$ are the perturbative and non-perturbative contributions to the decay, respectively. The latest predictions for the perturbative contributions are reaching the NNLO in QCD; combining them with the most recent estimate for the non-perturbative ones [38], this yields to $\mathcal{B}_{s\gamma} = (3.40 \pm 0.17) \times 10^{-4}$ [39], where the uncertainty stems from interpolation in m_c ($\pm 3\%$), higher-order effects ($\pm 3\%$) and parametric non-perturbative effects ($\pm 2.5\%$), which are to be added in quadrature. This result is in perfect agreement with the latest experimental measurements.

The associated exclusive decays $B_q \to V\gamma$, with V being a vector meson like K^* or ϕ, are of analogous interest. The following observables can be defined for this class of decays:

$$\mathcal{B}(B_q \to V\gamma) = \tau_{B_q} \frac{G_F^2 e^2 |V_{tb}^* V_{tq}|^2 m_{B_q}^3 m_b^2}{128\pi^4} \left(1 - \frac{m_V^2}{m_B^2}\right)^3 (|C_7|^2 + \left|C_7'\right|^2) T_1(0)\,, \tag{22}$$

$$A_{\rm CP}(B_q(t) \to V\gamma) = \frac{\Gamma(\bar{B}_q(t) \to \bar{V}\gamma) - \Gamma(B_q(t) \to V\gamma)}{\Gamma(\bar{B}_q(t) \to \bar{V}\gamma) + \Gamma(B_q(t) \to V\gamma)}\,, \tag{23}$$

with C_7 being the WC of the operator $Q_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R$, which corresponds to the LO term of $P(E_0)$.

A combined analysis of both inclusive and exclusive radiative B decays has been performed in Ref. [40], where the overall agreement of all the SM predictions with experimental measurements put very stringent constraints of potential NP effects in such channels.

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