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On recent developments in the theory of relativistic dissipative fluids

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I give an overview about recent results on the well-posedness and breakdown of solutions for relativistic fluid equations.

Keywords: Relativistic dissipation, fluids, quark-gluon plasma, neutron star merger, causality, blowup of solutions

1. Relativistic fluids and heavy-ion collisions

The study of the quark-gluon plasma is provides a window into fundamental physics 10^{-6} seconds after the big bang.[1] On Earth, this state of matter is produced in collisions between heavy nuclei at relativistic speeds, creating a strongly interacting mixture of quarks and gluons. Current state-of-the art modeling of the quark-gluon plasma[1] is based on relativistic fluid equations, showing that the plasma behaves very much like an expanding liquid drop. Dissipative effects[1–3] have to be taken into account to make theoretical predictions match experimental data. Dissipative effects may also be relevant for the study of neutron star mergers.[4]

In Einstein's theory of relativity, a Lorentzian manifold (M, g) is taken as a model for spacetime. M can be thought of as the set of all events in the universe, characterized by *where and when* they happen. For simplicity consider a flat spacetime $M = \mathbb{R}^4 = \{x^\alpha := (x^0, \ldots, x^3)\}$, where $x^0 = ct$ is the time coordinate and c the speed of light. The Minkowski metric $g = \mathrm{diag}(-1, 1, 1, 1)$ describes distances of events between one another. We use standard tensor notation throughout (repeated greek indices summed from 0 to 3, latin indices from 1 to 3). For a textbook introduction, see Refs. 5–7.

The basic fields in a relativistic fluid description are (e, n, u^α), where the total energy density e, the particle number density n and the four-velocity $u^\alpha = \frac{1}{\sqrt{1-|\vec{v}|^2/c^2}}\left(1, \frac{\vec{v}}{c}\right)$ all depend on x^α. The four-velocity replaces the ordinary three-dimensional velocity $\vec{v}$ of a fluid and is a time-like vector satisfying $g_{\mu\nu}u^\mu u^\nu = -1$. We first review the perfect fluid equations:

$$\begin{aligned}
&\text{relativistic Euler/momentum equation: } (e+p)\dot{u}^\beta = -(g^{\beta\alpha} + u^\beta u^\alpha)\nabla_\alpha p \\
&\text{conservation of energy: } \dot{e} + (e+p)\nabla_\alpha u^\alpha = 0 \\
&\text{conservation of particle number: } \nabla_\alpha(n u^\alpha) = 0 \\
&\text{equation of state/pressure equation: } p = p(e, n)
\end{aligned} \tag{1}$$

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where a dot $\dot{f} = u^\alpha \nabla_\alpha f$ denotes the relativistic material derivative. The momentum equation is analogous to Newton's second law $m \cdot a = F$, as $(e + p)$ corresponds to relativistic mass density, $a^\beta := u^\alpha \nabla_\alpha u^\beta = \dot{u}^\beta$ is the acceleration and the gradient of the pressure is a force. Additionally the projection operator $h^{\beta\alpha} := g^{\beta\alpha} + u^\beta u^\alpha$ appears in the relativistic version. In the formal limit $c \to \infty$, the velocity field $\vec{v}$ will satisfy the non-relativistic Euler equation for perfect fluids with rest mass density ρ

$$\rho \left(\partial_t v^i + v^j \partial_j v^i \right) = -g^{ij} \partial_j p. \tag{2}$$

The usual way to derive the fluid equations (1) is to define the energy momentum tensor

$$T^{\mu\nu} = (e + p) u^\mu u^\nu + p g^{\mu\nu} \tag{3}$$

and to postulate the conservation of energy and momentum $\nabla_\mu T^{\mu\nu} = 0$. This formulation goes back to the work of Einstein and Schwarzschild.[8,9] The first and second equation of (1) are equivalent to $\nabla_\mu T^{\mu\nu} = 0$. While the relativistic Euler equations are a rich source of mathematical problems, they constitute a highly idealized description of a fluid. A physically more complete description includes dissipative processes such as viscosity, diffusion and heat conduction.

To model dissipative effects of viscosity and heat conduction, the energy-momentum tensor is extended to

$$T^{\mu\nu} = (e + p + \Pi) u^\mu u^\nu + (p + \Pi) g^{\mu\nu} + \pi^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu, \tag{4}$$

where q^ν is the heat-flux vector, Π the bulk viscosity, and $\pi^{\mu\nu}$ the tensor of shear viscosity. These quantities satisfy the orthogonality and symmetry constraints

$$q^\nu u_\nu = 0, \quad \pi^{\mu\nu} u_\mu = 0, \quad \pi^{\mu\nu} = \pi^{\nu\mu}, \quad \pi^{\mu\nu} g_{\mu\nu} = 0. \tag{5}$$

The equations of motion that result from conservation of energy, momentum and particle number are now

$$\begin{aligned}
&\text{momentum:} \;\; (e + p + \Pi)\dot{u}_\mu + h^\nu_\mu \nabla_\nu (p + \Pi) + h^\alpha_\mu \nabla_\nu \pi^\nu_\alpha \\
&\qquad\qquad\qquad\qquad + h_{\nu\mu} \dot{q}^\nu + \left(\nabla_\nu u_\mu + \nabla_\alpha u^\alpha h_{\mu\nu} \right) q^\nu = 0 \\
&\text{energy:} \;\; \dot{e} + (e + p + \Pi) \nabla_\mu u^\mu + q_\mu \dot{u}^\mu + \nabla_\mu q^\mu + \pi^{\mu\nu} \sigma_{\mu\nu} = 0 \\
&\text{particles:} \;\; \nabla_\alpha (n u^\alpha) = 0 \\
&\text{equation of state:} \;\; p = p(e, n)
\end{aligned} \tag{6}$$

To complete the system, additional dynamical equations for $(\Pi, q^\nu, \pi^{\mu\nu})$ have to be postulated. A large body of literature (see Ref. 7 and references therein) has been devoted to finding suitable dynamical equations for these quantities.

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For a fluid with a single species of particles, i.e. excluding mixtures and chemical reactions, these evolution equations in the Israel-Stewart model take the form

$$\begin{aligned} \tau_0 \dot{\Pi} + \Pi + \zeta \nabla_\alpha u^\alpha + \left[\frac{1}{2} \zeta T \nabla_\alpha \left(\frac{\tau_0}{\zeta T} u^\alpha \right) \Pi \right] &= 0 \\ \tau_1 h_\nu^\lambda \dot{q}_\lambda + q_\nu = -\kappa T (h_\nu^\lambda \nabla_\lambda \ln T + a_\nu) &- \left[\frac{1}{2} \kappa T^2 \nabla_\alpha \left(\frac{\tau_1}{\kappa T^2} u^\alpha \right) q_\nu \right] \\ \tau_2 h_\mu^\alpha h_\nu^\beta \dot{\pi}_{\alpha\beta} + \pi_{\mu\nu} = -2\eta \sigma_{\mu\nu} &- \left[\frac{1}{2} \eta T \nabla_\gamma \left(\frac{\tau_2}{\eta T} u^\gamma \right) \pi_{\mu\nu} \right] \end{aligned} \tag{7}$$

The appearance of the relaxation times $\tau_0, \tau_1, \tau_2 > 0$ is crucial for the stability and well-posedness of the model. The physical meaning of variables and various transport coefficients is as follows:

- Π bulk viscous pressure (isotropic force from fluid element shrinking/expanding).
- $\pi^{\mu\nu}$ shear viscous stresses (anisotropic force from transverse fluid layer movement).
- $p = p(e, n)$ hydrostatic pressure (isotropic gas pressure)
- e total energy density (rest energy density + internal energy density)
- n particle density (measures particles per volume)
- T absolute temperature
- $\zeta > 0, \eta > 0$ bulk/shear viscosity coefficient
- $\kappa > 0$ heat conductivity coefficient $\tau_0, \tau_1, \tau_2 > 0$ characteristic relaxation times for $\Pi, q^\nu, \pi^{\mu\nu}$
- $\sigma_{\mu\nu}$ kinematic shear tensor $\sigma_{\mu\nu} = h_\mu^\alpha h_\nu^\beta \frac{1}{2}(\nabla_\beta u_\alpha + \nabla_\alpha u_\beta)$, distortion of fluid element

I will refer to the set of equations (6), (7) as the full Israel-Stewart (IS) model. To see the mathematical structure of the equations (7) more clearly, the *truncated IS model*[7] (p. 302) is introduced. In (7) the divergence-type terms are dropped, leading to

$$\begin{aligned} \tau_0 \dot{\Pi} + \Pi &= -\zeta \nabla_\alpha u^\alpha, \\ \tau_1 h_\nu^\lambda \dot{q}_\lambda + q_\nu &= -\kappa T (h_\nu^\lambda \nabla_\lambda \ln T + a_\nu), \\ \tau_2 h_\mu^\alpha h_\nu^\beta \dot{\pi}_{\alpha\beta} + \pi_{\mu\nu} &= -2\eta \sigma_{\mu\nu} \end{aligned} \tag{8}$$

A formal non-relativistic limit $c \to \infty$, $\tau_i \to 0$ of (6), (8) recovers the classical compressible Navier-Stokes equations.[7] The equations (7) can be derived using a systematic procedure starting from the second law to thermodynamics (entropy production) (see Ref. 7). The full IS model can also be motivated by considerations from kinetic theory, which aims to derive macroscopic fluid equations by a moment expansion process (see Refs. 10–14).

The first dissipative relativistic fluid theories were formulated by Eckart[15] and Landau-Lifshitz.[16] These models are more direct covariant analogues of the non-relativistic Navier-Stokes equations. Consequently the equations of motion have a

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parabolic character and information can propagate at infinite speed, violating the most basic requirement of relativity.[17] In addition, the Eckart model can exhibit catastrophic instabilities.[18–20] As a reaction to these undesired properties, the IS model was developed in Refs. 21–25, with an initial contribution by Müller.[26] Partial differential operators with the property of finite speed of propagation are called *hyperbolic*[27] and the IS model is conjectured to have the even stronger property of *causality*. Causality requires both finite speed of propagation and that changes in initial data propagate at sub-light speeds, i.e. that the values of $(e, n, \Pi, u^\mu, q^\nu, \pi^{\mu\nu})$ at any point (t_0, x_0) depend only on the initial data given on the domain of dependence, i.e. inside the past-directed light cone.

The following heuristic argument illustrates why (7) should lead to finite speed of propagation: Consider the following modification of Fourier's law of heat conduction

$$\tau_0 \partial_t \vec{q} + \vec{q} = -\kappa \nabla T \tag{9}$$

with $\vec{q}(t, \cdot) : \mathbb{R}^3 \to \mathbb{R}^3$ being the three-dimensional heat flux. Assuming that the temperature T is proportional to the internal energy of the fluid and using conservation of energy leads to

$$\tau_0 \partial_t^2 T + \partial_t T - \kappa \Delta T = 0, \tag{10}$$

a generalized form of the heat equation, which is recovered upon setting $\tau_0 = 0$. (10) for $\tau_0 > 0$ is the telegraph equation[28] which has finite speed of propagation. The idea is therefore that the presence of relaxation times τ_i in (7) will have a similar effect and guarantee finite speed of propagation.

2. Local-wellposedness

The local well-posedness problems for the full and truncated systems (6),(7) (or (6), (8)) is still open. In Ref. 29, local-wellposedness and causality was shown for a related system used in heavy-ion collisions:

$$\begin{aligned} u^\alpha \nabla_\alpha e + (e + P + \Pi)\, \nabla_\alpha u^\alpha + \pi_\mu^\alpha \nabla_\alpha u^\mu &= 0, \\ (e + P + \Pi)\, u^\beta \nabla_\beta u_\alpha + c_s^2 h_\alpha^\beta \nabla_\beta e + h_\alpha^\beta \nabla_\beta \Pi + h_\alpha^\beta \nabla_\mu \pi_\beta^\mu &= 0, \\ \tau_\Pi u^\mu \nabla_\mu \Pi + \Pi &= -\zeta \nabla_\mu u^\mu - \delta_{\Pi\Pi} \Pi \nabla_\mu u^\mu - \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}, \\ \tau_\pi h_{\alpha\beta}^{\mu\nu} u^\lambda \nabla_\lambda \pi^{\alpha\beta} + \pi^{\mu\nu} &= -2\eta \sigma^{\mu\nu} - \delta_{\pi\pi} \pi^{\mu\nu} \nabla_\alpha u^\alpha - \tau_{\pi\pi} \pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} - \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}, \end{aligned} \tag{11}$$

which introduces new relaxation times and system coefficients $\tau_\pi, \lambda_{\pi\Pi}, \ldots$ etc. This system contains shear/bulk viscosity but no heat conduction. The equations for shear and bulk viscosity differ somewhat from the Israel-Stewart model, but follow the same general idea of introducing a relaxation mechanism.

Theorem 2.1. *Consider the Cauchy problem for equations* (11) *in Minkowski space, with initial data* $\mathring{\Psi} = (\mathring{e}, \mathring{u}^\nu, \mathring{\Pi}, \mathring{\pi}^{\mu\nu})$ *given on* $\{t = 0\}$*. There is an open set of inital data satisfying the constraints and that* $\mathring{\Psi} \in \mathcal{G}^\delta(\{t = 0\})$*, where* $1 \le \delta < 20/19$*. Then, there exist a* $T > 0$ *and a unique* $\Psi = (e, u^\nu, \Pi, \pi^{\mu\nu})$ *defined on* $[0, T) \times \mathbb{R}^3$

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such that Ψ *is a solution in* $[0,T)\times\mathbb{R}^3$ *and* $\Psi = \mathring{\Psi}$ *on* $\{t=0\}$. *Moreover, the solution* Ψ *is causal (i.e. domain of influence of initial data is contained in forward light-cones).*

The set of admissible initial data is physically reasonable and can be described by a set of inequalities involving ε, Π and the eigenvalues of π^μ_ν. The notation $\mathcal{G}^\delta(\{t=0\})$ refers to a Gevrey-type space, and we refer to Ref. 29 for more details.

3. Breakdown of solutions

Solutions to nonlinear partial differential equations may have a finite lifespan, the most elementary example being Burgers' equation.[30] For suitable C^∞ initial data, solutions develop a shock, i.e. a singularity in the first derivative after a finite time. A deeper question can also be asked: What precisely happens at breakdown time? Do derivatives or the fields themselves become infinite? For perfect relativistic fluids there exists a well-developed theory that proves shock formation[31–33] giving a detailed picture of the quantities that actually blow up. These results are very deep and rely on hidden geometric-analytic structures in the perfect fluid equations, which allow their formulation as quasilinear second-order covariant wave equations. It is a hard unsolved problem how to generalize these delicate hidden structures to dissipative fluids. Considering the available mathematical technology, asking for a detailed description of the singularity in the dissipative case seems to be out of reach for now. The investigation of these matters is clearly in its beginning stages, and in Ref.[34] we establish a first blowup result for a viscous dissipative fluid with only bulk viscosity (no shear viscosity or heat conduction). The basic ideas are based on the work by Sideris[35] and for relativistic fluids based on Guo-Tahvildar-Zadeh,[36] who developed a robust technique to show breakdown of smooth solutions for compressible fluids using the fundamental conservation $\nabla_\mu T^{\mu\nu}=0$. Novel aspects of this method continue being discovered.[34,37] At the core of this technique lies a type of virial identity involving the second moment of the energy density, from which differential inequalities are derived, leading to the blowup via a contradiction argument.

Theorem 3.1. *Consider the viscous equations with bulk viscosity (without shear viscosity) with given smooth constitutive functions* p, ζ, τ_0. *Assume that*

$$\mathcal{C} := \int_0^\infty \frac{1}{n} \sup_{\rho\geq 0} \frac{\zeta(\rho,n)}{\tau_0(\rho,n)}\, dn < \infty$$

and make further (mild) technical assumptions of the equation of state $p = p(e,n)$. *There exists a smooth initial data* $(\mathring{\rho}, \mathring{n}, \mathring{\Pi}, \mathring{\mathbf{u}})$ *with large velocities such that the local smooth solution breaks down in finite time.*

A detailed discussion of the model and the assumptions on equations of state and bulk viscosity can be found in Ref. 34. The viscous case requires novel estimates for Π. From its evolution, we have $\dot{\Pi} \sim \nabla_\alpha u^\alpha$, but the quantity $\nabla_\alpha u^\alpha$ is very hard to control in a non-constructive blowup proof. Instead, we use a hidden structure in the equations, namely an auxiliary transport equation in ρ, n, Π allowing us to get estimates for Π.

4. Conclusion

Including dissipation into models of relativistic fluids is necessary for a complete macroscopic description, as effects such as viscosity and heat conduction are ubiquitous in nature. Investigating dissipative effects in relativity has a long history, yet there is a gap in knowledge considering many fundamental questions about existence, uniqueness, blowup and decay properties of solutions. First breakthroughs have been made but many important problems remain open, for example to show well-posedness and singularity formation for the Israel-Stewart model (6), (7).

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