

# INCREASING THE DYNAMIC AND MOMENTUM APERTURES OF THE THOMX RING BY MEANS OF OCTUPOLE CORRECTORS

J. Zhang<sup>†</sup>, P. Bambade, A. Faus-Golfe\*, H. Monard  
 LAL, Université Paris 11, IN2P3/CNRS, 91898 Orsay, France  
 IFIC (CSIC-UV), Valencia, Spain\*  
 A. Loulergue, Synchrotron SOLEIL, St-Aubin, 91192 Gif-sur-Yvette, France

## Abstract

The electron ring of the compact Compton-backscattering X-ray source ThomX which is being built at LAL featured with a small circumference of 18 meters and a low beam energy 50-70 MeV, and its long term single particle dynamics is dominated by the non linear effects in the transverse and longitudinal planes. In this paper, we study the feasibilities to reduce the sextupole resonances and then increase the dynamic aperture and momentum aperture of the ThomX ring, using octupoles correctors.

## INTRODUCTION

The compact Compton-backscattering X-ray source ThomX is under construction by a collaboration of seven institutes and an industry partner at LAL-Orsay, France. The accelerator part of this X-ray source is composed of an electron photon-gun, a linac, and a ring; and is featured with the compact size of 10 m long and 7 m wide, and the high average flux of  $10^{11}$  to  $10^{13}$  photons/second. However, the small size of the ring of 18 circumference and the low electron beam energy 50-70 MeV determine that the beam dynamics in the ThomX ring is dominated by the non-linear effect, which is a common issue for the future generation circular accelerators, like the low emittance light sources and the high luminosity colliders. For such type of accelerators, the dynamic aperture (DA) is normally smaller than the vacuum chamber size, and the momentum aperture (MA) is limited by the non linear motions of the off momentum particles.

## DA AND MA OF THE THOMX RING

Quadrupole fringe fields (FFs) show no obvious effects on the beam dynamics, but the sextupole-like second order dipole FFs contribute greatly to the vertical chromaticities [1], and the sextupoles and the multipole field errors in the main magnets (dipoles, quadrupoles, sextupoles) in the ring are the main sources of the non linear effects of the ThomX machine, and they reduced the DA and the MA which have nontrivial effects on the injection efficiency and the beam storage time in the ring.

In the final version of the ThomX ring lattice, there are 12 sextupoles which are composed of one family focusing sextupoles and two families defocusing sextupoles. The DA

is reduced to around 15 mm in the horizontal plane  $x$  and 20 mm in the vertical plane  $z$ , which are respectively 30 times of the horizontal beam size  $\sigma_x$  and 57 times of the vertical beam size  $\sigma_z$  at the injection point of the ring; the MA is around  $\pm 3\%$ , which is much larger than the final beam energy spread 0.6% after 20 ms storage time.

Table 1: Relative systematic multipole field errors in the ThomX ring, with the unit  $10^{-4}$  at 18 mm radial position. From the OPERA-3D simulations.

Pole N	Inner dip.	Outer dip.	Quad.	Sext.
6	+8.5	-16	-	
8	+0.1	-0.7	-	
10	+3	-6	-	
12	-	-	+2	-
18	-	-	-	-4
20	-	-	-6	-
28	-	-	-9	-
30	-	-	-	-0.9

Furthermore, the engineering construction of the magnets introduces high order magnetic field errors, which deteriorate the DA and MA of the ThomX ring. With the systematic multipole field errors shown in Table 1 [2], the DA is reduced to around 13 mm (25 times of  $\sigma_x$ ) in  $x$  and to 10 mm (29 times of  $\sigma_z$ ) in  $z$ , which is smaller than the physical vacuum chamber size of 20 mm in  $x$  and 14 mm in  $z$ , while larger than the scaled chamber size of 12 mm in  $x$  and 7 mm in  $z$ ; the MA is reduced to  $-1.8\%$  and  $2\%$  (Fig. 1 and 2), which is comparable to the energy momentum acceptance  $\pm 2\%$  limited by the vacuum chamber size.

Although from the simulation, the DA and MA (Fig. 1 and 2) seems to be sufficient for the operation mode of the ThomX ring with sextupoles and systematic multipole field errors, in the real ThomX machine, there will be other sources of perturbations that can reduce the DA and MA, such as the random multipole field errors, the systematic and random misalignment errors, power supply ripples, ground vibrations, thermal expansions of the ring, etc. As a result, one needs to further optimize the DA and MA of the machine.

## HAMILTONIAN AND SEXTUPOLE RESONANCE TERMS

To optimize the DA and MA of the ThomX ring, one powerful method is the analysis of the Hamiltonian resonance

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<sup>†</sup> zhangj@lal.in2p3.fr

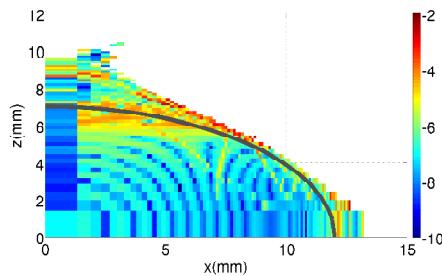


Figure 1: DA of the ThomX ring with dipole and quadrupole FFs, sextupoles and systematic multipole field errors at the injection position. The black line is the vacuum chamber scaled by  $\sqrt{\beta_{\max,x,z}/\beta_{\text{inj},x,z}}$ , where  $\beta_{\max,x,z}$  are the maximum beta functions and  $\beta_{\text{inj},x,z}$  are the beta functions at the injection position.

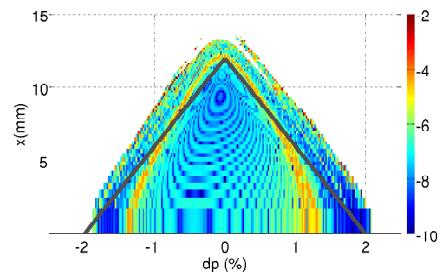


Figure 2: MA of the ThomX ring with dipole and quadrupole FFs, sextupoles and systematic multipole field errors at the injection position. The black line is the linear limit of the vacuum chamber scaled by the maximum dispersion  $\eta_{\max}$ .

components. Normally the resonances from the sextupoles are the main sources to introduce non linear beam motions and then limit the DA and MA. Consequently, one can use the octupoles to compensate the non linear resonances from the sextupoles and then increase the DA and MA [3].

Ignoring the fringe field of magnets, the particle motion in a conservative accelerator system without the energy loss and the energy gain, can be described by the expanded Hamiltonian,

$$H(x, p_x; z, p_z; -ct, \delta; s) \approx \frac{p_x^2 + p_z^2}{2} + A_s(x, z, s) \quad (1)$$

where  $x$  is the horizontal coordinate,  $z$  is the vertical coordinate,  $p_{x,z}$  are the canonical momentums;  $-ct$  is the longitudinal coordinate of the particle relative to the reference particle,  $\delta$  is the momentum offset; and  $A_s(x, z, s)$  is the longitudinal magnetic vector potential component at the longitudinal position  $s$ , and it is  $s$ -independent in the magnet body,

$$A_s(x, z, s) = \frac{b_3}{3}(x^3 - 3xy^2) \quad (\text{sextupole}) \quad (2)$$

$$A_s(x, z, s) = \frac{b_4}{4}(x^4 - 6x^2y^2 + y^4) \quad (\text{octupole}) \quad (3)$$

where  $b_3$  and  $b_4$  are respectively the magnetic coefficients of the sextupole and octupole.

To compensate the non linear resonances introduced by the sextupoles using octupole correctors, one can use the normal form [4], Lie algebra [5] and TPSA (Truncated Power Series Algebra) [6] to get the sextupole resonances components from Eqn. (1). Following the procedures in [7, 8], the Hamiltonian resonance terms of one sextupole can be expressed as  $h_{ijklm}$  which determine the linear chromaticities and tune shift with amplitude and energies, etc; the cross talk of two sextupoles determines the second order sextupole resonance terms  $h_{ijklm} * h_{ijklm}^*$ , which determine the second order chromaticities and other high order resonances. The linear resonance terms  $h_{ijklm}$  can be optimized by the adjustments of the optics and sextupoles, while the second order sextupole resonance terms can be compensated by the octupoles.

## DA AND MA OF THE THOMX RING WITH OCTUPOLES

The ThomX ring has 12 dipoles, 24 quadrupoles, 12 sextupoles, and 12 horizontal/vertical correctors which are integrated with the sextupoles. Due to its compact size, there are only 2 long straight sections of the length 1.6 meters in the ThomX ring, which are reserved for the installation of the septum and the RF cavity of the longitudinal feedback system. To compensate the second order sextupole resonance components, 2 identical defocusing octupoles are placed symmetrically beside the RF cavity, and 2 identical focusing octupoles are placed symmetrically beside the septum (Fig. 3). With such arrangements of the octupoles, the symmetry of the ring lattice can be kept, and the number of resonance lines that may deteriorate the DA and MA can be reduced.

The multipole field errors are first included in the lattice, and then the chromaticities are adjusted to zeros using two families sextupoles; finally the strengths and the positions of the octupoles are optimized using the code OPA [9, 10] to compensate the harmful sextupole resonances and then get the minimum total resonance strength. Finally, the DA is increased by 8 times of  $\sigma_x$  and 5 times of  $\sigma_z$ , and the MA is increased by 0.5% (Fig. 4 and 5), compared to the case without octupole correctors in figures 1 and 2.

## CONCLUSIONS

Due to the compact size and the low beam energy, the beam dynamics in the ThomX ring is dominated by the non linear effects. As a result, the DA and MA of the ring are limited by the sextupoles, multipole field errors and other perturbation sources.

In order to increase the DA and MA, two pairs of focusing and defocusing octupoles are placed in the two long straight sections of the ThomX ring. Without breaking the symmetry of the ring lattice, the octupole positions and strengths are optimized to reduce the second order sextupole resonances. The results show that the DA and MA of the

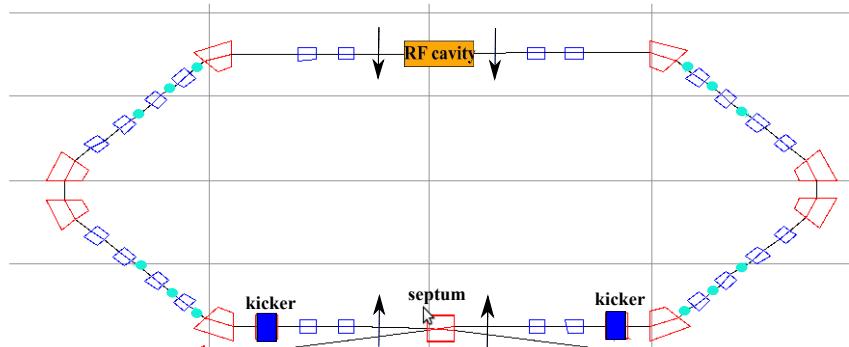


Figure 3: Layout of the ThomX ring with 4 octupoles to increase the DA and MA. The red trapezoids are dipoles; the blue rectangles are quadrupoles, the cyan dots are sextupoles; the down arrows are the defocusing octupoles, and the up arrows are the focusing octupoles.

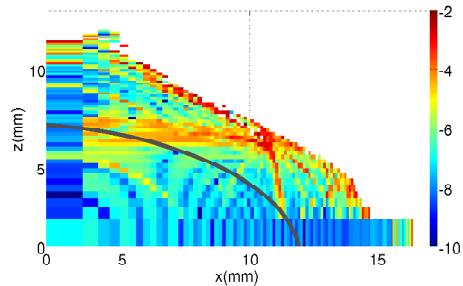


Figure 4: DA of the ThomX ring with dipole and quadrupole FFs, sextupoles, systematic multipole errors, and octupole correctors at the injection position. The black line is the vacuum chamber scaled by  $\sqrt{\beta_{\max,x,z}/\beta_{\text{inj},x,z}}$ , where  $\beta_{\max,x,z}$  are the maximum beta functions and  $\beta_{\text{inj},x,z}$  are the beta functions at the injection position.

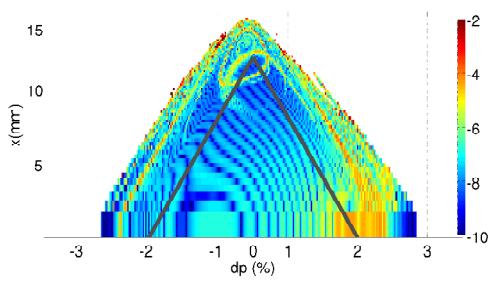


Figure 5: MA of the ThomX ring with dipole and quadrupole FFs, sextupoles, systematic multipole errors, and octupole correctors at the injection position. The black line is the linear limit of the vacuum chamber scaled by the maximum dispersion  $\eta_{\max}$ .

ThomX ring can be increased by the octupole correctors, which is positive for the future machine optimization since the MA is a weak point of the ThomX ring.

The DA and MA in this paper are tracked using the modified Tracy3 code [1, 11, 12] without the physical vacuum chamber size limitation for the nominal ThomX lattice of zero-dispersions in the straight section. In the future, the feasibility to increase the DA and MA using octupoles will be investigated for the ThomX lattice of non-zero dispersions in the straight sections.

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