LIST OF REFERENCES

1. L. D. Landau, Nucl. Phys. *13*, 181 (1959).
2. R. Karplus, C. M. Sommerfield, E. H. Wichman. Phys. Rev. *111*, 1187 (1958).
3a) T. Regge, Nuovo Cim. *14*, 951 (1959); *18*, 947 (1960).
b) V. N. Gribov, JETP, *41*, 667 (1961); *41*, 1962 (1961).
c) G. F. Chew, S. C. Frautschi, Phys. Rev. Letters, *5*, 580 (1961); *7*, 394 (1961).
d) R. Blankenbecler, M. L. Goldberger. Phys. Rev. *126*, 766 (1962).
e) S. C. Frautschi, M. Gell-Mann, F. Zachariasen (preprint).
f) S. C. Frautschi, G. F. Chew, S. Mandelstam (preprint).
4. V. N. Gribov, I. Ya. Pomeranchuk, JETP, *43*, 310 (1962).
5. V. N. Gribov, JETP, *41*, 1962 (1961).
6. E. Predazzi, T. Regge, Nuovo Cim. *24*, 518 (1962).

FERMION REGGE-POLES AND ASYMPTOTIC BEHAVIOUR OF MESON-NUCLEON LARGE-ANGLE SCATTERING

V. N. Gribov

Ioffe Physico-Technical Institute, Academy of Sciences of the USSR, Leningrad

(presented by V. N. Gribov)

1. INTRODUCTION

An extremely interesting and promising development of the theory of strong interactions has recently been proposed on the basis of a concept regarding moving poles of the scattering amplitudes as angular momentum functions, i.e., the concept of Regge-pole [1)] trajectories.

The attractiveness of this concept lies, first of all, in the fact that the Regge poles connect the spectra of particles and resonances to the asymptotic behaviour of high energy scattering. Besides, the asymptotic behaviour of the scattering, particularly in the region of small angles, proves to be comparatively simple and universal, though lacking any simple classical description. From the point of view of the Regge-pole concept, the pole trajectories, not the elementary particles, should become the main object of investigation in the theory of strong interactions.

The boson Regge-pole trajectories, particularly those of a pole having the quantum numbers of the vacuum (namely the Pomeranchuk trajectories), have been discussed in some detail [2, 3, 4, 5)].

Here we are going to show that the Regge-pole trajectories describing the fermion family (having a direct physical meaning at half-integer values of the angular momentum) possess a number of properties substantially different from those previously discussed and shown by non-relativistic quantum mechanics. Namely, we shall show that the poles of the scattering amplitudes $f^j_+(u)$ and $f^j_-(u)$ (referring to states with angular momentum j and parity $(-1)^{j\pm\frac{1}{2}}$) must coincide when the squared energy in the C.M. system u tends to zero, and become complex conjugate for $u<0$. This leads to an absolutely characteristic behaviour of meson-nucleon elastic scattering amplitudes in the region of angles close to 180° (formulae (9)-(11)).

The fact that the poles of $f^j_+(u)$ and $f^j_+(u)$ should coincide at $u = 0$ can be understood almost without calculations. Let us suppose that a pole of one of the amplitudes at $u = 0$ has $j = 1/2$; then there would be a corresponding particle with zero mass and ½ spin, i.e., a " neutrino ". A contribution to the meson-nucleon scattering amplitude due to such a particle is represented by the Feynman graph shown below (Fig. 1):

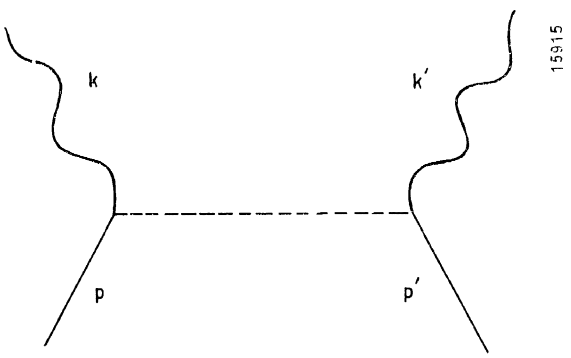

Fig. 1

With a non-gradient coupling this contribution is given by $\frac{1}{\hat{p}+\hat{k}}$ or $i\gamma_5\frac{1}{\hat{p}+\hat{k}}i\gamma_5$ depending on the parity of the " neutrino " with respect to the nucleon and the meson. In the former case there must be a pole in the amplitude $f^j_-(u)$ (a $S_{1/2}$-state), in the latter case the amplitude $f^j_+(u)$ must have a pole (a $P_{1/2}$-state). However, since $i\gamma_5\frac{1}{\hat{p}+\hat{k}}i\gamma_5=\frac{1}{\hat{p}+\hat{k}}$ the interaction in both cases is the same; consequently there must be poles in both the amplitudes $f^j_-(u)$ and $f^j_+(u)$. Thereby, a well-known circumstance is manifested, namely, because of γ_5-invariance of the Dirac equation for a zero mass particle, the neutrino parity concept has no meaning even in a theory with parity conservation. Since poles of $f^j_+(u)$ and $f^j_-(u)$ become complex conjugate for $u<0$, a close connection between these amplitudes is demonstrated. This connection is due to the kinematic singularity $\sqrt{u}$ occurring in the spinor amplitudes. $\sqrt{u}$ enters in the expressions for $f^j_\pm(u)$ in such a manner that it is possible to introduce the function $f^j(\sqrt{u})$ so that (*)

$$f^j_+(u)=f^j(\sqrt{u}), \qquad f^j_-(u)=f^j(-\sqrt{u}) \tag{1}$$

each pole of $f^j(\sqrt{u})$ in the variable $j=j(\sqrt{u})$ appearing in both amplitudes, and having simple properties as a function of $\sqrt{u}$.

Due to the explicit dependence of the equation determining the trajectory of the pole on $\sqrt{u}$, the momentum of the state will necessarily become complex when u is negative.

Thus, from purely kinematic considerations we conclude that there exist no fermion Regge-poles corresponding to states with real momenta and imaginary masses.

There arises a question whether, in the boson case, the momentum of states with imaginary mass must always be complex in spite of the absence of any kinematical reasons. We have no answer to this question for the present time. Let us only note that if such behaviour is supposed to be characteristic of the Pomeranchuk trajectory the partial wave with $l=0$ will have, generally speaking, complex non-physical poles as a function of energy.

2. FERMION REGGE-POLES CLOSE TO $u=0$

To prove the statements made let us consider meson-nucleon scattering. Regge-poles in the amplitudes of this process have been considered in a number of papers [3, 4, 5], where however the above situation has not been remarked. The scattering amplitude in a state of definite isotopic spin has the following form

$$F=a(u,t)+b(u,t)\tfrac{1}{2}(\hat{k}+\hat{k}') \tag{2}$$

$u=(p+k)^2$ is the square of the energy in the C.M. system, $-t=-(p'-p)^2$ is the square of the momentum transfer.

Considering the matrix elements $\langle\lambda'|F|\lambda\rangle$ between the states of nucleons with definite helicities λ and $\lambda'=$ either $+\lambda$ or $-\lambda$, it is easy to obtain the connection between $a(u,t)$, $b(u,t)$ and the partial amplitudes $\phi^j_{\lambda'\lambda}(u)$ [6].

$$A(u,t)=2ma(u,t)+(u^2-m^2-\mu^2)b(u,t)$$
$$=2\sum_j\phi^j_{\lambda\lambda}(u)[P'_{j+\frac{1}{2}}(z)-P'_{j-\frac{1}{2}}(z)]$$

$$B(u,t)=(u^2+m^2-\mu^2)a(u,t)+(u^2-m^2+\mu^2)mb(u,t)$$
$$=2\sqrt{u}\sum_j\phi^j_{-\lambda\lambda}(u)[P'_{j+\frac{1}{2}}(z)+P'_{j-\frac{1}{2}}(z)] \tag{3}$$

Here z is the cosine of the scattering angle:

$$z=1+2ut[u^2-2u(m^2+\mu^2)+(m^2-\mu^2)^2]^{-1}.$$

(*) I am indebted to V. M. Shekhter who has brought my attention to this point. The detailed proof of this formula and its application will be discussed elsewhere.

The "spiral" amplitudes $\phi^j_{\lambda'\lambda}(u)$ are connected to the amplitudes $f^j_\pm(u)$ with definite angular momentum and parity through the following relations

$$\phi^j_{\pm\lambda,\lambda}(u) = \tfrac{1}{2}[f^j_-(u) \pm f^j_+(u)] \qquad (4)$$

If we use the dispersion relations for $A(u, t)$ and $B(u, t)$ in the momentum transfer t at fixed u, by analogy with what was done in [3, 7] we can introduce the analytic functions of j, $\phi^{\pm j}_{\lambda'\lambda}(u)$, having a decreasing asymptotic behaviour for $j \to \infty$, satisfying the unitarity condition, and coinciding with the physical partial waves at even (+) and odd (−) j respectively. If, with the help of (3), we express $\phi^j_{\lambda'\lambda}(u)$ through $A(u, t)$ and $B(u, t)$, by continuing the obtained expressions to the point $u = 0$ and taking into consideration that $A(u, t)$ and $B(u, t)$ have no singularities at $u = 0$, we shall be led to the almost obvious conclusion (from the stand-point of (3)) that for any j, $\phi^j_{-\lambda\lambda}(u)$, tends to infinity or to zero at $u \to 0$, while $\phi^j_{\lambda\lambda}(u)$ remains finite. This is possible only under the condition that the singularities of $f^j_\pm(u)$ as functions of j coincide at $u = 0$. This statement holds irrespectively of the character of the singularities of $\phi^j_{\lambda'\lambda}$.

If one makes the conjecture that in any case, the nearest singularities on the side of large j are poles, one can trace their trajectories in more detail. In order to do that let us consider the asymptotic behaviour of $A(u, t)$, $B(u, t)$ at $u > 0$ and $t \to -\infty$, i.e. $s \to \infty$:

$$s = (p-k')^2, \qquad s+t+u = 2m^2+2\mu^2 .$$

By passing in the usual manner from the sum to the integral, assuming the poles at $u > 0$ to be on the real axis, and taking into account only the nearest poles in the amplitudes $\phi^{\pm j}_{\lambda'\lambda}$ we obtain

$$\begin{aligned} A^\pm(u,s) &= \tau^\pm_{\lambda\lambda}[s^{j-\frac{1}{2}} \mp (-s)^{j-\frac{1}{2}}]\frac{1}{\cos \pi j} \\ B^\pm(u,s) &= \sqrt{u}\,\tau^\pm_{-\lambda\lambda}[s^{j'-\frac{1}{2}} \pm (-s)^{j'-\frac{1}{2}}]\frac{1}{\cos \pi j'} \end{aligned} \qquad (5)$$

Here $j = j(u)$, $j' = j'(u)$ are the positions of the poles of $\phi^j_{+\lambda\lambda}(u)$ and $\phi^j_{-\lambda\lambda}(u)$, $\tau^\pm_{\lambda'\lambda}$ are the residues of the partial-wave amplitudes multiplied by

$$2^{j+1}\sqrt{2\pi}\frac{\Gamma(j+1)}{\Gamma(j+\frac{1}{2})}u[u^2-2u(m^2+\mu^2)+(m^2-\mu^2)^2]^{-1} .$$

It is easy to convince oneself [2] that the absorptive parts $A_1(u, s)$ and $B_1(u, s)$ are equal:

$$\begin{aligned} A^\pm_1(u,s) &= \pm\tau^\pm_{\lambda\lambda}(u)s^{j-\frac{1}{2}} \\ B^\pm_1(u,s) &= \pm\sqrt{u}\,\tau^\pm_{-\lambda\lambda}(u)s^{j'-\frac{1}{2}} \end{aligned} \qquad (6)$$

If we do not assume that there is a degeneration over parity, the poles $f^j_+(u)$ and $f^j_-(u)$ do not coincide for $u > 0$. Therefore, in the asymptotic behaviour of A and B the contribution is given only by the pole of one of the amplitudes, the one having greater j at given u. In this case, according to (4)

$$\tau_{\lambda\lambda} = \pm\tau_{-\lambda\lambda}; \qquad j' = j \qquad (7)$$

and consequently

$$B^\pm_1(u,s) = \pm\sqrt{u}A^\pm_1(u,s) \qquad (8)$$

At $u = 0$ the equality (8) makes no sense at all, since the poles of $f^j_+(u)$ and $f^j_-(u)$ coincide. If we consider that the poles of $f^j_+(u)$ and $f^j_-(u)$ remain on the real axis, for $u < 0$, as before, one of the poles will again become dominant and the expression (8) will be re-established. However, in this case because $\sqrt{u}$ is imaginary, (8) will be in contradiction with the reality of the functions $A_1(u, s)$ and $B_1(u, s)$ for $u < (m+\mu)^2$.

If the poles $f^j_+(u)$ and $f^j_-(u)$ retreat to the complex plane but do not remain complex conjugate, we shall again come to a contradiction with the reality of A_1 and B_1.

Thus, we come to the conclusion that the poles of the amplitude $f^j_+(u)$ and $f^j_-(u)$ should be complex conjugate for $u < 0$.

By the same reason the residues at the poles should also be complex conjugate.

3. THE ASYMPTOTIC BEHAVIOUR OF BACKWARD SCATTERING

Denoting the residues of $f^j_-(u)$ and f^j_+ multiplied by

$$2^{j+1}\sqrt{2\pi}\frac{\Gamma(j+1)}{\Gamma(j+\frac{1}{2})}u[u^2-2u(m^2+\mu^2)+(m^2-\mu^2)^2]^{-1}$$

by $\rho e^{\pm i\phi}$ we obtain

$$\begin{aligned} A^\pm_1(u,s) &= \pm\rho^\pm(u)s^{j'-\frac{1}{2}}\cos(j''\xi+\phi) \\ B^\pm_1(u,s) &= \pm\sqrt{-u}\,\rho^\pm(u)s^{j'-\frac{1}{2}}\sin(j''\xi+\phi) \end{aligned} \qquad (9)$$

where $\xi = \ln s$; $j' = j'(u)$, $j'' = j''(u)$ are the real and imaginary parts of the function $j = j(u)$ determining the position of the pole. The formulae (9) go into (6) for u positive so that A_1 and B_1 have no singularities at $u = 0$, if $j''(u) = \alpha\sqrt{-u}$ and $\phi(u) = \beta\sqrt{-u}$ for small u, where α and β have no singularity at $u = 0$.

The real parts of the amplitudes A and B at $u<0$ have the form

$$\operatorname{Re} A^{\pm} = \alpha_{\pm}\rho^{\pm}(u)\cos(j''\xi+\phi\mp\beta)s^{j'-\frac{1}{2}}$$

$$\operatorname{Re} B^{\pm} = \alpha_{\pm}\rho^{\pm}(u)\sqrt{-u}\sin(j''\xi+\phi\mp\beta)s^{j'-\frac{1}{2}}$$

$$\alpha_{\pm}^2 = \frac{\operatorname{ch}\pi j''\mp\sin\pi j'}{\operatorname{ch}\pi j''\pm\sin\pi j'},\quad \operatorname{tg}\beta = \frac{\operatorname{sh}\pi j''}{\cos\pi j'} \tag{10}$$

The expressions (9) and (10) define the asymptotic behaviour of meson-nucleon scattering in the channel where s is the energy and in the region of scattering angles close to 180°. Analogous formulae hold for the asymptotic behaviour of the process of a two-meson annihilation (where t is the energy).

The differential cross-section for the elastic scattering in the region of the angles close to 180° takes the form

$$\frac{d\sigma}{d\Omega} = \rho'^2(u)\left\{1+\sqrt{(1-\gamma^2)(1-\delta^2)}\cos 2(j''\xi+\phi'(u))\right\} \times\left(\frac{s}{m\mu}\right)^{2j'-1} \tag{11}$$

$$\delta = \frac{\alpha_{\pm}\sin\beta}{1+\alpha_{\pm}^2};$$

$$\gamma = \frac{4m^2u(m^2-\mu^2)+u(u+m^2-\mu^2)(u-3m^2-\mu^2)}{2m^2[u^2+(m^2-\mu^2)^2]-u[(u-m^2-\mu^2)^2+4m^2]}$$

As it was pointed out to the author by I. Ya. Pomeranchuk, the asymptotic behaviour (9)-(11) is due to an effective radius proportional to $\ln s$ if $j''(u)\sim\sqrt{-u}$. According to Froissart [8] this is the maximum possible increase of the effective radius of the interaction. On the other hand, the increase of the effective radius of interaction accounting for the diffraction scattering is proportional to $\ln^{\frac{1}{2}} s$ [2].

It should be noted that since the backward scattering amplitude does not increase with increasing energy, as is the case in forward scattering, the poles determining the asymptotic behaviour may, at small u, be in the region of $\operatorname{Re} j<0$. If the poles are in the region $\operatorname{Re} j<0$, the asymptotic behaviour may not be determined by them both because of the presence of the singularities of other types and because of the fact that for $\operatorname{Re} j<0$, the Legendre functions again become increasing with increasing z, and (9)-(11), generally speaking, do not follow from (3). To obtain these formulae it is convenient in this case to proceed by analogy with what was done by Mandelstam [9] for spinless particles.

The Mandelstam method consists of the following: by passing from the sum (3) to the integral and deforming the integration contour it is necessary to replace $P'_{j\pm\frac{1}{2}}(-z)$ according to the formula

$$-\frac{P'_{j\pm\frac{1}{2}}(-z)}{\cos\pi j} = \frac{1}{\pi\sin\pi j}\left[Q'_{j\pm\frac{1}{2}}(-z)-Q'_{-(j\pm\frac{1}{2})-1}(-z)\right] \tag{12}$$

The behaviour of $\phi^j_{\lambda'\lambda}$ for large j allows one, by calculating the contribution from the first term, to close the integration contour in the right-hand plane and to reduce it to the sum of the residues at the poles $1/\sin\pi j$. The second term for large z behaves like $z^{j\pm\frac{1}{2}-1}$, and therefore in the calculation of its contribution, it is helpful to deform the integration contour to the left-hand plane. Then it runs against the poles $1/\sin\pi j$. As a result, if the contour is deformed sufficiently to the left, the amplitudes A and B will, besides the conventional contribution from the poles of $\phi^j_{\lambda'\lambda}$, contain some additional terms of the form

$$\sum_{n=0}^{N}[\phi^n_{\lambda'\lambda}(u)+\phi^{-n}_{\lambda'\lambda}]Q'_{n-\frac{1}{2}}(-z) \tag{13}$$

which at large z behave like $z^{-(3/2+n)}$. The quantities $\phi^n_{\lambda'\lambda}+\phi^{-n}_{\lambda'\lambda}$ contained in (13) may be equal to zero, the asymptotic behaviour being determined only by the poles of $\phi^j_{\lambda'\lambda}$.

It may be shown that the amplitudes $\phi^j_{\lambda'\lambda}$ in the case of the scattering of a spin ½-particle on an external field will satisfy the condition $\phi^n_{\lambda'\lambda} = -\phi^{-n}_{\lambda'\lambda}$. We do not know whether this is valid in a real case. It may be only stated that if the partial waves have no singularities except the poles, $d\sigma/d\Omega$ at high energies either has the form (11), or the following one

$$\frac{d\sigma}{d\Omega} = C(u)\frac{1}{s^{n+3}} \tag{14}$$

where n is an integer number, or otherwise equals zero.

It should be noted that formulae (3) and (9)-(11) are, generally speaking, inapplicable in the region of such a small u that $us/(m^2-\mu^2)^2 \lesssim 1$, since in this region $z \lesssim 1$, and we have no reasons for confining ourselves to only one pole.

ERRATA ADDED IN PROOF
(received on October 1st 1962)

When calculating the elastic scattering differential cross-section (11) from formulae (9) and (10) for the scattering amplitudes an error was committed. The correct expression for $\frac{d\sigma}{d\Omega}$ has the form

$$\frac{d\sigma}{d\Omega} = \eta\left[|A|^2 - \frac{l}{u}|B|^2\right] = C(u)s^{2j'(u)-1}$$

$$C(u) = \eta\rho^2(1+\alpha^2),$$

$$\eta = -u\left[u^2 - 2u(m^2+\mu^2) + (m^2-\mu^2)^2\right]^{-1} \qquad (11')$$

It follows from this expression that $\frac{d\sigma}{d\Omega}$ does not oscillate as a function of the energy, despite the oscillations of the amplitudes A and B. All the other scattering characteristics are oscillating ones. For instance, the nucleon polarization ζ for the scattering on the non-polarized target does not decrease but oscillates as a function of the energy.

$$\zeta = \left(1 - \frac{\sin^2\pi j'}{\mathrm{ch}^2\pi j''}\right)^{\frac{1}{2}} \sin\left[2j''(u)\xi + 2\omega(u) \mp \beta(u)\right]$$

LIST OF REFERENCES

1. T. Regge, Nuovo Cimento *14*, 951 (1959); *18*, 947 (1960).
2. V. N. Gribov, JETP *41*, 667, 1962 (1961).
3. G. F. Chew, S. C. Frautschi, Phys. Rev. Lett. *7*, 394 (1961).
4. S. C. Frautschi, M. Gell-Mann, F. Zachariasen (preprint).
5. V. N. Gribov, I. Ya. Pomeranchuk, JETP (in press).
6. M. Jacob, G. C. Wick, Ann. Phys. *7*, 404 (1959).
7. V. N. Gribov, JETP (in press).
8. M. Froissart, Phys. Rev. *123*, 1053 (1961).
9. S. Mandelstam (preprint).

DISCUSSION

LOVELACE: Can you say anything about the slope of the Fermion trajectories? The experimentalists have suggested that there may be a very sharp backward π^+p peak sharper than the forward peak.

GRIBOV: The differential cross-section at small backward angles is that given by Equation (11) of the paper.

The behaviour of the cross-section for backward angles is of this type, see Fig. A, The cross-section decreases and shows oscillations. The width of the peak is of the order of $\mu/p \ln^{\frac{1}{2}} s$. The width of the oscillations is of the order or $\mu/p \ln s$. The oscillation has a smaller range than the decrease of the cross-section. The radius ϱ of this interaction is growing faster than in forward scattering;

$$\varrho_{\text{forw}} \ln^{\frac{1}{2}} \propto s, \quad \varrho_{\text{back}} \propto \ln s.$$

This is the maximum growth of the interaction radius according to Froissart.

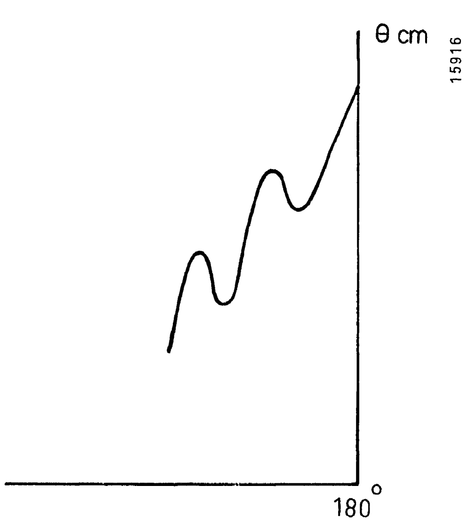

Fig. A