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Complementary Continuous-Discrete Time, Chronon Layering and Temporal Folding

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Abstract

Within the framework of a discrete-time chronon model, we consider a dual description of physical time. In this description, macroscopic time is a continuous parameter, while a microscopic integer chronon index labels elementary updates of the system. On this basis, a hierarchy of temporal layers Ch_N (Chronon) is introduced. The simple layers Ch2, Ch3 and Ch4 are analysed, and it is shown that they naturally support $U(1)$ (Unitary group), $SU(3)$ (Special Unitary group) and a pair-locked $SU(2)$ (Special Unitary group) symmetry, respectively. Special attention is paid to the twelve-slot layer Ch12. This layer is the minimal one which simultaneously separates partitions into four triads and three quartets. For Ch12, we demonstrate that the intersection of the corresponding commutants in $\mathbb{C}^3 \otimes \mathbb{C}^4$ reproduces the Standard Model gauge algebra $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the pattern of hypercharges and anomaly cancellation. The appearance of three fermion generations is interpreted in terms of three inequivalent embeddings of a triad into the dodecad which preserve the quartet structure. Possible connections of the chronon dynamics with the hierarchy of masses (via Floquet-type quasi-energies), with dark sectors associated with misaligned layers, and with a temporal interpretation of entanglement are briefly discussed on a qualitative level. These questions are formulated as open problems for further study.

Keywords: emergent gravity; foundations of time; discrete time; chronon; multimetric gravity; entanglement; Floquet dynamics

1. Introduction

The problem of reconciling General Relativity (GR), where time is a dynamical coordinate on a Lorentzian manifold [1,2], with Quantum Field Theory (QFT), where time appears as an external parameter for unitary evolution, is still open. Different approaches to quantum gravity and to the microscopic structure of spacetime are actively discussed in the literature. Among them, one can mention Loop Quantum Gravity and spin-foam models [3–6], Causal Set Theory [7–11], Causal Dynamical Triangulations [12–15], as well as the idea of asymptotic safety [16] and various scenarios where gravity is an emergent macroscopic phenomenon [17–22].

In parallel, there exist many attempts to understand the internal symmetries and matter content of the Standard Model directly from some underlying discrete or algebraic structure [23–25]. In such works, the gauge group and the representation content are expected to follow from the construction and not to be postulated by hand. In the present paper, we do not try to compete with these approaches on the level of dynamics. Our

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aim is closer to kinematics and to show that a very simple discrete time organisation of microscopic evolution is already sufficient to encode several characteristic features of the Standard Model spectrum.

We consider a dual description of time. On the one hand, there is a continuous macroscopic time t, which appears in the usual way in effective field theory. On the other hand, there is a discrete chronon index $n \in \mathbb{Z}$ describing a fundamental update step of the total system. The microscopic dynamics on the base layer (Ch1) is assumed to be a stochastic process on a hyperfinite Borel structure [26–28]. Coarse-graining over N steps defines an effective layer Ch_N with N internal temporal "slots". The algebraic structure of the effective Hamiltonian on a given layer is restricted by permutations of these slots which preserve the coarse-grained measure.

It turns out that already the first non-trivial layers are important. The layers Ch2, Ch3 and Ch4 support $U(1)$, $SU(3)$ and a pair-locked $SU(2)$ structures, respectively. The least common multiple $N = 12$ is the smallest period which allows us to accommodate simultaneously triadic and quartic patterns. In the dodecad layer Ch12, we obtain an effective Hilbert space $\mathcal{H}_{12} \simeq \mathbb{C}^{12} \simeq \mathbb{C}^3 \otimes \mathbb{C}^4$, and the intersection of the commutants of triad and quartet permutations in this space leads to the gauge algebra $SU(3)_C \times SU(2)_L \times U(1)_Y$. In the same combinatorial picture, the cancellation of gauge anomalies reduces to simple integer relations, and three generations of fermions correspond to three inequivalent embeddings of a triad into the dodecad which preserve the quartet partition.

The chronon hierarchy naturally suggests to use a Floquet-type language for describing the coarse-grained evolution [29–34]. In such a description, quasi-energies provide a convenient way to parameterise mass scales and possible small periodic corrections to the effective metric. However, in the present work, we restrict ourselves to qualitative comments in this direction and do not attempt a quantitative phenomenological analysis. The same remark applies to the possible interpretation of entanglement as a manifestation of temporal correlations between different layers.

Scope and Status of Claims

The core results of this work are kinematic: starting from a discrete chronon update index and the coarse-grained hierarchy of layers Ch_N, we constrain the algebra of the effective stroboscopic dynamics by slot-permutation invariances and compute commutants/intersections in finite-dimensional slot spaces. The identification with $SU(3)_C \times SU(2)_L \times U(1)_Y$ refers to the global internal symmetry algebra of the effective theory on Ch12; we do not derive a UV completion nor the full gauge-boson dynamics here. Remarks on gravity, dark sectors and entanglement (Sections 9 and 10) are qualitative and stated as open problems.

The paper is organised as follows. In Section 2 we introduce the basic definitions of the chronon hierarchy and the notion of a layer Ch_N. In Sections 3 and 4 we analyse the algebraic structure of the layers Ch2, Ch3 and Ch4. Section 5 is devoted to the special role of the dodecad layer Ch12. In Sections 6 and 7 we show how the Standard Model gauge algebra, charges and generation structure arise in this framework. Section 8 contains brief remarks about Higgs and neutrino sectors. In Sections 9 and 10 we discuss the effective field-theoretic description, gravity and entanglement. Section 11 contains a summary of the results and a list of open problems.

2. Foundational Framework

The dual-time framework is constructed to ensure compatibility with standard QFT while introducing a discrete substructure.

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2.1. Dual Representation and Matching

We assume distinguishable histories differ by integer multiples of action, $S(t_2, t_1) = m\hbar$, which introduces a characteristic microscopic timescale τ_1 without imposing a global lattice on t. Here the "dual-time" terminology does *not* mean two physical clock times. There is one macroscopic time parameter t used in the effective field theory (EFT) description, while $n \in \mathbb{Z}$ is a microscopic update counter that allows us to define stroboscopic coarse-graining and the layers Ch_N. The "quantum of action" assumption $S(t_2, t_1) = m\hbar$ is a modelling postulate that introduces a microscopic update scale τ_1 (as a minimal phase/action increment) without imposing a rigid lattice on the macroscopic time t. It is not meant as a new Standard-Model field degree of freedom by itself. The microscopic evolution is described by a sequence of graphs $G_n = (V_n, E_n)$, where edges carry spin-action labels satisfying the local balance condition:

$$\sum_{e \in E_{\mathrm{in}}(v)} \mathbf{J}(e) = \sum_{e \in E_{\mathrm{out}}(v)} \mathbf{J}(e). \tag{1}$$

To link the discrete and continuous sectors, we impose a matching condition. Every chronon edge e corresponds to a smooth embedding $\iota_e : [0, 1] \to M$ into the Lorentzian manifold. The endpoints are constrained to lie within a tolerance $\delta \ll \tau_1$ of the integer time steps $n\tau_1$. This ensures that the causal order defined by the discrete updates is causally consistent with the geometric time t.

2.2. Borel Structure and Renormalization

To rigorously define the layers, we employ the formalism of descriptive set theory. Let $X = Y^{\mathbb{Z}}$ be the configuration space of chronon labels. The shift operator T generates an orbit equivalence relation E_T, which is known to be hyperfinite. This property allows E_T to be approximated by finite relations E_n, providing a mathematical basis for our layering hypothesis.

We define the factor map

$$\pi_N : X \to X, \quad (\pi_N x)_k = x_{kN}. \tag{2}$$

This map induces a coarse-graining of the measure $\mu \mapsto \mu_N = \mu \circ \pi_N^{-1}$.

Definition (Temporal Layer Ch_N and Slot Space)

Given the base system $(X, \mathcal{B}, \mu, T)$, the block map π_N induces $\mathcal{B}_N = \pi_N^{-1}(\mathcal{B})$, $\mu_N = \mu \circ \pi_N^{-1}$ and the stroboscopic dynamics T^N. We refer to the corresponding factor description as the temporal layer Ch_N (sampling at resolution $\tau(N) = N\tau_1$). A single Ch_N cell contains N internal slots. Its internal slot Hilbert space is $H_N \simeq \mathbb{C}^N$, on which slot permutations act naturally. For the rigorous measure-theoretic construction, see Appendix B.

The full hierarchy of different layers is shown on Table 1.

Physically, this represents a Renormalization Group (RG) transformation $\mathcal{R}_N$. (Assumption.) A temporal layer Ch_N is dynamically distinguished if μ_N is a fixed point/attractor of the temporal RG flow. The effective dynamics on a layer Ch_N is governed by a stroboscopic unitary operator

$$U_N \approx \prod_{k=1}^{N} U_1^{(k)}, \tag{3}$$

which allows us to define an effective Hamiltonian $H_{\mathrm{eff}}^{(N)}$. The symmetry of this Hamiltonian is constrained by the permutations of the N internal slots that leave the coarse-grained measure invariant.

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Table 1. Layer hierarchy, minimal assumptions, and the resulting internal symmetry structure.

Layer	Minimal Structural Input (Assumption/Model Choice)	Slot Space	Resulting Internal Symmetry (Interpretation)
Ch_2	Two-slot cell, global phase invariance of $H^{(2)}_{\text{eff}}$	$H_2 \simeq \mathbb{C}^2$	$U(1)$ (seed for hypercharge)
Ch_3	Full slot equivalence, commutant constraint, isotropic/degenerate triplet sector	$H_3 \simeq \mathbb{C}^3$	$SU(3)$ (color sector, modulo phase)
Ch_4	Pair partition + pair-locking (coherent doublets)	$H_4 \simeq \mathbb{C}^4$	$SU(2)$ (weak isospin from locked doublet)
Ch_{12}	Simultaneous triad/quartet invariances, intersection of commutants in adapted factorization	$H_{12} \simeq \mathbb{C}^3 \otimes \mathbb{C}^4$	$SU(3)_C \times SU(2)_L \times U(1)_Y$, charges/anomalies; 3 compatible triad embeddings

3. Spin-Action Structure and Elementary Layers

The hierarchy begins with the elementary loop Ch1, carrying a single quantum of action. The first non-trivial structures emerge at $N = 2$ and $N = 3$.

The Doublet (Ch2) This layer consists of two slots. The effective Hamiltonian $H^{(2)}_{\text{eff}} = \delta\,\sigma_3 + \kappa\,\sigma_1$ describes a two-level system. The canonical commutation relation $[x_a, p_b] = i\hbar\delta_{ab}$ implies a minimal $(2+1)$-dimensional kinematics. Crucially, the Hamiltonian commutes with the global phase shift $\exp(i\alpha)\mathbb{1}_2$, identifying the internal symmetry group as $G(\text{Ch2}) = U(1)$. This Abelian factor is the seed for the hypercharge.

The Triplet (Ch3): Coarse-graining over three steps yields a three-slot cell with $H_3 \simeq \mathbb{C}^3$. Assuming full statistical equivalence of the slots (invariance under a transitive slot-permutation action), the effective Hamiltonian is constrained to lie in the commutant of the corresponding permutation representation on H_3. At a maximally isotropic fixed point (or within a fully degenerate triplet sector), the continuous unitary symmetry is enhanced to $U(3)$ (and hence to $SU(3)$ modulo an overall phase). A short explicit toy example of the commutant/degeneracy mechanism is given in Appendix B.

These elementary symmetries, $U(1)$ from the doublet and $SU(3)$ from the triplet, are the algebraic building blocks. Their combination requires a higher-order layer that can accommodate both structures simultaneously.

4. The Ch4 Layer and Pair-Locked $SU(2)$

The layer Ch4 ($\tau_4 = 4\tau_1$) introduces a new structural element which is the pair-locking mechanism. The four slots naturally partition into two pairs, $(1,3)$ and $(2,4)$, inherited from the underlying Ch2 blocks. Imposing symmetry under the permutation P that exchanges these pairs will force the Hamiltonian to be block-diagonal. A stronger physical condition is pair-locking, where the two doublets evolve coherently. This is formalized by requiring $[H^{(4)}_{\text{eff}}, \mathcal{L}] = 0$, where $\mathcal{L}$ swaps the pairs. The resulting Hamiltonian takes the tensor product form:

$$H^{(4)}_{\text{eff}} = A \otimes \mathbb{1}_2. \tag{4}$$

The operators $\sigma_i \otimes \mathbb{1}_2$ commute with this structure, generating an $SU(2)$ algebra. Thus, $G(\text{Ch4}) = SU(2)$. This derivation suggests that the weak isospin symmetry is not fundamental in the same sense as color, but is an emergent property of pair-correlated chronon dynamics. The effective geometry of this layer, after integrating out relative degrees of freedom, retains the $(2+1)$ Lorentzian signature.

5. The Dodecad: Structure and Stability of Ch12

We now turn to the central result of this framework. We seek the minimal temporal layer capable of sustaining the symmetries of the lower layers (Ch2, Ch3, Ch4) in a unified structure. Intermediate layers like Ch6 and Ch9 fail to produce the Standard Model group uniquely. Ch6 yields independent triadic and doublet sectors $[SU(3) \times SU(2) \times U(1)]$, but lacks the geometric locking to enforce a single gauge structure. Ch9 leads to an excessive $SU(3) \times SU(3)$ symmetry.

The layer Ch12 ($\tau_{12} = 12\tau_1$) is unique. It admits simultaneous partitions into four triads and three quartets. Concretely, labeling the 12 slots by $k \in \mathbb{Z}_{12}$, the triads can be taken as $\{\, i, i+4, i+8 \,\}$ for $i = 0,1,2,3$, while the quartets can be taken as $\{\, a, a+3, a+6, a+9 \,\}$ for $a = 0,1,2$. Equivalently, writing $k = 4a + i$ with $a \in \{0,1,2\}$ and $i \in \{0,1,2,3\}$ makes the compatibility with both partitions explicit and motivates the adapted identification $H_{12} \simeq \mathbb{C}^3 \otimes \mathbb{C}^4$ used in the commutant intersection. The effective Hamiltonian must satisfy:

$$[H_{\text{eff}}^{(12)}, S_3] = 0 \quad \text{and} \quad [H_{\text{eff}}^{(12)}, S_4] = 0, \tag{5}$$

where S_3 and S_4 generate triad and quartet permutations. The algebraic intersection of their commutants in $\mathcal{H}_{12} \simeq \mathbb{C}^3 \otimes \mathbb{C}^4$ is generated by $\lambda^a \otimes \mathbb{1}_4$, $\mathbb{1}_3 \otimes \sigma_i$, and the global phase. This yields precisely the group:

$$G(\text{Ch12}) = SU(3) \times SU(2) \times U(1). \tag{6}$$

Dynamical Stability: The preference for $N = 12$ is not merely combinatorial but dynamical. In the language of coupled oscillators, a stable periodic orbit requires phase locking of subharmonics [35]. $N = 12$ is the first "highly composite number" that supports the simultaneous locking of modes 2ω (doublet), 3ω (triplet), and 4ω (quartet). An effective Landau functional for the chronon phases, $\mathcal{V}_N \sim \sum_{m|N} \alpha_m [1 - \cos(m\theta)]$, is minimized when N has a maximal density of divisors. This identifies Ch12 as a deep local minimum of the vacuum energy, an attractor of the temporal RG flow.

6. Emergence of Gauge Symmetry and Charges

Having established the symmetry group, we define the physical charges as integer-valued projections on the slot indices. Let ρ_k be the integer label of the k-th slot.

Hypercharge Y is the unique linear combination invariant under triad permutations:

$$Y = \frac{1}{6} \sum_{i=0}^{3} C_i, \tag{7}$$

where C_i are triad sums. The factor $1/6$ arises naturally from the normalization of the integer lattice.

Weak Isospin T_3 arises from the quartet structure, specifically the difference between the first and second pairs in the quartets:

$$T_3 = \frac{1}{2}[(\rho_1 + \rho_2) - (\rho_3 + \rho_4)]. \tag{8}$$

Electric Charge follows as $Q = T_3 + Y$. These definitions are not ad hoc. They are the only linear invariants consistent with the Ch12 partitions. This implies that charge quantization is a direct consequence of the discrete slot structure.

7. Generation Structure and Anomaly Cancellation

The phenomenon of three fermion generations finds a topological explanation in the Ch12 geometry. A fermion field corresponds to an embedding of a triadic pattern, which represents color, into the 12-slot cycle. There are four possible positions for a triad. However, we must impose compatibility with the quartet structure, that is with weak isospin. The subgroup of cyclic shifts preserving the quartets is $\Gamma = \{0, 4, 8\}$. The quotient space of triad embeddings under this subgroup consists of exactly three equivalence classes: $\mathcal{P}_0, \mathcal{P}_4, \mathcal{P}_8$. The fourth shift, $\mathcal{P}_3$, is incompatible with the quartet partition. Thus, there are exactly three inequivalent ways to embed a colored fermion into the dodecad vacuum. These correspond to the three generations $\psi_{(0)}, \psi_{(1)}, \psi_{(2)}$.

Crucially, this geometric construction enforces an anomaly cancellation. The integer identities associated with the triad and quartet partitions ensure that

$$\sum_X Y^{(X)} = 0, \quad \sum_X [Y^{(X)}]^3 = 0. \tag{9}$$

For clarity, in the standard left-handed Weyl convention (i.e., right-handed fields represented by left-handed conjugates), the resulting $U(1)_Y$ assignment matches the usual SM normalization, and one finds within one generation $\mathrm{Tr}\, Y = 0$ and $\mathrm{Tr}\, Y^3 = 0$ explicitly; the mixed anomalies cancel for the same reason: triad and quartet contributions enter the relevant traces in cancelling combinations fixed by the partition invariants.

The mixed anomalies vanish similarly because weak doublets and color triplets contribute to the hypercharge sum in cancelling pairs. This renders the theory consistent at the quantum level without fine-tuning.

8. Higgs and Neutrino Sectors

The scalar sector is constrained by the decomposition of the internal Hilbert space $\mathcal{H}_{12} \simeq \mathbb{C}^3 \otimes \mathbb{C}^4$. The quartet factor decomposes as $\mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1}$ under $SU(2)$. The unique doublet component $\mathbf{2}$ is identified as the Higgs field H. Its potential $V(H)$ is the standard quartic invariant. For neutrinos, the triadic factor contributes a threefold degenerate subspace in the absence of mixing. This degeneracy explains why there are three light neutral modes. The mixing term δH_{mix} between the triad and quartet sectors lifts this degeneracy, generating small neutrino masses via a mechanism analogous to the seesaw, but purely structural. The suppression comes from the hierarchy of scales between the fundamental chronon and the effective layer period.

9. Effective Field Theory and Gravity

The coarse-graining of the chronon dynamics leads to an effective field theory (EFT) with the standard Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{ferm}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{Y}}. \tag{10}$$

The mass parameters in this EFT are determined by the quasi-energies of the Floquet Hamiltonian H_F.

In the gravitational sector, the metric $g_{\mu\nu}$ emerges as the statistical expectation value of chronon embeddings. In this sense, our description is naturally phrased in the language of covariant effective actions for gravity, where quantum and microscopic corrections are encoded in higher-derivative and nonlocal terms in the metric sector [36]. A novel prediction of this framework is the Floquet modulation of the metric:

$$g_{\mu\nu}(t) \approx \bar{g}_{\mu\nu} + \varepsilon_{\mu\nu} \cos\left(\frac{2\pi t}{\tau_{12}}\right). \tag{11}$$

Any such modulation, if present, would have to be extremely suppressed and consistent with existing observational bounds. In the present paper, we do not compute $\varepsilon_{\mu\nu}$ nor derive quantitative constraints; we only note this as a possible phenomenological direction for future study.

Furthermore, the framework offers a candidate for Dark Matter. Excitations in layers like Ch6 or Ch9 are "misaligned" with the Ch12 gauge group. Their projections $\Pi_{b\to 12}$ are gauge-neutral but possess energy-momentum. Consequently, they interact only gravitationally. The smoothing effect of the projection naturally leads to "cored" density profiles ($\rho \sim (1 + r/r_c)^{-2}$), potentially resolving the cusp-core problem in galactic halos.

10. Entanglement as Temporal Contextuality

We propose a reinterpretation of quantum entanglement based on the non-commutative geometry of the layers. The global condition for a closed history in the dodecad layer is $C^{12} = \mathbb{1}$, where C is the joint shift operator. This defines a global projector P_{ctx}. Local observables $A_a \otimes \mathbb{1}$ do not necessarily commute with P_{ctx}. This implies that "entanglement" is the manifestation of a global temporal constraint that prevents the assignment of local hidden variables. The violation of Bell inequalities is thus a result of temporal contextuality in the Kochen–Specker sense [37–39] rather than superluminal influence. The overlap integral I_{AB} of the layer activation profiles quantifies the capacity of the system to sustain this contextuality against decoherence.

11. Discussion and Open Problems

The proposed chronon framework shows that a rather simple discrete-time organisation of microscopic dynamics can reproduce several structural properties of the Standard Model. The dodecad layer Ch12 simultaneously contains four triads and three quartets, and this fact is sufficient to obtain the gauge algebra $SU(3)_C \times SU(2)_L \times U(1)_Y$, the usual pattern of hypercharges and anomaly cancellation, and the existence of three fermion generations. Only three inequivalent triad embeddings remain compatible with the quartet partition, which we interpret as three generations. In this sense, the model is close in spirit to other algebraic constructions of the Standard Model, but the main organising principle here is the hierarchy of temporal layers.

At the same time, a number of important questions remain open.

First, although the chronon picture suggests that fermion and boson masses should be related to Floquet-type quasi-energies, in the present work no concrete mass ratios or mixing angles are calculated. The mixing Hamiltonian δH_{mix}, which couples different sectors, is introduced only schematically and is effectively a free parameter. To speak about phenomenology, one needs at least one explicit example where δH_{mix} is derived from the microscopic dynamics and leads to realistic spectra.

Second, the ultraviolet completion of the theory is not constructed. The base dynamics on the fundamental layer (Ch1) is described only in general terms as a stochastic process on a hyperfinite Borel structure. We assume that this process has an attractor and that the effective description flows towards the dodecad layer Ch12, but a direct derivation of such a flow is absent. A more detailed study of the Borel dynamics and of the conditions under which the hierarchy Ch_N is realised is required.

Third, the idea to identify certain "shadow" layers (for example Ch6 and Ch9) with dark sectors is attractive, since excitations on these layers are neutral with respect to the effective gauge group but still gravitate. However, in the present work, only a qualitative argument is given that such layers may produce approximately cored profiles. A realistic comparison with astrophysical data would require to compute relic densities and halo profiles in a fully dynamical model.

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We do not address inflation or late-time accelerated expansion in this work; the "misaligned layer" remark is only a qualitative dark-matter-motivated possibility (SM-neutral but gravitating excitations). A concrete particle/cosmological model (spectrum, stability, relic abundance) requires specifying the microscopic update dynamics and is left for future work.

Finally, small periodic corrections to the effective metric, which appear naturally in the chronon picture, are expected to be strongly suppressed. Nevertheless, they can in principle lead to weak frequency dependence in the propagation of gravitational waves or to scale dependence of the effective Newton constant. Existing experimental and observational bounds on Lorentz invariance and equivalence principle violations may therefore place rather strict constraints on the fundamental time scale τ_1 [40–42]. A careful analysis of these constraints is left for future work.

12. Conclusions

In this paper, a chronon model with a dual description of time has been considered. The starting point is a microscopic discrete-time dynamics with an integer chronon index and a coarse-graining procedure which produces a hierarchy of temporal layers Ch_N. Each layer contains N internal slots, and the symmetries of the effective Hamiltonian are restricted by permutations of these slots which preserve the coarse-grained measure.

It has been shown that the simple layers Ch2, Ch3 and Ch4 already reproduce structures similar to $U(1)$, $SU(3)$ and a pair-locked $SU(2)$. The special role belongs to the dodecad layer Ch12, which is the minimal layer containing simultaneously triads and quartets. In this case, the effective Hilbert space can be written as $\mathcal{H}_{12} \simeq \mathbb{C}^3 \otimes \mathbb{C}^4$, and the intersection of the commutants of triad and quartet permutations leads to the gauge algebra $SU(3)_C \times SU(2)_L \times U(1)_Y$. Within the same combinatorial scheme the standard hypercharge assignments and the cancellation of anomalies follow from simple integer relations, and three fermion generations correspond to three inequivalent embeddings of a triad in the dodecad which preserve the quartet structure.

Several more speculative elements of the construction have also been indicated. The layered chronon dynamics naturally suggests a Floquet description of the effective evolution, in which mass scales are related to quasi-energies. Misaligned layers can play the role of dark sectors which interact only gravitationally, and entanglement can be interpreted as a consequence of temporal correlations between different layers. In the present work these topics are discussed only qualitatively and are formulated as directions for further research.

Thus, the main result of the paper is kinematical. A very simple discrete-time organisation of microscopic updates is sufficient to encode the Standard Model gauge algebra, the pattern of charges and anomalies, and the number of generations. Whether such a chronon hierarchy is really realised in Nature can only be decided by a more detailed dynamical analysis and by comparison with experiment, but even in its present form the construction may be useful as a compact way to organise several features of the Standard Model within a single combinatorial scheme.

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Appendix A. The Relativity of Temporal Resolution and RG-Flow Transformations

Appendix A.1. Beyond Kinematic Relativity

In the main text we treated the chronon index n and the coarse-grained layers Ch_N largely kinematically. Here we collect the simple assumptions that underlie this picture and clarify in which sense it is meaningful to speak of a relativity of temporal resolution.

Appendix A.2. Temporal Frames of Reference

We model the fundamental dynamics (Ch1) as an ergodic measure-preserving system (X, μ, T) on a standard Borel space of configurations $X \subseteq Y^{\mathbb{Z}}$, where Y is a countable alphabet of local labels and T is the left shift. Coarse-graining over N chronon steps is described by the block-factor map

$$(\pi_N x)_k = x_{kN}, \qquad \mu_N = \mu \circ \pi_N^{-1}. \tag{A1}$$

A temporal frame of reference at scale N, denoted $\mathrm{TFR}(N)$, is the description of physics obtained when all observables are constructed from the coarse-grained process (X_N, μ_N, T_N) induced by π_N. Two temporal frames $\mathrm{TFR}(N)$ and $\mathrm{TFR}(M)$ are related by a change of temporal resolution realised by a block map $\pi_{M/N}$ whenever M is a multiple of N.

The collection of all $\mathrm{TFR}(N)$, equipped with these block transformations, forms a simple version of a renormalization-group (RG) flow in temporal resolution. In contrast to spatial RG, which coarse-grains over length scales, here the flow coarse-grains over microscopic temporal structure.

Appendix A.3. Signature and Local Lorentz Invariance

Assume that on a given layer Ch_N the long-wavelength dynamics of small perturbations around homogeneous states is described by a quadratic action with one distinguished macroscopic time coordinate t_0 and N emergent orthogonal directions t_i associated with the internal slots. The most general isotropic quadratic form then reads

$$d\tau^2 = dt_0^2 + B \sum_{i=1}^{N} dt_i^2, \tag{A2}$$

for some constant B.

Appendix A.4. Foundational Postulates

We posit a framework based on a hybrid ontology $\mathcal{S} = (\mathcal{M}, G, \mathcal{A})$, where a continuous Lorentzian manifold $\mathcal{M}$ acts as a macroscopic arena for a fundamental discrete structure G. In this picture, the microscopic evolution is quantized: the elementary update step τ_1 corresponds physically to the minimal quantum of action $\hbar$, mediated by a unitary operator $\hat{U}_1$. We formalize the discrete sector as a stochastic dynamical system on a standard Borel space $X \subseteq Y^{\mathbb{Z}}$, requiring the resulting orbit graph to be hyperfinite to admit a rigorous layering procedure. The connection to gravity is established through dynamic coupling, where the statistical average of the graph dynamics, $\langle \hat{T}_{\mu\nu}(G) \rangle$, acts as the source term for the curvature of $\mathcal{M}$. Finally, to preserve causality across the discrete-continuous boundary, we impose the principle of Temporal Dominance: the effective proper time τ is treated as an invariant measure of evolution that must never exceed the rate of the fundamental global coordinate time t_0, ensuring the strict bound $d\tau \le |dt_0|$.

Appendix A.5. The Extended Principle of Relativity

We extend the foundational core postulates of the framework with the following principles governing the relationship between different TFRs.

Appendix A.5.1. Principle of Temporal Relativity

The fundamental laws of physics (the dynamics on the base configuration space X) are invariant with respect to the choice of the Temporal Frame of Reference. However, the effective description of these laws (the dynamics on the factor space X_N) is covariant with respect to the scale of temporal resolution N. All TFRs are equivalent for the formulation of fundamental laws.

Appendix A.5.2. Layered Covariance and Invariants

1. Local Lorentz Invariance Within any TFR(N), the effective laws of physics are locally invariant under the Poincaré group SO(N, 1) of the emergent (N+1)D spacetime (a consequence of the Signature Theorem).
2. Invariance of Action: The fundamental quantum of action $\hbar$ is invariant across all TFRs.

Appendix A.5.3. Law of Transformation: RG-Flow

The transformation between different Temporal Frames of Reference, TFR$(N) \to$ TFR(M), is mathematically described by the operator of the Renormalization Group (RG) flow, $RG_{N\to M}$.

Notation (No Multiple Physical Times)

The internal slot directions are denoted by t_i in this appendix only as auxiliary coordinates in the long-wavelength quadratic form. The Signature Theorem implies these directions are space-like ($B < 0$), and we explicitly rescale and reinterpret them as spatial coordinates $x_i = \sqrt{|B|}\, t_i$ (Equations (A18)–(A20)). Thus there is one macroscopic time coordinate t_0 and N space-like directions, not N additional clock times.

Appendix A.6. The Signature Theorem

On each layer Ch$_N$ we have one distinguished macroscopic time coordinate t_0, representing the global tick of the fundamental update process, and N internal coordinates $t_1, \dots, t_N$ associated with the N slots of the layer. Proper time τ is defined along worldlines—the proper time of any physical system never runs faster than the coordinate time of the fundamental update process. Equivalently, for every admissible displacement one has

$$d\tau \ge 0 \quad \text{and} \quad 0 \le d\tau \le |dt_0|. \tag{A3}$$

The effective proper-time element on the layer is modelled by a symmetric bilinear form

$$d\tau^2 = G_{ab}\, dt^a dt^b, \qquad a, b = 0, 1, \dots, N. \tag{A4}$$

Two kinematical facts about the microscopic dynamics strongly constrain the form of G_{ab}:

- **Internal statistical isotropy:** all internal slots are statistically equivalent, hence the restriction of G_{ab} to the internal subspace $\mathrm{span}\{t_1, \ldots, t_N\}$ must be proportional to the identity.
- **Synchronous update:** the fundamental rule updates all internal slots simultaneously with the global tick t_0. The generating vector field $\partial/\partial t_0$ is therefore orthogonal (with respect to G_{ab}) to all internal directions $\partial/\partial t_i$, and mixed terms $dt_0\, dt_i$ are absent in the effective line element.

Consequently the most general form of $d\tau^2$ compatible with these symmetries is

$$d\tau^2 = G_{00}\,(dt_0)^2 + B\sum_{i=1}^{N}(dt_i)^2, \tag{A5}$$

with real constants G_{00} and B, identical for all internal directions.

Signature Theorem: Let the effective proper-time interval on a given layer Ch_N be described by (A5), and assume that:

(i) Internal statistical isotropy and synchronous update hold, so that (A5) is valid;
(ii) Temporal Dominance: along every physically admissible worldline one has $d\tau \geq 0$ and $0 \leq d\tau \leq |dt_0|$ proper time never runs faster than the global coordinate time;
(iii) Dynamic non-triviality of internal directions: the quadratic form restricted to the internal coordinates is non-zero, i.e., there exist free excitations propagating along the layer with $dt_i/d\lambda \neq 0$ for some i in some affine parameterisation λ.

Then:

(a) the metric G_{ab} is non-degenerate;
(b) its signature is Lorentzian of type $(1, N)$, one positive and N negative eigenvalues;
(c) after a suitable rescaling of coordinates one can write

$$d\tau^2 = (dt_0)^2 - \frac{1}{c^2}\sum_{i=1}^{N}(dt_i)^2, \tag{A6}$$

for some constant $c > 0$. The linear group preserving (A6) is $\mathrm{SO}(N, 1)$.

Let us show that the coefficient of $(dt_0)^2$ is positive. Consider a physical worldline along which the internal configuration does not change, so $dt_i = 0$ for all i. Then (A5) reduces to

$$d\tau^2 = G_{00}\,(dt_0)^2. \tag{A7}$$

By Temporal Dominance (A3) we have $d\tau \geq 0$ and $d\tau \leq |dt_0|$, hence

$$0 \leq d\tau^2 \leq (dt_0)^2. \tag{A8}$$

For $dt_0 \neq 0$ this implies $0 \leq G_{00} \leq 1$. The case $G_{00} = 0$ would mean that a system at rest in the internal directions experiences no proper time at all, which is physically unreasonable. We therefore assume $G_{00} > 0$. Rescaling the unit of proper time $\tau \mapsto \tau' = \sqrt{G_{00}}\,\tau$ we may, without loss of generality, set

$$d\tau^2 = (dt_0)^2 + B\sum_{i=1}^{N}(dt_i)^2. \tag{A9}$$

This choice fixes the macroscopic time direction t_0 to have a *positive* eigenvalue while the opposite overall sign would correspond to reversing the orientation of proper time.

For an arbitrary physical displacement we again use (A3), which after squaring gives

$$0 \le d\tau^2 \le (dt_0)^2. \tag{A10}$$

Substituting (A9) yields

$$0 \le (dt_0)^2 + B \sum_{i=1}^{N} (dt_i)^2 \le (dt_0)^2. \tag{A11}$$

The upper bound implies

$$B \sum_{i=1}^{N} (dt_i)^2 \le 0 \tag{A12}$$

for all admissible displacements. By assumption (iii) there exist displacements with $\sum_i (dt_i)^2 > 0$, so necessarily

$$B \le 0. \tag{A13}$$

If we had $B > 0$, any displacement with non-zero dt_i would give $d\tau^2 > (dt_0)^2$ and thus $d\tau > |dt_0|$, contradicting Temporal Dominance. Hence

$$B \le 0. \tag{A14}$$

If $B = 0$, the internal coordinates t_i do not enter the line element at all:

$$d\tau^2 = (dt_0)^2. \tag{A15}$$

In that case the quadratic form restricted to the internal subspace is identically zero, and the metric has rank one. This contradicts assumption (iii) that the internal directions are dynamically non-trivial. Therefore

$$B \ne 0, \tag{A16}$$

and, together with $B \le 0$, we obtain

$$B < 0. \tag{A17}$$

With $B < 0$ the matrix representing the quadratic form (A9) in the basis $(t_0, t_1, \ldots, t_N)$ is diagonal with eigenvalues

$$\lambda_0 = 1, \qquad \lambda_i = B < 0 \quad (i = 1, \ldots, N). \tag{A18}$$

All eigenvalues are non-zero, so the metric is non-degenerate. There is exactly one positive eigenvalue and N negative ones, hence the signature is Lorentzian of type $(1, N)$ rather than $(N, 1)$. The reverse signature would require $\lambda_0 < 0$, but this is excluded by Step 1, where the orientation of t_0 was fixed by the requirement that proper time increases along future-directed worldlines.

Define rescaled internal coordinates

$$x_i := \sqrt{|B|}\, t_i, \qquad i = 1, \ldots, N. \tag{A19}$$

In terms of x_i the line element becomes

$$d\tau^2 = (dt_0)^2 - \sum_{i=1}^{N} (dx_i)^2. \tag{A20}$$

Restoring a conventional notation by writing $c = 1/\sqrt{|B|} > 0$ and returning to the original internal coordinates, we can equivalently express this as

$$d\tau^2 = (dt_0)^2 - \frac{1}{c^2}\sum_{i=1}^{N}(dt_i)^2, \tag{A21}$$

which is precisely (A6). The group of linear transformations preserving this quadratic form is isomorphic to $\mathrm{SO}(N, 1)$.

This completes the proof.

Maximal signal speed and dimensions. The inequality $d\tau^2 \geq 0$ implies

$$(dt_0)^2 \;\geq\; \frac{1}{c^2}\sum_{i=1}^{N}(dt_i)^2. \tag{A22}$$

For a null displacement $d\tau = 0$ this is saturated and defines an effective maximal coordinate speed of propagation:

$$\sum_{i=1}^{N}\left(\frac{dt_i}{dt_0}\right)^2 = c^2. \tag{A23}$$

Thus the constant c that emerged from the coarse-grained metric is precisely the invariant speed of massless excitations in the effective theory. When one reinterprets the internal coordinates t_i as spatial coordinates x^i with dimensions of length, while t_0 carries dimensions of time, c acquires its usual dimension of velocity.

Causal structure. The causal cones defined by (A6) induce a partial order on events, and the corresponding automorphism group of this order is the inhomogeneous Lorentz group, as in the classical results of Zeeman and Malament [43,44]. Additional related discussions include descriptive set theory for equivalence relations [45,46], stochastic gravity and QFT in curved spacetime [47–51], discrete and relativistic quantum walks [52–56], time-crystal proposals and realizations [57,58], and renormalization-group foundations [59]. In this sense, the emergence of a Lorentzian metric is equivalent to the emergence of a well-defined causal structure on the coarse-grained manifold.

Put differently, relativity of simultaneity and the Lorentzian signature are not independent postulates here, they are inevitable consequences of the fact that a fundamentally discrete universe updates all its parts strictly simultaneously with a global tick t_0, while local subsystems have limited bandwidth to reconfigure themselves between successive global updates.

Appendix A.7. RG Flow and Its Heuristic Relation to Boosts

The block transformation $\mu \mapsto \mu_N$ defines a flow in the space of effective theories, parametrised by temporal resolution. It is often convenient to visualise this flow using language borrowed from special relativity, by introducing a "temporal rapidity"

$$\theta_N \equiv \ln\left(\frac{N}{N_0}\right), \tag{A24}$$

for some reference scale N_0, and thinking of RG steps as motions along this axis in theory space.

This analogy can be suggestive because both Lorentz boosts and temporal coarse-graining change which events are grouped together into a single instant of effective time. However, it is important not to overstate the correspondence: RG transformations act in the space of couplings and probability measures, not on spacetime coordinates, and they are not represented by unitary operators on the microscopic Hilbert space.

For this reason we replace the strong correspondence postulate of the main text by a weaker, explicitly heuristic correspondence. Changes in temporal resolution along the RG flow can mimic some of the kinematic effects associated with Lorentz boosts, because both rearrange how microscopic temporal events are grouped into macroscopic ones. Nevertheless, RG transformations and Lorentz transformations are conceptually distinct operations and need not coincide.

In this sense, the relativity of simultaneity in the emergent spacetime can be seen as one manifestation of the coarse-grained loss of information about microscopic causal order, without implying any strict identification between boosts and RG steps.

Appendix B. Borel Structure, Hyperfiniteness and Renormalisation

Let Y be a finite alphabet encoding the chronon labels and define the configuration space

$$X = Y^{\mathbb{Z}} \tag{A25}$$

of bi-infinite sequences $x = (x_k)_{k\in\mathbb{Z}}$ with $x_k \in Y$. Equip X with the product σ-algebra

$$\mathcal{B} = \bigotimes_{k\in\mathbb{Z}} \mathcal{P}(Y), \tag{A26}$$

where $\mathcal{P}(Y)$ denotes the power set of Y. A convenient measurable basis is provided by cylinder sets

$$\begin{aligned}&[a_{k_1}, \dots, a_{k_m}] = \\ &\{x \in X : x_{k_j} = a_{k_j} \text{ for all } j = 1, \dots, m\},\end{aligned} \tag{A27}$$

which generate $\mathcal{B}$.

Fix a probability vector $p = (p_y)_{y\in Y}$ with $p_y \geq 0$ and $\sum_{y\in Y} p_y = 1$. We define the product probability measure

$$\mu = \bigotimes_{k\in\mathbb{Z}} p \tag{A28}$$

on $(X, \mathcal{B})$ by

$$\mu([a_{k_1}, \dots, a_{k_m}]) = \prod_{j=1}^{m} p_{a_{k_j}}. \tag{A29}$$

The microscopic shift of the chronon labels is represented by the $\mathbb{Z}$-action

$$T : X \to X, \qquad (Tx)_k = x_{k+1}. \tag{A30}$$

The quadruple $(X, \mathcal{B}, \mu, T)$ is a standard measure-preserving dynamical system, i.e.,

$$\mu(T^{-1}A) = \mu(A), \qquad \forall\, A \in \mathcal{B}. \tag{A31}$$

The orbit-equivalence relation generated by T,

$$E_T = \{(x, y) \in X \times X : \exists\, n \in \mathbb{Z},\ y = T^n x\}, \tag{A32}$$

is a countable Borel equivalence relation. It is known [28] that E_T is hyperfinite. There exists an increasing sequence of Borel sub-equivalence relations

$$E_1 \subset E_2 \subset \cdots \subset E_T \tag{A33}$$

such that

$$E_T = \bigcup_{n=1}^{\infty} E_n, \tag{A34}$$

and every equivalence class of E_n is finite. An explicit approximating sequence is given by

$$(x, y) \in E_n \quad \Longleftrightarrow \quad \exists\, k \in \mathbb{Z},\ |k| \le n,\ y = T^k x, \tag{A35}$$

for which each E_n-class has cardinality at most $2n + 1$.

The temporal layers can be realised as measure-theoretic factors of $(X, \mathcal{B}, \mu, T)$. For each $N \in \mathbb{N}$ define the factor map

$$\pi_N : X \to X, \qquad (\pi_N x)_k = x_{kN}, \tag{A36}$$

and let

$$\mathcal{B}_N = \pi_N^{-1}(\mathcal{B}), \qquad \mu_N = \mu \circ \pi_N^{-1}. \tag{A37}$$

Then $(X, \mathcal{B}_N, \mu_N, T^N)$ is a factor of the original dynamical system and represents the chronon process observed at the coarser temporal resolution $\tau_{(N)} = N\tau_1$. For any integrable observable $f \in L^1(X, \mathcal{B}, \mu)$, its coarse-grained version at layer ChN is the conditional expectation

$$f^{(N)} = \mathbb{E}\big[f \,\big|\, \mathcal{B}_N\big], \tag{A38}$$

which satisfies

$$\int_A f^{(N)}\, d\mu = \int_A f\, d\mu, \qquad \forall\, A \in \mathcal{B}_N. \tag{A39}$$

In this way the hyperfinite Borel structure encoded in E_T and its finite approximants E_n provides a mathematically controlled realisation of the temporal hierarchy: each layer ChN corresponds to a factor $(X, \mathcal{B}_N, \mu_N, T^N)$ obtained by coarse-graining along the orbits of the shift action.

Appendix B.1. Temporal Layers

For each $N \in \mathbb{N}$ we define the layer

$$\mathrm{Ch}_N \tag{A40}$$

as the description of the chronon process sampled at intervals of $\tau_{(N)} = N\tau_1$. A Ch_N cell contains N internal slots, and its internal Hilbert space is

$$\mathcal{H}_N \simeq \mathbb{C}^N \tag{A41}$$

with an effective Hamiltonian defined by the stroboscopic relation

$$U_N = \exp\!\left(-\frac{i}{\hbar} H_{\mathrm{eff}}^{(N)} \tau_{(N)}\right). \tag{A42}$$

The operator $H_{\mathrm{eff}}^{(N)}$ is determined by slot-permutation symmetries and the statistics of the chronon update rule described below.

We introduce a minimal stochastic update rule acting on G_n:

$$G_{n+1} = \mathcal{U}[G_n;\, \xi_n], \tag{A43}$$

where ξ_n are i.i.d. random variables drawn from a finite set describing allowed microscopic transitions (creation, annihilation or redirection of chronon edges), subject to the local constraint (1). The induced dynamics on X is stationary and compatible with the shift action.

The effective operator U_N arises when the update rule is statistically homogeneous:

$$\mathbb{E}[\mathcal{U}[G_n; \xi_n]] = \mathbb{E}[\mathcal{U}[G_{n+k}; \xi_{n+k}]], \qquad k \in \mathbb{Z}, \tag{A44}$$

and correlations decay sufficiently fast. Under these conditions, a Floquet-type approximation

$$U_N \approx \prod_{k=1}^{N} U_1^{(k)} \tag{A45}$$

leads to the representation (A42).

Every chronon edge $e \in E_n$ is associated with a smooth embedding

$$\iota_e : [0,1] \to M, \qquad \lambda \mapsto x_e^{\mu}(\lambda), \tag{A46}$$

where M is a smooth Lorentzian manifold equipped with metric $g_{\mu\nu}$. The discrete and continuous times are matched through

$$t(x_e^{\mu}(0)) \in [n\tau_1 - \delta, n\tau_1 + \delta], \tag{A47}$$

$$t(x_e^{\mu}(1)) \in [(n+1)\tau_1 - \delta, (n+1)\tau_1 + \delta], \tag{A48}$$

for a tolerance $\delta \ll \tau_1$. This identifies the chronon boundaries with geometric-time thresholds, ensuring a single physical notion of temporal ordering.

Appendix B.2. Toy Example: A Bernoulli Base Process, the Slot Hilbert Space, and Permutation Constraints

To illustrate the meaning of the internal "slot" space $H_N \simeq \mathbb{C}^N$ and the role of slot permutations, consider the simplest stationary base dynamics on $X = Y^{\mathbb{Z}}$ with $Y = \{0,1\}$ equipped with a Bernoulli product measure μ as in Equations (A24)–(A30). The factor map π_N in Equation (A35) produces the coarse-grained system $(X, \mathcal{B}_N, \mu_N, T^N)$, which by Definition (A39) is the layer Ch_N observed with temporal resolution $\tau(N) = N\tau_1$.

Slot Hilbert space.

A single Ch_N cell contains N internal slots. Kinematically we represent the slot label by the orthonormal basis $\{|1\rangle, \ldots, |N\rangle\}$ of the finite-dimensional space

$$H_N \;\cong\; \mathbb{C}^N. \tag{A49}$$

A permutation $\sigma \in S_N$ acts on H_N by the permutation representation

$$P(\sigma)|k\rangle \;=\; |\sigma(k)\rangle, \qquad P(\sigma) \;=\; \sum_{k=1}^{N} |\sigma(k)\rangle\langle k|. \tag{A50}$$

This representation encodes the statement that swapping slot labels is a symmetry operation whenever the coarse-grained statistics do not distinguish the slots.

Commutant constraint on $H_{\mathrm{eff}}^{(N)}$.

Assume that on a given layer the induced measure μ_N is invariant under a subgroup $G \subseteq S_N$ of slot permutations (a modelling assumption about the coarse-grained

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fixed-point/attractor statistics). Then the effective Hamiltonian in Equation (A41) is constrained by

$$P(g)\, H_{\rm eff}^{(N)}\, P(g)^{-1} \;=\; H_{\rm eff}^{(N)} \quad \forall\, g \in G, \qquad \text{i.e.,} \qquad H_{\rm eff}^{(N)} \in \mathrm{Comm}\big(P(G)\big). \tag{A51}$$

For example, for $N = 3$ and $G = S_3$ (full slot-exchange symmetry), one finds that the commutant is two-dimensional:

$$\mathrm{Comm}\big(P(S_3)\big) = \mathrm{span}\{I_3, J_3\}, \qquad J_3 := \sum_{i,j=1}^{3} |i\rangle\langle j|. \tag{A52}$$

Equivalently, in the slot basis $\{|1\rangle, |2\rangle, |3\rangle\}$, $H_{\rm eff}^{(3)}$ must have equal diagonal entries and equal off-diagonal entries:

$$H_{\rm eff}^{(3)} \;=\; a\, I_3 + b\, J_3. \tag{A53}$$

Its spectrum splits into a singlet $|s\rangle = (|1\rangle + |2\rangle + |3\rangle)/\sqrt{3}$ with eigenvalue $a + 3b$ and a doubly-degenerate orthogonal subspace with eigenvalue a. Importantly, although the imposed symmetry is discrete (S_3), the degeneracy implies an enhanced continuous unitary invariance of $H_{\rm eff}^{(3)}$, namely $U(2)$ acting within the degenerate subspace (and in the special case $b = 0$, $H_{\rm eff}^{(3)} \propto I_3$ and the invariance is enlarged to $U(3)$). In this way continuous internal symmetries appear as symmetry groups of degenerate effective dynamics, while the discrete slot permutations act as the underlying kinematical constraints selecting the allowed form of $H_{\rm eff}^{(N)}$.

The same logic applies for larger N: one first specifies (or motivates dynamically) the slot-permutation invariances of μ_N, which fixes the commutant algebra on H_N. Then, depending on degeneracy/locking patterns, the symmetry of the effective Hamiltonian is given by the corresponding unitary group on invariant or degenerate sectors.

Appendix C. Floquet-Type Temporal Modulation and the Effective Gravitational Constant

In the main text the effective continuum description on a temporal layer Ch_N was obtained by temporal coarse-graining of a fundamentally discrete update process. If a given layer represents a dynamically preferred attractor of the temporal renormalization flow, in particular the dodecad Ch12, its long-wavelength dynamics admits a natural stroboscopic, Floquet-type description. The purpose of this Appendix is to formulate the resulting residual temporal structure in the effective background geometry and in the gravitational coupling in a manner consistent with relativistic causality, local covariance, and current experimental bounds. The discussion should be understood as establishing a consistent parameterization and constraint framework, rather than as an explanation of presently observed anomalies.

Let $\tau_N = N\tau_1$ denote the characteristic period associated with the layer Ch_N. On time scales large compared to τ_1, the effective metric may be written as

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}^{\rm Fl}(\chi), \tag{A54}$$

where $\bar{g}_{\mu\nu}$ is the smooth background metric of general relativity, $h_{\mu\nu}^{\rm Fl}$ encodes residual temporal structure inherited from the discrete dynamics, and χ denotes a convenient macroscopic clock variable. The parameter χ may be identified with a chosen macroscopic slicing coordinate or, equivalently, with the proper time of a specified family of fiducial observers; observable statements are formulated in terms of local clock comparisons and do not depend on this choice. The Floquet regime corresponds to the periodicity condition

$$h_{\mu\nu}^{\rm Fl}(\chi + \tau_N) = h_{\mu\nu}^{\rm Fl}(\chi), \tag{A55}$$

which implies the Fourier expansion

$$h_{\mu\nu}^{\rm Fl}(\chi) = \sum_{k\neq 0} \epsilon_{\mu\nu}^{(k)} \exp\left(2\pi i k \frac{\chi}{\tau_N}\right), \qquad |\epsilon_{\mu\nu}^{(k)}| \ll 1. \tag{A56}$$

The spectrum consists of discrete frequencies $f_k = k/\tau_N$. These oscillatory contributions are not to be interpreted as propagating gravitational waves sourced by localized stress-energy. Rather, they represent a stationary narrowband component of the effective background, expressed in a convenient macroscopic slicing. Since this structure is not an operationally controllable, locally switchable degree of freedom, it cannot be used to transmit information or to define an absolute synchronization procedure, and therefore does not entail any violation of relativistic causality.

The same temporal coarse-graining that gives rise to the effective metric also defines the gravitational coupling entering the macroscopic field equations. At the effective level, the Bianchi identities $\nabla^\mu G_{\mu\nu}[\bar{g}] = 0$ require covariant conservation of the total source. A robust formulation is therefore to treat the coarse-grained fields as an open subsystem of an underlying discrete temporal dynamics and to write schematically

$$G_{\mu\nu}[\bar{g}] = 8\pi G_* \left(\langle T_{\mu\nu} \rangle_N + T_{\mu\nu}^{\rm res} \right) + \mathcal{O}(\epsilon^2), \tag{A57}$$

where $T_{\mu\nu}^{\rm res}$ denotes an effective contribution of the microscopic temporal degrees of freedom, defined such that

$$\nabla^\mu \left(\langle T_{\mu\nu} \rangle_N + T_{\mu\nu}^{\rm res} \right) = 0. \tag{A58}$$

Formally, this structure is reminiscent of unimodular and stochastic gravity approaches, in which coarse-grained or constrained degrees of freedom contribute effective source terms without violating geometric identities. In a phenomenological description, the combined effect of the reservoir contribution can be parameterized as a weak modulation of an effective coupling,

$$G_{\rm eff}(\chi) \simeq G_N \left[1 + \gamma \cos\left(\frac{2\pi\chi}{\tau_N} + \phi_0 \right) \right], \qquad |\gamma| \ll 1, \tag{A59}$$

with G_N the effective coupling associated with the temporal resolution τ_N. This parametrization should not be interpreted as a literal time variation of a fundamental constant in isolation, but as a compact encoding of the linear response of the coarse-grained gravitational sector after microscopic degrees of freedom have been integrated out.

The interpretation of this modulation is constrained by scale separation. If τ_N is microscopic, direct sampling of the phase of $G_{\rm eff}(\chi)$ by laboratory experiments lasting minutes to days is not expected, as simple time averaging suppresses any oscillatory contribution. Possible protocol dependence, if present, should instead be understood in terms of frequency response and temporal filtering. Any measurement procedure effectively implements a weighting functional $W(\chi)$, so that the inferred coupling is determined by a functional of $G_{\rm eff}$ rather than by its instantaneous value. Protocols involving narrowband demodulation, internal resonant elements, or band-limited readout can, in principle, place bounds on high-frequency periodic components, while purely slow averaging cannot. In this sense, the relevant observable is a frequency-dependent susceptibility of the gravitational sector.

Operationally, the presence of $h_{\mu\nu}^{\mathrm{Fl}}$ corresponds to a coherent narrowband "background hum" of the metric. Existing precision experiments already impose strong restrictions on admissible amplitudes. Comparisons of state-of-the-art atomic clocks and interferometric measurements constrain any periodic modulation of proper time and path length in the relevant bands, and the absence of correlated residuals in ground-based gravitational-wave detectors places stringent bounds on narrowband strain-like backgrounds in the $\sim$10–10^3 Hz range. These null results imply that, if a Floquet-type background component exists, its amplitude must lie many orders of magnitude below current sensitivity in those frequency bands. Accordingly, the discussion above should be read as providing a systematic way to translate such null results into constraints on the parameters $\epsilon_{\mu\nu}^{(k)}$ and γ, rather than as a claim of imminent detectability.

Finally, the framework remains compatible with high-energy astrophysical bounds. In ultra-relativistic regimes, periodic microstructure is efficiently averaged out in observables integrated over long propagation times and broad bandwidths, leading to very strong effective suppression of any residual modulation. By contrast, dedicated resonant or narrowband laboratory searches, operating outside the most tightly constrained frequency windows, offer a controlled setting in which chronon-induced narrowband effects may be constrained or, in principle, explored.

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