String Dualities at Order α'^3

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We compute the cosmological reduction of the fourth powers of the Riemann tensor claimed to arise in string theory at order α'^3, with an overall coefficient proportional to $\zeta(3)$, and show that it is compatible with an $O(9,9)$ symmetry. This confirms the general result in string theory, due to Sen [$O(d)\times O(d)$ symmetry of the space of cosmological solutions in string theory, scale factor duality and two-dimensional black holes, Phys. Lett. B **271**, 295 (1991)], that classical string theory with d-dimensional translation invariance admits an $O(d,d)$ symmetry to all orders in α'.

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Introduction.—String theory continues to be a most promising framework for a consistent theory of quantum gravity. At low energies, string theories are described by Einstein's theory of general relativity coupled to matter fields, which universally include an antisymmetric tensor (B-field) and a scalar (dilaton). Intriguingly, however, even classical string theory modifies general relativity in two important respects: it includes an infinite number of higher-derivative corrections governed by the (inverse) string tension α' [1,2], and it permits dualities identifying solutions that are drastically different in standard geometry. These general features suggest promising scenarios for cosmology [3–7], but an immediate obstacle is that the explicit form of the α' corrections is, at best, known to the first few orders. For Type II string theories not even the first nontrivial higher-derivative corrections, which arise at order α'^3, are known completely. Moreover, notwithstanding early important work in [8–10], the compatibility of α' corrections with string dualities such as T-duality has only in recent years become the focus of attention. The "space of duality invariant cosmologies" has been explored to all orders in α' and shown to permit novel features [11,12], but it is not known which points in this theory space actual string theories inhabit. It is thus a matter of some urgency to find efficient methods to deal with α' corrections.

In this Letter, we investigate T-duality at order α'^3. Our goal is to analyze whether the eight-derivative corrections (quartic in the Riemann tensor) with overall coefficient proportional to the transcendental $\zeta(3)$ are compatible with the $O(9,9)$ T-duality invariance upon reduction to one dimension (cosmic time). An important issue is that one should allow for the $O(d,d)$ transformations themselves to receive α' corrections [13]. It indeed follows from Meissner's work on the cosmological reduction to first order in α' that the T-duality transformations in terms of standard supergravity fields are α'-deformed [9,14,15]. However, it is possible to find new field variables for which the T-duality transformations take the standard form. A general framework was developed in [11,12] that systematically uses field redefinitions to bring both the dimensionally reduced action and the most general $O(d,d)$ invariant action to a form that involves only first order derivatives. The claim is that this procedure eliminates all ambiguities resulting from the freedom to perform integrations by part and to use lower-order equations of motion to modify higher-derivative terms. Upon passing to this canonical field basis, the $O(d,d)$ invariance, if present, should take the standard form. This yields a systematic procedure to test the reduced actions for $O(d,d)$ invariance, which has been successfully applied to first order in α' in cosmological reductions and, more recently, for general torus compactifications [16]. While the complete higher-derivative corrections at order α'^3 are not known, the eight-derivative terms involving only the metric are believed to be known completely. We will see that this is sufficient to show compatibility with $O(d,d)$, which in turn determines the B-field and dilaton couplings that survive upon cosmological reduction.

Apart from potential applications in cosmology, this result is of conceptual interest in view of these eight-derivative corrections being proportional to the transcendental number $\zeta(3)$. The transcendentality implies that these corrections cannot be linked by conventional symmetry transformations to corrections with rational coefficients, as present in bosonic and heterotic string theory starting at first order in α'. It is sometimes questioned whether the $\zeta(3)$ couplings are compatible with the continuous $O(d,d,\mathbb{R})$. In a previous version of this Letter, we arrived at the incorrect conclusion that these couplings are not compatible with $O(d,d,\mathbb{R})$ invariance, but this would have been in quite serious conflict with basic principles of string theory. As shown by Sen [17], classical (tree-level) string theory truncated to states of zero momentum along d directions admits an $O(d,d,\mathbb{R})$ invariance to all orders in α' [17]. While the original proof was couched in the language of string field theory, the argument only relies on the symmetries of the S matrix of this consistently truncated sector. At tree level, holomorphic factorization yields two independent manifest $O(d,\mathbb{R})$ symmetries, and combining this $O(d,\mathbb{R})\times O(d,\mathbb{R})$ invariance with the $GL(d,\mathbb{R})$ symmetry following from diffeomorphism invariance and constant shifts of the B-field implies $O(d,d,\mathbb{R})$ invariance [18]. Thus, the tree-level corrections at order α'^3 proportional to $\zeta(3)$ really ought to be consistent with $O(d,d,\mathbb{R})$, and indeed they are.

Cosmological reduction.—We now review the leading corrections in Type II string theory and compute the minimal form of the one-dimensional effective action obtained after a cosmological reduction. The α' corrections in Type II string theory begin at α'^3. The couplings for the gravitational sector were originally computed from four-point scattering amplitudes [1] and later from the sigma-model β function [19–21]. They take the compact form

$$J(c)\equiv t_8t_8R^4+\frac{c}{8}\epsilon_{10}\epsilon_{10}R^4, \tag{1}$$

where $c=1$ has been determined in the literature, but here we keep it more general in order to see whether this is fixed by duality arguments. The first term in Eq. (1) is

$$\begin{aligned} t_8t_8R^4 &= t^{\mu_1\cdots\mu_8}t_{\nu_1\ldots\nu_8}R_{\mu_1\mu_2}{}^{\nu_1\nu_2}R_{\mu_3\mu_4}{}^{\nu_3\nu_4}R_{\mu_5\mu_6}{}^{\nu_5\nu_6}R_{\mu_7\mu_8}{}^{\nu_7\nu_8}\\ &= 3\cdot 2^7\bigg[R_{\alpha\beta\mu\nu}R^{\beta\gamma\nu\rho}R^{\sigma\mu}{}_{\gamma\delta}R^{\delta\alpha}{}_{\rho\sigma}+\frac{1}{2}R_{\alpha\beta\mu\nu}R^{\beta\gamma\nu\rho}R_{\gamma\delta\rho\sigma}R^{\delta\alpha\sigma\mu}\\ &\quad-\frac{1}{2}R_{\alpha\beta\mu\nu}R^{\beta\gamma\mu\nu}R_{\gamma\delta\rho\sigma}R^{\delta\alpha\rho\sigma}-\frac{1}{4}R_{\alpha\beta\mu\nu}R^{\beta\gamma\rho\sigma}R^{\mu\nu}{}_{\gamma\delta}R^{\delta\alpha}{}_{\rho\sigma}\\ &\quad+\frac{1}{16}R_{\alpha\beta\mu\nu}R^{\beta\alpha\rho\sigma}R^{\gamma\delta\mu\nu}R_{\delta\gamma\rho\sigma}+\frac{1}{32}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}R_{\gamma\delta\rho\sigma}R^{\gamma\delta\rho\sigma}\bigg], \end{aligned} \tag{2}$$

where the t_8 tensor can be defined by its action over generic matrices [22,23]

$$\begin{aligned} t^{\alpha\beta\gamma\delta\mu\nu\rho\sigma}M^1_{\alpha\beta}M^2_{\gamma\delta}M^3_{\mu\nu}M^4_{\rho\sigma} &= 8\mathrm{Tr}\{M^1M^2M^3M^4+M^1M^3M^2M^4+M^1M^3M^4M^2\}\\ &\quad-2(\mathrm{Tr}\{M^1M^2\}\mathrm{Tr}\{M^3M^4\}+\mathrm{Tr}\{M^1M^3\}\mathrm{Tr}\{M^2M^4\}+\mathrm{Tr}\{M^1M^4\}\mathrm{Tr}\{M^2M^3\}). \end{aligned} \tag{3}$$

For the second term in Eq. (1), we have the following convention for the Levi-Civita tensor:

$$\begin{aligned} \epsilon_{10}\epsilon_{10}R^4 &= \epsilon^{\alpha\beta\mu_1\ldots\mu_8}\epsilon_{\alpha\beta\nu_1\ldots\nu_8}R_{\mu_1\mu_2}{}^{\nu_1\nu_2}R_{\mu_3\mu_4}{}^{\nu_3\nu_4}R_{\mu_5\mu_6}{}^{\nu_5\nu_6}R_{\mu_7\mu_8}{}^{\nu_7\nu_8}\\ &= -2\cdot 8!R_{[\alpha\beta}{}^{\alpha\beta}R_{\gamma\delta}{}^{\gamma\delta}R_{\mu\nu}{}^{\mu\nu}R_{\rho\sigma]}{}^{\rho\sigma}\\ &= 3.2^{10}\bigg[R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\nu}{}_{\mu\beta}R^{\sigma\mu}{}_{\alpha\rho}R^{\delta\rho}{}_{\sigma\nu}+R^{\gamma\delta}{}_{\alpha\beta}R^{\mu\nu}{}_{\gamma\delta}R^{\alpha\rho}{}_{\sigma\mu}R^{\sigma\beta}{}_{\nu\rho}\\ &\quad-\frac{1}{2}R_{\alpha\beta\mu\nu}R^{\beta\gamma\nu\rho}R_{\gamma\delta\rho\sigma}R^{\delta\alpha\sigma\mu}+\frac{1}{2}R_{\alpha\beta\mu\nu}R^{\beta\gamma\mu\nu}R_{\gamma\delta\rho\sigma}R^{\delta\alpha\rho\sigma}\\ &\quad-\frac{1}{16}R_{\alpha\beta\mu\nu}R^{\beta\alpha\rho\sigma}R^{\gamma\delta\mu\nu}R_{\delta\gamma\rho\sigma}-\frac{1}{32}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}R_{\gamma\delta\rho\sigma}R^{\gamma\delta\rho\sigma}+\cdots\bigg], \end{aligned} \tag{4}$$

where the dots stand for terms containing Ricci tensors and scalars, which can be eliminated by using field redefinitions at the expense of introducing dilaton couplings that we will ignore at the moment.

The couplings given by t_8t_8 have a nonzero contribution at the four-graviton level [1], while the $\epsilon_{10}\epsilon_{10}$ interactions have nonzero contributions starting only at the five-graviton level [24]. The presence of this term in the tree-level effective action was inferred by the β-function approach in [19–21], predicting $c=1$. This prediction was confirmed in [25] through sphere-level scattering amplitudes of five gravitons. The literature also suggests that this value for c is required by supersymmetry [26,27] and the emergence of T-duality symmetry in a circle compactification [28,29]. For the specific value $c=1$, it can be shown using Bianchi identities that the corrections are given by only two terms [20]

$$\begin{aligned} J(1)=-3\cdot 2^6[R^{\alpha\beta\mu\nu}R_{\mu\nu}{}^{\gamma\delta}R_{\alpha\gamma}{}^{\rho\sigma}R_{\rho\sigma\beta\delta}\\ -4R_{\alpha\beta}{}^{\gamma\delta}R_{\delta\mu}{}^{\alpha\nu}R_{\nu\rho}{}^{\beta\sigma}R_{\sigma\gamma}{}^{\mu\rho}]. \end{aligned} \tag{5}$$

The consensus is that these are the unique purely gravitational terms appearing in the leading α' corrections in Type II string theory.

The simplest way to test for $O(d,d)$ invariance is by performing a cosmological reduction in which the $D=10$ dimensional target space splits into a single temporal external direction and $d=9$ internal ones. The fields only depend on time, and we use the following ansatz:

$$G_{\mu\nu} = \mathrm{diag}(-n^2, g_{ij}), \qquad \phi = \frac{1}{2}\Phi + \frac{1}{2}\log(\sqrt{g}),$$
$$B_{\mu\nu} = \mathrm{diag}(0, b_{ij}), \tag{6}$$

where μ, ν are $D=10$ indices and i, j are $d=9$ indices. All partial derivatives but $\partial_0\Psi = \partial_t\Psi \equiv \dot{\Psi}$ are set to zero. After the reduction, the effective one-dimensional action can be cast in terms of the following quantities:

$$L^i{}_j \equiv g^{ik}\dot{g}_{kj}, \qquad M^i{}_j \equiv g^{ik}\dot{b}_{kj}, \tag{7}$$

plus the lapse function n, the lower-dimensional dilaton Φ, and their time derivatives.

A method to bring the effective action to a minimal form that makes it systematic to assess its $O(9,9)$ invariance was introduced in [11,12]. The idea is that the lower-dimensional equations of motion (where we have gauge fixed $n=1$ after varying the action),

$$\dot{L} = M^2 + \dot{\Phi}L,$$
$$\dot{M} = ML + \dot{\Phi}M,$$
$$\ddot{\Phi} = \frac{1}{2}\left[\dot{\Phi}^2 + \frac{1}{4}\mathrm{Tr}(L^2 - M^2)\right],$$
$$\dot{\Phi}^2 = \frac{1}{4}\mathrm{Tr}(L^2 - M^2), \tag{8}$$

can be combined with integrations by part to remove all higher-derivative terms containing dilatons and also allow one to remove the derivatives from L and M, leaving a final minimal form containing only powers of L and M. It was then shown which of these interactions can be cast in terms of the generalized metric

$$\mathcal{S} \equiv \mathcal{H}\eta^{-1} = \begin{pmatrix} bg^{-1} & g - bg^{-1}b \\ g^{-1} & -g^{-1}b \end{pmatrix}, \tag{9}$$

so as to make the $O(d,d)$ symmetry manifest, if present. We refer to [11,12] for details on this procedure.

In the two-derivative case, the parent action

$$S_0 = \int d^Dx\sqrt{-G}e^{-2\phi}\left[R + 4(\nabla\phi)^2 - \frac{1}{12}H^2\right], \tag{10}$$

compactifies to an action where the $O(9,9)$ symmetry is manifest [9,11,12]:

$$S_0 = \int dt e^{-\Phi}\left[-\dot{\Phi}^2 - \frac{1}{8}\mathrm{Tr}(\dot{\mathcal{S}}^2)\right], \tag{11}$$

where we used that $\mathrm{Tr}(\dot{\mathcal{S}}^2) = 2\mathrm{Tr}(M^2 - L^2)$.

In the following, we will simplify the problem by setting the B-field to zero, which is sufficient in order to display the cosmological effective action in a duality invariant form. In this case, the zeroth order equations of motion (EOMs), Eq. (8), allow for the redefinitions

$$\dot{L} \to \dot{\Phi}L, \qquad \ddot{\Phi} \to \frac{1}{2}\left[\dot{\Phi}^2 + \frac{1}{4}(L^2)\right], \qquad \dot{\Phi}^2 \to \frac{1}{4}(L^2), \tag{12}$$

where from now on we will denote the traces of $d\times d$ matrices by parenthesis, i.e.,

$$L^i{}_i = \mathrm{Tr}(L) \equiv (L), \qquad (L^2)^i{}_i = (L^2), \ldots \tag{13}$$

but we will keep the Tr notation for the duality covariant $2d\times 2d$ matrix $\mathcal{S}$. Indices are raised and lowered with g, namely $L_{ij} = g_{ik}L^k{}_j = \dot{g}_{ij}$.

In the simplified case with vanishing two-form, the generalized metric is related to L by (with M, N denoting doubled internal indices)

$$\mathcal{S}_M{}^N = \begin{pmatrix} 0 & g_{ij} \\ g^{ij} & 0 \end{pmatrix},$$
$$(\dot{\mathcal{S}}^{2m})_M{}^N = \begin{pmatrix} (-1)^m(L^{2m})_i{}^j & 0 \\ 0 & (-1)^m(L^{2m})^i{}_j \end{pmatrix}, \tag{14}$$

and so

$$\mathrm{Tr}(\dot{\mathcal{S}}^{2m}) = 2(-1)^m(L^{2m}), \qquad \mathrm{Tr}(\dot{\mathcal{S}}^{2m-1}) = 0, \qquad m\in\mathbb{N}. \tag{15}$$

This shows that only traces containing even powers of L can be written in terms of the generalized metric; those involving odd powers do not admit a duality covariant expression.

In this language, the reduced Riemann tensor reads

$$R_{ijkl} = \frac{1}{2}L_{i[k}L_{l]j}, \qquad R_{i0j0} = -\frac{1}{2}\dot{L}_{ij} - \frac{1}{4}(L^2)_{ij}. \tag{16}$$

The reduced action takes the form

$$S = \int dt\, e^{-\Phi}\left[-\dot{\Phi}^2 - \frac{1}{8}\mathrm{Tr}(\dot{\mathcal{S}}^2) + \alpha'^3\frac{\zeta(3)}{3\cdot 2^{14}}J(c)\right], \tag{17}$$

where the normalization of [2] is recovered upon setting $\alpha'=1$. Here $J(c)$, which was defined in Eq. (1), is evaluated using Eq. (16). We now briefly explain how this computation is performed following the general procedure of [11,12]. Inserting Eq. (16) into Eq. (1), one obtains terms involving traces of products of L and $\dot{L}$. One may then systematically eliminate all terms that contain $\dot{L}$ and (L^2) as follows: First, one uses the EOM, Eq. (12), to replace $\dot{L} \to \dot{\Phi} L$, leaving terms involving traces of products of L and powers of $\dot{\Phi}$. Even powers of $\dot{\Phi}$ can then be eliminated by use of the third equation in Eq. (12), $\dot{\Phi}^2 \to \frac{1}{4}(L^2)$. Those containing odd powers of $\dot{\Phi}$ vanish. [To see this, use repeatedly the substitution $\dot{\Phi}^2 \to \frac{1}{4}(L^2)$ to arrive at terms with a single $\dot{\Phi}$, then integrate it by parts to get $\int dte^{-\Phi}\dot{\Phi}X(L) = \int dte^{-\Phi}\dot{X}(L)$. Now using $\dot{L} \to \dot{\Phi} L$ gives the first term back with a different coefficient, thus proving that these terms vanish.] At this point, we are left with traces of powers of L, and we now argue that terms containing (L^2) vanish,

$$\int dte^{-\Phi}(L^2)X(L) = 0. \tag{18}$$

This is proved as follows. One uses the lapse EOM to replace (L^2) by $4\dot{\Phi}^2$, after which one integrates by part one $\dot{\Phi}$ factor, using $e^{-\Phi}\dot{\Phi} = -(d/dt)(e^{-\Phi})$. This creates terms with $\ddot{\Phi}$ and $\dot{L}$, for which ones uses again the EOM, Eq. (12), to write the result in terms of $\dot{\Phi}^2$ and (L^2). Finally, one replaces $\dot{\Phi}^2$ by $\frac{1}{4}(L^2)$ using the lapse equation. This reproduces the original integral, but with a different coefficient, hence proving that it is zero.

The upshot of the above procedure is that we may ignore from the beginning all $\dot{L}$ terms, setting $R_{i0j0} = -\frac{1}{4}(L^2)_{ij}$, and then eliminate all (L^2) contributions at the end of the computation. For the two contributions to $J(c)$, one then finds

$$t_8 t_8 R^4 \simeq \frac{9}{4}(L^8) + \frac{51}{16}(L^4)^2 - 6(L^3)(L^5),$$
$$e_{10} e_{10} R^4 \simeq -90(L^8) + \frac{45}{2}(L^4)^2 + 48(L^3)(L^5), \tag{19}$$

where the symbol $\simeq$ indicates that these equalities hold up to EOMs and integration by parts inside the integral $\int dte^{-\Phi}$. We then find that the reduction of Eq. (1) is given by

$$J(c) \simeq \frac{1}{4}(9 - 45c)(L^8) + \frac{1}{16}(51 + 45c)(L^4)^2 - 6(1 - c)(L^3)(L^5). \tag{20}$$

As explained, while the first two terms can be written in a $O(9,9)$ invariant form using the identities, Eq. (15), the last one involves traces of odd powers of L and is then noninvariant. These contributions come from the second term in the second line of Eq. (2) and the first term in the fourth line of Eq. (4), respectively.

Given that these contributions are unambiguous, meaning that they cannot be modified through EOMs or integrations by parts, T-duality then fixes the coefficient to its expected value $c = 1$, for which

$$J(1) \simeq -9(L^8) + 6(L^4)^2 = -\frac{9}{2}\mathrm{Tr}(\dot{\mathcal{S}}^8) + \frac{3}{2}[\mathrm{Tr}(\dot{\mathcal{S}}^4)]^2. \tag{21}$$

Using this in Eq. (17), we then finally arrive at the minimal form of the cosmological effective action written in a manifestly $O(9,9)$ invariant way:

$$S = \int dt\, e^{-\Phi}\Big\{-\dot{\Phi}^2 - \frac{1}{8}\mathrm{Tr}(\dot{\mathcal{S}}^2) + \frac{\alpha'^3\zeta(3)}{2^{15}}[-3\mathrm{Tr}(\dot{\mathcal{S}}^8) + \mathrm{Tr}(\dot{\mathcal{S}}^4)\mathrm{Tr}(\dot{\mathcal{S}}^4)]\Big\}. \tag{22}$$

Let us finally point out that, while the above result was computed for vanishing B-field and dilaton, by duality invariance Eq. (22) must be the complete cosmological action including B-field and dilaton. One might be worried that there could be dilaton couplings in higher dimensions that upon cosmological reduction and field redefinitions contribute to the gravitational terms and change the above coefficients, but one may convince oneself that this cannot happen. Following the steps outlined above, it can be seen that a generic term of the form $\nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_n}\phi X^{\mu_1\mu_2\ldots\mu_n}$, where X is a tensor depending on G and ϕ, compactifies to terms that can be redefined away, up to terms containing traces (L), which violate duality invariance.

Discussion.—In this Letter, we have analyzed the compatibility of the continuous $O(d,d,\mathbb{R})$ symmetry with the eight-derivative couplings quartic in the Riemann tensor proportional to $\zeta(3)$ that arises in string theory at order α'^3. We have shown that demanding $O(9,9,\mathbb{R})$ invariance upon cosmological reduction to one timelike dimension fixes the relative coefficient between the couplings $t_8 t_8 R^4$ and $\epsilon_{10}\epsilon_{10}R^4$ uniquely to the value determined independently by other methods. This result illustrates the strength of duality invariance, for it allows one to reconstruct from the $t_8 t_8 R^4$ term, which was computed by Gross and Witten from the four-point amplitude [2], the complete gravitational couplings in ten dimensions, in addition to determining the B-field and dilaton couplings that survive upon cosmological reduction. This gives strong constraints on the possible B-field and dilaton couplings in ten dimensions, but we do not expect these to be determined completely since there could be couplings that disappear upon reduction.

The results presented here are useful, in particular, in that they determine the first two nontrivial coefficients for Type II string theory in the general cosmological classification to all orders in α' [11]. There have already been a number of papers exploring cosmological consequences of this

α'-complete cosmology (see [30–34]), and here we have further constrained the "space of duality covariant string cosmologies." Moreover, our results provide a nontrivial test for the core assumption underlying this classification: that there is a field basis for which the $O(d,d,\mathbb{R})$ symmetry, expected to exist in string theory to all orders in α', takes the standard form. This is in contrast to double field theory [35] and conventional dimensional reduction with a generic number of external dimensions [16,36], where a Green-Schwarz-type mechanism needs to be invoked that can be viewed as α'-deforming the $O(d,d)$ transformations.

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Note added.—Upon completion of this Letter, we became aware of the results in [37], which exhibit obstacles for a double field theory formulation of the $\zeta(3)$ couplings in ten dimensions. The first version of this Letter also identified an obstacle for the conventional realization of $O(d,d)$ in dimensional reduction, but this was due to a computational mistake, and the corrected results presented here fully confirm the presence of $O(d,d)$. We therefore expect that there is also a double field theory formulation of the $\zeta(3)$ couplings, perhaps involving novel structures.

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[1] D. J. Gross and E. Witten, Superstring modifications of Einstein's equations, Nucl. Phys. **B277**, 1 (1986).

[2] D. J. Gross and J. H. Sloan, The quartic effective action for the heterotic string, Nucl. Phys. **B291**, 41 (1987).

[3] R. H. Brandenberger and C. Vafa, Superstrings in the Early Universe, Nucl. Phys. **B316**, 391 (1989).

[4] A. A. Tseytlin and C. Vafa, Elements of string cosmology, Nucl. Phys. **B372**, 443 (1992).

[5] G. Veneziano, Scale factor duality for classical and quantum strings, Phys. Lett. B **265**, 287 (1991).

[6] K. A. Meissner and G. Veneziano, Symmetries of cosmological superstring vacua, Phys. Lett. B **267**, 33 (1991).

[7] K. A. Meissner and G. Veneziano, Manifestly O(d,d) invariant approach to space-time dependent string vacua, Mod. Phys. Lett. A **06**, 3397 (1991).

[8] E. Bergshoeff, B. Janssen, and T. Ortin, Solution generating transformations and the string effective action, Classical Quantum Gravity **13**, 321 (1996).

[9] K. A. Meissner, Symmetries of higher order string gravity actions, Phys. Lett. B **392**, 298 (1997).

[10] N. Kaloper and K. A. Meissner, Duality beyond the first loop, Phys. Rev. D **56**, 7940 (1997).

[11] O. Hohm and B. Zwiebach, Duality invariant cosmology to all orders in α', Phys. Rev. D **100**, 126011 (2019).

[12] O. Hohm and B. Zwiebach, T-duality constraints on higher derivatives revisited, J. High Energy Phys. 04 (2016) 101.

[13] A. A. Tseytlin, Duality and dilaton, Mod. Phys. Lett. A **06**, 1721 (1991).

[14] J. Panvel, Higher order conformal invariance of string backgrounds obtained by O(d,d) transformations, Phys. Lett. B **284**, 50 (1992).

[15] I. Jack and S. Parsons, O(d,d) invariance at two loops and three loops, Phys. Rev. D **62**, 026003 (2000).

[16] C. Eloy, O. Hohm, and H. Samtleben, Green-Schwarz Mechanism for String Dualities, Phys. Rev. Lett. **124**, 091601 (2020).

[17] A. Sen, $O(d)\times O(d)$ symmetry of the space of cosmological solutions in string theory, scale factor duality and two-dimensional black holes, Phys. Lett. B **271**, 295 (1991).

[18] O. Hohm, A. Sen, and B. Zwiebach, Heterotic effective action and duality symmetries revisited, J. High Energy Phys. 02 (2015) 079.

[19] M. T. Grisaru, A. van de Ven, and D. Zanon, Four-loop β-function for the $N=1$ and $N=2$ supersymmetric nonlinear sigma model in two dimensions, Reports No. HUTP-86/A020, HUTP-86/A027, 1986.

[20] M. T. Grisaru and D. Zanon, Sigma-model superstring corrections to the Einstein-Hilbert action, Phys. Lett. B **177**, 347 (1986).

[21] M. D. Freeman, C. N. Pope, M. F. Sohnius, and K. S. Stelle, Supersymmetry in compactifications of the heterotic string, Phys. Lett. B **178**, 199 (1986).

[22] M. B. Green and J. H. Schwarz, Superfield theory of type II superstrings, Nucl. Phys. **B181**, 502 (1981); Nucl. Phys. **B198**, 441 (1982).

[23] J. H. Schwarz, Superstring theory, Phys. Rep. **89**, 223 (1982).

[24] B. Zumino, Gravity theories in more than four dimensions, Phys. Rep. **137**, 109 (1986).

[25] M. R. Garousi, S-duality invariant dilaton couplings at order α'^3, J. High Energy Phys. 10 (2013) 076.

[26] K. Peeters, P. Vanhove, and A. Westerberg, Supersymmetric higher derivative actions in ten-dimensions and eleven-dimensions, the associated superalgebras and their formulation in superspace, Classical Quantum Gravity **18**, 843 (2001).

[27] M. de Roo, H. Suelmann, and A. Wiedemann, The supersymmetric effective action of the heterotic string in ten-dimensions, Nucl. Phys. **B405**, 326 (1993).

[28] H. Razaghian and M. R. Garousi, R^4 terms in supergravities via T-duality constraint, Phys. Rev. D **97**, 106013 (2018).

[29] M. R. Garousi, Effective action of type II superstring theories at order α'^3: NS-NS couplings, J. High Energy Phys. 02 (2021) 157.

[30] C. Krishnan, de Sitter, α'-corrections & duality invariant cosmology, J. Cosmol. Astropart. Phys. 10 (2019) 009.

[31] P. Wang, H. Wu, H. Yang, and S. Ying, Non-singular string cosmology via α' corrections, J. High Energy Phys. 10 (2019) 263.

[32] H. Bernardo, R. Brandenberger, and G. Franzmann, String cosmology backgrounds from classical string geometry, Phys. Rev. D **103**, 043540 (2021).

[33] H. Bernardo, R. Brandenberger, and G. Franzmann, Solution of the size and horizon problems from classical string geometry, J. High Energy Phys. 10 (2020) 155.

[34] C. A. Núñez and F. E. Rost, New non-perturbative de Sitter vacua in α'-complete cosmology, J. High Energy Phys. 03 (2021) 007.

[35] O. Hohm and B. Zwiebach, Double field theory at order α', J. High Energy Phys. 11 (2014) 075.

[36] W. H. Baron, J. J. Fernandez-Melgarejo, D. Marques, and C. Nunez, The odd story of α'-corrections, J. High Energy Phys. 04 (2017) 078.

[37] S. Hronek and L. Wulff, O(D,D) and the string α' expansion: An obstruction, arXiv:2012.13410.