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3/88

EFFECT OF COIL-TO-COLLAR FRICTION ON LONGITUDINAL COIL DISPLACEMENTS

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SUMMARY

This is a zeroth-order whack at the subject.

Under the influence of the longitudinal Lorentz forces applied at the ends of the coil, and assuming the coil ends are unconstrained (they aren't completely, but lets play the game anyhow), the coil stretches. If it were unrestrained, it would stretch about 0.3 in. But frictional forces between the coil and the collars restrict this stretching to 23% of that, or .067 in. when the force relaxes, half of that gets locked in, and subsequent loading produces a stretching of .033 in.

The fraction of the zero-friction stretch that the friction permits, and the fraction that becomes locked in, depend on the ratio of a "penetration length" to the collar-package length. The penetration length is simply T_0/F, where T_0 is the longitudinal force, and F is the frictional force per unit length.

Frictional heating was investigated only to the extent of getting an upper limit to the temperature rise under severe stick-slip conditions. Under such conditions, the temperature rise is calculated to be 0.13 K, resulting in a decrease in critical current of 6%.

ANALYSIS

The following assumptions, none of which represent reality particularly well, are invoked.

1 Poisson's ratio is zero; radial forces on the coil produce no longitudinal coil strain.

2 The magnitude of the longitudinal friction force per unit length, F, between the coil and the collar is constant.

3 Transverse planes through the coil remain plane under all load conditions; the longitudinal stress in the coil is uniform over the coil cross section.

4 The collar is rigid; longitudinal friction forces create no strain in the collar.

5 The collar is free to move longitudinally.

Initially, the collar is clamped to the coil, and there is no longitudinal force in the coil. As the Lorentz tension T_0 is applied, frictional forces between the coil and the collar are developed near the end of the collar. We temporarily regard the collar as infinitely long. The state of forces in the coil is illustrated in Fig. 1.

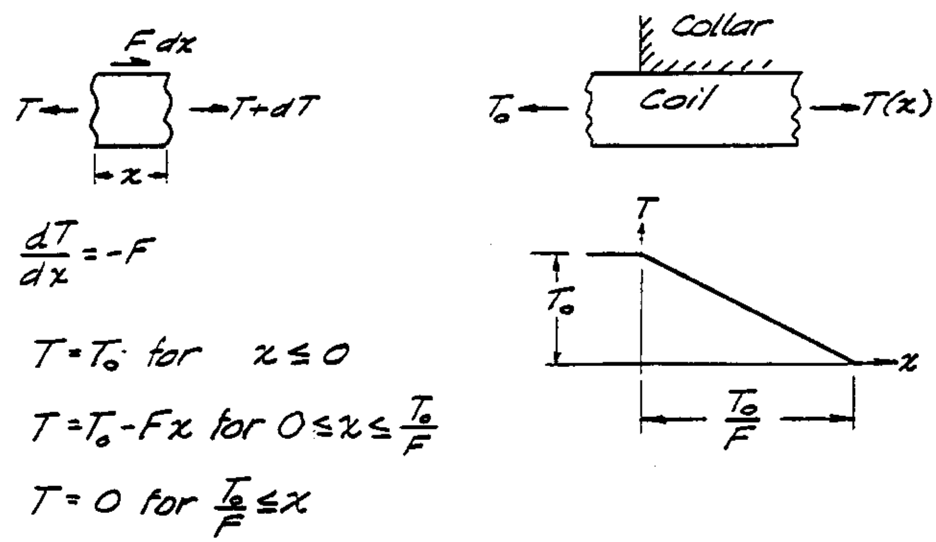

Fig. 1   Friction effects during loading.

When the tension is relaxed, the coil pulls itself back into the collar, and the direction of the friction forces reverses, resulting in the state of forces illustrated in Fig. 2.

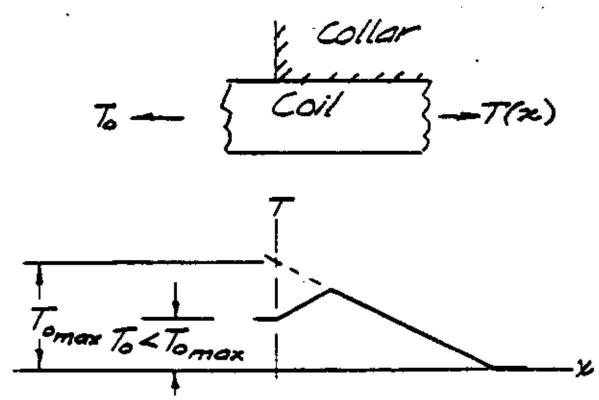

Fig. 2   Friction effects during unloading.

If the length of the collar is less than twice the penetration distance T_0/F, and the collar floats on the coil so that the tension T_0 is developed at both ends of the collar, then the stress patterns developed at the ends interfere at the center of the collar.

Three distinct kinds of force patterns can be developed depending on the relationship of the penetration distance T_0/F to the collar length L:

Mode I: $0 \leq T_0/F \leq L/2$

Mode II: $L/2 \leq T_0/F \leq L$

Mode III: $L \leq T_0/F$

The "film strips" in Fig. 3 show how the force pattern develops, for each of the three modes, upon initial loading, unloading, and re-loading. In all three cases, the force pattern of the first loading is different from the first, but the pattern repeats thereafter.

During loading, the part of the coil within the collar stretches, but not as much as if there were no friction. Upon unloading, the part of the coil within the collar remains under tension, and so it is longer than if there were no friction -- there is a permanent set.

The increase of length of the part of the coil within the collar is

$$\Delta = \int_0^x \epsilon \, dx = \frac{1}{AE} \int_0^x T \, dx$$

which is proportional to the area under the T-vs-x curve. The loaded and locked-in stretch are easily calculated from the diagrams of frames 4 and 8 of Fig. 3.

The loaded and locked-in stretch, and their difference, for each mode is presented in the following table, together with normalized values. Y is the increase in length of the part of the coil normalized to the same quantity for zero friction, and X is the ratio of the penetration distance to the collar length. There are other ways of normalizing things; this is a handy one. The results are plotted in Fig. 4.

An example, roughly approximating the SSC dipoles, follows. Following that, a crude analysis of frictional heating of the coil is presented.

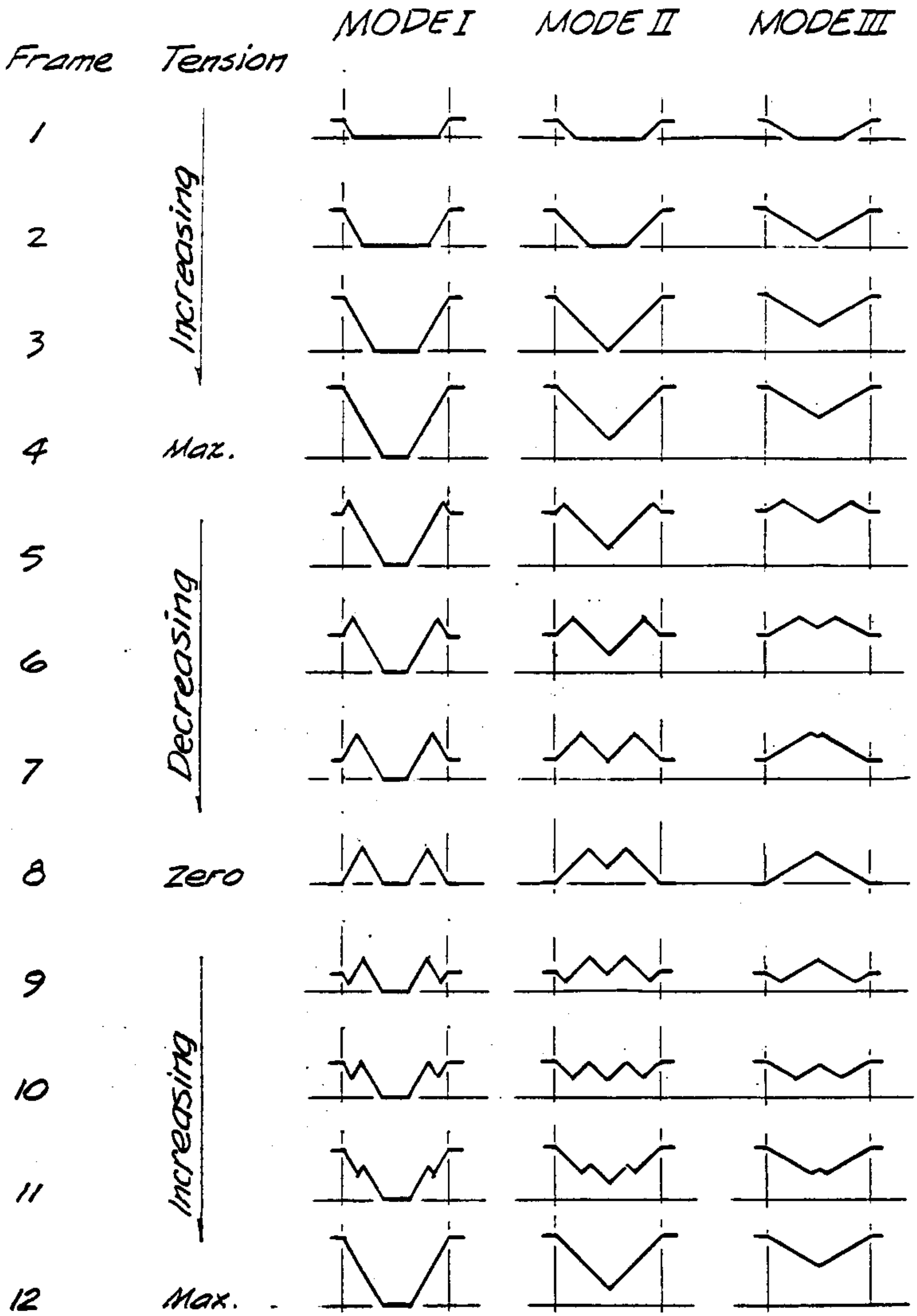

Fig. 3 Force diagrams for various stages of loading and unloading.

Mode	Range	Increase in length, Δ		
		Loaded	Unloaded	Difference
I	$0 \le \frac{T_0}{F} \le \frac{L}{2}$	$\frac{L}{AE}\cdot\frac{T_0^2}{F}$	$\frac{L}{AE}\cdot\frac{T_0^2}{2F}$	$\frac{L}{AE}\cdot\frac{T_0^2}{2F}$
II	$\frac{L}{2} \le \frac{T_0}{F} \le L$	$\frac{L}{AE}\cdot\left(T_0L-\frac{FL^2}{4}\right)$	$\frac{L}{AE}\cdot\left(T_0L-\frac{FL^2}{4}-\frac{T_0^2}{2F}\right)$	$\frac{L}{AE}\cdot\frac{T_0^2}{2F}$
III	$L \le \frac{T_0}{F}$	$\frac{L}{AE}\cdot\left(T_0L-\frac{FL^2}{4}\right)$	$\frac{L}{AE}\cdot\frac{FL^2}{4}$	$\frac{L}{AE}\cdot\left(T_0L-\frac{FL^2}{2}\right)$

Mode	Range	Normalized increase in length, Y		
		Loaded	Unloaded	Difference
I	$0 \le X \le \frac{1}{2}$	X	$\frac{X}{2}$	$\frac{X}{2}$
II	$\frac{1}{2} \le X \le 1$	$1-\frac{1}{4X}$	$1-\frac{1}{4X}-\frac{X}{2}$	$\frac{X}{2}$
III	$1 \le X$	$1-\frac{1}{4X}$	$\frac{1}{4X}$	$1-\frac{1}{2X}$

$$X = \frac{T_0/F}{L} \qquad Y \equiv \frac{\Delta}{T_0L/AE}$$

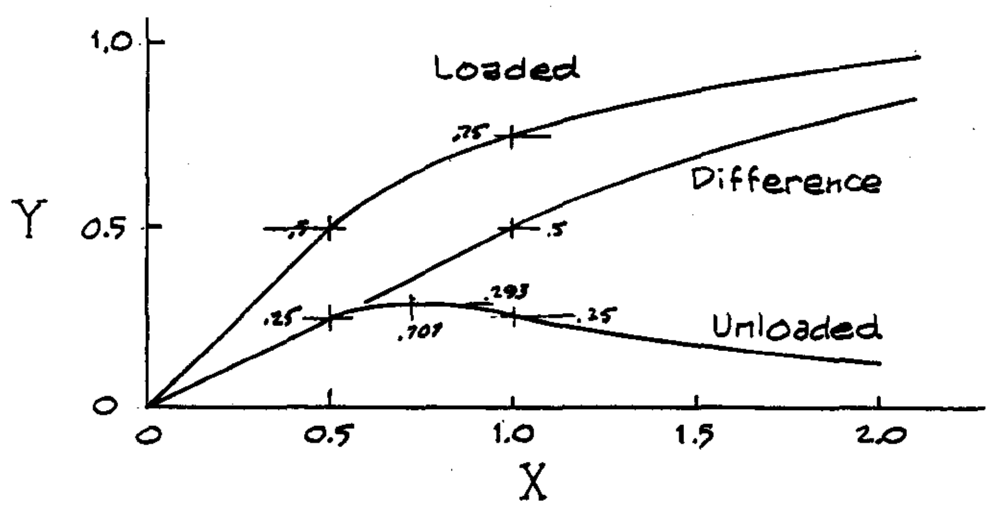

Fig. 4 Fraction of free stretch permitted by frictional forces during loading and unloading. Y is the fraction of free-stretch, and X is the ratio of the penetration distance, T /F to the collar-package length.

EXAMPLE - SSC DIPOLES

Maximum longitudinal force in coil	$T_o = 15000$ lb
Cross-sectional area of coil, incl. metal, insulation, and empty space, but not wedges	$A = 3.36$ in^2
Elastic modulus based on above area	$E = 10\times10^6$ psi
Coil length	$\Sigma L = 650$ in.
Collar length	$L = 6$ in.
Circumferential prestress	$\sigma_{ps} = 8000$ psi
Coefficient of friction	$\mu = 0.3$
Coil outside radius	$a = 1.6$ in.
Coil thickness	$h = 0.8$

Radial pressure on outside of coil =

$$= \sigma_{ps}\frac{h}{a} = 8000\,\frac{0.8}{1.6} = 4000 \text{ psi}$$

Force on coil, one quadrant:

Circum. subject to σ_{ps} = 0.8 in.

Force $8000 \times 0.8 =$ 6400 lb/in.

Circum subject to radial press = 1.82 in.

Force $4000 \times 1.82 =$ 7300 lb/in.

Total force = 13700 lb/in.

Force on coil, 4 quadrants $13700 \times 4 =$ 54800 lb/in.

Longitudinal friction force $F = 0.3 \times 54800 =$ 16400 lb/in

Penetration distance = $T_o/F = 15000/16400 = 0.91$ in

$X = \frac{T_o/F}{L} = 0.91/4 = 0.23$

$\therefore\ Y_{loaded} = 0.23$ (From graph or table)

$Y_{unloaded} = 0.115$

UPPER LIMIT TEMP RISE DURING STICK-SLIP

Change in length of part of coil within one half-length of a collar

Without friction: $\frac{T_0 \frac{L}{2}}{AE} = \frac{15000\,(6/2)}{3.36\times 10\times 10^6} = 0.00134$ in.

With friction $\quad 0.23\times .00134 = 0.00031$ in.

Difference $\quad 0.0010$ in.

The frictional force on .0010 in. of coil length is (since .001 is small compared to the penetration distance T_0/F) is just .001F
= 16.4 lb

so the energy deposited in that .001 in. of coil is 0.0164 in-lb $\left(\times\frac{1}{12}\times 1.36\right) = 0.0019$ J.

If that energy gets deposited in one strand of each outer cable the temp. rise is as follows

No of exposed strands 96

Cross section per strand

$\sim \frac{\pi}{4}.030^2 = 0.71\times10^{-3}$ in.^{2}

Volume for .001 in. of coil $= 0.71\times10^{-6}$ in^3

Corresponding mass $= 0.71\times10^{-6}\,\text{in}^3 \times 0.3\,\text{lb/in}^3$

$= 2.1\times10^{-7}$ lb

$\times .454 = 0.095$ kg

Specific ht. at ~4K (very ~) $= .15\times10^{-3}\,\frac{J}{gm\cdot K}$

$= .15\ \frac{J}{kg\cdot K}$

$$\Delta T = \frac{.0019\,J}{.15\,\frac{J}{kg\text{-}K}\times .095\,kg} = 0.13\ K$$

The corresponding decrease in j_c is about 6%. I doubt if the cable is that stable.