SOFT-PHOTON CONTRIBUTION TO ELECTRO-DYNAMICAL CROSS-SECTIONS AT VERY HIGH ENERGIES

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The aim of this paper is to discuss some characteristic features of the infrared divergence phenomena such as the soft photon contribution to the radiative corrections in the domain of very high energies. The discussion will procede as follows: a typical electrodynamic process such as the electron-positron annihilation into photons will be considered and the results of a complete perturbation calculation of order α^3 will be discussed briefly. Then an attempt will be made to generalize the rather interesting suggestions deriving from that calculation to every order of α.

ERIKSSON, in his lecture [5], has explained what is meant by infrared divergence and how the soft photon contribution works in its elimination. Let us briefly recall and apply those considerations to the annihilation process.

Let us try to calculate the higher order corrections in α to the Born approximation represented by the simple graph in Fig. 1.

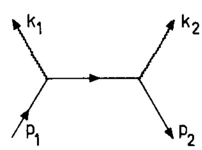

Fig. 1

As is well known, the relevant corrections are obtained by inserting one internal photon line into the above skeleton graph in all possible ways. This situation is shown in Fig. 2 in terms of Feynmann's diagrams. (Obviously there are as many as graphs deriving from the exchange of the two final photons $k_1 \leftrightarrow k_2$.)
All these terms diverge when we integrate over the energy of the virtual photon down to the limit zero and we find something like this:

$$\int_0 d\omega/\omega .$$

In order to avoid this divergence we regularize the above integrals by ascribing a fictious, non-zero mass Λ to the photon so that we finally get the Λ-dependent (but Lorentz-invariant) result:

$$d\sigma_v^{(\alpha^3)} = d\sigma_0^{(\alpha^2)} \{1 + (\alpha/\pi)[F(\gamma)\ln(\Lambda/m) + f_1(\gamma, \theta)]\} \quad (1)$$

where $d\sigma_v^{(\alpha^3)}$ is the differential cross-section for the annihilation process corrected by virtual photons and $d\sigma_0^{(\alpha^2)}$ is the corresponding quantity calculated in Born approximation $\gamma = \epsilon_+/m$, and θ is the scattering angle. Note

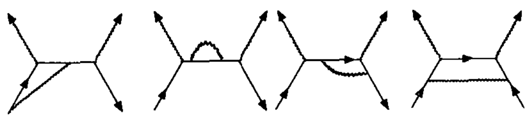

Fig. 2

the isotropic coefficient $F(\gamma)$ of the Λ dependent part. This fact can be easily understood by direct inspection of the Feynmann's graphs but we will not go into the details here.

Equation (1) is not a physically meaningful result owing to the Λ dependence. In order to remove it, the consideration is made that a scattering process (in a broad sense) can be never considered as purely elastic. In our case, for instance, we cannot ensure experimentally that the annihilation of the pair leads to a final state with two photons only. In fact there is always an inelastic contribution because of the emission of some supplementary real photons. They are not detected if their total energy is less than an upper limit ΔE which can be taken as the resolving power of the experiment. However, although not experimentally distinguishable from the fundamental process, this sort of background effect cannot be omitted. So we have to add the cross-section for the emission of one additive photon (to α_3 order), with an energy under the threshold of detection ΔE, to Eq. (1). It is possible to eliminate the infrared divergence, taking into account that the emission probability of soft photons diverges in the limit $\omega \to 0$. Usually the procedure followed in the calculation is to consider the inelastic contribution from real soft photons whose energy is less than a given quantity $\Delta \ll m$, in a particular reference frame.

In this way we get the cross-section for annihilation into three photons, one of which with energy $\leq \Delta \ll m$

$$\begin{aligned} d\sigma_{soft}^{(\alpha^3)} &= d\sigma_0^{(\alpha^2)} \, (\alpha/\pi) \, [F(\gamma) \ln(2\Delta/\Lambda) + \varphi(\gamma)] \\ &= d\sigma_0^{(\alpha^2)} \, (\alpha/\pi) \, [F(\gamma) \ln(\Delta/\gamma\Lambda) + \varphi(\gamma)]. \end{aligned} \tag{2}$$

In order to calculate Eq. (2) we have taken into account only the infrared part of the above cross-section that corresponds to the soft photon emission from external electron lines only. This causes the appearing of a contribution only γ dependent.

In the C.M.S. system we get

$$\begin{aligned} \varphi(\gamma) &= 2 \ln^2(2\gamma) - \pi^2/3, \\ F(\gamma) &= 2[\, 2 \ln(2\gamma) - 1] \,. \end{aligned} \tag{3}$$

By combination of Eqs. (1) and (2) we get the Λ independent result

$$d\sigma_{v+s}^{(\alpha^3)} = d\sigma_0 \left\{1 + \frac{\alpha}{\pi}\left[\varphi(\gamma) + F(\gamma)\ln\frac{\Delta}{m\gamma} + f_1(\gamma,\theta)\right]\right\}. \tag{4}$$

While the introduction of soft photons only is sufficient to cancel out the infrared divergence it is not entirely realistic. In fact at high energies the actual resolving power ΔE of the experimental device does not fulfill the condition $\Delta E \ll m$ and generally has an angular dependence. So in order to compare the theoretical calculations with the experimental results one ought also to take into account the photons (not detected) in the range $\Delta \to \Delta E$. They are usually called hard photons. If we want a result independent of ΔE we can go to the limit of no resolving power by allowing the energy of the additive photon to reach the maximum value given by the conservation laws. In so doing the total correction is the sum of the virtual plus the inelastic part and we might call it a radiative correction in a broader sense.

Going back to our particular process this pattern of thought gives as a final result the cross-section for annihilation of a pair into two and three photons up to α^3 order.

The cross-section for annihilation of a pair into three hard photons (with energies larger than Δ) is

$$d\sigma_{hard}^{(\alpha^3)} = d\sigma_0(\alpha/\pi)[G(\gamma)\ln(m\gamma/\Delta) + g_1(\gamma,\theta)]. \tag{5}$$

It is verified that $G(\gamma) = F(\gamma)$ so the combination of Eqs. (5) and (4) gives

$$\begin{aligned} d\sigma^{(\alpha^3)} &= d\sigma_0\{1 + (\alpha/\pi)[\varphi(\gamma) + f_1(\gamma,\theta) + g_1(\gamma,\theta)]\} \\ &= d\sigma_0\{1 + \delta(\gamma,\theta)\}. \end{aligned} \tag{6}$$

In Eq. (6) φ, g_1, f_1 represent the soft photon, the virtual photon and the hard photon contribution to the total correction δ, respectively. (Really this division is rather arbitrary and not unique depending on the used regularization procedure and not invariant owing to the not covariant definition of soft photons.)

For computational reasons it is simpler to discuss the integrated correction $\delta(\gamma) = \int d\Omega\,\delta(\gamma,\theta)$, or the total cross-section for annihilation into photons up to α^3 order.

$$\sigma = \sigma_0\{1 + (\alpha/\pi)[\varphi(\gamma) + f(\gamma) + g(\gamma)]\} = \sigma_0\{1 + \delta(\gamma)\}. \tag{7}$$

In this case it is possible to write the final result in the extreme relativistic case

$$\sigma_0 = (\pi r_0^2/2\gamma^2)[2\ln(2\gamma) - 1], \tag{8}$$

$$\delta(\gamma) = (\alpha/12\pi)\left[8\ln^2(2\gamma) - 2\ln(2\gamma) + 4\pi^2 - 13 + \frac{11 - 5\pi^2}{2\ln(2\gamma) - 1}\right], \tag{9}$$

$\gamma \gg 1$ (C.M.S. energy).

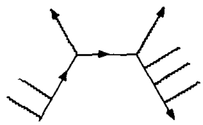

Fig. 3

The relevant point in Eq. (9) is the ln (2γ) behaviour of the total correction. If in order to understand its origin we perform the usual division between virtual soft and hard photon contribution, we find in the C.M.S.:

$$\sigma_{v+\Delta} = \sigma_0 \{1 + (\alpha/12\pi)[8\ln^2(2\gamma) + 22\ln(2\gamma) + \ldots + F(\gamma)\ln(\Delta/m\gamma)]\}$$

$$\sigma_{hard} = \sigma_0(\alpha/12\pi)[-24\ln(2\gamma) + \ldots F(\gamma)\ln(m\gamma/\Delta)],$$

so that we can ascribe the $\ln^2(2\gamma)$ contribution to the virtual plus soft photon part. Hard photons do not contribute to the dominant behaviour in the very high energy case, so that, from this point of view, we can forget their existence. To get a further insight into the $\ln^2(2\gamma)$ derivation, let us go back to the differential cross-section of Eq. (4):

$$d\sigma_{v+s} = d\sigma_0\{1 + \delta_{v+s}(\gamma,\theta,\Delta)\}.$$

Some general previsions can be made on the high energy behaviour of δ_{v+s}. A quite general theorem by ERIKSSON and PETERMANN [2] states that for large values of the momentum transfer (in the C.M.S.), $q^2 \gg m^2$, δ_{v+s} behaves at most like $(\alpha/\pi)\ln(q^2/m^2)$ to order α^n. More precisely, the first correction can be written in the form

$$\delta_{v+s} = (\alpha/\pi)[c_1\ln(q^2/m^2)\ln(\Delta/E) + c_2\ln(q^2/m^2) + c_3\ln(\Delta/E) + c_n].$$

The validity of the Petermann theorem can also be verified in our case. This means that, being the soft photon part isotropic and always $\approx \ln^2(2\gamma)$, the contribution of the virtual photon for large momentum transfer is such as to compensate those $\ln^2(2\gamma)$ terms. Let us look at another boundary condition, the region of small momentum transfer, where

$$q^2 = (p_1 - k_1)^2 = -m^2 + 2m^2\gamma^2(1 - \beta\cos\theta) \sim m^2\gamma^2\theta$$

or

$$q^2 \lesssim m^2, \quad \theta \lesssim 1/\gamma.$$

In this situation we find that the virtual photon contribution is small (no $\ln^2(2\gamma)$ terms) so that there is no more compensation and the $\ln^2(2\gamma)$ from the soft part is still present and dominant. The subsequent integration and the addition of the hard photons do not cancel the $\ln^2(2\gamma)$.

Obviously these are considerations whose validity is limited to the explicit α^3 calculation. Let us try to generalize those results: let us consider, for instance, the α^n situation (cross-section to α^{n+2} order). The correction

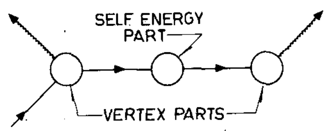

Fig. 4

is the result of n virtual photons, n - 1 virtual + one soft, ... n soft photons. We can make the following assumption based on the ideas of our previous calculation: if we limit ourselves to the small momentum transfer region (in S. C. M.) the virtual photon contribution is small, i. e. not of the $\alpha^n \ln^{2n}(2\gamma)$ type but at most of $\alpha^n \ln^{2n-1}(2\gamma)$ type. So the relevant term in the asymptotic limit comes practically from the graph with n soft photons. In this way the result is obtained at one and looks like

$$d\sigma^{(n+z)} \simeq d\sigma_0(\alpha/\pi)^n\ (1/m!)[\varphi(\gamma)+F(\gamma)\ \ln(\Delta/m\gamma)]^n \tag{10}$$

corresponding to the graph in Fig. 3.

If we sum up, using Eq. (3):

$$\begin{aligned} d\sigma &\sim d\sigma \sum_{0}^{\infty} {}_n\ (\alpha/\pi)^n\ (1/n!)\ [\varphi(\gamma)+F(\gamma)\ \ln(\Delta/m\gamma)]^n \\ &= d\sigma_0\ e^{\frac{\alpha}{\pi}\varphi(\gamma)}\ e^{(\alpha/\pi)\,F(\gamma)\ln\Delta/m\gamma} \\ &= d\sigma_0\ e^{\frac{2\alpha}{\pi}\ell n^2(2\gamma)}\ (\Delta/m\gamma)^{(4\alpha/\pi)\ln(2\gamma)} \end{aligned} \tag{11}$$

This is only s rough evaluation but it can give an idea of the philosophy we shall follow. Obviously a more rigorous derivation is possible.

To this purpose we will use in a slightly modified form the general result by ERIKSSON [3]. That is

$$P(\Delta) = \frac{e^{-\gamma c}}{\Gamma(1+c)}\left(\frac{\Delta}{m\gamma}\right)^c e^{B+A}\left(\frac{m}{\Lambda}\right)^c |M|^2 \tag{12}$$

where the symbols are those of Eriksson except A which is defined as

$$A = \frac{\alpha}{\pi}\left\{-\sum_i Q_i^2 + 2\sum_{i<j}\mathrm{Re}\int_0^1 g[x(p_i\epsilon_i + p_j\epsilon_j) - \epsilon_j p_j]\,dx\right\}$$

(for a full understanding of the notations see [3].

$|M|^2$ is the squared matrix element corrected by all the virtual photons. It is infrared divergent but we can regularize it with the fictitious photon mass Λ. Furthermore, the general theory of infrared divergences allows us to say that the dependence of Λ cancel out with $(m/\Lambda)^c$. Eq. (12) is a correct result, though a not covariant one, owing to the presence of Λ. Let us consider the annihilation case where an explicit calculation gives:

$$A = (2\alpha/\pi)\ln^2(2\gamma),$$

$$B = (4\alpha/\pi)\ln(2\gamma), \tag{13}$$

$$C = (4\alpha/\pi)\ln(2\gamma).$$

Note again the characteristic isotropy of the soft photon part for the annihilation process. Thus the angular dependence is contained in $|M|^2$. If we assume the validity of the Eriksson and Petermann theorem for $q^2 \gg m^2$ $(\theta \sim \pi/z)$, $|M|^2$ has to behave like $e^{-2\alpha/\pi \ln^2(2\gamma)}$ in order to compensate the soft photon part. What happens for $q^2 \sim 0$? If we put

$$(m/\Delta)^c\ |M|^2 = |M_0|^2\, \chi\ (p_i,\ k_i),$$

our assumption is that for $q^2 \to 0$, $\chi\ (p_i\ ,\ k_i)$ goes to a constant or, more

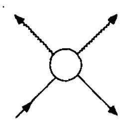

Fig. 5

generally, it behaves in a simple logarithmic and not a $(\log)^2$ manner. In this way, neglecting all the logarithmic terms, the result is

$$d\sigma \sim d\sigma_0\ e^{(2\alpha/\pi)\ \ln^2(2\gamma)\ (\Delta/m\gamma)\ (4\alpha/\pi)\ \ln(2\gamma)}$$

which is Eq. (11). So our general conclusion is that it is possible in forward annihilation to discriminate in a very clear out way between the virtual and soft photon contribution. Really this result may seem a rather academic one owing to the Δ term. So our perturbative result could lead us to another hypothesis, i. e. that the hard photons do not contribute $\ln^2(2\gamma)$ terms. Consequently we eliminate the Δ dependence by adding the hard photons while the dominant behaviour remains still

$$d\sigma \sim d\sigma_0\ \ e^{(2\alpha/\pi)\ \ln^2(2\gamma)} \tag{14}$$

with $d\sigma$ forward (or nearly forward) cross-section for annihilation into photons.

It is necessary to find out if our hypotheses are verified. The problem is not difficult for the virtual photon contribution. We all know that there are two classes of diagrams which contribute to M. They can be represented by the reducible graphs typified by Fig. 4 and the irreducible ones typified by Fig. 5. Though many photons exchange, as in Fig. 6.

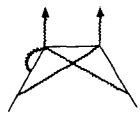

Fig. 6

As long as we confine ourselves to the logarithmic terms, it is possible to show by direct inspection(*) of the matrix elements to every order of α, that our first hypothesis works very well for both classes, that is

$$|M|^2 \, (m/\Lambda)^c \underset{q^2\to 0}{\to} |M_0|^2.$$

So Eq. (11) is correct.

The hard photon part is more complicated to hand. The problem is under study and we hope to be able to prove that Eq. (14) is also true soon.

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* By using for instance, the method of SUDAKOV [4].