

The investigations of the P -wave B_s states combining quark model and lattice QCD in the coupled channel framework

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ABSTRACT: Combining the quark model, the quark-pair-creation mechanism and $B^{(*)}\bar{K}$ interaction, we have investigated the near-threshold P -wave B_s states in the framework of the Hamiltonian effective field theory. With the heavy quark flavor symmetry, all the parameters are determined in the D_s sector by fitting the lattice data. The masses of the bottom-strange partners of the $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$ are predicted to be $M_{B_{s0}^*} = 5730.2_{-1.5}^{+2.4}$ MeV and $M_{B_{s1}^*} = 5769.6_{-1.6}^{+2.4}$ MeV, respectively, which are well consistent with the lattice QCD simulation. The two P -wave B_s states are the mixtures of the bare $\bar{b}s$ core and $B^{(*)}\bar{K}$ component. Moreover, we find a crossing point between the energy levels with and without the interaction Hamiltonian in the finite volume spectrum in the 0^+ case, which corresponds to a CDD (Castillejo-Dalitz-Dyson) zero in the T -matrix of the $B\bar{K}$ scattering. This CDD zero will help deepen the insights of the near-threshold states and can be examined by future lattice calculation.

KEYWORDS: Hadronic Spectroscopy, Structure and Interactions, Properties of Hadrons, Effective Field Theories of QCD

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1 Introduction

Significant progress has been made in hadron spectroscopy since 2003. A number of new hadrons involving heavy quarks have been discovered. However, after 20 years, their properties are still poorly understood [1]. Among them, the $D_{s0}^*(2317)$ [2] and $D_{s1}^*(2460)$ [3] are of great interest since they are much lighter than the quark model predictions [4]. These two states have been widely investigated by both theoretical and experimental sides, see reviews [5–8] for more details. Various proposals regarding their nature are proposed, including the quenched and unquenched $c\bar{s}$ quark models [4, 9–18], the molecule model [19–42], the tetraquark model [43–47], and the $c\bar{s}$ plus tetraquark model [12, 48–51].

The debates on the inner structures of $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$ really deepen our understanding on the formation of a physical state. At first, the pure quark model gives predictions of four P -wave $c\bar{s}$ mesons with the spin-parity as $J^P = 0^+$ (D_{s0}^*), 1^+ (D_{s1}^*), 1^+ ($D_{s1}'^*$) and 2^+ (D_{s2}^*). The predicted masses of the higher $D_{s1}'^*$ and D_{s2}^* are well consistent with the experimental data, while the two lower ones are not [1, 4]. The $J^P = 0^+$ state around 2480 MeV was predicted in the $c\bar{s}$ sector [4], which was obviously higher than the state $D_{s0}^*(2317)$ discovered by the experiment later. A similar situation happened to the $D_{s1}^*(2460)$. What makes the obscure is that they are very close to the $D^{(*)}K$ threshold. Using the scattering potential of the light pseudoscalar meson off the heavy meson based on the Chiral effective field theory (χ -EFT), the $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$ were described as the dynamically generated bound states [25, 26, 31], which indicates that the interaction at the hadronic level plays an important role to form these states. In an alternative view, the predictions of the $c\bar{s}$ state in the quark model should provide useful information for the D_s^* states due to its great success in describing the other conventional hadrons. However, it should be emphasized that the current quark model is not perfect for the real world even in a phenomenology study, because the interaction with the hadron channel is absent there. Such the hadron channel will play an important role in the near-threshold state, which is known as the coupled-channel effect. This effect may shift the masses of the near-threshold hadrons sizably [52–57]. From this point of view, in a previous paper [58], we stuck to

the conventional quark model and considered the coupled-channel effects from the S -wave $D^{(*)}K$ channels to investigate the $D_{s_0}^*(2317)$ and $D_{s_1}^*(2460)$ states. The higher $D_{s_1}(2536)$ and $D_{s_2}^*(2573)$ mainly coupled to the D -wave $D^{(*)}K$ channels. It turns out that the bare states play an extremely important role in the formation of the physical states.

The extended Hamiltonian effective field theory (HEFT) [59–62] can be used to quantitatively calculate the energy levels and scattering amplitudes in terms of the hadronic degrees of freedom, which naturally includes the coupled channel effects of the bare states and various channels. In this framework [58], the bare states and the mesons in threshold channels are well-defined in the quark model based on the well-established ground hadron spectrum. Their coupling potentials can be described by the quark-pair-creation (QPC) model [63] and the channel-channel interactions can be induced by exchanging the light mesons. All the parameters, such as the cutoff and channel coupling constants will be determined by fitting the lattice simulation results. This framework can connect various physical information rather than just fit the limited data, therefore sufficiently reducing the number of free parameters.

The bottom analogues of the $D_{s_0}^*(2317)$ and $D_{s_1}^*(2460)$ are still absent in experiments. Within the heavy quark symmetry, they are directly related to the D_s states and the predictions are usually obtained as a by-product in the theoretical study of the D_s states (more details referred to reviews [5–8, 64]). Thus, the investigations of the bottom analogs can not only enrich the hadron spectroscopy, but also can be used to examine the theoretical studies of the near-threshold hadrons. Nowadays, they have attracted more and more interest. Their masses have been studied in several scenarios, such as the $\bar{b}s$ meson in constituent quark model [65–68], the $B^{(*)}\bar{K}$ molecules [19, 25, 26, 31, 69–71], and the $\bar{b}s$ plus $B^{(*)}\bar{K}$ molecules [72]. In this work, we will use the extended Hamiltonian effective field theory (HEFT), which has been used in the D_s sector, to study the bottom analogs. In our previous work [58], it was shown that the heavy quark symmetry is a good symmetry in the D_s sector. Within the heavy quark symmetry, we will use the same coupling constants and cutoff parameters in the bottom-strange sector and obtain the predictions of the spectra of the P -wave B_s states.

This paper is arranged as follows. In section 2, we present the HEFT framework in the finite volume and the T -matrix in the infinite volume. In section 2.1, we demonstrate the study on the bottom-strange bare state in the quark model and its coupling with the nearby threshold channels. The channel-channel interaction is illustrated in section 2.2. In section 3, we obtain the mass spectra of the 0^+ and 1^+ B_s states, and compare our predicted energy levels with those from lattice simulation. At last, a summary is given in section 4.

2 The Extended Hamiltonian effective field theory

For a physical hadron with multiple components, the energy-independent Hamiltonian reads

$$H = H_0 + H_I, \tag{2.1}$$

where H_0 is the non-interacting Hamiltonian,

$$H_0 = \sum_b |b\rangle m_b \langle b| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|. \tag{2.2}$$

	parameter	this work	GI [4]
Masses	$\frac{1}{2}(m_u + m_d)$	264 MeV	220 MeV
	m_s	497 MeV	419 MeV
	m_c	1720 MeV	1628 MeV
	m_b	5065 MeV	4977 MeV
Potentials	b	0.18 GeV ²	0.18 GeV ²
	c	-426 MeV	-253 MeV
Relativistic effects	σ_0	1.45 GeV	1.80 GeV
	s	1.55	1.55
	ϵ_c	-0.194	-0.168
	ϵ_t	-0.016	0.025
	$\epsilon_{\text{so}(V)}$	-0.277	-0.035
	$\epsilon_{\text{so}(S)}$	-0.289	0.055

Table 1. The free parameters in the potential quark model.

Here b represents a bare $\bar{b}s$ core with a mass m_b , which is defined in the quark model. α denotes the $B^{(*)}\bar{K}$ channels, and $E_\alpha(\vec{k}) = \sqrt{m_K^2 + \vec{k}^2} + \sqrt{m_{B^{(*)}}^2 + \vec{k}^2}$ is the kinematic energy with \vec{k} the relative momentum. The interacting Hamiltonian is $H_I = g + v$, where g and v are the potential between the bare $\bar{b}s$ core and $B^{(*)}\bar{K}$ channels, and the potential within the $B^{(*)}\bar{K}$ channels, respectively. Their explicit forms will be illustrated in the following sections.

2.1 The bare state $\bar{b}s$

Since the quark model gains a great success in explaining the properties of the low-lying mesons, the predicted $\bar{b}s$ state is naturally expected to exist. In this work, we adopt the relativized quark model proposed by Godfrey-Isgur (GI) [4] to determine the $\bar{b}s$ state. The GI model provided a successful description of the mass spectra of the low-lying mesons, from the pion to the bottomonium [4].

In the quark model, the Hamiltonian describing quark-antiquark interaction reads,

$$\mathcal{H}_{q\bar{q}} = \sqrt{\mathbf{p}^2 + m_q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + \frac{\lambda_q \lambda_{\bar{q}}}{2} \mathcal{V}_{q\bar{q}}, \quad (2.3)$$

where the $\lambda_{q(\bar{q})}$ and $m_{q(\bar{q})}$ are the color matrix and the mass of the constituent quark (anti-quark), respectively. The effective potential $\mathcal{V}_{q\bar{q}}$ contains the one-gluon-exchange interaction and linear confinement interaction. Its explicit form can be found in ref. [4].

In ref. [58], we used the masses of the well-established mesons which locate far away from the two-meson thresholds to update the parameters in the GI model. Their values

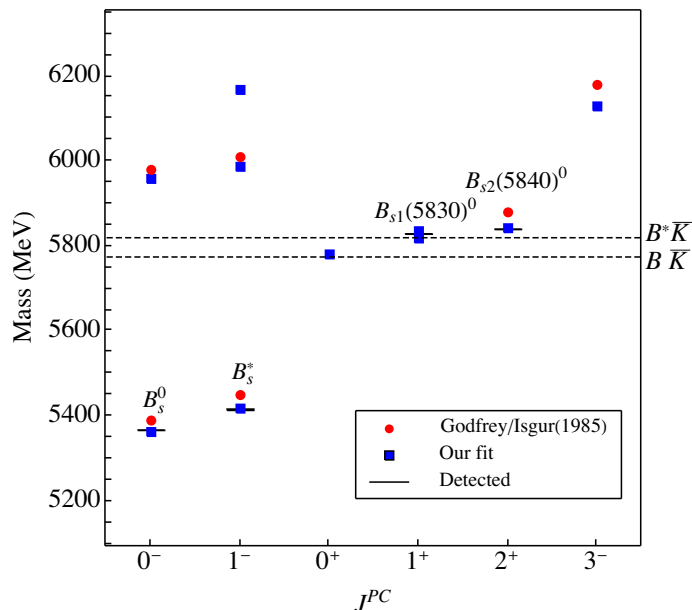


Figure 1. Mass spectrum of bare $\bar{b}s$ mesons within the relativized quark model. The circles and squares are the results predicted in ref. [4] and our new fit, respectively. The shaded areas represent the experimental masses and their uncertainties [1].

are summarized in table 1. The mass spectrum is better fitted to the experimental data than that in ref. [4]. In figure 1, we present the comparison of the mass spectrum for the bottom-strange mesons using the original and updated set of parameters. In the quark model, there are four P -wave $\bar{b}s$ mesons. These bare $\bar{b}s$ states are in the vicinity of the $B^{(*)}\bar{K}$ channels, which resembles the charm-strange case. Thus, the coupled-channel effects may significantly shift the masses of these B_s states. The possible coupling channels are shown in table 2. Similar to the D_s states, the lighter and heavier bare 1^+ states are almost on the heavy quark spin bases, which implies a good heavy quark symmetry. Thus, the bare 0^+ and the lighter bare 1^+ states are expected to mainly couple with the S -wave $B\bar{K}$ and $B^*\bar{K}$ channels, respectively, while the heavier 1^+ and the 2^+ bare states mainly couple with the D -wave $B^*\bar{K}$ or $B\bar{K}$ channels within good heavy quark symmetry. The D -wave coupling is significantly suppressed in the vicinity of the thresholds. Therefore, the mass shifts of the heavier bare 1^+ and 2^+ states can be neglected. Indeed, their bare masses, 5835.6 MeV and 5842.7 MeV, are very close to the experimental ones 5828.70 ± 0.20 MeV and 5839.86 ± 0.12 MeV [1].

In this work, the interactions between $\bar{b}s$ and $B^{(*)}\bar{K}$ can be written as

$$g = \sum_{\alpha,b} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha b}(\vec{k}) \langle b| + h.c. \right\}, \quad (2.4)$$

where $g_{\alpha b}(\vec{k})$ can be calculated from its partial wave expansion $g_{\alpha b}^{L_\alpha}(|\vec{k}|)$. The $\bar{b}s$ state couples with the $B^{(*)}\bar{K}$ channels through the creation of a light quark-antiquark pair with a quantum number $J^{PC} = 0^{++}$, as shown in figure 2. From the phenomenological QPC model [73–79], $g_{\alpha b}^{L_\alpha}(|\vec{k}|)$ reads

$$g_{\alpha b}^{L_\alpha}(|\vec{k}|) = \gamma I_{\alpha b}^{L_\alpha}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}. \quad (2.5)$$

	$b(^{2S+1}L_J\rangle)$	$b(\text{mass})$	α	L
B_{s0}^*	$ ^3P_0\rangle$	5780.9	$B\bar{K}$	S
B_{s1}^*	$-0.74 ^1P_1\rangle + 0.67 ^3P_1\rangle$ $= 0.98\phi_s - 0.22\phi_d$	5818.5	$B^*\bar{K}$	S, D
$B_{s1}^{*'}$	$0.67 ^1P_1\rangle + 0.74 ^3P_1\rangle$ $= 0.22\phi_s + 0.98\phi_d$	5835.6	$B^*\bar{K}$	S, D
$B_{s2}^{*'}$	$ ^3P_2\rangle$	5842.7	$B\bar{K}, B^*\bar{K}$	D

Table 2. The related bare $\bar{b}s$ cores (b) and the $B^{(*)}\bar{K}$ (α) channels in the Hamiltonians of the physical B_s states. The wave functions and mass spectrum (MeV) of the bare states are shown. $\phi_s = |\frac{1}{2}_l \otimes \frac{1}{2}_h\rangle$ and $\phi_d = |\frac{3}{2}_l \otimes \frac{1}{2}_h\rangle$ are the heavy quark symmetry bases, where h and l are the heavy and light degrees of freedom, respectively. The script L in the last column denotes the orbital excitation in the $B^{(*)}\bar{K}$ channels.

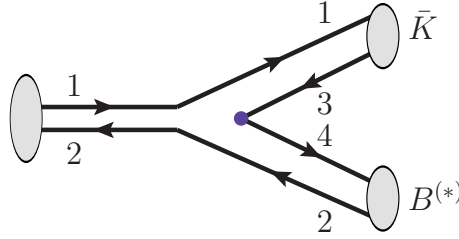


Figure 2. The diagram contributes to the process of the bare state coupling to $B^{(*)}\bar{K}$ channel in the QPC model. The first and second quarks are anti-strange and bottom quarks, respectively. The third and fourth are the light quarks created from a vacuum.

Here, γ is related to the creation probability of the quark pair. The exponential form factor with the cutoff Λ' is introduced to truncate the hard vertices [78, 79]. The spatial transform factor $I_{\alpha b}^{L_\alpha}(|\vec{k}|)$ can be calculated with the exact wave functions obtained from the quark model as

$$\begin{aligned}
 I_{\alpha b}^{L_\alpha}(|\vec{k}|) &= -\frac{\sqrt{4\pi(2L_\alpha + 1)}}{2J_b + 1} \sum_{M_{J_{\alpha_1}} M_{J_{\alpha_2}}} C_{L_\alpha 0; J_{\alpha_1 2}(M_{J_{\alpha_1}} + M_{J_{\alpha_2}})}^{J_b(M_{J_{\alpha_1}} + M_{J_{\alpha_2}})} C_{J_{\alpha_1} M_{J_{\alpha_1}}; J_{\alpha_2} M_{J_{\alpha_2}}}^{J_{\alpha_1 2}(M_{J_{\alpha_1}} + M_{J_{\alpha_2}})} \\
 &\times \sum \mathcal{A}(M_{J_{\alpha_1}}, M_{J_{\alpha_2}}, M_{S_b}, M_{L_b}, M_{L_{\alpha_1}}, M_{S_{\alpha_1}}, M_{L_{\alpha_2}}, M_{S_{\alpha_2}}, M) \\
 &\times \int d^3p \psi_{n_{\alpha_1} L_{\alpha_1} M_{L_{\alpha_1}}}^* \left(\frac{m_3}{m_2 + m_3} \vec{k} + \vec{p} \right) \psi_{n_{\alpha_2} L_{\alpha_2} M_{L_{\alpha_2}}}^* \left(\frac{m_3}{m_1 + m_3} \vec{k} + \vec{p} \right) \\
 &\times \psi_{n_b L_b(M_{L_{\alpha_1}} + M_{L_{\alpha_2}})}(\vec{k} + \vec{p}) \mathcal{Y}_{LM}(\vec{p}), \tag{2.6}
 \end{aligned}$$

where the $S_b(M_{S_b})$, $L_b(M_{L_b})$, and J_b are the intrinsic spin, the orbital angular momentum (third direction component) and the total spin of the b -th bare core, respectively. $S_{\alpha_{1/2}}$

$(M_{S_{\alpha_{1/2}}}), L_{\alpha_{1/2}}(M_{L_{\alpha_{1/2}}}),$ and $J_{\alpha_{1/2}}(M_{J_{\alpha_{1/2}}})$ are corresponding quantum numbers of the two mesons in the α -th $B^{(*)}\bar{K}$ channels, respectively. C is the 3-j Clebsch-Gordan coefficients. The two Clebsch-Gordan coefficients show the couplings $\vec{J}_{\alpha_{12}} = \vec{J}_{\alpha_1} + \vec{J}_{\alpha_2}$ and $\vec{J}_b = \vec{J}_{\alpha_{12}} + \vec{L}_\alpha$ (L_α being the orbital excitation between two mesons in the α -th channel). $L(M)$ denotes the relative orbital angular momentum between the third and the fourth quarks created in the vacuum as illustrated in figure 2. The coefficient \mathcal{A} denotes a series of Clebsch-Gordan coefficients for the coupling of the spin and orbital angular momenta (details are referred to refs. [80, 81]). In the spatial integrals, the $\mathcal{Y}_{LM}(\vec{p}) = |\vec{p}|^L Y_{LM}(\hat{p})$ (p is the relative momentum) is the relative wave function between the third and the fourth quarks created in the vacuum. Y_{LM} is the spherical harmonics wave function. The ψ_{nlm} is the spatial wave function of the meson and reads [82]

$$\psi_{nlm}(\vec{p}) = N_n |\vec{p}|^l \sqrt{\frac{4\pi}{(2l+1)!!}} Y_{lm}(\hat{p}) \exp\left\{-\frac{\vec{p}^2}{2n\beta^2}\right\},$$

where β is the oscillating parameters, and n is related to the radial excitation. The normalization factor is

$$N_n = \left(\frac{1}{\pi n \beta^2}\right)^{3/4} \frac{1}{(2n\beta^2)^{l/2}}.$$

The $\bar{b}s$ may also couple with the other channels. However, the couplings $\bar{b}s \rightarrow B_s^* \pi$ or $\bar{b}s \rightarrow B_s \gamma$ can be neglected, since the strengths of the isospin-breaking and electromagnetic vertices are significantly weaker than the strong one. Other possible strongly coupled channels, such as the $B_s \eta$ for B_{s0}^* , are located far away from the physical states and therefore are not considered.

2.2 Two-body potential

The potential within the two-body channels reads

$$v = \sum_{\alpha, \beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V_{\alpha, \beta}(\vec{k}, \vec{k}') \langle \beta(\vec{k}')|, \quad (2.7)$$

where $V_{\alpha, \beta}(\vec{k}, \vec{k}')$ can be straightforwardly obtained by the Lagrangian [83–85],

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{PPV} + \mathcal{L}_{VVV} \\ &= ig_v \text{Tr}(\partial^\mu P [P, V_\mu]) + ig_v \text{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu]), \end{aligned} \quad (2.8)$$

where g_v is an overall coupling constant. In the SU(4) flavor symmetry [86], the P and V , respectively, represent the 4×4 pseudoscalar and vector meson matrices:

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3\eta_c}{\sqrt{12}} \end{pmatrix}, \quad (2.9)$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & \rho^+ & K^{*+} & D^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & K^{*0} & -\sqrt{\frac{2}{3}}\omega' + \frac{J/\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3J/\psi}{\sqrt{12}} \end{pmatrix}. \quad (2.10)$$

Considering the sizable SU(4) symmetry breaking, we do not use the same coupling constant for the $D^{(*)}D^{(*)}V$ and the KKV vertices in the $D^{(*)}K \rightarrow D^{(*)}K$ progresses. The related coupling constant g_c ($\propto g_{D^{(*)}D^{(*)}V}g_{KKV}$) was determined by fitting data in the D_s sectors, which will be illustrated in section 3. For the interactions in the $B^{(*)}\bar{K} \rightarrow B^{(*)}\bar{K}$ scattering here, we use the same formula as those in the charmed strange sector, since the heavy b and c quarks are both nice spectators. To include the effects of the hadron structures, we introduce a form factor with a cutoff parameter Λ for the interaction vertex,

$$\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2} \right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2} \right)^2, \quad (2.11)$$

where p_i and p_f are the relative momenta of initial and final particles in the $B^{(*)}K \rightarrow B^{(*)}K$ process, respectively.

In the calculation, we need the partial wave expansion of $V_{\alpha,\beta}(\vec{k}, \vec{k}')$, i.e., $V_{\alpha,\beta}^{JL_\alpha L_\beta}(|\vec{k}|, |\vec{k}'|)$. Here J is the total angular momentum, which equals the spin of the corresponding bare state. L_α and L_β are the partial wave quantum numbers of α and β channels, respectively.

2.3 The Hamiltonian in the finite volume

In a box with length L , the possible momentum values are the integral multiples of the lowest non-trivial momentum $2\pi/L$ in any one dimension, i.e. $k_n = \sqrt{n} \frac{2\pi}{L}$, $\sqrt{n} = \sqrt{n_x^2 + n_y^2 + n_z^2}$, $n_{x,y,z} = 0, \pm 1, \pm 2, \dots$. The potentials are then transformed into the discretized forms

$$\begin{aligned} \tilde{g}_{\alpha b}^{L_\alpha}(k_n) &= \sqrt{\frac{C_3(n)}{4\pi}} \left(\frac{2\pi}{L} \right)^{3/2} g_{\alpha b}^{L_\alpha}(k_n), \\ \tilde{v}_{L_\alpha L_\beta}^{J, \alpha, \beta}(k_{n_1}, k_{n_2}) &= \sqrt{\frac{C_3(n_1)}{4\pi}} \sqrt{\frac{C_3(n_2)}{4\pi}} \left(\frac{2\pi}{L} \right)^3 V_{\alpha, \beta}^{J L_\alpha L_\beta}(k_{n_1}, k_{n_2}), \end{aligned} \quad (2.12)$$

where a factor $C_3(n)$ is introduced and it denotes the number of choices of n_x, n_y, n_z to form n .

With the kinematic energy and the discretized potentials, the finite-volume Hamiltonian matrix can be easily obtained.

0⁺: one bare state and one channel. In this case, the Hamiltonian contains the $\bar{b}s$ core (3P_0) and the $B\bar{K}$ channel as well as their interactions. For simplicity, we denote the potential $\tilde{g} = \tilde{g}_{B\bar{K}{}^3P_0}^S$ and $\tilde{v} = \tilde{v}_{SS}^{0, B\bar{K}, B\bar{K}}$. Solving the Schrödinger equation $H|\Psi_E\rangle = E|\Psi_E\rangle$ is equivalent to finding the solutions of the following matrix equations

$$\det \left([H_0]_{N+1} + [H_I]_{N+1} - E[I]_{N+1} \right) = 0, \quad (2.13)$$

where \det represents taking the determinant of the matrix and $[I]_{N+1}$ is an $(N+1) \times (N+1)$ unit matrix. N must be large enough until the results are stable, here we use $N = 600$. The non-interacting and interacting Hamiltonian matrices can be written as

$$H_0 = \begin{pmatrix} m_b & 0 & 0 & \cdots \\ 0 & \sqrt{k_0^2 + m_B^2} + \sqrt{k_0^2 + m_K^2} & 0 & \cdots \\ 0 & 0 & \sqrt{k_1^2 + m_B^2} + \sqrt{k_1^2 + m_K^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (2.14)$$

and

$$H_I = \begin{pmatrix} 0 & \tilde{g}(k_0) & \tilde{g}(k_1) & \cdots \\ \tilde{g}(k_0) & \tilde{v}(k_0, k_0) & \tilde{v}(k_0, k_1) & \cdots \\ \tilde{g}(k_1) & \tilde{v}(k_1, k_0) & \tilde{v}(k_1, k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (2.15)$$

respectively.

1⁺: two bare states and one channel. There are two bare cores with the spin-parity $J^P = 1^+$ in the quark model, i.e., 1P_1 and 3P_1 . They will couple with the S -wave and D -wave $B^*\bar{K}$ channels. For simplicity, we denote the potential $\tilde{g}_{iL'} = \tilde{g}_{B^*\bar{K}i}^{L'}$ and $\tilde{v}_{L'L''} = \tilde{v}_{L'L''}^{1, B^*\bar{K}, B^*\bar{K}}$. Here $i(=1, 2)$ denotes the bare 1P_1 and 3P_1 states. L' and L'' could be S or D . This leads to a $(2N+2) \times (2N+2)$ Hamiltonian matrix

$$\det \left([H_0]_{2N+2} + [H_I]_{2N+2} - E[I]_{2N+2} \right) = 0, \quad (2.16)$$

with the explicit forms as

$$H_0 = \begin{pmatrix} m_{b1} & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & m_{b2} & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & E(0) & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & E(0) & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & E(1) & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & E(1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (2.17)$$

where $E(n) = \sqrt{k_n^2 + m_{B^*}^2} + \sqrt{k_n^2 + m_K^2}$, and

$$H_I = \begin{pmatrix} 0 & 0 & \tilde{g}_{1S}(k_0) & \tilde{g}_{1D}(k_0) & \tilde{g}_{1S}(k_1) & \tilde{g}_{1D}(k_1) & \cdots \\ 0 & 0 & \tilde{g}_{2S}(k_0) & \tilde{g}_{2D}(k_0) & \tilde{g}_{2S}(k_1) & \tilde{g}_{2D}(k_1) & \cdots \\ \tilde{g}_{1S}(k_0) & \tilde{g}_{2S}(k_0) & \tilde{v}_{SS}(k_0, k_0) & \tilde{v}_{SD}(k_0, k_0) & \tilde{v}_{SS}(k_0, k_1) & \tilde{v}_{SD}(k_0, k_1) & \cdots \\ \tilde{g}_{1D}(k_0) & \tilde{g}_{2D}(k_0) & \tilde{v}_{DS}(k_0, k_0) & \tilde{v}_{DD}(k_0, k_0) & \tilde{v}_{DS}(k_0, k_1) & \tilde{v}_{DD}(k_0, k_1) & \cdots \\ \tilde{g}_{1S}(k_1) & \tilde{g}_{2S}(k_1) & \tilde{v}_{SS}(k_1, k_0) & \tilde{v}_{SD}(k_1, k_0) & \tilde{v}_{SS}(k_1, k_1) & \tilde{v}_{SD}(k_1, k_1) & \cdots \\ \tilde{g}_{1D}(k_1) & \tilde{g}_{2D}(k_1) & \tilde{v}_{DS}(k_1, k_0) & \tilde{v}_{DD}(k_1, k_0) & \tilde{v}_{DS}(k_1, k_1) & \tilde{v}_{DD}(k_1, k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (2.18)$$

The energy levels in the finite volume correspond to the eigenvalues of the Hamiltonian matrix, which can be used to determine the parameters by fitting the lattice data. For the eigenvectors of the Hamiltonian matrix, the squares of its coefficients represent the probabilities $P(\alpha)$ ($\alpha = \bar{b}s, B^{(*)}\bar{K}$) of the bare $\bar{b}s$ and $B^{(*)}\bar{K}$ components [60].

With the parameters determined, we will return to the infinite limit to study the physical properties of the scattering T -matrix by solving the relativistic Lippmann-Schwinger equation [60, 62, 87, 88],

$$T_{\alpha,\beta}^{JL_\alpha L_\beta}(k, k'; E) = \mathcal{V}_{\alpha,\beta}^{JL_\alpha L_\beta}(k, k'; E) + \sum_{\alpha' L_{\alpha'}} \int q^2 dq \times \mathcal{V}_{\alpha,\alpha'}^{JL_\alpha L_{\alpha'}}(k, q; E) \frac{1}{E - E_{\alpha'}(q) + i\epsilon} T_{\alpha',\beta}^{JL_{\alpha'} L_\beta}(q, k'; E), \quad (2.19)$$

where the effective potential $\mathcal{V}_{\alpha,\beta}(k, k'; E)$ is related to the interaction Hamiltonian,

$$\mathcal{V}_{\alpha,\beta}^{JL_\alpha L_\beta}(k, k'; E) = \sum_b g_{\beta b}^{L_\beta *}(k') \frac{1}{E - m_b} g_{\alpha b}^{L_\alpha}(k) + V_{\alpha,\beta}^{JL_\alpha L_\beta}(k, k'). \quad (2.20)$$

The bound states or resonances are obtained by searching for the poles of the T -matrix in the complex plane.

3 Predictions of 0^+ and 1^+ B_s states

In the extended HEFT framework, there are four undetermined parameters: the γ and the cutoff parameter Λ' in the QPC model, the coupling constant g_c which combines the $B^{(*)}B^{(*)}V$ and KKV vertices, as well as the cutoff Λ from the $B^{(*)}\bar{K}$ interactions. In our previous work [58], we constructed the Hamiltonians for the charm-strange mesons with $J^P = 0^+$ and 1^+ to simultaneously fit two sets of lattice data from refs. [89, 90]. When the cutoff Λ was taken as 1 GeV, the other parameters were fitted as

$$g_c = 4.2_{-3.1}^{+2.2}, \quad \Lambda' = 0.323_{-0.031}^{+0.033} \text{ GeV}, \quad \gamma = 10.3_{-1.0}^{+1.1}. \quad (3.1)$$

These values are consistent with those in other phenomenological investigations [91, 92]. For the Λ dependence, it can be absorbed by the renormalization of the interaction kernel. With different Λ employed, the final results remained the same as shown in the supplemental material of ref. [58].

With the heavy quark symmetry, we take the same parameters for the bottom-strange mesons as those in the charm-strange sector. The predicted energy levels of $J^P = 0^+$ (left) and 1^+ (right) bottom-strange states are presented in figure 3. For comparison, we also present the lattice energy levels from ref. [93]. As shown in figure 3, our predictions are well consistent with the lattice simulation.

In figure 3 (left), there is a special crossing of energy levels (marked as E_c) between the free (dashed lines) and total Hamiltonians (solid lines) in the 0^+ sector around $L = 3.7$ fm. In principle, the free energy levels in the finite volume correspond to the phase shifts of the scattering being 0 or π in the infinite volume, i.e. there is no interaction at all. Now the

J^P		0^+	1^+
mass [MeV]	rel. quark model [65]	5804	5842
	rel. quark model [66]	5833	5865
	rel. quark model [67]	5830	5858
	nonrel. quark model [68]	5788	5810
	quark model (KKMT) [94]	5719	5765
	LO $\chi - \text{SU}(3)$ [19]	5643	5690
	Bardeen, Eichten, Hill [95]	5718 ± 35	5765 ± 35
	LO UChPT [25, 26]	5725 ± 39	5778 ± 7
	NLO UHMChPT [31]	$5696 \pm 20 \pm 30$	$5742 \pm 20 \pm 30$
	NLO UHMChPT [96]	5720^{+16}_{-23}	5772^{+15}_{-21}
	HQET + ChPT [69]	5706.6 ± 1.2	5765.6 ± 1.2
	Covariant ChPT [70]	5726 ± 28	5778 ± 26
	local hidden gauge [71]	$5475.4 \sim 5457.5$	$5671.2 \sim 5663.6$
	heavy meson chiral unitary [72]	5709 ± 8	5755 ± 8
	lattice QCD [97]	$5752 \pm 16 \pm 5 \pm 25$	$5806 \pm 15 \pm 5 \pm 25$
	lattice QCD [93]	$5713 \pm 11 \pm 19$	$5750 \pm 17 \pm 19$
		this work	$5730.2^{+2.4}_{-1.5}$
$P(\bar{b}s)$ [%]	heavy meson chiral unitary [72]	$48.2 \pm 1.5/54.2 \pm 1.1$	$50.3 \pm 1.4/51.7 \pm 1.3$
	this work	$54.7^{+5.2}_{-4.1}$	$56.7^{+4.6}_{-3.7}$

Table 3. The comparison of the B_s pole masses (MeV) and the contents of bare cores extracted in this work with those from other theoretical works and lattice QCD. In this work, the content of the bare $\bar{b}s$ cores in the B_s states, denoted as $P(\bar{b}s)$, is extracted at $L = 5$ fm. The errors on our masses and probabilities are obtained from the errors of the parameters in eq. (3.1).

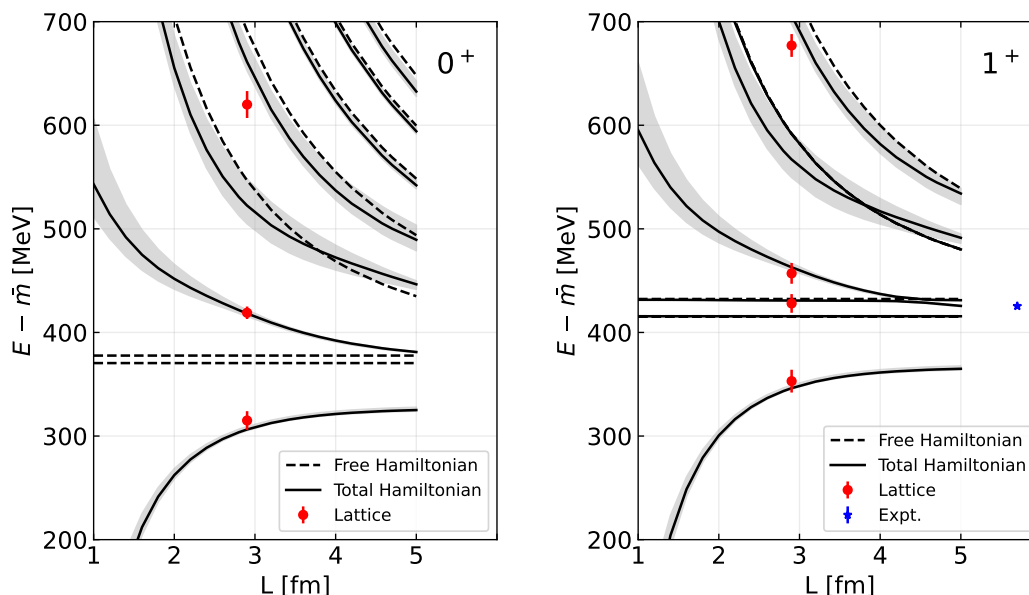


Figure 3. The comparison of the predicted energy levels for the B_{s0}^* (left), the $B_{s1}^{*(\prime)}$ (right) states with lattice simulation. The \bar{m} is defined as $\frac{1}{4}(m_{B_s} + 3m_{B_s^*}) = 5403.3$ MeV. The red dots with the error bar are the lattice energy levels from ref. [93], while the blue star is the experimental mass of B_{s1}^{\prime} . The black curves and the dashed lines are the predictions in a finite volume with and without interacting Hamiltonian, respectively.

energy levels with and without the interaction of Hamiltonian share the same eigen-energy, which means that the system feels no interaction at E_c and correspondingly, we should have $T(E_c) = 0$. This is confirmed as shown in figure 4, where we present the dependence of phase shift on the center of mass energy and the red star shows $\delta(E_c) = 0$. Such energy E_c is known as the Castillejo-Dalitz-Dyson (CDD) zero [98].

The appearance of the CDD zero, which is strong evidence of the cancellation in the potential, is promising for us to understand the physical picture of the bound state or resonance. However, the details of such cancellations depend on the parameterization of the potential. In our framework, the contributions from the coupling of the bare state with the threshold channels and the channel-channel potentials canceled at the CDD zero. Hence, it indicates the existence of the bare state and also the important role of threshold channel components as discussed in various references [99–105]. However, until now, there is no exact and convincing evidence for the existence of CDD zero. The main reason is that the scattering T -matrix of $2 \rightarrow 2$ process in the heavy quark sector cannot be obtained directly in experiments. Here, we provide a novel method to search for a CDD zero in lattice QCD, checking the crossing point of the energy levels with and without interaction Hamiltonian in the finite volume. As shown in our model, the spectrum of the B_{s0}^* state provide a golden platform to confirm the existence of the CDD zero.

In ref. [72], the authors also considered the bare $\bar{b}s$ core and the $B^{(*)}\bar{K}$ component with a different potential parametrization, where two sets of bare masses were extracted from other quark model calculations and the other two referred parameters were determined by fitting six data points from lattice QCD [93]. In this work, we directly employ the parameters

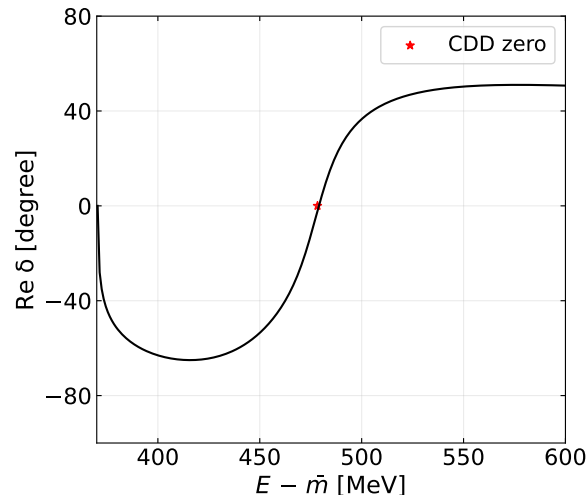


Figure 4. The phase shift of $B\bar{K}$ scattering, in which the red star denotes the possible CDD zero. The \bar{m} is defined as $\frac{1}{4}(m_{B_s} + 3m_{B_s^*}) = 5403.3$ MeV.

determined in the D_s sector to study the B_s sector with the heavy quark flavor symmetry. The predicted energy levels are surprisingly consistent with both the 0^+ and 1^+ lattice QCD energy levels at low energy. With the two different potential parameterizations, the mass spectra and components of the two B_s states are similar to each other as summarized in table 3. However, the two parameterizations provide different results for the CDD pole. Our model predicts the existence of a CDD zero in the 0^+ sector, while it is absent in ref. [72]. The third lattice data in the 0^+ sector corresponds to the third energy level in ref. [72], while it is close to the fourth one in our work. As described in the lattice paper [93], three S -wave meson-meson operators were included with two different momentum sets, $\vec{p} = 0$ and $\vec{p} = \pm(2\pi/L)\hat{e}_{x,y,z}$, labeled by $B(0)\bar{K}(0)$ and $B(1)\bar{K}(-1)$, respectively. The lowest energy level is dominated by several quark-antiquark operators and energy level 2 by the one of $B(0)\bar{K}(0)$ operator and another quark-antiquark operator. The highest energy level is purely dominated by the $B(1)\bar{K}(-1)$ operator. The energy shift between the lattice level and the corresponding $B\bar{K}$ free energy is due to the coupled channel effect, which should be smaller with increasing p of $B\bar{K}$. However, the shift between the third lattice energy level and the $B(1)\bar{K}(-1)$ free energy is even larger than that between the second lattice energy level and the $B(0)\bar{K}(0)$ free energy. Therefore, it might miss another energy level between the highest two lattice data. More lattice QCD calculations are expected in the future to give more constraint to the parameterizations and examine the CDD pole's existence.

At last, we obtain the pole masses of the T -matrix and list them in table 3, together with the masses from other phenomenological studies and lattice QCD calculation for comparison. The pole positions of B_{s0}^* and B_{s1}^* are located in the first Riemann-sheet at $5730.2_{-1.5}^{+2.4}$ MeV of the $B\bar{K}$ channel and $5769.6_{-1.6}^{+2.4}$ MeV of the $B^*\bar{K}$ channel, respectively. Both the 0^+ and the lighter 1^+ bare $\bar{b}s$ core have a significant mass shift due to the S -wave interactions with $B^{(*)}\bar{K}$ channel. The coupled channel effects also make them a mixture of the bare $\bar{b}s$ core and $B^{(*)}\bar{K}$ components. By analyzing the eigenvectors, the $P(\alpha)$ shows

that the two components are significant and essential for the B_{s0}^* and B_{s1}^* states. The bare $\bar{b}s$ core in the B_{s0}^* accounts for around $54.7_{-4.1}^{+5.2}\%$ at $L = 5$ fm, while the $B\bar{K}$ component occupies around 45.3%. The bare $\bar{b}s$ core in the B_{s1}^* accounts for around $56.7_{-3.7}^{+4.6}\%$ at $L = 5$ fm, while the $B^*\bar{K}$ component occupies around 43.3%.

In contrast, the D -wave interaction around the threshold is significantly suppressed at $\mathcal{O}(k^2)$ compared with the S -wave one. Therefore, the energy level of the B_{s1}^{*f} almost keeps stable, and its bare $\bar{b}s$ core dominates.

4 Summary

In summary, we have investigated the 0^+ and 1^+ bottom-strange mesons with the framework which incorporates the quark model, the QPC model, and the coupled-channel unitary approach into the HEFT framework. This framework has been successfully used to describe both the lattice QCD data and the experimental mass spectra of the $D_{s0}^*(2317)$, $D_{s1}^*(2460)$, $D_{s1}^*(2536)$, and $D_{s2}^*(2573)$ states. Here, we employed the same parameters determined by fitting the lattice energy levels of the D_s states. The predicted energy levels of the 0^+ and 1^+ B_s states are well consistent with the lattice QCD simulation at low energy. Moreover, a very clear physical picture emerges from our results for the 0^+ and 1^+ B_s states, i.e., they are the mixture of the bare $\bar{b}s$ and $B^{(*)}\bar{K}$ components. The bare masses are shifted by tens of MeV due to the coupled-channel effects with the S -wave $B\bar{K}$ and $B^*\bar{K}$ channels, respectively.

The extracted pole masses from the T -matrix are also consistent with the results from the lattice QCD and other phenomenological models as shown in table 3. In addition, we predict a CDD zero in the $B\bar{K}$ scattering through the finite volume spectrum. It can be used to examine the potential as well as the inner structures of the physical states as pointed out in ref. [106]. Therefore, future investigations from a theoretically motivated model and lattice QCD simulation for the $B\bar{K} \rightarrow B\bar{K}$ process is necessary and expected.

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