THE ANATOMY OF ε'/ε BEYOND LEADING LOGARITHMS WITH IMPROVED HADRONIC MATRIX ELEMENTS[1]

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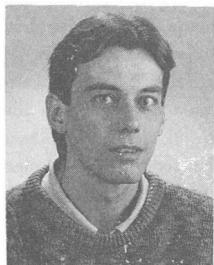
Abstract

Results of a recent calculation of the effective $\Delta S = 1$ Hamiltonian at the next-to-leading order will be presented[2]. These, together with an improved treatment of hadronic matrix elements, are used to evaluate the measure for direct CP-violation, ε'/ε, at the next-to-leading order. Taking $m_t = 130\,GeV$, $\Lambda_{\overline{MS}} = 300\,MeV$ and calculating $\langle Q_6 \rangle$ and $\langle Q_8 \rangle$ in the $1/N$ approach, we find in the NDR scheme $\varepsilon'/\varepsilon = (6.7 \pm 2.6) \times 10^{-4}$ in agreement with the experimental findings of E731. We point out however that the increase of $\langle Q_6 \rangle$ by only a factor of two gives $\varepsilon'/\varepsilon = (20.0 \pm 6.5) \times 10^{-4}$ in agreement with the result of NA31. The dependencies of ε'/ε on $\Lambda_{\overline{MS}}$, m_t, and some B-parameters, parameterizing hadronic matrix elements, are briefly discussed.

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1 Introduction

In recent years, determinations of the measure for direct CP-violation in $K \rightarrow \pi\pi$ decays, ε'/ε, both on the experimental and on the theoretical side, have experienced great improvement, though the situation, nevertheless, is not yet conclusive. The experimental researchers, after heroic efforts on both sides of the atlantic, find [1, 2],

$$Re(\varepsilon'/\varepsilon) = \begin{cases} (23 \pm 7) \cdot 10^{-4} & \text{NA31} \\ (7.4 \pm 6.0) \cdot 10^{-4} & \text{E731} \end{cases}, \tag{1}$$

clearly indicating a non-zero ε'/ε in the case of NA31, whereas the value of E731 is still compatible with superweak theories in which $\varepsilon'/\varepsilon = 0$.

Theoretically, ε'/ε is governed by penguin contributions with QCD penguins dominating for values of $m_t \lesssim 150 \, GeV$, but if the top quark mass would turn out to be as large as $\mathcal{O}(200 \, GeV)$, as has been first pointed out be Flynn and Randall [3], QED penguins become important and tend to cancel the QCD contribution, yielding ε'/ε close to zero.

At the leading order a detailed anatomy of ε'/ε in the presence of a heavy top quark has been performed by the authors of ref. [4], which has subsequently been corroborated in [5] and [6]. Since the outcome of the fight between QCD and electroweak penguins is rather sensitive to the various approximations used in [4, 5, 6], it is very important to improve the theoretical calculations both on the short-distance side (Wilson coefficient functions $C_i(\mu)$) and on the long distance side (hadronic matrix elements $\langle Q_i(\mu) \rangle$). These improvements will be outlined in the following.

2 Coefficient Functions

In the framework of the operator product expansion, $\Delta S = 1$ weak transitions are described by the effective Hamiltonian

$$\mathcal{H}_{eff}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu), \tag{2}$$

with $C_i(\mu)$ being the Wilson coefficient functions. The operators Q_i can be classified into $Q_{1,2}$ being current-current, Q_{3-6} QCD penguin, and Q_{7-10} electroweak penguin operators [4].

The next-to-leading order calculation of the coefficient functions $C_i(\mu)$ now consists of the following steps:

- calculation of the 10×10 $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha\alpha_s)$ anomalous dimension matrices which govern the next-to-leading order renormalization group evolution from some high energy scale $\mathcal{O}(M_W)$ down to the low energy scale μ [7, 8],

- calculation of the initial conditions $C_i(M_W)$ which are used as a starting point for the renormalization group evolution [9, 10],

- and solving the renormalization group equation with inclusion of both $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha\alpha_s)$ corrections [9, 10].

An independent calculation of the $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha\alpha_s)$ anomalous dimension matrices has also been performed by the authors of [11], finding agreement with our final results.

Writing $C_i(\mu)$ as $\lambda_u z_i(\mu) - \lambda_t y_i(\mu)$, with $\lambda_i = V_{id}V_{is}^*$, V_{ij} being Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, splits $C_i(\mu)$ into the major real part $\sim z_i(\mu)$, describing CP-conserving transitions, and the imaginary part $\sim y_i(\mu)$, being responsible for CP-violating transitions. z_i and y_i depend on $\Lambda_{\overline{MS}}$, the renormalization scale μ, the renormalization scheme, and in the case of y_i also on m_t. A thorough discussion of all these dependencies has been presented in ref. [10]. As an example, we explicitly give in tab. 1 the coefficient functions for $\Lambda_{\overline{MS}} = 300\,GeV$, $\mu = 1\,GeV$, a renormalization scheme with anticommuting γ_5 (NDR), and $m_t = 130\,GeV$.

z_1	z_2	z_3	z_4	z_5	z_6	z_7/α	z_8/α	z_9/α	z_{10}/α
-0.486	1.262	0.013	-0.035	0.007	-0.035	0.008	0.016	0.016	-0.009

y_1	y_2	y_3	y_4	y_5	y_6	y_7/α	y_8/α	y_9/α	y_{10}/α
0	0	0.029	-0.052	0.001	-0.099	-0.080	0.095	-1.225	0.502

Table 1: The coefficient functions $z_i(1\,GeV)$ and $y_i(1\,GeV)$.

3 Hadronic Matrix Elements

Besides the coefficient functions $y_i(\mu)$, for the calculation of ε'/ε we also need the matrix elements $\langle \pi\pi | Q_i(\mu) | K \rangle_{0,2}$, where the subscript denotes the isospin of the final state pions. Since a direct calculation of the matrix elements involves long-distance dynamics in QCD, and is therefore rather difficult and uncertain, in [10] we advocated a more phenomenological approach. For further references on other non-perturbative approaches to hadronic matrix elements see also [10].

Imposing experimental data on CP-conserving $K \to \pi\pi$ decays and some plausible properties of the hadronic matrix elements which are fulfilled by all common non-perturbative methods, we can fix part of the matrix elements completely and express the rest in terms of the three parameters $B_2^{(1/2)}(\mu)$, $B_6^{(1/2)}(\mu)$, and $B_8^{(3/2)}(\mu)$. In addition, from experimental data we can deduce $B_2^{(1/2)}(m_c) = 6.7 \pm 0.9$ in the NDR scheme. In our approach it is most convenient to evaluate the matrix elements at the scale m_c.

4 Main Results for ε'/ε

Using the results for the coefficient functions y_i and the hadronic matrix elements from sects. 2 and 3 respectively [10], we can write the CP-violating quantity ε'/ε in the following form,

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4} \left[\frac{Im\lambda_t}{1.7 \cdot 10^{-4}} \right] \left[P^{(1/2)} - P^{(3/2)} \right] . \qquad (3)$$

We have factored out the central value of $Im\lambda_t$ in the range for the CKM-matrix elements considered in [10], such that ε'/ε (in units of 10^{-4}) is directly given by $P^{(1/2)}$ and $P^{(3/2)}$. The contribution from $\Delta I = 1/2$, $P^{(1/2)}$, is dominated by QCD penguins, whereas the $\Delta I = 3/2$ part mainly comes from electroweak penguins. In terms of the B-parameters introduced in the last section, their expressions are:

$$P^{(1/2)} = a_0^{(1/2)} + a_2^{(1/2)} B_2^{(1/2)} + a_6^{(1/2)} B_6^{(1/2)} , \qquad (4)$$

$$P^{(3/2)} = a_0^{(3/2)} + a_8^{(3/2)} B_8^{(3/2)} . \qquad (5)$$

The coefficients a_i depend on $\Lambda_{\overline{MS}}$, the renormalization scheme considered, and the top quark mass. Again, as an example, in tab. 2 we give the values of the a_i for $\Lambda_{\overline{MS}} = 300\,MeV$, the NDR scheme, and $m_t = 130$, 150, and $170\,GeV$.

$m_t[GeV]$	$a_0^{(1/2)}$	$a_2^{(1/2)}$	$a_6^{(1/2)}$	$a_0^{(3/2)}$	$a_8^{(3/2)}$
130	-7.61	0.54	11.68	-1.21	3.02
150	-7.24	0.51	11.77	-1.40	4.59
170	-6.84	0.47	11.85	-1.59	6.38

Table 2: Coefficients in the expansion of $P^{(1/2)}$ and $P^{(3/2)}$.

Using these coefficients together with the factorization values for B_6 and B_8, $B_6^{(1/2)}(m_c) = B_8^{(3/2)}(m_c) = 1$, and $m_t = 130\,GeV$, our final results for ε'/ε are

$$\varepsilon'/\varepsilon = \begin{cases} (6.7 \pm 2.6) \times 10^{-4} & 0 < \delta \le \frac{\pi}{2}, \\ (4.8 \pm 2.2) \times 10^{-4} & \frac{\pi}{2} < \delta < \pi, \end{cases} \qquad (6)$$

in perfect agreement with the E731 result. However, for $B_6^{(1/2)}(m_c) = 2$ and $B_8^{(3/2)}(m_c) = 1$ we find

$$\varepsilon'/\varepsilon = \begin{cases} (20.0 \pm 6.5) \times 10^{-4} & 0 < \delta \le \frac{\pi}{2}, \\ (14.4 \pm 5.6) \times 10^{-4} & \frac{\pi}{2} < \delta < \pi, \end{cases} \qquad (7)$$

in agreement with the findings of NA31. Here, δ is the complex phase in the CKM matrix, and the explicit errors include variation of the CKM-matrix elements and $B_2^{(1/2)}$ as given in [10] and sect. 3 respectively.

Let us briefly point out further dependencies of ε'/ε. For a thorough discussion the reader is again referred to ref. [10].

- ε'/ε decreases if m_t increases but becomes zero only for $m_t \gtrsim 200\,GeV$.

- ε'/ε increases if $\Lambda_{\overline{MS}}$ increases. This dependence is illustrated in fig. 1 for $m_t = 130\,GeV$ and $B_6 = B_8 = 1$.

Similar final results for ε'/ε were also obtained in ref. [12]. Taken separately however, some contributions to ε'/ε differ notably from the values obtained in our analysis [10].

5 Conclusions

Despite the achievements in recent years, the fate of theoretical estimates of ε'/ε in the years to come depends crucially on whether it will be possible to further reduce the uncertainties in m_t, $\Lambda_{\overline{MS}}$, $B_6^{(1/2)}$, $B_8^{(3/2)}$, and the CKM-matrix elements.

On the other hand, with the new generation of measurements, in a few years of time hopefully also the experimental situation will be clarified. Nevertheless, if the top quark turns out to be too heavy, this may become difficult.
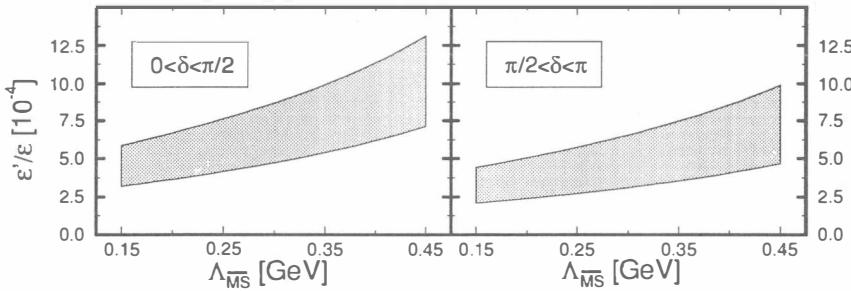
Figure 1: ε'/ε as a function of $\Lambda_{\overline{MS}}$ for $m_t = 130\,GeV$ and $B_6 = B_8 = 1$.

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