Modified generalized liquid drop model (MGLDM) with statistical preformation probability

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Introduction

The modified generalized liquid drop model (MGLDM) with a new statistical preformation probability has been used to study cluster radioactivity. The spontaneous emission of particles or clusters, such as C, O, F, Ne, Mg, Si etc. heavier than alpha particles termed as cluster radioactivity was first proposed theoretically in 1980 by Sandulescu et al. [1]. In 1984 Rose and Jones [2] observed the emission of ^{14}C clusters from ^{223}Ra with branching ratio relative to alpha particles as $8.5 \pm 2.5 \times 10^{-10}$. Since then the emission of various clusters like ^{18,20}O, ^{23}F, $^{22,\,24\text{-}26}$Ne, $^{28\text{-}30}$Mg, ^{34}Si has also been observed from different parent nuclei ranging from ^{221}Fr to ^{242}Cm [3]. The doubly closed ^{208}Pb or nearby one nuclei the daughter/residual nuclei usually produced in the observed cluster decay of ^{221}Fr to ^{242}Cm.

Modified generalized liquid drop model (MGLDM)

In MGLDM, the macroscopic energy for a deformed nucleus is defined as,

$$E = E_V + E_S + E_C + E_R + E_P\,, \qquad (1)$$

where E_V, E_S, E_C, E_R and E_P are the volume-, surface-, Coulomb-, rotational- and proximity energy respectively.

$$E_V = -15.494[(1-1.8I_1^2)A_1 + (1-1.8I_2^2)A_2]$$

$$E_S = 17.9439[(1-2.6I_1^2)A_1^{2/3} + (1-2.6I_2^2)A_2^{2/3}]$$

$$E_C = \frac{0.6e^2Z_1^2}{R_1} + \frac{0.6e^2Z_2^2}{R_2} + \frac{e^2Z_1Z_2}{r}\,. \qquad (2)$$

Here A_i, Z_i, R_i and I_i are the charges, radii and relative neutron excess of the two nuclei and r is the distance between the mass centers. The nuclear proximity energy E_P given by Blocki et al. [4] and the barrier penetrability P is calculated using the following integral

$$\mathrm{P} = \exp\left\{-\frac{2}{\hbar}\int_{R_{in}}^{R_{out}} \sqrt{2B(r)[E(r)-E(sphere)]}\,dr\right\} \quad (3)$$

Here B(r) is the mass inertia taken as the reduced mass. The partial half-life is calculated using $T_{1/2}=\ln 2/\nu P_c P$, where ν is the assault frequency and P_c is the statistical cluster preformation probability.

Statistical Preformation factor

In a parent nucleus exhibiting cluster radioactivity the daughter nuclei, with mass $A_1=Z_1+N_1$, is always a closed shell or near closed shell spherical nuclei. Inside parent nuclei the daughter nuclei can be considered as a core and cluster, with mass $A_2=Z_2+N_2$, orbiting around the core. The volume v inside the parent nucleus is regarded as a system and the remaining part of the parent nucleus is regarded as a particle reservoir. If this volume v contains Z_2 protons and N_2 neutrons we can consider the cluster formed inside the nucleus. The probability of finding N_2 neutrons and Z_2 protons inside the volume v can be obtained by grand-canonical distribution or the cluster preformation probability P_C given as,

$$\log_{10} P_C = \frac{(1-x+\ln x)A_2}{\ln 10}\,. \qquad (4)$$

The dimensionless quantity x, the density ratio reflects the nuclear density where the cluster forms.

Results and discussion

Cluster decay of various $^{216\text{-}229}$Ra isotopes emitting ^{14}C clusters has been studied within the MGLDM with statistical preformation factor. Dong et al [5] in 2009 introduced a relation for cluster preformation probability within the framework of statistical physics. The density ratio χ in cluster preformation probability is obtained in present study by the method of least square fitting to available experimental cluster decay data. The obtained value shows that the density is 18% less than central nuclear matter density. In cluster radioactivity the cluster forms at the nuclear surface where the nuclear matter has a lower density.

The cluster decay is energetically possible only if $Q > 0$. The decay energy or the Q value for ^{14}C emission is given by the equation,

$$Q = \Delta M_p - (\Delta M_c + \Delta M_d), \quad (5)$$

where ΔM_p, ΔM_c and ΔM_d are the mass excess of parent, cluster and daughter nuclei respectively. The mass excess values are added from the recent mass table of Wang et al., [6].

Table 1 The Computed Q value and half-lives for ^{14}C emission from $^{216-229}$Ra isotopes.

Parent nuclei	Daughter nuclei	Q value (MeV)	$\log_{10}[T_{1/2}(s)]$ Present
^{216}Ra	^{202}Pb	26.205	27.933
^{217}Ra	^{203}Pb	27.648	24.233
^{218}Ra	^{204}Pb	28.740	21.622
^{219}Ra	^{205}Pb	30.144	18.506
^{220}Ra	^{206}Pb	31.038	16.623
^{221}Ra	^{207}Pb	32.395	13.954
^{222}Ra	^{208}Pb	33.049	12.706
^{223}Ra	^{209}Pb	31.828	14.928
^{224}Ra	^{210}Pb	30.535	17.446
^{225}Ra	^{211}Pb	29.465	19.661
^{226}Ra	^{212}Pb	28.196	22.478
^{227}Ra	^{213}Pb	27.343	24.478
^{228}Ra	^{214}Pb	26.102	27.602
^{229}Ra	^{215}Pb	25.063	30.405

Table 1 gives the half-lives computed for the emission of ^{14}C clusters from $^{216-229}$Ra isotopes. The first three columns give the parent nuclei, daughter nuclei and computed Q values. The half-lives computed using the MGLDM with statistical preformation probability are listed in the fourth column. From the table it can be seen that half-life decreases with increasing mass number and reaches a minimum value ($T_{1/2}$=5.08x10^{12}s) for the parent nuclei ^{222}Ra and then increases with increasing mass number. In cluster radioactivity the minimum in half-life represents the stability of daughter nuclei ^{208}Pb which is due to shell closure of both protons and neutrons (Z=82, N=126). So the role of doubly magic nuclei in cluster radioactivity is revealed in our study. We have compared our predicted values with available experimental data and found both are in good agreement.

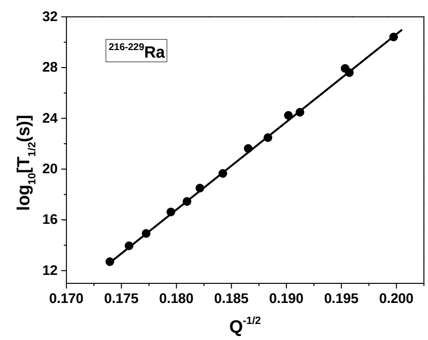

Figure 1 Geiger-Nuttall plot for ^{14}C radioactivity

We have studied the Geiger-Nuttall (GN) plot connecting logarithm of half-lives and $Q^{-1/2}$ and the plot is found to be linear. The linear nature of GN plots stress the reliability of present calculation.

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