Galactic dark matter in Newtonian theory

Kirill Vankov[1], Anatoli Vankov[1]
Independent researcher
`kirill.vankov@gmail.com`

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Abstract. This article presents the results of our research on a multifaceted Centennial problem of the apparent inability of Newton's gravity to treat observations of galaxies and their hierarchical structure by Kepler's laws. The suspected cause is "Dark Matter" (further, Galactic Dark Matter – GDM), which is invisible but interacts with ordinary matter through Newton's gravity. However, misapplying Newton's theory is not excluded: there are cases demonstrating galactic observations in terms of GDM-free Keplerian orbits. We revisited this problem in all aspects and scrutiny. As a result, we found hidden roots of compromised conceptions of Kepler's orbits and mass-to-light ratio Υ. Then we developed an approximate algorithm for treating galactic observations in the Newtonian gravitation framework and demonstrated its success in examples of the Milky Way and other galaxies. Now astronomers believing in the deficiency of Newton's gravity theory can verify its asserted ability to treat galactic observations free of the GDM. We also considered observations on a cosmic scale for a possible interplay of GDM with Cold Dark Matter (CDM) and other parameters of the Lambda-CDM Cosmology. The conclusion is made that GDM and CDM were introduced for different reasons and our results do not necessarily affect the Cosmological Model.

Key words: Galactic Dark Matter, Centennial problem, apparent failure of Kepler's laws, Mass-to-light ratio problem, controversies, suggested solution

1 Introduction

Next, we use the generalized term "astronomers" meaning specialists directly or indirectly involved in the Galactic Dark Matter (GDM) issue, which has a long history dating back to the 1930s. At that time, astronomers encountered difficulties with the suspiciously fast motion of stars in galactic structures. The main argument was around the apparent inability of Kepler's laws to explain a flat rotational curve (RC): the constancy of galactic disc rotation within a significant radial range. These galaxies were referred to as "non-Keplerian," in contrast to "Keplerian" systems where velocities decrease with radius similar to the Solar planetary system.

Specifically, the problem arose due to an apparent discrepancy between observed flat RCs and their theoretical predictions. Roughly, it can be thought that a centrally dominated mass calculated from the observed luminous material was kinetically insufficient to produce the conditions necessary for a flat RC in a galactic disc. The "missing mass" was named Galactic Dark Matter (GDM).

At the end of the last century, a status of "missing mass" can be seen from the textbook by Prof. Kaufmann [1]. There, interesting data are presented with the example of the Milky Way. Namely, the enclosed mass within the radius $R = 8\,\mathrm{kpc}$ (the radius of the Sun's orbit around the center of the Milky Way) is estimated from observations as $M_0 = 9.4 \times 10^{10}\, M_\odot$ while the mass required to explain the GDM presence must be at least $M_0 = 6.0 \times 10^{11}\, M_\odot$. Today, our current assessment is $M_0 = 1.0 \times 10^{11}\, M_\odot$, which is close to the value they wanted, so what is the problem? This fact characterizes a deep historical confusion in posing the GDM problem.

Nowadays, astronomers have a lot of experience in the treatment of GDM with phenomenological parametric models, like Dark Halo and Modified Newtonian Dynamics (MOND). Numerous attempts to directly detect it have, so far, failed. They concluded that the GDM has a hypothetical form of gravitating substance devoid of atomic and nuclear structure, cannot emit or absorb light, and is collisionless as being long-range interacting. It should not be mixed up with Cold Dark Matter (CDM) in cosmology.

2 Criticism of models

2.1 DM Halo model

There are at least two competing models that stand out among other proposals: MOND and the GDM Halo model. The fact that there are two rival models is strange. Only one of them could be true if both were not false.

Let us start with a brief discussion of the prevailing Dark Halo model using Newton's potential

$$\Phi(r) = G\,M\,m/r = m\,V(r)^2\,, \quad F(s) = G\,m_1\,m_2/s^2. \tag{1}$$

Here m must be a mass $m \ll M$ of a test particle in a one-body approximation of gravitational dynamics for a bounded material system having a mass distribution $M(r)$ and rotating about the center of mass. However, we shall see, that in the model, it is used in a narrow meaning of Statics outside Dynamics regime. A circular rotation is a particular case of the static Newton's Universal law of attracting force $F(s)$ between two bodies of masses m_1 and m_2, where s is a distance between them (two-body case). The one-body case follows if one particle has a negligibly small mass in comparison with the other. The potential exhibits equality of the effects of gravity with centripetal acceleration reaction in a circle rotation, while the Dynamics theory describes equations of orbital motion governed by conservation laws.

In the GDM Halo model, the concept of Potential is the basis of a phenomenological model designed to simulate the GDM distribution $M_{dm}(r)$ in "non-Keplerian" galaxies. "Keplerian" ones are considered in analogy to Solar planets' orbits. The potential superposition technique is used to model space-distributed sources made of GDM and ordinary matter. Those sources must support the rotation of a disc made of GDM and ordinary matter, which should be separated. The main purpose is to probe the GDM distribution $M_{dm}(r)$ to make disc rotation faster with increasing radius, resulting in a "non-Keplerian" (flat) RC. Using the superposition principle and introducing the RC velocity components, they intend to decompose the radial distribution of mass profile into main sources in the disc, like Black Hole, Bulge, Disc, and finally Dark Matter, if present:

$$V(r)^2 = V_{\mathrm{bh}}(r)^2 + V_{\mathrm{b}}(r)^2 + V_{\mathrm{d}}(r)^2 + V_{\mathrm{h}}(r)^2, \tag{2}$$

where, bh - Black Hole, b - Bulge, d - Disc, h - Halo, following the article [2].

Recently, the above decomposition was thoroughly used, see [3]. They claim an essentially improving interpretation of the Milky Way's flat RC in terms of the DM-halo model compared to the MOND. It was achieved by using the latest empirical database Gaia DR3. The work is also valuable as a large review of the problem. The problem was recently studied in [4], where the updated Tully-Fisher relation between the luminosity and rotational velocity in the Milky Way and other galaxies was used.

There are a lot of assumptions, calculations, and simulations there, but the absence of an algorithm explaining “non-Keplerian” galaxies and large statistical uncertainties are disappointing. We argue that the GDM problem in the Dark Halo model in terms of Newtonian Gravitation Dynamics is ill-posed. Obviously, all physical laws are broken there, which makes physical research meaningless. The formula (2), would have a sense if the sources are attached to a rigid frame like charges in Electrostatic but not in a galaxy wrapped in Dark Halo. It is not clear, which parts of Dark Halo are mixed with ordinary matter and participate in the rotation of the disc. Neither is it clear how GDM as the attractor could remain still. The required amount of GDM is huge, most in the outer Halo. If so, a galaxy should rotate about a center of mass somewhere shifted from the origin $r = 0$ to some much greater position.

Genuine Newton’s theory can tell us if the input data (observations) are right or wrong but it cannot distinguish between gravitational identical matter painted in dark or white. There is a rich literature on Classical Mechanics, for example, an excellent textbook “Classical Mechanics”, 3d Editions [5] demanded since 1950 till today and usable in classical astronomy.

2.2 Competing MOND

To reiterate, the MOND is an alternative to the Dark Halo model having a different explanation of flat RCs in spiral galaxies. Proponents of MOND seem to be in agreement with our thesis about the identity of dark and white matter in Newton’s theory, the validity of which is respected by the Dark Halo model, where “dark matter” is called “missing (hidden) matter”. So MOND denies the real existence of the GDM and, at the same time, denies the validity of Newton’s theory and deals with a modification of it. Consequently, the “missing matter” argument is not used there. The common reason in both models is *discrepancy of observations from a theory*, namely, both question how to boost the rotational speed at high radii (law dynamic acceleration) to fit the observed flat RCs in spiral galaxies.

Indeed, both are phenomenological parametric models, - Newton’s mimics but not theories. Basic physical principles and conservation laws are broken there. The intentions are similar: to explain flat RC as a sign of a “non-Keplerian galaxy” at large radii where dynamic acceleration is very weak. In the Dark Halo model, it is invisible GDM making rotation faster to the extent of RC flatness. In MOND the fast rotation is made by the introduction of a new fundamental constant a_0 boosting Newton’s acceleration to squared g_N in the interpolating regime, as fast as $g \propto a^2/a_0$ at $g_N > a_0$.

Thus, the two models explain the discrepancy differently, but only one could be correct. The conflict is resolved if both are wrong, except Newton’s theory of gravity. The only question of primary importance remains open: *discrepancy of observations from what theory?*, definitely not Newton’s.

The spectacular success of the MOND is due to Professor Milgrom, who suggesteffd a hypothesis about modifying Newton’s 2d law in 1983 [6]. His idea is suitable for astronomical objects besides spiral galaxies. In particular, it is tested on binaries. The model is essentially attractive because it denies the existence of absurd Dark Matter. Besides, it was presented as a possible alternative to the Dark Matter halo admitting the possibility of misapplying Newton's theory. Not surprisingly, the MOND revealed controversies in practice deserving short comments. Later, Professor Peebles pointed out in his book [7, page 418], the possibility of misapplying Newtonian gravitation, actually, in both Models. We concur with this observation and admit that it is likely moving in the right direction.

Next, allow us a few comments on MOND practice in the area of binaries. In the notable work [8] the author simulated a virtual Newtonian world by analyzing binary samples. They were selected from the database GAIA DR3 provided large statistics of wide binaries below and above accelerations $a_0 = 1.2 \times 10^{-10}\,\mathrm{m/s^2}$ (MOND parameter). As a result, he stated a strong evidence supported MOND's validity under the assumption of Dark Matter non-existence.

At the same time, in another research on binaries [9], the results were quite the opposite picture – agreement with Newton's theory. There, the authors selected samples carefully, paying much attention to the internal kinematics of wide binaries, especially, orbits of each star with separation above $s = 2000\,\mathrm{au}$ and accelerations below $2\,a_0$.

In a recent paper by Banik with coauthors [10], the Model was tested on the WBS database (Wide Binary Stars). They concluded that the Model is unable to explain a great variety of observations.

Not surprisingly, both Models have critical supporters and opponents. Now, we want to return to the question: discrepancy of observations from what theory? It looks like all of them point at Newton's theory but for different reasons.

3 Roots of the problem: revelation

The Dark Matter model is based on the concept of $1/r$ potential and must be consistent with Kepler's-Newton's theory of orbits, so comparison of prediction regardless of observations must not be affected by the practical luminosity of matter. We argue that, in reality, astronomers deal with not a theory but a phenomenological Model deviated from observations, rather than verification of the real theory. This is the primary cause of the GDM advent, understanding of this fact is in the air.

Numerous textbooks are presenting theoretical fundamentals of GDM treatments, for example, an excellent textbook [11] gives the basic principle of GDM concept. There, the role of potential is presented in section 2.7, "The Potential of our Galaxy", p. 110-111. It gives us a revelation of astronomers' failure in applying Newton's theory and its "dynamical tracers", see extract:

Ideally, we should rely solely on dynamical tracers, such as the velocity fields of gas and stars and observations of gravitational lensing, to map out the distribution of mass in the Galaxy. Sadly, at present, such a project is unfeasible. Since we are not yet in a position to model the Galactic density and gravitational field in a purely dynamical way, we flesh out the available dynamical constraints with photometric information. In particular, we simply *assume that each component has a mass-to-light ratio Υ that is independent of position. For the reason given above, this procedure is arbitrary and unsatisfactory, but it yields concrete Galactic potentials, which make testable predictions regarding the kinematics of stars and gas.*

Naturally, Newton's theory allows us to treat galaxies independently of luminosity measurements. Historically, a wrong option was chosen: the usage of "arbitrary and unsatisfactory photometric information from luminosity measurements" with fuzzy Υ criterion. In textbook [12], readers find astronomers' dealing with Dark Matter modeling.

We state that the GDM conception proved to be fictitious, the proof means that the Centennial GDM crisis is over. It should take time to return to the practice of physical science of GDM-free astronomy respecting classical heritage.

The further successful demonstration of Newtonian Gravitation Dynamics applied to Galaxies will complete the purpose of this article.

4 Newtonian Gravitation Dynamics of galaxies

Kepler's laws are phenomenological, but they hold significant historical and pedagogical value when illustrating planetary orbits in their elliptical form described in terms of two geometric parameters: eccentricity and semi-latus rectum. Kepler's laws do not lead to equations of orbital motion as in contemporary Classical Gravitation Dynamics. The latter is governed by the conservation laws for total energy and angular momentum in isolated gravitational systems of material bodies moving through spacetime. The conservation laws are formulated in terms of physical parameters and are underpinned by Emmy Noether's renowned Space-Time Symmetry Theorem. Additionally, the Virial Theorem is relevant to the dynamic stability of complex N-body systems.

In equations, we use the concept of Standard Test Particle (STP) of mass m moving in space-time in GM/R potential field in one-body approximation. By definition, the STP mass m is negligibly small compared with the source mass $m \ll M$, because it is not explicitly present in equations of orbital motion.

Therefore, readers will come across novelty in our presentation of elliptic orbits in the Newtonian framework in the *advanced form.* Instead of dealing with two geometric parameters, we adopt a single dimensionless physical parameter denoted as σ in equations of motion. This is possible in the approximation of Special Relativity Dynamics (SRD), as shown by Synge in [13,14]. We followed it and developed the equations of orbital motion in One-Body SRD approximation in quadratic form. Our novel approach remarkably streamlines orbit classification and enhances the physical interpretation of solutions to problems, as seen in the following (in short).

Denotations:

- radial speed $\beta_r = \mathrm{d}r/\mathrm{d}(c_0 t)$;
- angular speed β_θ;
- angle of rotation θ;
- radial (inverted) coordinate $\xi = r_0/r$;
- mass of source M;
- gravitational radius $r_g = G\,M/c_0^2$;
- parameter of orbit type $\sigma = (r_g/r_0)/\beta_0^2$.

The initial conditions in equations of STP motion: $r = r_0$, $\beta = \beta_0$, $\theta = 0$.

The internal STP energy of rest mass is $m\,c_0^2$, where it will be taken $m = 1$, and the speed of light $c_0 = 1$. The nature of an orbit is contingent upon initial conditions (input data), which are provided below alongside the conservation laws and equations of motion.

Conventional expression of the total energy of STP E_t includes the scalar sum of potential and kinetic energies (the sum of radial and angular ones) $E_t = -G\,M\,m + (1/2)\,m\,V_r^2 + (1/2)\;V_\theta^2$. In the orthogonal basis, the squared kinetic energy should be the sum of the squared components $E_k^2 = V_r^2 + V_\theta^2$, where the angular velocity is constrained by the conserved angular momentum $l_0 = m\,V_\theta\,r$. So $V_\theta^2 = l^2/r_2$. Consequently, the squared potential energy includes the STP rest mass energy $m\,c_0^2 - G\,M\,m/r$.

In the result, the following equations of motion express the conservation of total energy ϵ_0 in dimensionless form is given by

$$\epsilon_0^2 = 1 - 2\frac{r_g}{r} + \beta_r^2 + \frac{l_0^2}{r^2}\,, \quad l_0 = r\beta_\theta = r_0\beta_0\,. \tag{3}$$

The above physical concept of ϵ is very convenient because it shows a balance of energies in different types of orbits. For example, $\epsilon < 1$ characterizes a depth of boundedness. The system is unbounded if $\epsilon > 1$.

For further work, one needs some ingenuity to introduce for convenience the inverted variable $\xi = r_0/r$ and a parameter $\sigma = r_g/r\,\beta^2$ in the equation of motion:

$$(\mathrm{d}\xi/\mathrm{d}\theta)^2 = 1 - 2\,\sigma + 2\,\sigma\xi - \xi^2\,, \tag{4}$$

The solution

$$r(\theta)/r_0 = (\sigma + (1-\sigma)\,\cos\theta)^{-1}\;. \tag{5}$$

See more details in [15], [16].

Given initial conditions, the equations of STP motion describe all possible classical orbits in Newton's One-Body approximation. The one-parameter classification is illustrated in the Fig. 1. There are 5 types of them ranked by β_0 in the picture: a circle (2), $\sigma = 1$, elliptic sub-circle (1), $1 < \sigma < \infty$ and over-circle (3), $0.5 < \sigma < 1$, parabolic (unstable) (4), $\sigma = 0.5$, and hyperbolic (unbounded) (5), $\sigma < 0.5$. We recommend astronomers abandon Kepler's geometrical orbits in favor of the above physical one-parameter equations.

The σ criterion has a remarkable feature of symmetry $G\,M/r = V^2$, for example, a proportionality $M \propto r$ gives the same solution with the orbiting speed V unchanged (flat RC).

The theorems and laws altogether constitute the basis of Classical Gravitational Dynamics. As noted, in the advanced form it can be extended to the SRD in Minkowski space. Using the SRD, one can assess the relativistic effects of high speed and strong field. Sadly, the SRD is almost forgotten or ignored in Modern Physics.

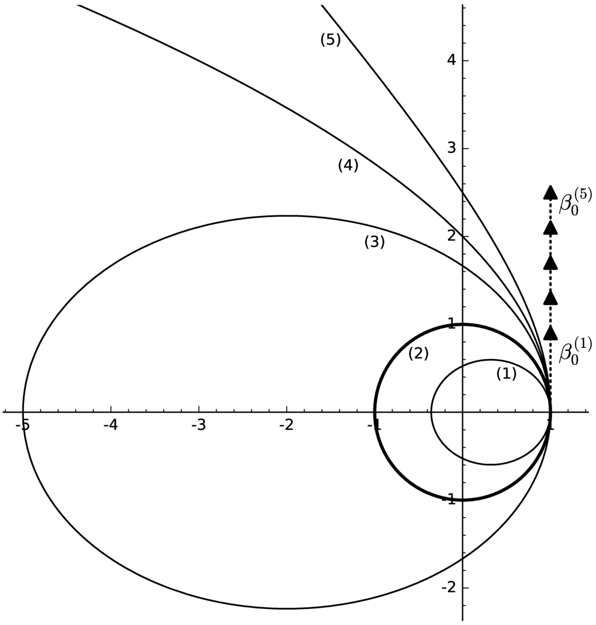

Fig. 1. Five possible classical orbits in the framework of One-Body Newtonian Gravitation.

5 Galactic structure in spiral galaxies

5.1 The Core and the Disc

Observations of the Core with a central SBH are usually treated in the Newtonian approach, complications arise with galactic structure as a whole. As an example, the Milky Way is a bar-shaped spiral galaxy having a bulge and the Core, which is a very small part of it, while the mass of SBH SgrA* in the Core is comparatively negligible. Astronomers' works are focused on the determination of integral mass distribution $M(r)$ mass and density $D(r_i)$ beyond the Core and its interplay with the dynamics of the disc. Can Newton's gravity be applied without invoking the GDM?

To answer the question, we first suggest introducing a zone of Core-to-disc transition, the Core being described in spherical geometry transferring to cylindrical coordinates in the disc. Strictly speaking, this is an N-body problem requiring a numerical simulation, which is a topic of separate study. However, we can assess the role of the Core in the formation of RC empirically. In the Milky Way, astronomers observe a large area of flat RC with a rotation rate of about $V_1 = 240\,\mathrm{km/s}$ starting at $R_1 = 1\,\mathrm{kpc}$. One could say that the rotation is energized by the "critical mass" about $M_1 = 5 \times 10^{10} M_\odot = 1 \times 10^{41}\,\mathrm{kg}$, as shown in [2]. Before applying Newton's dynamics to galactic observations, we need to introduce some new concepts.

5.2 Concepts of rings, self-sustained rotation, and standard test particle

Our novel idea is to consider a disc composed of rings R_i of arbitrary small thickness $\mathit{\Delta} r$. A deferential mass distribution $\mathit{\Delta} M_i = 2\,\pi\,H\,D(r_i)\,\mathit{\Delta} r$ can serve a role of rings adjacent to each other at r_i, for $i = 1, 2, 3....$ Each ring contains all kinds of galactic materials rotating with the same speed $V_i(r)$ in accordance with the Equivalence Principle. Therefore, the mass density $D(r_i)$ must be averaged there.

Suppose, we cut off the disc leaving the Core of radius r_1 with the first ring R_1 remaining in the place to rotate. The galaxy can continue evolving if the Core catches from the neighborhood one by one more rings R_i. Together with the Core mass, inner disc masses within radius r_i play the role of sources supporting a rotation of each ring $r > r_i$ in the outer disc area. A final type of galaxy, its characteristics such as RC, integral and differential mass distributions, and others will depend on initial conditions in the Core with a central Black Hole and disc area near the Core: it should have sufficient attracting power (critical mass) to rotate adjacent rings with given constant velocities.

In the following, we present Newton's galactic dynamics in one-body approximation with the potential, from formula (1), reduced to cylindrical geometry. The concept of rings playing the role of Standard Test Particle (STP) will be used. In the case of the Milky Way, it is seen that the flat RC condition requires the source mass proportionally increasing with radius $M(r) \propto r$, then the mass density will decrease inversely proportional $D(r) \propto 1/r$. Cutting off or adding parts of the disc does not make a change. A slight influence of the outer part of the disc on the rotation of inner rings can be neglected to the next-order precision. This fact verifies the validity of our model approximation. We call the above model of galactic disc with flat RC *the Self-Sustained Rotating Disc* as a natural concept in the Newtonian model of galaxies. It tells that the RC type is tightly constrained by geometry and differential mass distribution.

5.3 Black holes and the principle of ultimate gravitational compression

Astronomers used to notice but did not pay much attention to that the average density of a black hole inside the Schwarzschild sphere is inversely proportional to the square of its mass, see [17]. However, this happens under special conditions of the Event Horizon in a non-rotating black hole: the greater the size and mass, the smaller the density. Let us call it the *Principle of ultimate compression*, which tells us that the critical (nuclear) density, say, $d_{\mathrm{cr}} \approx 1 \times 10^{19}\,\mathrm{kg/m^3}$ cannot be physically exceeded. Ideally, it occurs when a natural radius of a solid sphere with an observed border R approaches a theoretical radius $r_g = G\,M/c_0^2$, while the speed of STP approaches infinity. Notice that we use the Gravitational radius r_g, which is half the Schwarzschild radius. This brings us to an interesting result.

Assuming that BH mass M is measured, one can calculate the theoretical values of the other two parameters d and R. We have

$$R = r_g = \frac{G\,M}{c_0^2}\,, \quad d = \frac{3\,c_0^2}{4\,\pi\,G\,R^2}\,, \tag{6}$$

where $R \sim M$, $M \sim R^3\,d$, and $d \sim R^{-2}$ as a mean ball density. The equality $R = r_g$ imposes a constraint on the 3 parameters R, M, d. Given any of them, the other two can be calculated using the above proportionality rules. If you double the mass, the radius is doubled too, volume jumps cubic, hence, density falls square. Such a scheme of mass compression in BH is physically meaningful in a certain range of masses above the Solar mass $M_\odot$. The smallest one is the Neutron star (NS), which can be considered the lightest stellar BH.

The proposed Principle of Ultimate Gravitational Compression to the critical density d_{cr} is principally different from the conventional concept of gravitational collapse, and it changes our understanding of Galactic Dynamics.

In the case of the Milky Way, the measured mass of SgrA* is $M = 4.1 \times 10^6\,M_\odot$. Then, the calculation gives values of

- $R = 6.0 \times 10^9\,\mathrm{m}$ and
- $d = 6.0 \times 10^6\,\mathrm{kg/m^3}$;
- the actually measured radius is about $R = 2.4 \times 10^{10}\,\mathrm{m}$.

In another example of galaxy NGC 1052-DF2 of Ultra Diffuse type, the measured SBH of mass is $M = 1.5 \times 10^8\,M_\odot$. The calculated values are:

- $R = 2.3 \times 10^{11}\,\mathrm{m}$
- $d = 6000\,\mathrm{kg/m^3}$

that is, the mass density drastically drops.

The heaviest SBH is identified in the center of Messier 87 galaxy at a distance about 16 Mpc with the measured BH mass about $M = 6 \times 10^9\,M_\odot$ (1500 times heavier than SgrA*). One can imagine a “devouring monster”. Surprisingly, by the above proportionality rules, the calculated quantities are $R = 9 \times 10^{12}\,\mathrm{m}$, and the mean density of the monster comes to the level of air $d = 1.2 \times 10^3\,\mathrm{kg/m^3}$, which is not real. The UGC Principle tells us that, when SBH can hardly exist in a *stable spherical* form, It is rather highly flattened by rotation, consequently, restoring high density but always less than critical. If so, Black Hole observations and treatments of them could be confusing and misinterpreted.

Back to the Neutron Star. Assume that the measured parameters of SgrA* are reasonably true, particularly, mean mass density $d_{\mathrm{sgr}} = 9.0 \times 10^6\,\mathrm{kg/m^3}$. Then, one can assess the NS parameters by taking one of them given and finding the rest using the proportionality rule. For example, let us take the critical density $d_{\mathrm{cr}} = 1.0 \times 10^{19}\,\mathrm{kg/m^3}$ to compare it with d_{sgr}. From the square proportionality factor $k^2 = 1.1 \times 10^6$, the values of radius and mass of the Neutron Star follow $R = 5.67 \times 10^3\,\mathrm{m}$, $M = 3.84\,M_\odot$, what is physically reasonable. In our approach, one can assess any BH and NS case individually or in comparison.

There are numerous publications devoted to the ultimate density of stars and other cosmological objects, among them, this fundamental physical problem is studied in depth by Krizek [18].

5.4 RC unfolding algorithm

Next, we present a scheme of algorithm unfolding the main physical characteristics of galaxies having a disc with arbitrary RCs. Principally, one can treat each "observed ring" using equations (3, 4). The main purpose is to calculate the integral radial mass distribution using measured RC velocities $V(r_i)$, which are empirical data. Recall, that we use dimensionless representation, where $V(r) = \mathrm{d}r/\mathrm{d}(c_0 t)$, $c_0 = 1$ speed of light. Hence, output calculations can also be a set of numbers. We use an approximate model of a disc having a constant height H and a radial dependent mean density $D(r_i)$. Output data include a mass integral distribution $M(r_i)$. It should be found as a function of measured velocities $V(r_i)$ in terms of ring concept and STPs of mass $m \ll M_i$

$$M(r_i) = \frac{1}{G}\, r_i\, V(r_i)^2\,. \tag{7}$$

The differential radial mass distribution, by definition, should be a plot of $\varDelta M(r_i)/\varDelta r$ consistent with the equation (7). Then, the distribution of mass density $D(r_i)$ can be determined

$$\frac{\varDelta M(r_i)}{\varDelta r} = 2\,\pi\, H\, D(r_i)\, r_i = \frac{1}{G}\,(V(r_i)^2 + 2\, r_i\, V(r_i))\frac{\varDelta V(r_i)}{\varDelta r}\,, \tag{8}$$

$$D(r_i) = \frac{1}{2\, G\, \pi\, H\, r}(V(r_i)^2 + 2\, r_i\, V(r_i))\frac{\varDelta V(r_i)}{\varDelta r}\,. \tag{9}$$

Next, we sum up our findings and demonstrate the power and elegance of Newton's physics in the simplified example of Milky Way (MW) Galactic Dynamics.

5.5 GDM-free Milky Way

We define the differential mass increment in the galactic disc in relationship with the concepts of R_i-ring as the standard test particle of mass $m(r) \ll M(r_i)$

$$\varDelta M(r) = 2\pi\, H\, r\, D(r)\, \varDelta\, r\,. \tag{10}$$

In the RC flat region, the mass of ring $m(r) = \varDelta M(r)$ is constant. Indeed, the enclosed mass $M(r_i)$ within r_i increases with radius $M(r) \sim r$ making $G\, M(r)/r = \mathrm{Const}$ while the mean density of matter $D(r)$ must decrease inversely proportional to radius $D(r) \sim 1/r$.

Measuring velocities $V(r)$ in the RC flat region of the Milky Way, for example, at the Solar place, astronomers usually choose the reference quantities V_0 at $r = R_0$, in addition to the measured disc of constant thickness H. Having done this and using the above proportionality rules in the formulas, they can define and calculate other quantities in the flatness range, including gravitational radius r_g, total and kinetic energies, and angular momentum of any part of the galaxy. The mean mass density can be assessed from proportionality $D(r) = R_c\, D_c/r$ with respect to the starting point in the Core area R_1.

So we have the following MW characteristics at our (Solar) position in the galaxy

- $R_0 = 8\,\mathrm{kpc} = 2.47 \times 10^{20}\,\mathrm{m}$;

- $V_0 = 2.40 \times 10^5\,\mathrm{m/s}$;
- $M_0 = 2.16 \times 10^{41}\,\mathrm{kg} = 1.0 \times 10^{11}\,M_\odot$;
- $H = 0.3\,\mathrm{kpc} = 9.3 \times 10^{18}\,\mathrm{m}$;
- $D_0 = 5.5 \times 10^{-19}\,\mathrm{kg/m^3} = 9.2\,M_\odot/\mathrm{pc}^3$;
- $r_g = 1.6 \times 10^{14}\,\mathrm{m}$.

The extrapolation to the Core radius $R_1 = 1\,\mathrm{kpc}$ gives the mass $M_1 = 2.7 \times 10^{40}\,\mathrm{kg} = 1.3 \times 10^{10}\,M_\odot$. This is consistent with observations and the requirement of Core engagement in self-supporting disc rotation: mass and density in the Core area should grow with radius faster than in the RC flat region. Strictly speaking, the RC curve has some humps, which we ignore in our model.

Having measured R_0, V_0, H and using the above proportionality rules in the range of RC flatness, one can calculate other galactic quantities. In particular, one can calculate a time period P of orbit rotation at any radius R. This quantity implicitly reveals the 3d Kepler's law $P^2 \sim R^3$ in the one-body approximation. At Sun's position $r = R_0$, the period is $P_0 = 2.3 \times 10^8$ years. Using this proportionality, one can calculate $P(R_i)$ for any circle at any R_i. Also, one can derive the third Kepler's law $P^2 = 4\pi^2(R^3/G\,M)$ knowing that potential energy is equal to doubled kinetic energy $K = m\,V^2/2$. In elliptic orbits, one should use the semi-major axis a instead of radius R.

To continue, let us consider how strong the MW galaxy is bounded by the dimensionless criterion of total energy ϵ: (3), the lower its value the stronger it is bounded, it is unbounded when $\epsilon > 1$. For every Ring the value of ϵ is less than one by about 1.0×10^{-6}, on the edge of stability. Notably, the potential (non-dimensional) function of enclosed masses in a flat RC area has the same constant value on the outer surface of each ring:

$$\varPhi = r_g/R = 1.3 \times 10^{-6} = \mathrm{const}\,. \tag{11}$$

At the same time, the *dynamic* acceleration $g(R)$ of rings decreases with the radius, $g(R) = V_0^2/R^2$. At $R = 8\,\mathrm{kpc}$, it is $g = 2.36 \times 10^{-10}\,\mathrm{m/s^2}$, and at $R = 1\,\mathrm{kpc}$, it is $g = 3 \times 10^{-9}\,\mathrm{m/s^2}$. Small accelerations are due to extremely low mass density $D(r)$ spreading over a large radius range. Compare it with the *static* acceleration near Earth's surface $g = 10\,\mathrm{m/s^2}$.

The area of RC flatness in the Milky Way continues approximately till about 25 kpc when velocities begin to decline. There is no clearly defined end, the boundaries of the disc are more diffused with radius. Consequently, instability increases leading to the development of spiral arms, when orbits become over-circle or hyperbolic ($\sigma < 0.5$). Then disc materials will flow out through the spiral arms. This phenomenon is related to the worsening ratio of angular momentum $L(r) \propto r^2$ to the constant rotational energy, while a local acceleration decreases $g(r) \propto 1/r$. This factor enables a growth of velocity dispersion. Astronomers used the effect for identifying the ages of stars and other physical properties depending on angular momentum $L(r)$, see [19].

5.6 Other galaxies

Thus, we have introduced new concepts in Galactic Dynamics, such as R_i-rings, which play the role of STP in the One-Body approximation. We also explore the notion of self-sustained disc rotation and, finally, the Principle of Ultimate Compression of Black Hole mass in the Core. These concepts form the basis of our methodology for interpreting galactic observations within the framework of Newtonian Gravitation applied to the GDM problem. We need to develop algorithms for determining and calculating the main characteristics of a galactic disc, specifically the integral and differential radial mass distributions $M(r)$ and $\varDelta M(r_i)$, as well as the mass density distribution $D(r)$. This is provided that the disc's thickness H and orbiting velocities $\mathrm{RC}(r)$ are measured at a reference point R_0. For now, we will not delve into the study of brightness and mass-luminosity ratios.

In the treatment of galactic observations, careful consideration must be given to methodological issues. One such issue is the bottom-up evolution of galaxies. The focal point here is the physical process of forming a bound system of matter rotating around the center of mass. Another area of interest pertains to the ages of galaxies. A newly formed "baby galaxy" could potentially contain a stellar Black Hole. The subsequent evolution hinges significantly on the amount of matter in the surrounding space, or the Halo. As we have realized, mature spiral galaxies have Supermassive Black Holes (SMBH) within a constrained mass range. The heaviest of these galaxies become flattened by rotation, leading to complications in the Core's structure, which serves as a primary factor in galactic evolution.

Morphological issues also arise due to the interactions between galaxies in various environments. A larger galaxy might collide with a massive object, giving rise to the creation of "unconventional" galaxies, such as irregular dwarf galaxies and others. The stochastic nature of galactic evolution should manifest as irregularities and anomalies in the radial mass density distribution $D(r)$ within a galactic disc. On the contrary, sparser environments might yield galaxies with lower masses, lacking distinct spirals and visible discs. These galaxies could fall into the elliptic or "irregular" categories, making it challenging to precisely define their degree of gravitational binding. We will delve further into this issue later. With an understanding of these matters, we are now prepared to discuss algorithms for Newton's investigation into the purported presence of GDM in galactic structures.

5.7 Non-flat RC

The majority of galaxies seem to have non-flat RCs. This is because of failure during the course of evolution to satisfy Core conditions providing the self-sustained disc rotation. Usually, big galaxies like the MW, have super-heavy central BH with density close to the critical, a large Halo, spiral arms, and no distinct edge. They could be mature, bounded systems in an environment rich in material. Our analysis of RC formation looks like an imitation of "bottom to top" evolution,– the larger, the older. But accidental events could happen. Theoretically, cutting off some outer part of a large disc would leave the galaxy stable. Galaxies as giant as the MW can host inside a smaller ring galaxy, most likely, as a result of galactic collision. A similar observed phenomenon is known as Hoag's object, see [20]. The variety of environments is a good reason for the diversity of galaxies, particularly, in their morphology. Some examples are given next.

5.8 Spiral galaxy Messier-33

The galaxy M-33 is one among others revealing a specific stage of cosmological evolution. It is half the size of MW and presents puzzles: it has neither a visible SBH nor a Bulge. The RC measured in the range up to about 15 kpc shows the proportionality $V(r) \sim r^{1/2}$ that is,

$$V(r) = \left(\frac{G\,M(r)}{r}\right)^{1/2} . \tag{12}$$

According to the above-discussed proportionality rules, the disc density should be constant, which makes the mass proportional to the square of r, $M(r) \sim r^2$. How could it be possible with no SBH?

Our explanation goes to the *Principle of Ultimate Gravitational Compression* discussed above.

From assessment based on the proportionality rules, the approximate value of M at $R = 8\,\mathrm{kpc}$ is $M = 5 \times 10^{10}\,M_\odot$, about half that in MW. One can expect that the RC above $R > 15\,\mathrm{kpc}$ will approach a maximal value and then slowly decline without forming a self-sustained rotating disc. Based on the current RC data, astronomers decided a need for the GDM Halo in this galaxy.

5.9 Elliptical Messier-87 galaxy

Those galaxies have already been discussed above in relation to the SBH. The M-87 galaxy has some similarities with M-33. As shown, it has the estimated heaviest SBH of mass about 1500 times that of SgrA* mass calculated density is about the Earth's air one. Similarly to the case of M-33, the SBH of such a low density cannot exist as a spherical mass. Instead, it should be flattened by rotation to the ellipsoidal shape containing significantly suppressed mass. If so, we observe a solid galactic Core, whose visible size extends up to 200 kpc of diameter.

The M-87 galaxy is approximately double the size of the MW, and it has a substantially larger mass. It is considered elliptic, possibly surrounded by a huge matter Halo. It cannot be characterized by the RC. This is the case when numerical simulations of observations are needed. Anyway, some astronomers try to find signs of the GDM existence in the hypothetical Halo there, for example, see [21].

Recently, NASA astronomers demonstrated breakthrough images of SBH in the M-87 and MW galaxies from the Event Horizon Telescope (EHT), which is a system of several telescopes at different locations. The project cost dozens of millions of dollars before starting. The idea is a reconstructing virtual image of the BH using information from many images, see [22]. *The Event Horizon* is a hypothetical phenomenon when the gravity about a Black Hole is so strong that nothing can escape, not even light. According to General Relativity, it is *the apparent horizon* unlike *the absolute event horizon* in Cosmology. They say, "notion of a horizon" in General Relativity is subtle and depends on "fine distinctions".

According to [23], the BH images turned out to be not true, but rather the result of an incorrect reconstruction procedure. This is not surprising in view of our treatment of the M-87 galaxy and its SBH.

5.10 Ultra-diffuse galaxies

To astronomers' surprise, they observe galaxies apparently lacking the GDM, in particular, in ultra-diffuse galaxies (UDG) having low density. Often, they have an elliptic form and a non-flat RC looking "Keplerian", meaning no GDM. In terms of Newtonian Dynamics, their disc rotation is not self-sustained due to the Core mass smallness. The mass and its density are not sufficient to keep the disc rotation at the maximal speed reached in the Core zone. Consequently, the RC(r) is going down with the radius. Unfortunately, observations of the Core with a central BH in such galaxies are aggravated by the extremely low luminosity of UDGs.

In recent observations of the gas-rich ultra-diffuse galaxy AGC 114905, see [24], the authors managed to get high-resolution precision allowing them to determine the parameters of RC and the Halo. They concluded that the galaxy definitely does not have the GDM. This is not an exclusion from a long list of UDF galaxies lacking GDM: see discussed above the UDG DF2 galaxy.

At this point, our main mission to demonstrate a successful treatment of galaxies in the Newtonian Gravitation framework is completed.

6 Summary and conclusion

This work is the result of our research aimed at the explanation of the Centennial Galactic DM puzzle. From the viewpoint of the Dark Halo model, observations of flat RC in spiral galaxies contradict Kepler's laws and Newton's theory. We proved the opposite, albeit the technique is not trivial. The following innovations were introduced:

- Current Newton's theory of gravitational dynamics is based on Kepler's *geometrical* concept of two-parameter elliptic orbits. We suggest formulating one-parameter Newton's *physical* theory of orbits, as an approximation of Special Relativity dynamics. There, the equation of motion and its solution depend on initial conditions constrained by conservation laws in One-Body approximation;
- Suggested: 1) concepts of Rings (Standard Test Particle) and Self-Sustained disc rotation in galaxies forming flat RCs; 2) transition from polar coordinates in Core area to cylindrical geometry; 3) Principle of Ultimate Gravitational Compression of ordinary matter; 4) treatment of observations with RC unfolding algorithm;
- We demonstrate the validity of Newton's theory proving GDM being fictitious. Thus, we reconcile all astronomical community, - Dark Halo supporters, - believing in the validity of Newton's theory, and MOND supporters believing in the non-existence of Dark Matter.

There could be arguments that, besides galactic measurements, evidence for GDM also comes from observations on a large cosmic scale, in particular, for the early Universe, but we argue.

Dark Matter notions in Cosmology in terms of Cold Dark Matter (CDM) along with Dark Energy (DE) were introduced for reasons very different from GDM:

- CDM as a special parameter of the Lambda-CDM Model in the group of few other main parameters imposing constraints on fitting the Model to observations to explain metric space Hubble expansion involving receding galaxies.
- The Lambda-CDM Model requires the next modification in view of revolutionary Webb images of the Early Universe. We state that our new findings about GDM explanation are fundamentally important regardless of Lambda-CDM Model status, and even could be useful for a reinterpretation of model parameters. So, the GDM/CDM issue deserves separate work.

All things considered, we want our results to rapidly reach out to the scientific community. We call upon astronomers to accept the physical nonexistence of the Dark Matter phenomenon and return to Astronomy respecting classical heritage.

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