CHARGED HADRON MULTIPLICITIES: A PROBABILISTIC PARTON BRANCHING MODEL

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We make the case that parton branching is the dominant dynamical influence on the shape of charged hadron KNO multiplicity distributions. For e^+e^- data, the KNO distribution is well fit by a two quark jet evolution with a simple hadronization scheme. For $\bar{p}p$ data, we predict a flattening of the KNO distribution due to jets by SSC energies.

Introduction

Multiplicity distributions represent averaged, gross observables, not event-by-event observables. The probability P_n for observing n charged hadrons in an e^+e^- or $\bar{p}p$ collision, with mean number $\bar{n}$, is a statistical distribution. The question I am addressing is whether the statistics of large samples of many-particle events washes out the underlying dynamics.

The popular negative binomial distribution $P_n^k(\bar{n})$ is an example of a statistical distribution with no satisfactory dynamical basis. This distribution was used by the UA(5) collaboration to fit pp or $\bar{p}p$ data from 10 GeV to 900 GeV center-of-mass energy. The parameter k varied over that energy range from about 20 to about 3, decreasing with increasing energy and $\bar{n}$. One of the original applications of the negative binomial distribution was to quantum optics, where k has the interpretation of number of sources, an integer expected to increase with energy, not decrease as in the hadronic data.

The work I will present, done in collaboration with a graduate student, Robert O. Knuteson, is a study of parton branching as the dominant underlying dynamics of hadron multiplicities, from a probabilistic point of view. We find distributions which are in excellent agreement with e^+e^- data and in plausible agreement with $\bar{p}p$ data.

In e^+e^- hadroproduction three processes take place which must be modeled believably. There is hard scattering, generally leading to a quark-antiquark $Q\bar{Q}$ parton pair, possibly also including one or more gluons G. There is unresolvable gluon production, with gluons emitted along a flux tube between the Q and $\bar{Q}$ or along the line of motion of the partons. These gluons are not resolvable due to either too little energy or too small an angle relative to a hard parton. Then there is hadronization of the partons when their invariant mass reaches some low value Q_o^2.

The hard partons branch resolvably, with vertex probabilities given by the QCD LLA (leading logarithm approximation) Altarelli-Parisi splitting functions. This is what we have extensively studied. Both the unresolvable gluons and the hadronization involve invariant masses below the cutoff Q_o^2 and cannot be studied in perturbative QCD. We model these processes in a plausible way. The soft or collinear gluons are non-jet partons. We represent them by a low energy multiplicity distribution taken from data just below the jet threshold energy. We model the hadronization by letting some percentage of final gluons become two hadrons and some percentage four, and of quarks some percentage one and some percentage three. This conserves charge.

For $\bar{p}p$ collisions there is the further complication that the initial parton-parton collision yields branching jet partons which depend on the structure functions of the initial hadrons. We must fold together the momentum distributions of partons inside the hadrons to get a starting point for the jet partons. In e^+e^- collisions, we can assume that the Q and $\bar{Q}$ carry equal energy, one-half the center-of-mass energy minus the amount in unresolvable gluons.

Two positive features of our model are that it is probabilistically correct, with detailed perturbative QCD branching probabilities, and that it is very efficient to implement by computer. It does not involve detailed randomized event generation as do Monte Carlo programs.

Probabilistic Parton Branching

For the perturbatively tractable part of hadron production we assume that every jet is initiated by a hard parton, Q or G (we let antiquarks be represented by Q). The allowed branching processes are $G \to GG$, gluon bremsstrahlung; $Q \to QG$, quark bremsstrahlung; and $G \to QQ$, gluon pair production. These processes take place with probabilities which are related to the Altarelli-Parisi splitting functions,

$$
\begin{aligned}
P_{GGG} &= 2N_c \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right] \\
P_{QQG} &= C_F \left[\frac{1+z^2}{1-z}\right] \\
P_{GQQ} &= \frac{N_f}{2}\left[z^2 + (1-z)^2\right].
\end{aligned}
$$

N_c and N_f are the number of colors (3) and flavors (4), C_F is a color factor (4/3), and z is the ratio of the daughter energy to the parent energy in the lab, E_1^{lab}/E^{lab}.

The most general master equation from the theory of branching processes which is suitable for this problem is

$$
\frac{dP}{dt}(\vec{k},t) = -\int_{\text{“lost”}} d\tau \sum_{\vec{k}'} w(\vec{k},t|\vec{k}',\tau)P(\vec{k},t) + \int_{\text{“gained”}} d\tau \sum_{\vec{k}'} w(\vec{k}',\tau|\vec{k},t)P(\vec{k}',\tau).
$$

Here P is the probability and the regions of integration are those into which probability is lost and from which probability is gained, determined kinematically below. The parameter t, corresponding to time in some applications of this master equation, is invariant mass, which decreases as a jet evolves. The vector $\vec{k}$ stands for a set of discrete variables, while t is continuous. The probability w is related to the Altarelli-Parisi splitting functions.

We factorize the probability w,

$$w(\vec{k},t|\vec{k}',\tau) = K(\vec{k}|\vec{k}')P(t|\tau).$$

The notation of a bar | means conditional probability. The set of discrete variables $\vec{k}$ contains gluon number n and quark number m. If we let ABC stand for three parent-daughter combinations, then

$$\begin{aligned} K(\vec{k}|\vec{k}') &= n\delta_{m',m}\delta_{n',n+1}\ \delta_{ABC},\ _{GGG} \\ &+ m\delta_{m',m}\delta_{n',n+1}\ \delta_{ABC},\ _{QQG} \\ &+ n\delta_{m',m+2}\delta_{n',n-1}\ \delta_{ABC},\ _{GQQ}. \end{aligned}$$

$P(t|\tau)$ includes two factors itself,

$$P(t|\tau) = P(t|\text{split at } t)P(\text{split at } |\tau),$$

the product of the probability $P(t|\text{split at } t)$ that a parton with invariant mass will branch and the probability $P(\text{split at} t|\tau)$ that the daughter parton will have invariant mass in the range τ to $\tau + d\tau$. Then

$$P_{ABC}(t|\text{split at } t) = \frac{\alpha_s(t)}{2\pi}(\frac{1}{t})\gamma_{ABC}(t).$$

Here γ_{ABC} is an integral over the Altarelli-Parisi splitting functions,

$$\gamma_{ABC}(t) = \int\limits_{z_{min}(t)}^{1-z_{min}(t)} dz P_{ABC}(z),$$

and the coupling is

$$\alpha_s(t) = \frac{1}{b_o ln(t|\Lambda^2)},$$

where Λ is the QCD scale parameter and

$$b_o = \frac{1}{12\pi}(11N_c - 2N_f).$$

The probability to end at τ is proportional to one over the kinematic range for daughters,

$$P(\text{split at } t|\tau) = \frac{1}{t - 4Q_o^2}.$$

To save space I will not write out the combined $w(\vec{k}, t|\vec{k}'\tau)$ nor the related $w(\vec{k}', \tau|\vec{k}, t)$.

Kinematic constraints give the τ regions "lost" and "gained" in terms of fixed t as

$$\begin{aligned} \text{“}lost\text{”} &= [Q_o^2, t - 3Q_o^2] \\ \text{“}gained\text{”} &= [t + 3Q_o^2, T_{\max}], \end{aligned}$$

where $T_{\max}$ is determined by $\sqrt{s}$ in e^+e^- scattering and by folding parton structure functions in $\bar{p}p$ scattering. The final version of the branching equation is

$$\begin{aligned} \frac{d}{dt}P_{mn}(t) = &- n\frac{\alpha_s(t)}{2\pi}\left(\frac{1}{t}\right)\gamma_{GGG}(t)P_{mn}(t) \\ &- m\frac{\alpha_s(t)}{2\pi}\left(\frac{1}{t}\right)\gamma_{QQG}(t)P_{mn}(t) \\ &- n\frac{\alpha_s(t)}{2\pi}\left(\frac{1}{t}\right)\gamma_{GQQ}(t)P_{mn}(t) \\ &+ (n-1)\int_{t+3Q_o^2}^{T_{max}} d\tau\frac{\alpha_s(\tau)}{2\pi}\left(\frac{1}{\tau}\right)\left(\frac{1}{\tau - 4Q_o^2}\right)\gamma_{GGG}(\tau)P_{mn-1}(\tau) \\ &+ m\int_{t+3Q_o^2}^{T_{max}} d\tau\frac{\alpha_s(\tau)}{2\pi}\left(\frac{1}{\tau}\right)\left(\frac{1}{\tau - 4Q_o^2}\right)\gamma_{QQG}(\tau)P_{mn-1}(\tau) \\ &+ (n+1)\int_{t+3Q_o^2}^{T_{max}} d\tau\frac{\alpha_s(\tau)}{2\pi}\left(\frac{1}{\tau}\right)\left(\frac{1}{\tau - 4Q_o^2}\right)\gamma_{GQQ}(\tau)P_{m-2n+1}(\tau). \end{aligned}$$

This equation can be solved by computer by replacing the continuous variable t by a discrete lattice of points t_k,

$$t_k \in [Q_o^2, T_{\max}], \;\; t_o = T_{\max}.$$

The notation for the discrete version of $P_{mn}(t)$ is thus P^k_{mn}. The computer program requires input of $\Lambda, N_c, N_f, T_{\max}, Q_o^2, \Delta t, \Delta\tau, P^o_{mn} = \delta_{m_o n_o}$. Then the lattice of points T_k is computed, the discrete equation is iterated until $t_k \leq T_{min} = 4Q_o^2$, and the output is $P_{mn}(Q_o^2)$, the probability of having m quarks and n gluons all with invariant mass between $T_{min} = 4Q_o^2$ and Q_o^2.

Complete details of our investigation are given in R.O.Knuteson's Ph.D. dissertation and are being written for publication. For this brief paper I have selected three figures showing the results of our calculations. Figure 1 shows the KNO "shape" of a pure two-quark and a pure two-gluon evolution. The gluons give a narrower distribution. Figure 2 shows our fit to a subset of e^+e^- data, though the curves shown resulted from fitting more data than these. This fit incorporated our models for soft processes and for hadronization. Figure 3 shows the hard parton branching contribution to the $\bar{p}p$ KNO distribution, as we predict it for three energies. There are also diffractive and soft processes contributing to the KNO distribution, not included in these curves.

Figure Captions

Fig. 1. At $T_{max} = 100\,GeV^2$ for each parton, the KNO plot for two initial gluons is narrower than the KNO plot for two initial quarks. These jets evolved to $Q_o^2 = 1\,GeV^2$, with $\Lambda = 0.3\,GeV$.

Fig. 2. The KNO plots for e^+e^- data at three energies from TASSO are well fitted by two quark $(q\bar{q})$ jets evolving to $Q_o^2 = 3\,GeV^2$, with $\Lambda = 0.3\,GeV$. Hadronization is modeled by letting each final quark become one, and each final gluon, two, hadrons. Soft processes are modeled from e^+e^- data at 12 GeV from JADE.

Fig. 3. There are several contributions to the $\bar{p}p$ KNO plots. This figure shows only the contribution from parton-parton collisions, with subsequent branching. Two jets are initiated by each parton-parton collision. Between 2 and 40 TeV center-of-mass energy, the KNO plot flattens.

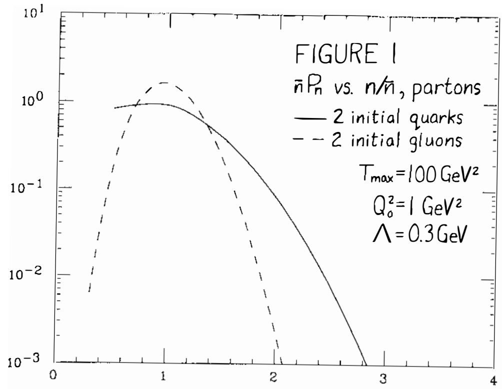
FIGURE 1
$\bar{n}P_n$ vs. $n/\bar{n}$, partons
— 2 initial quarks
– – 2 initial gluons
T_{max}=100 GeV²
Q_o^2=1 GeV²
Λ = 0.3 GeV
10¹
10⁰
10⁻¹
10⁻²
10⁻³
0
1
2
3
4

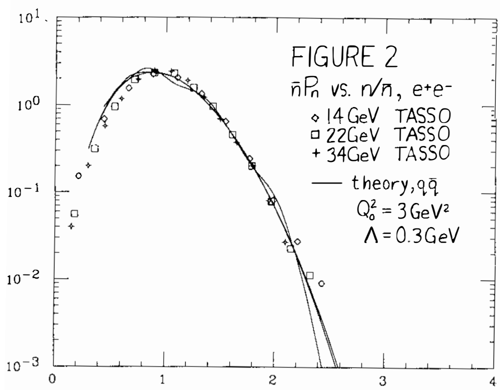
FIGURE 2
$\bar{n}P_n$ vs. $n/\bar{n}$, e+e-
◇ 14 GeV TASSO
□ 22GeV TASSO
+ 34GeV TASSO
— theory, $q\bar{q}$
Q_o^2 = 3 GeV²
Λ = 0.3GeV
10¹
10⁰
10⁻¹
10⁻²
10⁻³
0
1
2
3
4

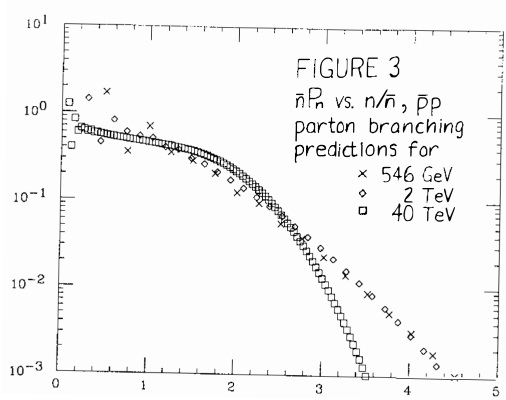
FIGURE 3
$\bar{n}P_n$ vs. $n/\bar{n}$, $\bar{p}p$
parton branching
predictions for
× 546 GeV
◇ 2 TeV
□ 40 TeV
10¹
10⁰
10⁻¹
10⁻²
10⁻³
0
1
2
3
4
5