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Research Article

Superfield Approach to Nilpotency and Absolute Anticommutativity of Conserved Charges: 2D Non-Abelian 1-Form Gauge Theory

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We exploit the theoretical strength of augmented version of superfield approach (AVSA) to Becchi-Rouet-Stora-Tyutin (BRST) formalism to express the nilpotency and absolute anticommutativity properties of the (anti-)BRST and (anti-)co-BRST conserved charges for the two $(1 + 1)$-dimensional (2D) non-Abelian 1-form gauge theory (without any interaction with matter fields) in the language of superspace variables, their derivatives, and suitable superfields. In the proof of absolute anticommutativity property, we invoke the strength of Curci-Ferrari (CF) condition for the (anti-)BRST charges. No such *outside* condition/restriction is required in the proof of absolute anticommutativity of the (anti-)co-BRST conserved charges. The latter observation (as well as other observations) connected with (anti-)co-BRST symmetries and corresponding conserved charges are *novel* results of our present investigation. We also discuss the (anti-)BRST and (anti-)co-BRST symmetry invariance of the appropriate Lagrangian densities within the framework of AVSA. In addition, we dwell a bit on the derivation of the above fermionic (nilpotent) symmetries by applying the AVSA to BRST formalism, where *only* the (anti)chiral superfields are used.

1. Introduction

The principle of *local* gauge invariance is at the heart of standard model of particle physics, where there is a stunning degree of agreement between theory and experiment. One of the most elegant approaches to covariantly quantize the above kinds of gauge theories (based on the principle of *local* gauge invariance) is Becchi-Rouet-Stora-Tyutin (BRST) formalism, where *each* local gauge symmetry is traded with *two* nilpotent symmetries that are christened as the BRST and anti-BRST symmetries. The latter symmetries are the *quantum* version of the gauge symmetries and their very existence ensures the covariant canonical quantization of a given gauge theory. The decisive features of the above quantum (anti-)BRST symmetries are the observations that (i) they are nilpotent of order two and (ii) they are absolutely anticommuting in nature. In the language of theoretical physics, the nilpotency property ensures the fermionic (supersymmetric-type) nature of the (anti-)BRST symmetries and the linear independence of BRST and anti-BRST symmetries is encoded in the property of absolute anticommutativity of the above (anti-)BRST symmetries.

The superfield approach to BRST formalism [1–8] provides the geometrical basis for the properties of nilpotency and absolute anticommutativity which are associated with the (anti-)BRST symmetries. In the above *usual* superfield approach [1–8], the celebrated horizontality condition (HC) plays a key and decisive role. The HC leads, however, to the derivation (as well as geometrical interpretation) of the (anti-)BRST symmetries that are associated with the gauge and corresponding (anti)ghost fields *only*. It does *not* shed any light on the (anti-)BRST symmetries that are associated with the *matter* fields in a given *interacting* gauge theory. In a set of papers [9–12], the above *usual* superfield formalism has been systematically generalized so as to derive the (anti-)BRST symmetries for the gauge, *matter*, and (anti)ghost fields *together*. The latter superfield approach [9–12] has been christened as the augmented version of superfield approach to BRST formalism, where, consistent with the HC, additional restrictions (i.e., gauge invariant conditions) are also invoked. We shall exploit the latter superfield approach [9–12] to discuss a few key features of the 2D non-Abelian 1-form gauge theory (without any interaction with matter fields) which have already been discussed within the framework of BRST formalism [13–16].

To be more specific, in the above works [13–16], we have shown the existence of the nilpotent (anti-)BRST as well as (anti-)co-BRST symmetry transformations for the 2D non-Abelian 1-form gauge theory. The central theme of our present investigation is to capture the nilpotency and absolute anticommutativity of the (anti-)BRST and (anti-)co-BRST conserved charges [13–16] within the framework of augmented version of superfield approach (AVSA) to BRST formalism. In the proof of the absolute anticommutativity of the (anti-)BRST charges (within the framework of AVSA), we invoke the CF condition to recast the expressions for these charges in an appropriate form and, then only, the superfield formalism is applied. However, in the case of the *above* proof of the (anti-)co-BRST charges, we do *not* invoke any CF-type restrictions. In our present investigation, we have proven the nilpotency and absolute anticommutativity of the conserved (anti-)BRST and (anti-)co-BRST charges that have been derived from *two* sets of coupled Lagrangian densities (cf. (1) and (39) below) for our 2D non-Abelian 1-form theory.

In the BRST approach to a given gauge theory, the existence of the (anti-)BRST symmetries and their conserved charges is well known. However, we have been able to establish the existence of (anti-)co-BRST symmetry transformations (in addition to the nilpotent (anti-)BRST symmetry transformations) in the case of a toy model of a rigid rotor in one $(0+1)$ dimension of spacetime [17]. Furthermore, we have demonstrated the existence of such (i.e., (anti-)co-BRST) symmetries in the cases of Abelian p-form ($p = 1, 2, 3$) gauge theories in the two $(1+1)$ dimensions, four $(3+1)$ dimensions, and six $(5+1)$ dimensions of spacetime (see, e.g., [18] and references therein). In other words, we have established that the nilpotent (anti-)co-BRST symmetries exist for any arbitrary Abelian p-form ($p = 1, 2, 3, \ldots$) gauge theory in $D = 2p$ dimensions of spacetime [18]. One of the decisive features of the (anti-)co-BRST symmetries is the observation that it is the gauge-fixing term that remains invariant under these transformations (unlike the kinetic term that remains invariant under the (anti-)BRST transformations). The geometrical origin for these observations has been provided in our review article (see, e.g., [18] and references therein).

We concentrate on the 2D non-Abelian theory (without any interaction with matter fields) because this theory has been shown [13] to be a perfect model of Hodge theory as well as a *new* model of topological field theory (TFT) which captures a few aspects of Witten-type TFTs [19] and some salient features of Schwarz-type TFTs [20]. The equivalence of the coupled Lagrangian densities of this 2D theory with respect to the (anti-)co-BRST symmetries has been established in our recent publication [14]. We have also discussed the CF-type restrictions for this theory within the framework of superfield approach [15], where we have demonstrated the existence of a tower of CF-type restrictions. This happens for *this* theory because it is a TFT where there are *no* physical propagating degrees of freedom for the 2D gauge field. In another work [16], we have derived all the conserved currents and charges for this 2D theory and shown their algebraic structure that is found to be reminiscent of the Hodge algebra [21–24]. In other words, we have provided the physical realizations of the de Rham cohomological operators of differential geometry (and their algebra) in the language of the continuous (as well as discrete) symmetries, corresponding conserved charges, and their algebra in operator form.

We have exploited the key ideas of AVSA to BRST formalism to derive the (anti-)BRST and (anti-)co-BRST symmetry transformations by using HC and dual-HC (DHC) as well as the (anti)chiral superfield approach to BRST formalism (see Appendices A and B below) in the context of our present 2D non-Abelian 1-form gauge theory. In our earlier works [9–12], we have *never* been able to capture the nilpotency as well as absolute anticommutativity properties of the (anti-)BRST and (anti-)co-BRST charges. The central objective of our present paper is to achieve this goal in the case of 2D non-Abelian 1-form gauge theory. To the best of our knowledge, this issue is being pursued for the first time in our present endeavor. Thus, the *novelty* in our present investigation is the observation that the nilpotency of the fermionic symmetry transformations and CF-type restrictions play a decisive role in capturing the nilpotency and absolute anticommutativity properties of the conserved (anti-)BRST and (anti-)co-BRST charges in the ordinary 2D spacetime (see Section 5 below). However, it is the nilpotency of the translational generators (along the Grassmannian directions) that plays a crucial role for the *same* purpose within the framework of AVSA to BRST formalism on the supermanifold (see Section 6 below).

The following key factors have spurred our curiosity to pursue our present investigation. First, to add some *new* ideas to the existing technique(s) of the superfield formalism is a challenging problem. In this context, we have expressed the fermionic charges (i.e., nilpotent (anti-)BRST and (anti-)co-BRST) in the language of the superfields and derivatives defined on the $(2, 2)$-dimensional supermanifold. Second, in our earlier works [14, 15], we have derived the expressions for the conserved fermionic charges in the ordinary 2D space. It is a challenging problem to express their nilpotency and absolute anticommutativity properties in terms of the quantities that are defined on the $(2, 2)$-dimensional supermanifold. Third, it is also an interesting as well as *novel* idea to discuss various aspects of the (anti-)co-BRST charges within the framework of AVSA to BRST formalism. Finally, the insights and understandings, gained in our present investigation, would turn out to be useful when we shall discuss the 4D Abelian 2-form and 6D Abelian 3-form gauge theories within the framework of AVSA to BRST formalism. In fact, we have already shown, in our earlier works [25, 26], that the above 4D and 6D Abelian 2-form and 3-form gauge theories are the models for the Hodge theory and they *do* support the existence of the (anti-)BRST and (anti-)co-BRST symmetries (as well as their corresponding conserved charges) in addition to the *other* continuous symmetries (and corresponding charges). There exist discrete symmetries, too, in these theories [25, 26]. All these symmetries (and corresponding conserved charges) are required for the proof that the above models are the tractable field theoretic examples of Hodge theory.

Our present paper is organized as follows. In Section 2, we discuss the nilpotent (fermionic) (anti-)BRST and (anti-)co-BRST symmetries in the Lagrangian formulation. Section 3 is devoted to the discussion of horizontality condition (HC) that leads to the derivation of (anti-)BRST symmetries for the gauge field and corresponding fermionic (anti)ghost fields along with the CF condition. Section 4 deals with the dual-HC (DHC) which enables us to derive the (anti-)co-BRST symmetries that exist for the 2D non-Abelian 1-form gauge theory. The subject matter of Section 5 concerns itself with the discussion of nilpotency and absolute anticommutativity properties of the fermionic charges within the framework of BRST formalism in 2D ordinary spacetime. In Section 6, we discuss the nilpotency and absolute anticommutativity of the fermionic charges within the framework of AVSA to BRST formalism on a $(2, 2)$-dimensional supermanifold, where the CF condition plays an important role for (anti-)BRST charges. Finally, we discuss the key results of our present investigation in Section 7, where we point out a few possible theoretical directions that might be pursued for future investigations.

In Appendices A and B, we derive the (anti-)BRST and (anti-)co-BRST symmetry transformations by exploiting the ideas of (anti)chiral superfield approach to BRST formalism which match with the *ones* derived in the main body of the text. We express the (anti-)BRST and (anti-)co-BRST invariance of the Lagrangian densities in the language of the AVSA to BRST formalism in Appendix C.

We note that the theoretical materials, contained in Sections 5 and 6, are deeply interrelated. In fact, sometimes, it is due to our observations in Section 5 that we have been able to express the nilpotency and anticommutativity properties of the charges in Section 6 within the framework of AVSA to BRST formalism. On the other hand, at times, it is our knowledge of the AVSA to BRST formalism (cf. Section 6) that has turned out to be handy for our derivations of the above properties in 2D ordinary space (cf. Section 5).

Convention and Notations. We take the 2D ordinary Minkowskian background spacetime to be *flat* with a metric tensor $\eta_{\mu\nu} = \mathrm{diag}(+1, -1)$, where the Greek indices $\mu, \nu, \lambda, \ldots = 0, 1$ correspond to the time and space directions, respectively. We choose 2D Levi-Civita tensor $\varepsilon_{\mu\nu}$ to obey the properties: $\varepsilon_{\mu\nu}\varepsilon^{\mu\nu} = -2!$, $\varepsilon_{\mu\nu}\varepsilon^{\nu\lambda} = \delta_\mu^\lambda$, $\varepsilon_{01} = +1 = \varepsilon^{10}$, and so forth. In 2D, the curvature tensor (i.e., field strength tensor) $F_{\mu\nu}$ has only one existing component $F_{01} = E = -\varepsilon^{\mu\nu}[\partial_\mu A_\nu + (i/2)(A_\mu \times A_\nu)]$ because $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i(A_\mu \times A_\nu)$. Here, in the $SU(N)$ Lie algebraic space, we have adopted the notations $A \cdot B = A^a B^a$ and $(A \times B)^a = f^{abc} A^b B^c$ for the nonnull vectors A^a and B^a, where $a, b, c = 1, 2, \ldots, N^2 - 1$ and f^{abc} are the structure constants in the $SU(N)$ Lie algebra $[T^a, T^b] = f^{abc} T^c$ for the generators T^a, which are present in the definition of 1-form potential $A_\mu = A_\mu \cdot T = A_\mu^a T^a$ and curvature 2-form field strength tensor $F_{\mu\nu} = F_{\mu\nu} \cdot T = F_{\mu\nu}^a T^a$ and so forth. Throughout the whole body of our text, we denote the (anti-)BRST and (anti-)co-BRST fermionic ($s_{(a)b}^2 = s_{(a)d}^2 = 0$) symmetry transformations by $s_{(a)b}$ and $s_{(a)d}$, respectively.

2. Preliminaries: Nilpotent (Fermionic) Symmetries

We discuss here the (anti-)BRST and (anti-)co-BRST symmetries (and derive their corresponding conserved charges) in the Lagrangian formulation of the 2D non-Abelian 1-form ($A^{(1)} = dx^\mu A_\mu = dx^\mu A_\mu \cdot T$) gauge theory within the framework of BRST formalism. The starting coupled Lagrangian densities, in the Curci-Ferrari gauge [27, 28], are

$$\begin{aligned}
\mathscr{L}_B &= \mathscr{B} \cdot E - \frac{1}{2}\mathscr{B} \cdot \mathscr{B} + B \cdot \left(\partial_\mu A^\mu\right) \\
&\quad + \frac{1}{2}\left(B \cdot B + \overline{B} \cdot \overline{B}\right) - i\partial_\mu \overline{C} \cdot D^\mu C, \\
\mathscr{L}_{\overline{B}} &= \mathscr{B} \cdot E - \frac{1}{2}\mathscr{B} \cdot \mathscr{B} - \overline{B} \cdot \left(\partial_\mu A^\mu\right) \\
&\quad + \frac{1}{2}\left(B \cdot B + \overline{B} \cdot \overline{B}\right) - iD_\mu \overline{C} \cdot \partial^\mu C,
\end{aligned} \tag{1}$$

where $\mathscr{B}$, B, and $\overline{B}$ are the Nakanishi-Lautrup type auxiliary fields that have been invoked for various purposes. For instance, $\mathscr{B}$ is introduced in the theory to linearize the kinetic term $(-(1/4)F_{\mu\nu} \cdot F^{\mu\nu} = (1/2)E \cdot E \equiv \mathscr{B} \cdot E - (1/2)\mathscr{B} \cdot \mathscr{B})$ and auxiliary fields B and $\overline{B}$ satisfy the Curci-Ferrari restriction: $B + \overline{B} + (C \times \overline{C}) = 0$, where the (anti)ghost fields $\overline{C}$ and C are fermionic (i.e., $(C^a)^2 = (\overline{C}^a)^2 = 0$, $C^a\overline{C}^b + \overline{C}^b C^a = 0$, $C^a C^b + C^b C^a = 0$, $\overline{C}^a\overline{C}^b + \overline{C}^b\overline{C}^a = 0$, $\overline{C}^a C^b + C^b \overline{C}^a = 0$, etc.) in nature and they are required in the theory for the validity of unitarity. In the above, we have the covariant derivatives $[D_\mu C = \partial_\mu C + i(A_\mu \times C)$ and $D_\mu \overline{C} = \partial_\mu \overline{C} + i(A_\mu \times \overline{C})]$ on the (anti)ghost fields in the adjoint representation.

The Lagrangian densities in (1) respect the following off-shell nilpotent ($s_{(a)b}^2 = 0$) (anti-)BRST symmetries transformations ($s_{(a)b}$):

$$\begin{aligned}
s_b A_\mu &= D_\mu C, \\
s_b C &= -\frac{i}{2}(C \times C), \\
s_b \overline{C} &= iB, \\
s_b B &= 0, \\
s_b (\mathscr{B} \cdot \mathscr{B}) &= 0, \\
s_b \overline{B} &= i\left(\overline{B} \times C\right), \\
s_b E &= i(E \times C), \\
s_b \mathscr{B} &= i(\mathscr{B} \times C),
\end{aligned}$$

$$
\begin{aligned}
s_b(\mathscr{B}\cdot E) &= 0,\\
s_{ab}A_\mu &= D_\mu\overline{C},\\
s_{ab}\overline{C} &= -\frac{i}{2}\left(\overline{C}\times\overline{C}\right),\\
s_{ab}C &= i\overline{B},\\
s_{ab}\overline{B} &= 0,\\
s_{ab}(\mathscr{B}\cdot\mathscr{B}) &= 0,\\
s_{ab}E &= i\left(E\times\overline{C}\right),\\
s_{ab}\mathscr{B} &= i\left(\mathscr{B}\times\overline{C}\right),\\
s_{ab}B &= i\left(B\times\overline{C}\right),\\
s_{ab}(\mathscr{B}\cdot E) &= 0,
\end{aligned}
\tag{2}
$$

because the Lagrangian densities $\mathscr{L}_B$ and $\mathscr{L}_{\overline{B}}$ transform under $s_{(a)b}$ as

$$
\begin{aligned}
s_b\mathscr{L}_B &= \partial_\mu\left(B\cdot D^\mu C\right),\\
s_{ab}\mathscr{L}_{\overline{B}} &= -\partial_\mu\left(\overline{B}\cdot D^\mu\overline{C}\right),\\
s_{ab}\mathscr{L}_B &= -\partial_\mu\left[\left\{\overline{B}+\left(C\times\overline{C}\right)\right\}\cdot\partial^\mu\overline{C}\right]\\
&\quad+\left\{B+\overline{B}+\left(C\times\overline{C}\right)\right\}\cdot D_\mu\partial^\mu\overline{C},\\
s_b\mathscr{L}_{\overline{B}} &= \partial_\mu\left[\left\{B+\left(C\times\overline{C}\right)\right\}\cdot\partial^\mu C\right]\\
&\quad-\left\{B+\overline{B}+\left(C\times\overline{C}\right)\right\}\cdot D_\mu\partial^\mu C.
\end{aligned}
\tag{3}
$$

It should be noted that *both* the Lagrangian densities in (1) respect *both* (i.e., BRST and anti-BRST) symmetries on the constrained hypersurface, where the CF condition ($B+\overline{B}+(C\times\overline{C})=0$) is satisfied. In other words, we note that $s_b\mathscr{L}_{\overline{B}}=-\partial_\mu[\overline{B}\cdot\partial^\mu C]$ and $s_{ab}\mathscr{L}_B=\partial_\mu[B\cdot\partial^\mu\overline{C}]$ because of the validity of CF condition. As a consequence, the action integrals $S=\int d^2x\mathscr{L}_B$ and $S=\int d^2x\mathscr{L}_{\overline{B}}$ remain invariant under the (anti-)BRST symmetries on the above hypersurface located in the 2D Minkowskian spacetime manifold. It is interesting to point out that the absolute anticommutativity $\{s_b, s_{ab}\}=0$ is *also* satisfied on the above hypersurface, which is defined by the field equation: $B+\overline{B}+(C\times\overline{C})=0$.

According to the celebrated Noether's theorem, the above continuous symmetries lead to the derivations of conserved currents and charges. These (anti-)BRST charges, corresponding to the above continuous symmetries $s_{(a)b}$, are (see, e.g., [14] for details)

$$
\begin{aligned}
Q_{ab} &= \int dx\left[\dot{\overline{B}}\cdot\overline{C}-\overline{B}\cdot D_0\overline{C}+\frac{1}{2}\left(\overline{C}\times\overline{C}\right)\cdot\dot{C}\right],\\
Q_b &= \int dx\left[B\cdot D_0C-\dot{B}\cdot C-\frac{1}{2}\dot{\overline{C}}\cdot(C\times C)\right],
\end{aligned}
\tag{4}
$$

where a single dot on a field denotes the ordinary time derivative (e.g., $\dot{C}=\partial C/\partial t$).

The above conserved charges $Q_{(a)b}$ are nilpotent ($Q_b^2=Q_{ab}^2=0$) of order two and they obey absolute anticommutativity property (i.e., $Q_bQ_{ab}+Q_{ab}Q_b=0$). These properties can be mathematically expressed as follows:

$$
\begin{aligned}
s_bQ_b &= -i\left\{Q_b,Q_b\right\}=0\Longrightarrow\\
Q_b^2 &= 0,\\
s_{ab}Q_{ab} &= -i\left\{Q_{ab},Q_{ab}\right\}=0\Longrightarrow\\
Q_{ab}^2 &= 0,\\
s_{ab}Q_b &= -i\left\{Q_b,Q_{ab}\right\}=0\Longrightarrow\\
\left\{Q_b,Q_{ab}\right\} &= 0\Longleftrightarrow\\
s_bQ_{ab} &= -i\left\{Q_{ab},Q_b\right\}=0.
\end{aligned}
\tag{5}
$$

The preciseness of the above expressions can be verified by taking into account the nilpotent (anti-)BRST symmetry transformation $s_{(a)b}$ (cf. (2)) and expressions for the nilpotent (anti-)BRST charges from (4). It should be noted that the property of absolute anticommutativity of the (anti-)BRST charges (i.e., $\{Q_b,Q_{ab}\}=0$) is *true* only when we use the CF condition (i.e., $B+\overline{B}+(C\times\overline{C})=0$).

The Lagrangian densities (1) *also* respect the following off-shell nilpotent ($s_{(a)d}^2=0$) and absolutely anticommuting ($s_ds_{ad}+s_{ad}s_d=0$) (anti-)co-BRST [i.e., (anti)dual BRST] symmetry transformations ($s_{(a)d}$) (see, e.g., [13, 14]):

$$
\begin{aligned}
s_{ad}A_\mu &= -\varepsilon_{\mu\nu}\partial^\nu C,\\
s_{ad}C &= 0,\\
s_{ad}\overline{C} &= i\mathscr{B},\\
s_{ad}B &= 0,\\
s_{ad}\overline{B} &= 0,\\
s_{ad}E &= D_\mu\partial^\mu C,\\
s_{ad}\left(\partial_\mu A^\mu\right) &= 0,\\
s_{ad}\mathscr{B} &= 0,\\
s_dA_\mu &= -\varepsilon_{\mu\nu}\partial^\nu\overline{C},\\
s_d\overline{C} &= 0,\\
s_dC &= -i\mathscr{B},\\
s_dB &= 0,\\
s_d\overline{B} &= 0,\\
s_dE &= D_\mu\partial^\mu\overline{C},\\
s_d\left(\partial_\mu A^\mu\right) &= 0,\\
s_d\mathscr{B} &= 0,
\end{aligned}
\tag{6}
$$

because the above Lagrangian densities transform, under $s_{(a)d}$, as follows:

$$
\begin{aligned}
s_{ad}\mathscr{L}_{\overline{B}} &= \partial_\mu\left[\mathscr{B}\cdot\partial^\mu C\right],\\
s_{d}\mathscr{L}_{B} &= \partial_\mu\left[\mathscr{B}\cdot\partial^\mu \overline{C}\right],\\
s_{ad}\mathscr{L}_{B} &= \partial_\mu\left[\mathscr{B}\cdot D^\mu C+\varepsilon^{\mu\nu}\overline{C}\cdot\left(\partial_\nu C\times C\right)\right]+i\left(\partial_\mu A^\mu\right)\\
&\quad\cdot\left(\mathscr{B}\times C\right),\\
s_{d}\mathscr{L}_{\overline{B}} &= \partial_\mu\left[\mathscr{B}\cdot D^\mu \overline{C}-\varepsilon^{\mu\nu}C\cdot\left(\partial_\nu \overline{C}\times \overline{C}\right)\right]+i\left(\partial_\mu A^\mu\right)\\
&\quad\cdot\left(\mathscr{B}\times \overline{C}\right).
\end{aligned}
\tag{7}
$$

It is clear that *both* the Lagrangian densities respect *both* (i.e., co-BRST and anti-co-BRST) fermionic symmetry transformations on a hypersurface, where the CF-type restrictions $\mathscr{B}\times C=0$ and $\mathscr{B}\times\overline{C}=0$ are satisfied. We lay emphasis on the observation that absolute anticommutativity $\{s_d, s_{ad}\}=0$ is satisfied *without* any use of CF-type restrictions $\mathscr{B}\times C=0$ and $\mathscr{B}\times\overline{C}=0$. More elaborate discussions about these CF-type restrictions (and other related restrictions) can be found in our earlier works (see, e.g., [14, 16] for details).

The Noether conserved ($\dot{Q}_{(a)d}=0$) charges $Q_{(a)d}$, corresponding to the continuous and nilpotent symmetry transformations (6), are

$$
\begin{aligned}
Q_d &= \int dx\left[\mathscr{B}\cdot\dot{\overline{C}}+B\cdot\partial_1\overline{C}\right]\\
&\equiv \int dx\left[\mathscr{B}\cdot\dot{\overline{C}}-D_0\mathscr{B}\cdot\overline{C}+\left(\partial_1\overline{C}\times C\right)\cdot\overline{C}\right],\\
Q_{ad} &= \int dx\left[\mathscr{B}\cdot\dot{C}-\overline{B}\cdot\partial_1 C\right]\\
&\equiv \int dx\left[\mathscr{B}\cdot\dot{C}-D_0\mathscr{B}\cdot C-\left(\overline{C}\times\partial_1 C\right)\cdot C\right].
\end{aligned}
\tag{8}
$$

The above charges are found to be nilpotent ($Q_{(a)d}^2=0$) and absolutely anticommuting ($Q_dQ_{ad}+Q_{ad}Q_d=0$) in nature. These claims can be verified in a straightforward fashion by taking the help of symmetries (6) and expressions of the charges (8) as follows:

$$
\begin{aligned}
s_dQ_d &= -i\left\{Q_d,Q_d\right\}=0 \Longrightarrow\\
Q_d^2 &= 0,\\
s_{ad}Q_{ad} &= -i\left\{Q_{ad},Q_{ad}\right\}=0 \Longrightarrow\\
Q_{ad}^2 &= 0,\\
s_dQ_{ad} &= -i\left\{Q_{ad},Q_d\right\}=0 \Longleftrightarrow\\
s_{ad}Q_d &= -i\left\{Q_d,Q_{ad}\right\}=0 \Longrightarrow\\
\left\{Q_d,Q_{ad}\right\} &= 0.
\end{aligned}
\tag{9}
$$

In fact, in this simple proof, one has to verify the left-hand side of the above equations. In the forthcoming sections, we shall exploit the beauty and strength of the AVSA to BRST formalism to capture the above properties in a cogent and consistent manner.

3. Horizontality Condition: Off-Shell Nilpotent (Anti-)BRST Symmetry Transformations

We concisely mention here the key points associated with the geometrical origin of the nilpotent (anti-)BRST symmetries and existence of the CF condition within the framework of Banora-Tonin (BT) superfield formalism [4, 5]. In this connection, first of all, we generalize the 2D ordinary theory onto $(2,2)$-dimensional supermanifold, where the non-Abelian 1-form gauge field $A_\mu(x)$ and (anti)ghost fields $(\overline{C})C$ are generalized onto their corresponding superfields with the following expansions (incorporating the secondary fields $R_\mu, \overline{R}_\mu, S_\mu, B_1, B_2, \overline{B}_1, \overline{B}_2, s, \overline{s}$) on the $(2,2)$-dimensional supermanifolds [4, 5]:

$$
\begin{aligned}
&A_\mu(x)\longrightarrow\\
&B_\mu\left(x,\theta,\overline{\theta}\right)=A_\mu(x)+\theta\overline{R}_\mu(x)+\overline{\theta}R_\mu(x)+i\theta\overline{\theta}S_\mu(x),\\
&C(x)\longrightarrow\\
&F\left(x,\theta,\overline{\theta}\right)=C(x)+i\theta\overline{B}_1+i\overline{\theta}B_1+i\theta\overline{\theta}s(x),\\
&\overline{C}(x)\longrightarrow\\
&\overline{F}\left(x,\theta,\overline{\theta}\right)=\overline{C}(x)+i\theta\overline{B}_2+i\overline{\theta}B_2+i\theta\overline{\theta}\overline{s}(x),
\end{aligned}
\tag{10}
$$

where the supermanifold is characterized by the superspace coordinates $Z^M=(x^\mu,\theta,\overline{\theta})$. The 2D ordinary bosonic coordinates x^μ $(\mu=0,1)$ and the Grassmannian coordinates $(\theta,\overline{\theta})$ (with $\theta^2=\overline{\theta}^2=\theta\overline{\theta}+\overline{\theta}\theta=0$) specify the superspace coordinate Z^M and all the superfields, defined on the supermanifold, are function of them. The 2-form super curvature is

$$
\begin{aligned}
\widetilde{F}^{(2)} &= \left(\frac{dZ^M\wedge dZ^N}{2!}\right)\widetilde{F}_{MN}\left(x,\theta,\overline{\theta}\right)\\
&\equiv \widetilde{d}\widetilde{A}^{(1)}+i\left(\widetilde{A}^{(1)}\wedge\widetilde{A}^{(1)}\right),
\end{aligned}
\tag{11}
$$

where the super curvature tensor $\widetilde{F}_{MN}=(\widetilde{F}_{\mu\nu},\widetilde{F}_{\mu\theta},\widetilde{F}_{\mu\overline{\theta}},\widetilde{F}_{\theta\theta},\widetilde{F}_{\theta\overline{\theta}},\widetilde{F}_{\overline{\theta}\overline{\theta}})$. In the above equation, the ordinary exterior derivative $d=dx^\mu\partial_\mu$ and non-Abelian 1-form ($A^{(1)}=dx^\mu A_\mu$) gauge connection have been generalized onto the $(2,2)$-dimensional supermanifold as

$$
\begin{aligned}
d &= dx^\mu\partial_\mu\longrightarrow\\
\widetilde{d} &= dx^\mu\partial_\mu+d\theta\partial_\theta+d\overline{\theta}\partial_{\overline{\theta}},\\
\widetilde{d}^2 &= 0,\\
A^{(1)} &= dx^\mu A_\mu\longrightarrow\\
\widetilde{A}^{(1)} &= dx^\mu B_\mu\left(x,\theta,\overline{\theta}\right)+d\theta\overline{F}\left(x,\theta,\overline{\theta}\right)\\
&\quad+d\overline{\theta}F\left(x,\theta,\overline{\theta}\right),
\end{aligned}
\tag{12}
$$

where $(\partial_\mu, \partial_\theta, \partial_{\bar\theta})$ are the superspace derivatives (with $\partial_\mu = \partial/\partial x^\mu$, $\partial_\theta = \partial/\partial\theta$, and $\partial_{\bar\theta} = \partial/\partial\bar\theta$).

We have observed earlier that the kinetic term $(-(1/4)F_{\mu\nu}\cdot F^{\mu\nu} = \mathscr{B}\cdot E - (\mathscr{B}\cdot\mathscr{B})/2)$ of the Lagrangian densities (1) remains invariant under the (anti-)BRST symmetries (2) and it has its origin in the exterior derivative d (i.e., $F^{(2)} = dA^{(1)} + iA^{(1)} \wedge A^{(1)}$). This gauge invariant quantity should remain independent of the Grassmannian variables $(\theta, \bar\theta)$ as the latter are only *mathematical* artifacts and they cannot be physically realized. Thus, we have the following equality due to the gauge invariant restriction (GIR):

$$-\frac{1}{4}\widetilde{F}_{MN}(x,\theta,\bar\theta)\cdot\widetilde{F}^{MN}(x,\theta,\bar\theta) = -\frac{1}{4}F_{\mu\nu}(x)\cdot F^{\mu\nu}(x). \tag{13}$$

The celebrated horizontality condition (HC) requires that the Grassmannian components of $\widetilde{F}_{MN}(x,\theta,\bar\theta) = (\widetilde{F}_{\mu\nu}, \widetilde{F}_{\mu\theta}, \widetilde{F}_{\mu\bar\theta}, \widetilde{F}_{\theta\theta}, \widetilde{F}_{\theta\bar\theta}, \widetilde{F}_{\bar\theta\bar\theta})$ should be set equal to zero so that, ultimately, we should have the following equality:

$$-\frac{1}{4}\widetilde{F}_{\mu\nu}(x,\theta,\bar\theta)\cdot\widetilde{F}^{\mu\nu}(x,\theta,\bar\theta) = -\frac{1}{4}F_{\mu\nu}(x)\cdot F^{\mu\nu}(x). \tag{14}$$

The requirement of HC leads to the following [4, 5, 11, 12]:

$$\begin{aligned}
R_\mu(x) &= D_\mu C,\\
\bar{R}_\mu(x) &= D_\mu \bar{C},\\
s &= i\left(\bar{B}\times C\right),\\
\bar{s} &= -i\left(B\times\bar{C}\right),\\
S_\mu &= \left(D_\mu B + D_\mu C\times\bar{C}\right)\\
&\equiv -\left(D_\mu\bar{B} + C\times D_\mu\bar{C}\right),\\
B_1 &= -\frac{1}{2}(C\times C),\\
\bar{B}_2 &= -\frac{1}{2}\left(\bar{C}\times\bar{C}\right),\\
B_1 + B_2 &+ \left(C\times\bar{C}\right) = 0,
\end{aligned} \tag{15}$$

where the last entry is nothing but the celebrated CF condition $(B + \bar{B} + (C\times\bar{C}) = 0)$ if we identify $\bar{B}_1 = \bar{B}$ and $B_2 = B$. It is crystal clear that the HC leads to the derivation of the secondary fields in terms of the auxiliary and basic fields of the starting Lagrangian densities (1). The substitution of the above expressions for the secondary fields into the super expansion (10) leads to the following [4, 5, 11, 12]:

$$\begin{aligned}
B_\mu^{(h)}(x,\theta,\bar\theta) &= A_\mu(x) + \theta\left(D_\mu\bar{C}\right) + \bar\theta\left(D_\mu C\right)\\
&\quad + i\theta\bar\theta\left[D_\mu B + D_\mu C\times\bar{C}\right]\\
&\equiv A_\mu(x) + \theta\left(s_{ab}A_\mu\right) + \bar\theta\left(s_b A_\mu\right)\\
&\quad + \theta\bar\theta\left(s_b s_{ab} A_\mu\right),\\
F^{(h)}(x,\theta,\bar\theta) &= C(x) + \theta\left(i\bar{B}\right) + \bar\theta\left[-\frac{i}{2}(C\times C)\right]\\
&\quad + \theta\bar\theta\left(-\bar{B}\times C\right)\\
&\equiv C(x) + \theta\left(s_{ab}C\right) + \bar\theta\left(s_b C\right)\\
&\quad + \theta\bar\theta\left(s_b s_{ab} C\right),\\
\bar{F}^{(h)}(x,\theta,\bar\theta) &= \bar{C}(x) + \theta\left[-\frac{i}{2}\left(\bar{C}\times\bar{C}\right)\right] + \bar\theta(iB)\\
&\quad + \theta\bar\theta\left(B\times\bar{C}\right)\\
&\equiv \bar{C}(x) + \theta\left(s_{ab}\bar{C}\right) + \bar\theta\left(s_b\bar{C}\right)\\
&\quad + \theta\bar\theta\left(s_b s_{ab}\bar{C}\right),
\end{aligned} \tag{16}$$

where the superscript (h) on the superfields denotes the fact that these superfields have been obtained after the application of HC. A close look at the above expressions demonstrates that the coefficients of $(\theta, \bar\theta)$ are nothing but the anti-BRST and BRST transformations (2), respectively, which have been listed for the Lagrangian densities (1).

Due to application of HC, ultimately, we obtain the following expression for the super curvature tensor (as we have already set $\widetilde{F}_{\mu\theta} = \widetilde{F}_{\mu\bar\theta} = \widetilde{F}_{\theta\bar\theta} = \widetilde{F}_{\theta\theta} = \widetilde{F}_{\bar\theta\bar\theta} = 0$):

$$\widetilde{F}_{\mu\nu}^{(h)}(x,\theta,\bar\theta) = \partial_\mu B_\nu^{(h)} - \partial_\nu B_\mu^{(h)} + i\left(B_\mu^{(h)}\times B_\nu^{(h)}\right). \tag{17}$$

Substitution of the expression for $B_\mu^{(h)}(x,\theta,\bar\theta)$, from (16), yields

$$\begin{aligned}
\widetilde{F}_{\mu\nu}^{(h)}(x,\theta,\bar\theta) &= F_{\mu\nu}(x) + \theta\left(iF_{\mu\nu}\times\bar{C}\right) + \bar\theta\left(iF_{\mu\nu}\times C\right)\\
&\quad + \theta\bar\theta\left[-\left(F_{\mu\nu}\times C\right)\times\bar{C} - F_{\mu\nu}\times B\right]\\
&\equiv F_{\mu\nu}^{(h)} + \theta\left(s_{ab}F_{\mu\nu}\right) + \bar\theta\left(s_b F_{\mu\nu}\right)\\
&\quad + \theta\bar\theta\left(s_b s_{ab} F_{\mu\nu}\right),
\end{aligned} \tag{18}$$

which leads to the derivation of the (anti-)BRST symmetry transformations for $F_{\mu\nu}$ (cf. (2)). It is now crystal clear that the requirements of gauge invariant restrictions in (14) and (13) are satisfied due to HC and, in this process, we have obtained the (anti-)BRST symmetry transformations for *all* the fields (as well as the CF condition) for our theory. We have derived these (anti-)BRST symmetry transformations by exploiting the potential of (anti)chiral superfields approach to BRST formalism in Appendix A.

4. Dual Horizontality Condition: Nilpotent (Anti-)Co-BRST Symmetry Transformations

We exploit here the dual-HC (DHC) to derive the (anti-)co-BRST symmetry transformations for the (anti)ghost fields and basic tenets of AVSA to obtain the precise form of the (anti-)co-BRST symmetry transformations associated with the gauge field ($A_\mu = A_\mu \cdot T$) of our 2D non-Abelian theory. In this context, first of all, we note that the gauge-fixing term ($\partial_\mu A^\mu$) has its origin in the coexterior derivative ($\delta = - * d *$) of the differential geometry in the following sense (see, e.g., [21–24] for details):

$$\delta A^{(1)} = - * d * \left(dx^\mu A_\mu\right) = \partial_\mu A^\mu, \qquad \delta^2 = 0, \tag{19}$$

where $\delta = - * d*$ is the coexterior derivative and $*$ is the Hodge duality operator on 2D Minkowskian *flat* spacetime manifold. It is clear that the Lorentz gauge-fixing term ($\partial_\mu A^\mu$) is a 0-form which emerges out from the 1-form ($A^{(1)} = dx^\mu A_\mu$) due to application of the coexterior derivative ($\delta = - * d*$) which reduces the degree of a form by one.

We have seen that the gauge-fixing term ($\partial_\mu A^\mu$) remains invariant under the (anti-)co-BRST symmetry transformations (cf. (6)). We generalize this observation onto our chosen $(2,2)$-dimensional supermanifold as follows:

$$\widetilde{\delta}\widetilde{A}^{(1)} = \delta A^{(1)}, \qquad \widetilde{\delta} = - \star \widetilde{d} \star, \qquad \widetilde{\delta}^2 = 0 \qquad \widetilde{d}^2 = 0, \tag{20}$$

where $\widetilde{\delta}$ is the super coexterior derivative defined on the $(2,2)$-dimensional supermanifold and $\star$ is the Hodge duality operator on the $(2,2)$-dimensional supermanifold (see, e.g., [29] for details). The left-hand side of (20) has already been computed in our previous work [29]. We quote here the result of operation of $\widetilde{\delta}$ on $\widetilde{A}^{(1)}$ as 0-form; namely,

$$\partial_\mu B^\mu + \partial_\theta \overline{F} + \partial_{\overline{\theta}} F + s^{\overline{\theta}\overline{\theta}} \left(\partial_{\overline{\theta}} \overline{F}\right) + s^{\theta\theta} \left(\partial_\theta F\right) = \partial_\mu A^\mu, \tag{21}$$

where $s^{\theta\theta}$ and $s^{\overline{\theta}\overline{\theta}}$ appear in the following Hodge duality $\star$ operation:

$$\star \left(dx_\mu \wedge dx_\nu \wedge d\overline{\theta} \wedge d\overline{\theta}\right) = \varepsilon_{\mu\nu} s^{\overline{\theta}\overline{\theta}}, \qquad \star \left(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta\right) = \varepsilon_{\mu\nu} s^{\theta\theta}. \tag{22}$$

These factors (i.e., $s^{\theta\theta}$ and $s^{\overline{\theta}\overline{\theta}}$) are essential to get back the 4-forms (($dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta$) and ($dx_\mu \wedge dx_\nu \wedge d\overline{\theta} \wedge d\overline{\theta}$)) if we apply *another* $\star$ on (22). In other words, we have super Hodge duality $\star$ on the 0-form as follows:

$$\star \left(\varepsilon_{\mu\nu} s^{\theta\theta}\right) = \pm \left(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta\right), \qquad \star \left(\varepsilon_{\mu\nu} s^{\overline{\theta}\overline{\theta}}\right) = \pm \left(dx_\mu \wedge dx_\nu \wedge d\overline{\theta} \wedge d\overline{\theta}\right). \tag{23}$$

The equality in (21) ultimately leads to

$$\partial_\theta F = 0, \qquad \partial_{\overline{\theta}} \overline{F} = 0, \qquad \partial_\mu B^\mu + \partial_\theta \overline{F} + \partial_{\overline{\theta}} F = \partial_\mu A^\mu \tag{24}$$

because of the fact that there are *no* terms carrying the factors $s^{\theta\theta}$ and $s^{\overline{\theta}\overline{\theta}}$ on the right-hand side.

At this stage, we substitute the expressions of $B_\mu(x,\theta,\overline{\theta})$, $F(x,\theta,\overline{\theta})$, and $\overline{F}(x,\theta,\overline{\theta})$ into (24) to derive the following important relationships:

$$\partial_\mu R^\mu = 0, \qquad \partial_\mu \overline{R}^\mu = 0, \qquad \partial_\mu S^\mu = 0, \qquad s = 0, \qquad \overline{B}_1 = 0, \qquad B_2 = 0, \qquad \overline{s} = 0, \qquad B_1 + \overline{B}_2 = 0. \tag{25}$$

The last entry, in the above, is just like the CF-type restriction which is *trivial*. With the choices $B_1 = -\mathscr{B}$ and $\overline{B}_2 = \mathscr{B}$, we obtain the following expansions:

$$F^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right) = C(x) + \overline{\theta}(-i\mathscr{B}) \equiv C(x) + \overline{\theta}\left(s_d C\right), \qquad \overline{F}^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right) = \overline{C}(x) + \theta(i\mathscr{B}) \equiv \overline{C}(x) + \overline{\theta}\left(s_{ad}\overline{C}\right), \tag{26}$$

where the superscript (dh) denotes the expansions of the superfields after the application of DHC. It is self-evident that we have already obtained the (anti-)co-BRST symmetry transformation (4) for the (anti)ghost fields $(\overline{C})C$ of our theory as

$$s_d C = -i\mathscr{B}, \qquad s_{ad} C = 0, \qquad s_d \overline{C} = 0, \qquad s_{ad} C = i\mathscr{B}. \tag{27}$$

Thus, the DHC leads to the derivation of (anti-)co-BRST symmetry transformations for the (anti)ghost fields and very useful restrictions on the secondary fields in (25).

We are now in the position to derive the (anti-)co-BRST symmetry transformations $s_{(a)d}$ for the gauge field A_μ. We exploit here the idea of AVSA to BRST formalism, which states that the (anti-)co-BRST invariant quantities should be independent of the "soul" coordinates $(\theta,\overline{\theta})$. During the early days of the developments of superspace technique, the bosonic coordinates x^μ of the superspace coordinates $Z^M = (x^\mu,\theta,\overline{\theta})$ were called the "body" coordinates and the Grassmannian variables $(\theta,\overline{\theta})$ were christened as the "soul" coordinates. In this context, we observe that the following is true:

$$s_{(a)d}\left[\varepsilon^{\mu\nu} A_\nu \cdot \partial_\mu \mathscr{B} - i\partial_\mu \overline{C} \cdot \partial^\mu C\right] = 0. \tag{28}$$

Thus, we have the following equality due to AVSA to BRST formalism:

$$\begin{aligned} &\varepsilon^{\mu\nu} B_\nu \left(x,\theta,\bar\theta\right)\cdot\partial_\mu \mathscr{B}(x) - i\partial_\mu \bar F^{(\text{dh})}\left(x,\theta,\bar\theta\right) \\ &\quad\cdot \partial^\mu F^{(\text{dh})}\left(x,\theta,\bar\theta\right) \\ &\quad\equiv \varepsilon^{\mu\nu} A_\nu(x)\cdot\partial_\mu\mathscr{B}(x) - i\partial_\mu \bar C(x)\cdot\partial^\mu C(x). \end{aligned} \tag{29}$$

The substitution of the expansions from (26) yields the following:

$$\begin{aligned} &\varepsilon^{\mu\nu}\bar R_\nu + \partial^\mu C = 0, \\ &\varepsilon^{\mu\nu} R_\nu + \partial^\mu \bar C = 0, \\ &\varepsilon^{\mu\nu} S_\nu - \partial^\mu \mathscr{B} = 0. \end{aligned} \tag{30}$$

It is worthwhile to point out that we have *not* taken any super expansion of $\mathscr{B}(x)$ on the left-hand side in (29) because of the fact that $s_{(a)d}\mathscr{B}(x) = 0$. In other words, we have taken $\mathscr{B}(x)\to\widetilde{\mathscr{B}}(x,\theta,\bar\theta) = \mathscr{B}(x)$. Ultimately, the relation in (30) produces the following:

$$\begin{aligned} R_\mu &= -\varepsilon_{\mu\nu}\partial^\nu \bar C, \\ \bar R_\mu &= -\varepsilon_{\mu\nu}\partial^\nu C, \\ S_\mu &= \varepsilon_{\mu\nu}\partial^\nu \mathscr{B}. \end{aligned} \tag{31}$$

The substitution of these expressions into the super expansions of $B_\mu(x,\theta,\bar\theta)$ leads to the following (in terms of the (anti-)co-BRST symmetry transformations (6)):

$$\begin{aligned} B_\mu^{(\text{dg})}\left(x,\theta,\bar\theta\right) &= A_\mu(x) + \theta\left(\varepsilon_{\mu\nu}\partial^\nu C\right) + \bar\theta\left(-\varepsilon_{\mu\nu}\partial^\nu \bar C\right) \\ &\quad + \theta\bar\theta + \left(\varepsilon_{\mu\nu}\partial^\nu\mathscr{B}\right) \\ &\equiv A_\mu(x) + \theta\left(s_{ad}A_\mu\right) + \bar\theta\left(s_d A_\mu\right) \\ &\quad + \theta\bar\theta\left(s_d s_{ad} A_\mu\right). \end{aligned} \tag{32}$$

Here the superscript (dg) on $B_\mu(x,\theta,\bar\theta)$ denotes the expansion that has been obtained after the application of (anti-)co-BRST (i.e., dual gauge) invariant restriction (29). We end this section with the remark that we have obtained *all* the (anti-)co-BRST symmetry transformations for our 2D non-Abelian 1-form gauge theory by exploiting the theoretical strength of DHC and basic tenets of AVSA to BRST formalism.

5. Nilpotency and Absolute Anticommutativity of the Fermionic Charges: Ordinary 2D Spacetime

We, first of all, capture the nilpotency and absolute anticommutativity of the (anti-)BRST and (anti-)co-BRST charges in the *ordinary* space where the concepts/ideas behind the continuous symmetry and their generators (as well as the nilpotency of the (anti-)BRST and (anti-)co-BRST symmetry transformations) play very important roles. We would like to lay stress on the fact that some of the key results of our present section have been obtained due to our knowledge of the AVSA to BRST formalism that is contained in Section 6. Towards this goal in mind, we observe a few aspects of the conserved charges (listed in (4) and (8)) corresponding to the (anti-)BRST and (anti-)co-BRST symmetries of the Lagrangian densities (1). Using the (anti-)BRST and (anti-)co-BRST symmetry transformations of (2) and (6), we observe that the following are true:

$$\begin{aligned} Q_b &= s_b\left(\int dx\left[B\cdot A_0 + i\dot{\bar C}\cdot C\right]\right), \\ Q_{ab} &= s_{ab}\left(\int dx\left[i\bar C\cdot\dot C - \bar B\cdot A_0\right]\right), \\ Q_d &= s_d\left(\int dx\left[\mathscr{B}\cdot A_1 + B\cdot A_0\right]\right), \\ Q_{ad} &= s_{ad}\left(\int dx\left[\mathscr{B}\cdot A_1 - \bar B\cdot A_0\right]\right). \end{aligned} \tag{33}$$

It should be noted that we have expressed the conserved charges in (4) and (8) in terms of the transformations in (2) and (6). It is elementary now to check that the nilpotency of the charges is satisfied:

$$\begin{aligned} s_b Q_b &= -i\left\{Q_b, Q_b\right\} = 0, \\ s_{ab} Q_{ab} &= -i\left\{Q_{ab}, Q_{ab}\right\} = 0, \\ s_d Q_d &= -i\left\{Q_d, Q_d\right\} = 0, \\ s_{ad} Q_{ad} &= -i\left\{Q_{ad}, Q_{ad}\right\} = 0, \end{aligned} \tag{34}$$

due to the nilpotency properties of (anti-)BRST and (anti-)co-BRST symmetry transformations (i.e., $s^2_{(a)b} = 0$, $s^2_{(a)d} = 0$). In the above, we have used the basic principles behind the continuous symmetries and symmetry generators (as the conserved charges of the theory). We *also* point out that we have taken into account *one* of the expressions for $Q_{(a)b}$ and $Q_{(a)d}$ from (4) and (8) which have been explicitly derived in Section 2.

To prove the absolute anticommutativity properties of the (anti-)BRST and (anti-) co-BRST conserved charges, we note the following useful relationships:

$$\begin{aligned} Q_d &= s_{ad}\left[\int dx\left(-i\bar C\cdot\dot{\bar C} + \frac{\bar C}{2}\cdot\left(A_0\times\bar C\right)\right)\right], \\ Q_{ad} &= s_d\left[\int dx\left(iC\cdot\dot C - \frac{C}{2}\cdot\left(A_0\times C\right)\right)\right], \\ Q_b &= s_{ab}\left[\int dx\left(iC\cdot\dot C - \frac{C}{2}\cdot\left(A_0\times C\right)\right)\right], \\ Q_{ab} &= s_b\left[\int dx\left(-i\bar C\cdot\dot{\bar C} + \frac{\bar C}{2}\cdot\left(A_0\times\bar C\right)\right)\right], \end{aligned} \tag{35}$$

which establish the absolute anticommutativity properties of the (anti-)co-BRST and nilpotent (anti-)BRST charges as follows:

$$\begin{aligned} s_{ad} Q_d &= -i \{Q_d, Q_{ad}\} = 0, \\ s_d Q_{ad} &= -i \{Q_{ad}, Q_d\} = 0, \\ s_{ab} Q_b &= -i \{Q_b, Q_{ab}\} = 0, \\ s_b Q_{ab} &= -i \{Q_{ab}, Q_b\} = 0, \end{aligned} \quad (36)$$

due to, once again, the nilpotency ($s^2_{(a)b} = 0$, $s^2_{(a)d} = 0$) properties of the (anti-)BRST and (anti-)co-BRST symmetry transformations. It is interesting to point out that the expressions in the square brackets for the pair (Q_b, Q_{ad}) and the pair (Q_d, Q_{ab}) are *exactly* the *same* (as is evident from (35)). We would like to make a few remarks at this stage. A close look at (35) and (36) establishes one of the key observations that the nilpotency of symmetries *and* absolute anticommutativity properties of the conserved (anti-)BRST and (anti-)co-BRST charges are interrelated. Furthermore, we would like to mention that, in the expressions for $Q_{(a)d}$ in (35), we have dropped total space derivative terms in our computations. It is very important to emphasize here that, in the expressions for $Q_{(a)b}$ (cf. (4)), we have utilized the strength of CF condition ($B+\overline{B}+(C\times\overline{C}) = 0$) to recast these expressions in a *suitable* form before expressing them in the form in (35). To elaborate on it, we take a simple example where the expression for the BRST charges Q_b (cf. (4)), emerging from the Noether conserved current, is

$$Q_b = \int dx \left[B \cdot D_0 C - \dot{B} \cdot C - \frac{\dot{\overline{C}}}{2} \cdot (C \times C) \right]. \quad (37)$$

Using the CF condition $B+\overline{B}+(C\times\overline{C}) = 0$ (associated with the (anti-)BRST symmetries), we can recast the above expression in the following suitable form:

$$\begin{aligned} Q_b &= \int dx \left[\dot{\overline{B}} \cdot C - \overline{B} \cdot D_0 C - \left(C \times \overline{C}\right) \cdot D_0 C + \frac{\dot{\overline{C}}}{2} \right. \\ &\quad \left. \cdot (C \times C) + \left(\dot{C} \times \overline{C}\right) \cdot C \right] \\ &\equiv \int dx \left[\dot{\overline{B}} \cdot C - \overline{B} \cdot D_0 C + \frac{\dot{\overline{C}}}{2} \cdot (C \times C) - i \left(C \times \overline{C}\right) \right. \\ &\quad \left. \cdot (A_0 \times C) \right]. \end{aligned} \quad (38)$$

The above form of the BRST charge has been expressed in the anti-BRST *exact* form as given in (35). A similar kind of argument has gone into the expression for the anti-BRST charge Q_{ab} (cf. (35)), where we have been able to express it as the BRST *exact* form. No such kinds of arguments have been invoked in the cases of the (anti-)co-BRST charges (cf. (35)) which have been expressed as the co-BRST *exact* and anti-co-BRST *exact* forms.

We have modified the Lagrangian densities (1) in our earlier works [14, 16] by incorporating a couple of fermionic Lagrange multiplier fields (λ and $\overline{\lambda}$ with $\lambda^2 = \overline{\lambda}^2 = 0$ and $\lambda\overline{\lambda} + \overline{\lambda}\lambda = 0$) in such a manner that the modified Lagrangian densities [14, 16],

$$\begin{aligned} \mathscr{L}_B^{(\overline{\lambda})} &= \mathscr{B} \cdot E - \frac{1}{2} \mathscr{B} \cdot \mathscr{B} + B \cdot \left(\partial_\mu A^\mu\right) \\ &\quad + \frac{1}{2} \left(B \cdot B + \overline{B} \cdot \overline{B}\right) - i \partial_\mu \overline{C} \cdot D^\mu C + \overline{\lambda} \\ &\quad \cdot (\mathscr{B} \times C), \\ \mathscr{L}_{\overline{B}}^{(\lambda)} &= \mathscr{B} \cdot E - \frac{1}{2} \mathscr{B} \cdot \mathscr{B} - \overline{B} \cdot \left(\partial_\mu A^\mu\right) \\ &\quad + \frac{1}{2} \left(B \cdot B + \overline{B} \cdot \overline{B}\right) - i D_\mu \overline{C} \cdot \partial^\mu C + \lambda \\ &\quad \cdot \left(\mathscr{B} \times \overline{C}\right), \end{aligned} \quad (39)$$

respect the following *perfect* (anti-)co-BRST symmetries transformations:

$$\begin{aligned} s_{ad} A_\mu &= -\varepsilon_{\mu\nu} \partial^\nu C, \\ s_{ad} C &= 0, \\ s_{ad} \overline{C} &= i \mathscr{B}, \\ s_{ad} \mathscr{B} &= 0, \\ s_{ad} E &= D_\mu \partial^\mu C, \\ s_{ad} \left(\partial_\mu A^\mu\right) &= 0, \\ s_{ad} \lambda &= -i \left(\partial_\mu A^\mu\right), \\ s_{ad} \overline{\lambda} &= 0, \\ s_d A_\mu &= -\varepsilon_{\mu\nu} \partial^\nu \overline{C}, \\ s_d \overline{C} &= 0, \\ s_d C &= -i \mathscr{B}, \\ s_d \mathscr{B} &= 0, \\ s_d E &= D_\mu \partial^\mu \overline{C}, \\ s_d \left(\partial_\mu A^\mu\right) &= 0, \\ s_d \overline{\lambda} &= -i \left(\partial_\mu A^\mu\right), \\ s_d \lambda &= 0. \end{aligned} \quad (40)$$

It can be checked that the above (anti-)co-BRST symmetry transformations are off-shell nilpotent and absolutely anticommuting in nature (where we do not invoke any kinds of CF-type restrictions for its validity). We *also* note that the superscripts (λ) and $(\overline{\lambda})$ on the Lagrangian densities are logically correct because the Lagrange multipliers λ and

$\bar{\lambda}$ characterize these Lagrangian densities. Furthermore, we observe that these Lagrange multiplier fields carry the ghost numbers equal to (+1) and (−1), respectively. Finally, it can be explicitly checked that the following are true:

$$\begin{aligned} s_d \mathscr{L}_B^{(\bar{\lambda})} &= \partial_\mu \left[\mathscr{B} \cdot \partial^\mu \bar{C}\right], \\ s_{ad} \mathscr{L}_{\bar{B}}^{(\lambda)} &= \partial_\mu \left[\mathscr{B} \cdot \partial^\mu C\right], \\ s_d \mathscr{L}_{\bar{B}}^{(\lambda)} &= \partial_\mu \left[\mathscr{B} \cdot D^\mu \bar{C} - \varepsilon^{\mu\nu} \left(\partial_\nu \bar{C} \times \bar{C}\right) \cdot C\right], \\ s_{ad} \mathscr{L}_B^{(\bar{\lambda})} &= \partial_\mu \left[\mathscr{B} \cdot D^\mu C + \varepsilon^{\mu\nu} \bar{C} \cdot \left(\partial_\nu C \times C\right)\right], \end{aligned} \tag{41}$$

which demonstrate that the action integrals $S = \int d^2x \mathscr{L}_B^{(\bar{\lambda})}$ and $S = \int d^2x \mathscr{L}_{\bar{B}}^{(\lambda)}$ remain invariant under the (anti-)co-BRST symmetry transformations. We would like to lay emphasis on the fact that *both* the Lagrangian densities $\mathscr{L}_B^{(\bar{\lambda})}$ and $\mathscr{L}_{\bar{B}}^{(\lambda)}$ respect *both* the co-BRST and anti-co-BRST symmetries (cf. (40)) separately and independently.

A close look at the transformations (41) and (8) (cf. Section 2) demonstrates that the expressions for the charges $Q_d^{(\bar{\lambda})} = Q_d$ and $Q_{ad}^{(\lambda)} = Q_{ad}$ (cf. (8)) remain the *same* as far as the Lagrangian densities in (1) *and* $\mathscr{L}_B^{(\bar{\lambda})}$ as well as $\mathscr{L}_{\bar{B}}^{(\lambda)}$ are concerned. However, we note that the anti-co-BRST charge $Q_{ad}^{(\bar{\lambda})}$ (derived from the Lagrangian density $\mathscr{L}_B^{(\bar{\lambda})}$) and co-BRST charge $Q_d^{(\lambda)}$ (derived from the Lagrangian density $\mathscr{L}_{\bar{B}}^{(\lambda)}$) would be different from (8). These conserved charges and their expressions have been derived in our earlier work (see, e.g., [16] for details). We quote here these expressions explicitly:

$$\begin{aligned} Q_{ad}^{(\bar{\lambda})} &= \int dx \left[\mathscr{B} \cdot \dot{C} - \partial_1 B \cdot C + \bar{C} \cdot \left(\partial_1 C \times C\right)\right] \\ &\equiv \int dx \left[\mathscr{B} \cdot \dot{C} - D_0 \mathscr{B} \cdot C + \left(\partial_1 \bar{C} \times C\right) \cdot C + \bar{C} \right. \\ &\quad \left. \cdot \left(\partial_1 C \times C\right)\right] \equiv \int dx \left[\mathscr{B} \cdot \dot{C} - D_0 \mathscr{B} \cdot C - \bar{C} \right. \\ &\quad \left. \cdot \left(\partial_1 C \times C\right)\right], \\ Q_d^{(\lambda)} &= \int dx \left[\mathscr{B} \cdot \dot{\bar{C}} + \partial_1 \bar{B} \cdot \bar{C} - \left(\partial_1 \bar{C} \times \bar{C}\right) \cdot C\right] \\ &\equiv \int dx \left[\mathscr{B} \cdot \dot{\bar{C}} - D_0 \mathscr{B} \cdot \bar{C} - \left(\bar{C} \times \partial_1 C\right) \cdot \bar{C} \right. \\ &\quad \left. - \left(\partial_1 \bar{C} \times \bar{C}\right) \cdot C\right] \equiv \int dx \left[\mathscr{B} \cdot \dot{\bar{C}} - D_0 \mathscr{B} \cdot \bar{C} \right. \\ &\quad \left. + \left(\bar{C} \times \partial_1 \bar{C}\right) \cdot C\right]. \end{aligned} \tag{42}$$

In the above equivalent expressions, we have utilized the equations of motion (derived from the Lagrangian densities in (39)) and we have *also* dropped the total space derivative terms. To prove the nilpotency $[(Q_{ad}^{(\bar{\lambda})})^2 = 0,\ (Q_d^{(\lambda)})^2 = 0]$ of the above charges, we note that they can be expressed in terms of the (anti-)co-BRST transformations as

$$\begin{aligned} Q_{ad}^{(\bar{\lambda})} &= s_{ad} \left(\int dx \left[-i\bar{C} \cdot D_0 C + i\dot{\bar{C}} \cdot C\right]\right), \\ Q_d^{(\lambda)} &= s_d \left(\int dx \left[i\bar{C} \cdot \dot{C} - iD_0 \bar{C} \cdot C\right]\right). \end{aligned} \tag{43}$$

The above expressions for the (anti-)co-BRST charges produce the last entry in the expressions for the charges $Q_{ad}^{(\bar{\lambda})}$ and $Q_d^{(\lambda)}$ in (42). It can be now trivially checked that

$$\begin{aligned} s_{ad} Q_{ad}^{(\bar{\lambda})} &= -i \left\{Q_{ad}^{(\bar{\lambda})}, Q_{(ad)}^{(\bar{\lambda})}\right\} = 0 \Longleftrightarrow \\ s_{ad}^2 &= 0, \\ s_d Q_d^{(\lambda)} &= -i \left\{Q_d^{(\lambda)}, Q_d^{(\lambda)}\right\} = 0 \Longleftrightarrow \\ s_d^2 &= 0. \end{aligned} \tag{44}$$

Thus, we observe that the nilpotency of the charges $Q_{ad}^{(\bar{\lambda})}$ and $Q_d^{(\lambda)}$ is deeply connected with the nilpotency of the (anti-)-co-BRST symmetries (i.e., $s_{(a)d}^2 = 0$) when we exploit the beauty and strength of the connection between the continuous symmetries and their corresponding generators. We would like to state that the nilpotency of the charges $Q_d^{(\bar{\lambda})} = Q_d$ (cf. (8) and (9)) and $Q_{ad}^{(\lambda)} = Q_{ad}$ has already been proven in (9). This happens because of the fact that the expressions for $Q_{ad}^{(\lambda)}$ and $Q_d^{(\bar{\lambda})}$ are the same as given in (8) for the Lagrangian densities (1). Thus, we have proven the nilpotency of *all* the charges derived from the modified Lagrangian densities (39), where λ and $\bar{\lambda}$ are present.

We now focus on the proof of the property of absolute anticommutativity of the charges $Q_{ad}^{(\bar{\lambda})}$ and $Q_d^{(\lambda)}$, which are nontrivial (cf. (42)). In this connection, we would like to point out that the absolute anticommutativity of the charges $Q_d^{(\bar{\lambda})} = Q_d$ and $Q_{ad}^{(\lambda)} = Q_{ad}$ has already been proven in our present section itself. We note that the following are true:

$$\begin{aligned} Q_d^{(\lambda)} &= s_{ad} \left[\int dx \left(-i\bar{C} \cdot \dot{\bar{C}} + \frac{\bar{C}}{2} \cdot \left(A_0 \times \bar{C}\right)\right)\right], \\ Q_{ad}^{(\bar{\lambda})} &= s_d \left[\int dx \left(iC \cdot \dot{C} - \frac{C}{2} \cdot \left(A_0 \times C\right)\right)\right]. \end{aligned} \tag{45}$$

The above expressions demonstrate that the absolute anticommutativity property of the (anti-)co-BRST charges (i.e., $\{Q_d^{(\lambda)}, Q_{ad}^{(\bar{\lambda})}\} = 0$) is *true* and this property is primarily connected with the off-shell nilpotency ($s_{(a)d}^2 = 0$) of the (anti-)co-BRST symmetry transformations ($s_{(a)d}$) that are present in our 2D non-Abelian theory (cf. (40)). To corroborate the above statements, it is straightforward to note that

$$\begin{aligned} s_{ad} Q_d^{(\lambda)} &= -i \left\{Q_d^{(\lambda)}, Q_{ad}^{(\bar{\lambda})}\right\} = 0 \Longleftrightarrow \\ s_{ad}^2 &= 0, \end{aligned}$$

$$s_d Q_{ad}^{(\bar{\lambda})} = -i \left\{ Q_{ad}^{(\bar{\lambda})}, Q_d^{(\lambda)} \right\} = 0 \Longleftrightarrow$$
$$s_d^2 = 0. \tag{46}$$

From the above relationships, it is crystal clear that the absolute anticommutativity (i.e., $\{Q_d^{(\lambda)}, Q_{ad}^{(\bar{\lambda})}\} = 0$) for the (anti-)co-BRST charges is deeply connected with the nilpotency ($s_{(a)d}^2 = 0$) property of the (anti-)co-BRST symmetry transformations ($s_{(a)d}$) for the Lagrangian densities (39). We wrap up this section with the remark that we have proven the nilpotency and absolute anticommutativity properties of the (anti-)co-BRST charges for the Lagrangian densities (1) as well as (39) where we do *not* invoke any kinds of CF-type restrictions. This observation is *novel* and drastically different from the proof of the absolute anticommutativity property of the conserved and nilpotent (anti-)BRST charges where it is mandatory for us to invoke the CF condition.

6. Nilpotency and Absolute Anticommutativity of the Fermionic Charges: Superfield Approach

We express here the properties of nilpotency and absolute anticommutativity by exploiting the geometrical AVSA to BRST formalism. In this connection, first of all, we recall that the (anti-)BRST symmetry transformations $s_{(a)b}$ have been shown to be connected with the translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ along $(\theta, \bar{\theta})$-directions of the $(2,2)$-dimensional supermanifold through the following mappings:

$$s_b \longleftrightarrow \left.\frac{\partial}{\partial \bar{\theta}}\right|_{\theta=0},$$
$$s_{ab} \longleftrightarrow \left.\frac{\partial}{\partial \theta}\right|_{\bar{\theta}=0}. \tag{47}$$

We can very well choose the Grassmannian variables to be (θ_1, θ_2) and identify the nilpotent symmetries, $s_b \leftrightarrow \partial_{\theta_1}|_{\theta_2=0}$ and $s_{ab} \leftrightarrow \partial_{\theta_2}|_{\theta_1=0}$, because there are other nilpotent ($s_{(a)d}^2 = 0$) symmetries $s_{(a)d}$ in our theory, too. The *latter* nilpotent symmetries could be identified with translational generators as $s_d \leftrightarrow \partial_{\theta_3}|_{\theta_4=0}$ and $s_{ad} \leftrightarrow \partial_{\theta_4}|_{\theta_3=0}$, where we shall have another set of a pair of Grassmannian variables (θ_3, θ_4). However, for the sake of brevity, we have chosen only $(\theta, \bar{\theta})$ as the Grassmannian variables so that we could discuss the (anti-)BRST and (anti-)co-BRST symmetries separately and independently. The above mappings imply that the nilpotency of the (anti-)BRST symmetries (i.e., $s_{(a)b}^2 = 0$) is intimately connected with the nilpotency ($\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0$) of the translational generators $(\partial_\theta, \partial_{\bar{\theta}})$. This observation is utilized in expressing the expressions for the conserved and nilpotent (anti-)BRST charges in (33) as follows:

$$Q_{ab} = \frac{\partial}{\partial \theta} \int dx \left[i\bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot \dot{F}^{(h)}(x,\theta,\bar{\theta}) - \bar{B}(x) \cdot B_0^{(h)}(x,\theta,\bar{\theta}) \right]\Big|_{\bar{\theta}=0}$$
$$\equiv \int d\theta \int dx \left[i\bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot \dot{F}^{(h)}(x,\theta,\bar{\theta}) - \bar{B}(x) \cdot B_0^{(h)}(x,\theta,\bar{\theta}) \right]\Big|_{\bar{\theta}=0},$$
$$Q_b = \frac{\partial}{\partial \bar{\theta}} \int dx \left[B(x) \cdot B_0^{(h)}(x,\theta,\bar{\theta}) + i\dot{\bar{F}}^{(h)}(x,\theta,\bar{\theta}) \cdot F^{(h)}(x,\theta,\bar{\theta}) \right]\Big|_{\theta=0}$$
$$\equiv \int d\bar{\theta} \int dx \left[B(x) \cdot B_0^{(h)}(x,\theta,\bar{\theta}) + i\dot{\bar{F}}^{(h)}(x,\theta,\bar{\theta}) \cdot F^{(h)}(x,\theta,\bar{\theta}) \right]\Big|_{\theta=0}. \tag{48}$$

The above expressions establish the nilpotency of the (anti-)BRST charges $Q_{(a)b}$ because

$$\partial_\theta Q_{ab} = 0 \Longleftrightarrow$$
$$\partial_\theta^2 = 0 \Longleftrightarrow$$
$$s_{ab} Q_{ab} = -i \{Q_{ab}, Q_{ab}\} = 0,$$
$$\partial_{\bar{\theta}} Q_b = 0 \Longleftrightarrow$$
$$\partial_{\bar{\theta}}^2 = 0 \Longleftrightarrow$$
$$s_b Q_b = -i \{Q_b, Q_b\} = 0. \tag{49}$$

It should be noted that we do *not* invoke any kinds of CF-type restrictions for the proof of off-shell nilpotency of the above (anti-)BRST charges.

We capture now the absolute anticommutativity property of the (anti-)BRST symmetry generators $Q_{(a)b}$ in the language of the AVSA to BRST formalism. In this context, we concentrate on the expressions for (anti-)BRST charges that have been quoted in (35). It can be checked that we have the following expressions for these charges in the language of the AVSA to BRST formalism:

$$Q_{ab} = \frac{\partial}{\partial\bar{\theta}} \left[\int dx \left\{ -i\bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot \dot{\bar{F}}^{(h)}(x,\theta,\bar{\theta}) + \frac{1}{2}\bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot \left(B_0^{(h)}(x,\theta,\bar{\theta}) \times \bar{F}^{(h)}(x,\theta,\bar{\theta})\right) \right\} \right] \Big|_{\theta=0}$$
$$= \int d\bar{\theta} \left[\int dx \left\{ -i\bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot \dot{\bar{F}}^{(h)}(x,\theta,\bar{\theta}) + \frac{1}{2}\bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot \left(B_0^{(h)}(x,\theta,\bar{\theta}) \times \bar{F}^{(h)}(x,\theta,\bar{\theta})\right) \right\} \right] \Big|_{\theta=0},$$
$$Q_b = \frac{\partial}{\partial\theta} \left[\int dx \left\{ iF^{(h)}(x,\theta,\bar{\theta}) \cdot \dot{F}^{(h)}(x,\theta,\bar{\theta}) - \frac{1}{2}F^{(h)}(x,\theta,\bar{\theta}) \cdot \left(B_0^{(h)}(x,\theta,\bar{\theta}) \times F^{(h)}(x,\theta,\bar{\theta})\right) \right\} \right] \Big|_{\bar{\theta}=0}$$
$$= \int d\theta \left[\int dx \left\{ iF^{(h)}(x,\theta,\bar{\theta}) \cdot \dot{F}^{(h)}(x,\theta,\bar{\theta}) - \frac{1}{2}F^{(h)}(x,\theta,\bar{\theta}) \cdot \left(B_0^{(h)}(x,\theta,\bar{\theta}) \times F^{(h)}(x,\theta,\bar{\theta})\right) \right\} \right] \Big|_{\bar{\theta}=0}. \quad (50)$$

It is straightforward to note that the nilpotency properties of the translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ along the Grassmannian directions imply that

$$\partial_{\bar{\theta}} Q_{ab} = 0 \Longleftrightarrow$$
$$\partial_{\bar{\theta}}^2 = 0,$$
$$\partial_\theta Q_b = 0 \Longleftrightarrow$$
$$\partial_\theta^2 = 0. \quad (51)$$

The above observations lead us to draw the conclusion that the absolute anticommutativity $(Q_b Q_{ab} + Q_{ab} Q_b = 0)$ of the (anti-)BRST charges (cf. (36)) in the ordinary space can be captured in the language of the superfield approach to BRST formalism.

We briefly comment here on the expressions for the (anti-)co-BRST charges $Q_{(a)d}$ that have been expressed in *two* different ways in (33) and (35). We have established earlier that the following mappings are true in the cases of s_d and s_{ad}:

$$s_d \longleftrightarrow \lim_{\theta=0} \frac{\partial}{\partial\bar{\theta}},$$
$$s_{ad} \longleftrightarrow \lim_{\bar{\theta}=0} \frac{\partial}{\partial\theta}. \quad (52)$$

We can very well repeat here the previous footnote written in our manuscript. However, this would be *only* an academic exercise. The main issue is the fact that we discuss the (anti-)BRST and (anti-)co-BRST symmetries, within the framework of AVSA to BRST formalism, separately and independently. Thus, when we focus on (θ_1, θ_2), we do not bother about (θ_3, θ_4) and vice versa. This is precisely the reason why we have taken, for the sake of brevity, *only* the $(2,2)$-dimensional supermanifold for our discussion, where, at a time, only a pair of Grassmannian variables are taken into account. Thus, the nilpotency of the (anti-)co-BRST charges can be expressed in terms of the quantities on the $(2,2)$-dimensional supermanifold as follows:

$$Q_d = \frac{\partial}{\partial\bar{\theta}} \left[\int dx \left\{ \mathscr{B}(x) \cdot B_1^{(\mathrm{dg})}(x,\theta,\bar{\theta}) + B(x) \cdot B_0^{(\mathrm{dg})}(x,\theta,\bar{\theta}) \right\} \right] \Big|_{\theta=0}$$
$$\equiv \int d\bar{\theta} \int dx \left[\mathscr{B}(x) \cdot B_1^{(\mathrm{dg})}(x,\theta,\bar{\theta}) + B(x) \cdot B_0^{(\mathrm{dg})}(x,\theta,\bar{\theta}) \right] \Big|_{\theta=0}, \quad (53)$$

where the superscript (dg) denotes the superfields (cf. (32)) that have been obtained after the application of (anti-)co-BRST invariant restriction in (29). Similarly, we note that the following is correct:

$$Q_{ad} = \frac{\partial}{\partial\theta} \left[\int dx \left\{ \mathscr{B}(x) \cdot B_1^{(\mathrm{dg})}(x,\theta,\bar{\theta}) - \bar{B}(x) \cdot B_0^{(\mathrm{dg})}(x,\theta,\bar{\theta}) \right\} \right] \Big|_{\bar{\theta}=0} \quad (54)$$
$$\equiv \int d\theta \int dx \left[\mathscr{B}(x) \cdot B_1^{(\mathrm{dg})}(x,\theta,\bar{\theta}) - \bar{B}(x) \cdot B_0^{(\mathrm{dg})}(x,\theta,\bar{\theta}) \right] \Big|_{\bar{\theta}=0}.$$

It is crystal clear, from (53) and (54), that the following are true:

$$\partial_{\bar{\theta}} Q_d = 0 \Longleftrightarrow$$
$$\partial_{\bar{\theta}}^2 = 0,$$
$$\partial_\theta Q_{ad} = 0 \Longleftrightarrow$$
$$\partial_\theta^2 = 0. \quad (55)$$

The above relationships, in the ordinary 2D space, correspond to the following explicit expressions in the language of anticommutators:

$$s_d Q_d = -i\{Q_d, Q_d\} = 0 \Longleftrightarrow$$
$$Q_d^2 = 0 \Longleftrightarrow$$
$$s_d^2 = 0,$$
$$s_{ad} Q_{ad} = -i\{Q_{ad}, Q_{ad}\} = 0 \Longleftrightarrow$$
$$Q_{ad}^2 = 0 \Longleftrightarrow$$
$$s_{ad}^2 = 0. \quad (56)$$

Thus, we have captured the nilpotency property of the (anti-)co-BRST charges in the language of the quantities that are defined on the $(2,2)$-dimensional supermanifold. In fact, the nilpotency $(Q_{(a)d}^2 = 0)$ of the (anti-)co-BRST charges is deeply connected with the nilpotency $(\partial_\theta^2 = 0,\ \partial_{\bar{\theta}}^2 = 0)$ of the translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ along the Grassmannian directions $(\theta, \bar{\theta})$ of the $(2,2)$-dimensional supermanifold.

Now we dwell a bit on the absolute anticommutativity property of the (anti-)co-BRST charges $Q_{(a)d}$ that have been expressed in (35). Taking the inputs from (52), (32), and (26), we have the following:

$$
\begin{aligned}
Q_{ad} &= \frac{\partial}{\partial\bar\theta}\Big[\Big[\int dx\Big(iF^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big) - \frac{1}{2}F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big)\Big]\Big]\Big|_{\theta=0} \\
&\equiv \int d\bar\theta\Big[\Big[\int dx\Big(iF^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big) - \frac{1}{2}F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big)\Big]\Big]\Big|_{\theta=0}, \\
Q_{d} &= \frac{\partial}{\partial\theta}\Big[\Big[\int dx\Big(-i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big) + \frac{1}{2}\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big)\Big]\Big]\Big|_{\bar\theta=0} \\
&\equiv \int d\theta\int dx\Big[\Big[\int dx\Big(-i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big) + \frac{1}{2}\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big)\Big]\Big]\Big|_{\bar\theta=0}.
\end{aligned} \tag{57}
$$

Thus, the expressions for the (anti-)co-BRST charges (cf. (57)) imply that

$$
\begin{aligned}
\partial_\theta Q_d = 0 &\Longleftrightarrow \\
\partial_\theta^2 = 0 &\Longleftrightarrow \\
s_{ad} Q_d = -i\{Q_d, Q_{ad}\} &= 0, \\
\partial_{\bar\theta} Q_{ad} = 0 &\Longleftrightarrow \\
\partial_{\bar\theta}^2 = 0 &\Longleftrightarrow \\
s_d Q_{ad} = -i\{Q_{ad}, Q_d\} &= 0.
\end{aligned} \tag{58}
$$

The above expressions capture the absolute anticommutativity property of the (anti-)co-BRST charge (i.e., $\{Q_d, Q_{ad}\} = 0$) in the language of AVSA to BRST formalism. We observe, once again, that it is the nilpotency ($\partial_\theta^2 = \partial_{\bar\theta}^2 = 0$) of the translational generators $(\partial_\theta, \partial_{\bar\theta})$ that plays a decisive role in capturing the nilpotency as well as absolute anticommutativity properties of the (anti-)co-BRST charges in the terminology of AVSA to BRST formalism.

Finally, we would like to comment briefly on the nilpotency and absolute anticommutativity properties of the (anti-)co-BRST charges $(Q_d^{(\lambda)}, Q_{ad}^{(\bar\lambda)})$ that have been derived from the Lagrangian densities (39) and listed in (42) in different forms. We would like to lay emphasis on the fact that the Lagrangian densities (39) are very *special* in the sense that these Lagrangian densities respect proper (anti-)co-BRST symmetry transformations (listed in (40)) separately and independently (cf. (41)), where we do *not* invoke any kinds of CF-type restrictions from outside. Thus, as far as symmetry considerations are concerned, these Lagrangian densities are really beautiful from the point of view of the proper (anti-)co-BRST symmetry transformations (41). Within the framework of AVSA to BRST formalism, it can be checked that the expressions in (43) are

$$
\begin{aligned}
&Q_{ad}^{(\bar\lambda)} \\
&= \frac{\partial}{\partial\theta}\int dx\Big[-i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta) + \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big) + i\dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big]\Big|_{\bar\theta=0} \\
&\equiv \int d\theta\int dx\Big[-i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta) + \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big) + i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big]\Big|_{\bar\theta=0}, \\
&Q_{d}^{(\lambda)} \\
&= \frac{\partial}{\partial\bar\theta}\int dx\Big[i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta) - \dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot F^{(\mathrm{dh})}(x,\theta,\bar\theta) + \Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big)\cdot F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big]\Big|_{\theta=0} \\
&\equiv \int d\bar\theta\int dx\Big[i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta) - \dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot F^{(\mathrm{dh})}(x,\theta,\bar\theta) + \Big(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big)\cdot F^{(\mathrm{dh})}(x,\theta,\bar\theta)\Big]\Big|_{\theta=0},
\end{aligned} \tag{59}
$$

where the superfields with superscripts (dh) and (dg) have been explained in Section 4. It is clear, from the above expressions, that we have the following:

$$
\begin{aligned}
\partial_\theta Q_{ad}^{(\bar\lambda)} = 0 &\Longleftrightarrow \\
\partial_\theta^2 = 0 &\Longleftrightarrow \\
s_{ad} Q_{ad}^{(\bar\lambda)} = -i\Big\{Q_{ad}^{(\bar\lambda)}, Q_{ad}^{(\bar\lambda)}\Big\} &= 0, \\
\partial_{\bar\theta} Q_d^{(\lambda)} = 0 &\Longleftrightarrow
\end{aligned}
$$

$$\partial_{\bar\theta}^2 = 0 \Longleftrightarrow$$
$$s_d Q_d^{(\bar\lambda)} = -i\left\{Q_d^{(\lambda)}, Q_d^{(\lambda)}\right\} = 0. \tag{60}$$

The above relations prove the nilpotency of $Q_d^{(\lambda)}$ and $Q_{ad}^{(\bar\lambda)}$, which is also connected with the nilpotency of the translational generators $(\partial_\theta, \partial_{\bar\theta})$ along the Grassmannian directions $(\theta, \bar\theta)$ of the $(2,2)$-dimensional supermanifold on which our 2D *ordinary* theory is considered. To be more precise, the nilpotency of the above (anti-)co-BRST charges (which have been derived from the Lagrangian densities (39)) becomes very transparent when concentrating on the *third* and *sixth* lines in (60). In particular, the anticommutator of the conserved charges with *themselves* being zero immediately implies the nilpotency property (of these conserved charges). Let us now concentrate on the forms of the (anti-)co-BRST charges that have been written in (45). As is evident from (46), the absolute anticommutativity property of the (anti-)co-BRST charges is primarily hidden in (45) and is deeply connected with the nilpotency property of the (anti-)co-BRST symmetry transformations $(s_{(a)d})$. Thus, we express the forms of the (anti-)co-BRST charges (45) in the language of AVSA to BRST formalism as follows:

$$\begin{aligned}
Q_{ad}^{(\bar\lambda)} &= \frac{\partial}{\partial\bar\theta}\int dx\left[iF^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta) - \frac{1}{2}F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\left(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times F^{(\mathrm{dh})}(x,\theta,\bar\theta)\right)\right]\Big|_{\theta=0} \\
&\equiv \int d\bar\theta\int dx\left[iF^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot F^{(\mathrm{dh})}(x,\theta,\bar\theta) - \frac{1}{2}F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\left(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times F^{(\mathrm{dh})}(x,\theta,\bar\theta)\right)\right]\Big|_{\theta=0}, \\
Q_d^{(\lambda)} &= \frac{\partial}{\partial\theta}\int dx\left[-i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta) + \frac{1}{2}\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\left(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\right)\right]\Big|_{\bar\theta=0} \\
&\equiv \int d\theta\int dx\left[-i\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\dot{\bar F}^{(\mathrm{dh})}(x,\theta,\bar\theta) + \frac{1}{2}\bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\cdot\left(B_0^{(\mathrm{dg})}(x,\theta,\bar\theta)\times \bar F^{(\mathrm{dh})}(x,\theta,\bar\theta)\right)\right]\Big|_{\bar\theta=0},
\end{aligned} \tag{61}$$

where the superfields with superscripts (dh) and (dg) have already been explained in Section 4. It is straightforward to note, from the above equation, that

$$\begin{aligned}
\partial_\theta Q_d^{(\lambda)} &= 0 \Longleftrightarrow \\
\partial_\theta^2 &= 0 \Longleftrightarrow \\
s_{ad} Q_d^{(\lambda)} &= -i\left\{Q_d^{(\lambda)}, Q_{ad}^{(\bar\lambda)}\right\} = 0, \\
\partial_{\bar\theta} Q_{ad}^{(\bar\lambda)} &= 0 \Longleftrightarrow \\
\partial_{\bar\theta}^2 &= 0 \Longleftrightarrow \\
s_d Q_{ad}^{(\bar\lambda)} &= -i\left\{Q_{ad}^{(\bar\lambda)}, Q_d^{(\lambda)}\right\} = 0.
\end{aligned} \tag{62}$$

We end this section with the remark that the absolute anticommutativity property of the (anti-)co-BRST charges is deeply connected with the nilpotency property $(\partial_\theta^2 = \partial_{\bar\theta}^2 = 0)$ of the translational generators $(\partial_\theta, \partial_{\bar\theta})$ along the Grassmannian directions $(\theta, \bar\theta)$ of the $(2,2)$-dimensional supermanifold on which our 2D ordinary non-Abelian theory is generalized.

7. Conclusions

We have exploited the theoretical strength of the AVSA to BRST formalism to express the properties of the nilpotency and absolute anticommutativity of the fermionic conserved charges (i.e., (anti-)BRST and (anti-)co-BRST charges) of our self-interacting 2D non-Abelian theory (without any interaction with matter fields). We have *not* achieved this goal in our earlier works [9–12] on the AVSA to BRST formalism. Thus, the results in our present investigation are achieved for the first time. It is straightforward to express the nilpotency property of the fermionic (i.e., conserved (anti-)BRST and (anti-)co-BRST) charges in the language of the AVSA to BRST formalism. However, the property of absolute anticommutativity is captured, within the framework of AVSA to BRST formalism, by applying specific mathematical trick, where the CF condition plays a decisive role.

We would like to lay emphasis on the contents of Section 5, where we have been able to exploit the virtues of symmetry principles to express the (anti-)BRST and (anti-)co-BRST charges in various *exact forms*. These theoretical expressions have been exploited, in turn, to capture the nilpotency and absolute anticommutativity properties in the language of AVSA to BRST formalism in Section 6. We observe that the CF condition $(B + \bar B + (C \times \bar C) = 0)$ enables us to express the BRST charge as an anti-BRST *exact* form and anti-BRST charge as a BRST *exact* form. We would like to lay emphasis on the fact that the contents of Sections 5 and 6 are intertwined in an elegant manner. Although it appears, from our statements in this paragraph, that the contents of Section 5 have influenced our results in Section 6, we would like to stress that, many times, our understandings of the contents of Section 6 have influenced our results in Section 5. Thus, to be precise, the key results of our present endeavor are influenced by our knowledge of *both* the above sections which are interrelated. These results play an important role in establishing the absolute anticommutativity properties of the above fermionic charges. Thus, first of all, we have proven the nilpotency and absolute anticommutativity of the (anti-)BRST and (anti-)co-BRST charges in the language of symmetry properties (cf. Section 5). In particular, we have shown that it is the nilpotency of the (anti-)BRST and (anti-)co-BRST symmetry transformations that has played a decisive role in the proof of the above *properties* in the *ordinary* 2D space of our non-Abelian 1-form gauge theory. In fact, the results of Section 5 have been translated into the language of AVSA to BRST formalism in Section 6.

The proof of the nilpotency and absolute anticommutativity properties in the language of AVSA to BRST formalism for the fermionic (anti-)BRST and (anti-)co-BRST charges is a *novel* result because, in our earlier works on AVSA to BRST formalism [9–12], we have not achieved this goal. In our very recent works [30–32], we have captured the property of the absolute anticommutativity of nilpotent charges within the framework of (anti)chiral superfield approach to BRST formalism. However, we have *not* done *so* within the framework of AVSA to BRST formalism, where the *full* expansions of the superfields are taken into account. We plan to exploit our present idea to consolidate it by applying it to the cases of 1D toy model of the rigid rotor, 2D self-dual bosonic theory, modified versions of 2D Proca and anomalous gauge theory, and 4D Abelian 2-form and 6D Abelian 3-form gauge theories, where we have demonstrated the existence of (anti-)BRST and (anti-)co-BRST charges, as these theories are the models for the Hodge theory [18, 30–32].

Appendix

A. On the Derivation of (Anti-)BRST Symmetries

Here we derive the (anti-)BRST symmetries (cf. Section 2) by exploiting the simple (but fruitful) augmented version of (anti)chiral superfield approach (ACSA) to BRST formalism [30–32], where the (anti-)BRST invariant restrictions play very crucial roles. In this context, first of all, we generalize the basic 2D fields (e.g., A_μ, C, and $\bar{C}$) onto $(2,1)$-dimensional antichiral super-submanifold (of the general $(2,2)$-dimensional supermanifold) as

$$\begin{aligned} A_\mu(x) &\longrightarrow B_\mu(x,\bar{\theta}) = A_\mu(x) + \bar{\theta} R_\mu(x), \\ C(x) &\longrightarrow F(x,\bar{\theta}) = C(x) + i\bar{\theta} B_1(x), \\ \bar{C}(x) &\longrightarrow \bar{F}(x,\bar{\theta}) = \bar{C}(x) + i\bar{\theta} B_2(x), \end{aligned} \tag{A.1}$$

which are nothing but the limiting cases of the super expansions in (10) (that are on the general $(2,2)$-dimensional supermanifold). It is worthwhile to mention here that the Nakanishi-Lautrup auxiliary field $B(x)$ has *no* antichiral expansion (i.e., $B(x) \rightarrow \tilde{B}(x,\bar{\theta}) = B(x)$) because we note that $s_b B(x) = 0$. We further point out that $s_b(\bar{C} \times B) = 0$. This observation can be generalized onto the antichiral $(2,1)$-dimensional super-submanifold with the following restriction on the superfields due to the ACSA to BRST formalism:

$$\begin{aligned} \bar{F}(x,\bar{\theta}) \times \tilde{B}(x,\bar{\theta}) &= \bar{C}(x) \times B(x), \\ \tilde{B}(x,\bar{\theta}) &= B(x), \end{aligned} \tag{A.2}$$

which leads to $B_2 \times B = 0$. One of the nontrivial solutions is that B_2 is proportional to B. For the sake of brevity, however, we choose $B_2 = B$. The above restriction (A.2) is consistent with the basic tenets of AVSA/ACSA to BRST formalism, where we demand that the BRST invariant quantity should be independent of $\bar{\theta}$ variable. Thus, we have

$$\bar{F}^{(b)}(x,\bar{\theta}) = \bar{C}(x) + i\bar{\theta} B(x) \equiv \bar{C}(x) + \bar{\theta}(s_b \bar{C}), \tag{A.3}$$

where the superscript (b) denotes that the superfield $\bar{F}^{(b)}(x,\bar{\theta})$ has been obtained after the application of BRST invariant restriction (A.2). It goes without saying that (in the above process) we have derived the BRST transformation for $\bar{C}$ as $s_b \bar{C} = iB$ (cf. Section 2).

We carry out the above kinds of exercises to obtain the *other* BRST symmetry transformations associated with the other fields of the theory. In this context, first of all, we observe that the following are the useful BRST invariant quantities (in addition to the earlier BRST invariant quantities, $s_b B = 0$, $s_b(B \times \bar{C}) = 0$):

$$\begin{aligned} s_b(D_\mu C) &= 0, \\ s_b(C \times C) &= 0, \\ s_b(A^\mu \cdot \partial_\mu B + i\partial_\mu \bar{C} \cdot D^\mu C) &= 0. \end{aligned} \tag{A.4}$$

According to the AVSA/ACSA to BRST formalism, the above quantities can be generalized onto the $(2,1)$-dimensional antichiral super-submanifold and the corresponding superfields can be restricted to obey the following conditions:

$$\begin{aligned} &\partial_\mu F(x,\bar{\theta}) + i(B_\mu(x,\bar{\theta}) \times F(x,\bar{\theta})) = \partial_\mu C(x) \\ &\quad + i(A_\mu(x) \times C(x)), \\ &F(x,\bar{\theta}) \times F(x,\bar{\theta}) = C(x) \times C(x), \\ &B^\mu(x,\bar{\theta}) \cdot \partial_\mu B(x) + i\partial_\mu \bar{F}^{(b)}(x,\bar{\theta}) \cdot \partial^\mu F(x,\bar{\theta}) \\ &\quad - \partial_\mu \bar{F}^{(b)}(x,\bar{\theta}) \cdot (B^\mu(x,\bar{\theta}) \times F(x,\bar{\theta})) = A^\mu(x) \\ &\quad \cdot \partial_\mu B(x) + i\partial_\mu \bar{C}(x) \cdot \partial^\mu C(x) - \partial_\mu \bar{C}(x) \\ &\quad \cdot (A^\mu(x) \times C(x)). \end{aligned} \tag{A.5}$$

In other words, we demand that the left-hand side of the above equality should remain independent of "*soul*" coordinate $\bar{\theta}$. The above requirements lead to the following:

$$\begin{aligned} &D_\mu B_1(x) + R_\mu(x) \times C(x) = 0, \\ &B_1(x) \times C(x) = 0, \\ &R^\mu(x) \cdot \partial_\mu B(x) + \partial_\mu \bar{C}(x) \cdot (R^\mu(x) \times C(x)) \\ &\quad + \partial_\mu \bar{C}(x) \cdot D^\mu B_1(x) - \partial_\mu B(x) \cdot D^\mu C(x) = 0. \end{aligned} \tag{A.6}$$

We discuss here the solutions of the above conditions. It is clear that $B_1(x)$ is proportional to $(C(x) \times C(x))$ because we have obtained the condition $B_1(x) \times C(x) = 0$ in (A.6). Thus, the nontrivial expression for $B_1(x) = \kappa(C(x) \times C(x))$ where κ is a numerical constant. From the relation $D_\mu B_1(x) + R_\mu(x) \times C = 0$, it is clear that the choices,

$$\begin{aligned} B_1(x) &= -\frac{1}{2}(C(x) \times C(x)), \\ R_\mu(x) &= D_\mu C(x), \end{aligned} \tag{A.7}$$

satisfy the relation $B_1(x) \times C(x) = 0$ and $D_\mu B_1 + (R_\mu \times C) = 0$ *together*. It is gratifying to note that these conditions also satisfy the last relationship that has been quoted in (A.6). Thus, ultimately, we have obtained the following expansions (with superscript (b)):

$$\begin{aligned} B_\mu^{(b)}(x,\bar\theta) &= A_\mu(x) + \bar\theta(D_\mu C) \\ &\equiv A_\mu(x) + \bar\theta(s_b A_\mu(x)), \\ F^{(b)}(x,\bar\theta) &= C(x) + \bar\theta\left[-\frac{i}{2}(C \times C)\right] \\ &\equiv C(x) + \bar\theta(s_b C(x)), \\ \bar F^{(b)}(x,\bar\theta) &= \bar C(x) + \bar\theta(iB(x)) \\ &\equiv \bar C(x) + \bar\theta(s_b \bar C(x)). \end{aligned} \tag{A.8}$$

In other words, we have already derived the BRST symmetry transformations s_b for the basic fields $(A_\mu, C, \bar C)$ which are nothing but the coefficients of $\bar\theta$ in the superfields expansions (A.8). The sanctity of this statement can be checked from (2).

Finally, we comment on the derivation of the BRST symmetry transformations, $s_b\mathscr{B} = i(\mathscr{B} \times C)$, $s_b E = i(E \times C)$, and $s_b\bar B = i(\bar B \times C)$. In this context, we have the following generalizations on the $(2,1)$-dimensional antichiral super-submanifold:

$$\begin{aligned} \mathscr{B}(x) &\longrightarrow \tilde{\mathscr{B}}(x,\bar\theta) = \mathscr{B}(x) + \bar\theta P(x), \\ E(x) &\longrightarrow \tilde E(x,\bar\theta) = E(x) + \bar\theta Q(x), \\ \bar B(x) &\longrightarrow \tilde{\bar B}(x,\bar\theta) = \bar B(x) + \bar\theta S(x), \end{aligned} \tag{A.9}$$

where $(P(x), Q(x), S(x))$ are the fermionic secondary fields that have to be determined in terms of the basic and auxiliary fields of the theory. We note the following:

$$\begin{aligned} s_b(\mathscr{B} \times C) &= 0, \\ s_b(E \times C) &= 0, \\ s_b(\bar B \times C) &= 0. \end{aligned} \tag{A.10}$$

According to the basic tenets of AVSA/ACSA, we have the following equalities:

$$\begin{aligned} \tilde{\mathscr{B}}(x,\bar\theta) \times F^{(b)}(x,\bar\theta) &= \mathscr{B}(x) \times C(x), \\ \tilde E(x,\bar\theta) \times F^{(b)}(x,\bar\theta) &= E(x) \times C(x), \\ \tilde{\bar B}(x,\bar\theta) \times F^{(b)}(x,\bar\theta) &= \bar B(x) \times C(x), \end{aligned} \tag{A.11}$$

which show that the BRST invariant quantities of (A.10) should remain independent of the "soul" coordinate $\bar\theta$. This restriction yields the following:

$$\begin{aligned} P(x) &= i(\mathscr{B} \times C), \\ Q(x) &= i(E \times C), \\ S(x) &= i(\bar B \times C). \end{aligned} \tag{A.12}$$

Thus, ultimately, we have derived the following:

$$\begin{aligned} \tilde{\mathscr{B}}^{(b)}(x,\bar\theta) &= \mathscr{B}(x) + \bar\theta[i(\mathscr{B} \times C)] \\ &\equiv \mathscr{B}(x) + \bar\theta(s_b\mathscr{B}(x)), \\ \tilde E^{(b)}(x,\bar\theta) &= E(x) + \bar\theta[i(E \times C)] \\ &\equiv E(x) + \bar\theta(s_b E(x)), \\ \tilde{\bar B}^{(b)}(x,\bar\theta) &= \bar B(x) + \bar\theta[i(\bar B \times C)] \\ &\equiv \bar B(x) + \bar\theta(s_b\bar B(x)), \end{aligned} \tag{A.13}$$

where superscript (b) denotes the fact that the above super expansions have been derived after the application of the BRST invariant restrictions (A.10) and (A.11). From (A.13), we note that coefficients of $\bar\theta$ are nothing but the BRST symmetry transformations for $\mathscr{B}$, E, and $\bar B$ fields as given in (2) (cf. Section 2 for details).

We now focus on the derivation of the anti-BRST symmetry by chiral superfield approach to BRST formalism, where we have the following generalizations:

$$\begin{aligned} A_\mu(x) &\longrightarrow B_\mu(x,\theta) = A_\mu(x) + \theta\bar R_\mu(x), \\ C(x) &\longrightarrow F(x,\theta) = C(x) + i\theta\bar B_1(x), \\ \bar C(x) &\longrightarrow \bar F(x,\theta) = \bar C(x) + i\theta\bar B_2(x), \end{aligned} \tag{A.14}$$

where $(\bar R_\mu, \bar B_1, \bar B_2)$ are the secondary fields that have to be determined in terms of the basic and auxiliary fields of the theory by invoking the anti-BRST invariant restrictions. It goes without saying that the above expansions are the limiting cases of the super expansions in (10) when $\bar\theta = 0$. It can be checked that we have the following useful and interesting anti-BRST invariant quantities (cf. Section 2):

$$\begin{aligned} s_{ab}\bar B &= 0, \\ s_{ab}(\bar B \times C) &= 0, \\ s_{ab}(\bar C \times \bar C) &= 0, \end{aligned}$$

$$s_{ab}\left[A^{\mu}\cdot\partial_{\mu}\overline{B}-iD_{\mu}\overline{C}\cdot\partial^{\mu}C\right]=0,$$
$$s_{ab}\left(D_{\mu}\overline{C}\right)=0. \quad (A.15)$$

According to the basic tenets of AVSA/ACSA, we have to demand that the above quantities (when generalized onto (2, 1)-dimensional chiral supermanifold) should be independent of the Grassmannian variable θ. In other words, we have the following equalities:

$$\begin{aligned}&\overline{B}(x)\times F(x,\theta)=\overline{B}(x)\times C(x),\\&\overline{F}(x,\theta)\times\overline{F}(x,\theta)=\overline{C}(x)\times\overline{C}(x),\\&\partial_{\mu}\overline{F}(x,\theta)+iB_{\mu}(x,\theta)\times\overline{F}(x,\theta)=\partial_{\mu}\overline{C}(x)\\&\quad+i\left(A_{\mu}(x)\times\overline{C}(x)\right),\\&B^{\mu}(x,\theta)\cdot\partial_{\mu}\overline{B}(x)-i\partial_{\mu}\overline{F}(x,\theta)\cdot\partial^{\mu}F(x,\theta)\\&\quad+\left(B_{\mu}(x,\theta)\times\overline{F}(x,\theta)\right)\cdot\partial^{\mu}F(x,\theta)=A^{\mu}(x)\\&\quad\cdot\partial_{\mu}\overline{B}(x)-iD_{\mu}\overline{C}(x)\cdot\partial^{\mu}C(x).\end{aligned} \quad (A.16)$$

We note here that, because of $s_{ab}\overline{B}=0$, we have *no* chiral super expansion of $\overline{B}(x)$ (i.e., $\overline{B}(x)\rightarrow\widetilde{\overline{B}}(x,\theta)=\overline{B}(x)$). The above equalities lead to the following expressions for the secondary fields $(\overline{R}_{\mu},\overline{B}_1,\overline{B}_2)$ in terms of basic and auxiliary fields of our theory:

$$\begin{aligned}\overline{R}_{\mu}&=D_{\mu}\overline{C},\\\overline{B}_1&=\overline{B},\\\overline{B}_2&=-\frac{1}{2}\left(\overline{C}\times\overline{C}\right).\end{aligned} \quad (A.17)$$

Thus, we have obtained the following chiral super expansions:

$$\begin{aligned}B_{\mu}^{(ab)}(x,\theta)&=A_{\mu}(x)+\theta\left(D_{\mu}\overline{C}\right)\\&\equiv A_{\mu}(x)+\theta\left(s_{ab}A_{\mu}\right),\\F^{(ab)}(x,\theta)&=C(x)+\theta\left(i\overline{B}\right)\equiv C(x)+\theta\left(s_{ab}C\right),\\\overline{F}^{(ab)}(x,\theta)&=\overline{C}(x)+\theta\left[-\frac{i}{2}\left(\overline{C}\times\overline{C}\right)\right]\\&\equiv\overline{C}(x)+\theta\left(s_{ab}\overline{C}\right),\end{aligned} \quad (A.18)$$

where the superscript (ab) denotes the super expansions of the chiral superfields after the application of the anti-BRST invariant restrictions [cf. (A.15) and (A.16)]. A close look at (A.18) demonstrates that we have already obtained the anti-BRST symmetry transformations (cf. Section 2) for the basic fields $A_{\mu}(x)$, $C(x)$, and $\overline{C}(x)$ of our theory.

Now we dwell a bit on the derivation of the anti-BRST symmetry transformations: $s_{ab}B=i(B\times\overline{C})$, $s_{ab}E=i(E\times\overline{C})=0$, and $s_{ab}\mathscr{B}=i(\mathscr{B}\times\overline{C})$. In this connection, we note that the following are the useful anti-BRST invariant quantities for our further discussion:

$$\begin{aligned}s_{ab}\left(B\times\overline{C}\right)&=0,\\s_{ab}\left(E\times\overline{C}\right)&=0,\\s_{ab}\left(\mathscr{B}\times\overline{C}\right)&=0.\end{aligned} \quad (A.19)$$

According to the basic principles of AVSA/ACSA, the above quantities should be independent of the Grassmannian variable θ when they are generalized onto the (2, 1)-dimensional chiral super-submanifold. In other words, we have the following equalities:

$$\begin{aligned}\widetilde{\mathscr{B}}(x,\theta)\times\overline{F}^{(ab)}(x,\theta)&=B(x)\times\overline{C}(x),\\\widetilde{E}(x,\theta)\times\overline{F}^{(ab)}(x,\theta)&=E(x)\times\overline{C}(x),\\\widetilde{\mathscr{B}}(x,\theta)\times\overline{F}^{(ab)}(x,\theta)&\equiv\mathscr{B}(x)\times\overline{C}(x),\end{aligned} \quad (A.20)$$

where the expansion for the chiral superfield $\overline{F}^{(ab)}(x,\theta)$ has been given in (A.18) and the chiral super expansions of the other superfields are as follows:

$$\begin{aligned}B(x)&\longrightarrow\widetilde{B}(x,\theta)=B(x)+\theta\overline{P}(x),\\E(x)&\longrightarrow\widetilde{E}(x,\theta)=E(x)+\theta\overline{Q}(x),\\\mathscr{B}(x)&\longrightarrow\widetilde{\mathscr{B}}(x,\theta)=\mathscr{B}(x)+\theta\overline{S}(x).\end{aligned} \quad (A.21)$$

Hence, the fields $(\overline{P}(x),\overline{Q}(x),\overline{S}(x))$ are the fermionic secondary fields that are to be determined in terms of the basic and auxiliary fields of our 2D non-Abelian theory from the anti-BRST invariant restrictions (cf. (A.19) and (A.20)). Explicit substitution of expansions from (A.18) and (A.21) leads to the following very useful and interesting relationships:

$$\begin{aligned}\overline{P}(x)&=i\left(B(x)\times\overline{C}(x)\right),\\\overline{Q}(x)&=i\left(E(x)\times\overline{C}(x)\right),\\\overline{S}(x)&=i\left(\mathscr{B}(x)\times\overline{C}(x)\right).\end{aligned} \quad (A.22)$$

These relationships prove the fermionic nature of the secondary fields $(\overline{P}(x),\overline{Q}(x),\overline{S}(x))$, which is also evident from (A.21), due to the fermionic $(\theta^2=0)$ nature of θ. Thus, we have the following super expansions for the superfields in (A.21):

$$\begin{aligned}\widetilde{B}^{(ab)}\left(x,\overline{\theta}\right)&=B(x)+\theta\left[i\left(B\times\overline{C}\right)\right]\\&\equiv B(x)+\theta\left(s_{ab}B(x)\right),\\\widetilde{E}^{(ab)}\left(x,\overline{\theta}\right)&=E(x)+\theta\left[i\left(E\times\overline{C}\right)\right]\\&\equiv E(x)+\theta\left(s_{ab}E(x)\right),\\\widetilde{\mathscr{B}}^{(ab)}\left(x,\overline{\theta}\right)&=\mathscr{B}(x)+\theta\left[i\left(\mathscr{B}\times\overline{C}\right)\right]\\&\equiv\mathscr{B}(x)+\theta\left(s_{ab}\mathscr{B}(x)\right),\end{aligned} \quad (A.23)$$

where the superscript (ab) denotes the super expansions of the superfields after the application of the anti-BRST invariant restrictions (A.20). The coefficients of θ in (A.23) are nothing but the anti-BRST symmetry transformations for the fields $B(x)$, $E(x)$, and $\mathscr{B}(x)$. Thus, we have derived *all* the (anti-)BRST symmetry transformations of our non-Abelian theory by applying the (anti)chiral superfield approach to BRST formalism.

B. On the Derivation of (Anti-)Co-BRST Symmetries

We derive here the nilpotent and absolutely anticommuting (anti-)co-BRST symmetry transformations by exploiting the virtues of the (anti-)co-BRST invariant restrictions within the framework of the (anti)chiral superfield approach to BRST formalism. In this context, first of all, we take the antichiral super expansions (A.1) and (A.9) and focus on the following very useful and interesting co-BRST invariant quantities:

$$\begin{aligned} s_d\overline{C} &= 0, \\ s_d\left(\partial_\mu A^\mu\right) &= 0, \\ s_d\mathscr{B} &= 0, \\ s_d\left(D_\mu\partial^\mu\overline{C}\right) &= 0, \\ s_d B &= 0, \\ s_d\overline{B} &= 0, \\ s_d\left(C\times\mathscr{B}\right) &= 0, \\ s_d\left[\varepsilon^{\mu\nu}A_\nu\cdot\partial_\mu\mathscr{B} - i\partial_\mu\overline{C}\cdot\partial^\mu C\right] &= 0. \end{aligned} \tag{B.1}$$

It is crystal clear that the co-BRST invariant quantities when generalized onto the $(2,1)$-dimensional antichiral super-submanifold (of the general $(2,2)$-dimensional supermanifold) should be independent of the Grassmannian coordinate $\overline{\theta}$. Against this backdrop, it is very evident (from (A.1) and (A.9)) that the following are true:

$$\begin{aligned} &\overline{C}(x) \longrightarrow \\ &\overline{F}^{(d)}\left(x,\overline{\theta}\right) = \overline{C}(x) + \overline{\theta}(0) \Longrightarrow \\ &B_2 = 0, \\ &s_d\overline{C} = 0, \\ &\mathscr{B}(x) \longrightarrow \\ &\widetilde{\mathscr{B}}^{(d)}\left(x,\overline{\theta}\right) = \mathscr{B}(x) + \overline{\theta}(0) \Longrightarrow \\ &P(x) = 0, \\ &s_d\mathscr{B} = 0, \\ &\overline{B}(x) \longrightarrow \\ &\overline{B}^{(d)}\left(x,\overline{\theta}\right) = \overline{B}(x) + \overline{\theta}(0) \Longrightarrow \\ &S(x) = 0, \\ &s_d\overline{B}(x) = 0, \end{aligned} \tag{B.2}$$

where the superscript (d) on the superfields denotes that the above superfields have been derived after the application of co-BRST invariant restrictions (B.1), which demonstrate that the co-BRST invariant quantities should be independent of the soul coordinate $\overline{\theta}$ (due to the basic tenets of augmented version of (anti)chiral superfield approach to BRST formalism). Further, the other co-BRST invariant quantities in (B.1) imply that

$$\begin{aligned} &\partial_\mu\partial^\mu\overline{C}(x) + iB_\mu\left(x,\overline{\theta}\right)\cdot\partial^\mu\overline{C}(x) = D_\mu\partial^\mu\overline{C}(x) \Longrightarrow \\ &R_\mu\times\partial^\mu\overline{C} = 0, \\ &\partial_\mu B^\mu\left(x,\overline{\theta}\right) = \partial_\mu A^\mu(x) \Longrightarrow \\ &\partial_\mu R^\mu = 0. \end{aligned} \tag{B.3}$$

It is evident that the nontrivial co-BRST symmetry transformations are $s_d C = -i\mathscr{B}$ and $s_d A_\mu = -\varepsilon_{\mu\nu}\partial^\nu\overline{C}$. These can be derived from the co-BRST invariant restrictions:

$$\begin{aligned} &\widetilde{\mathscr{B}}^{(d)}\left(x,\overline{\theta}\right)\times F\left(x,\overline{\theta}\right) = \mathscr{B}(x)\times C(x), \\ &\varepsilon^{\mu\nu}B_\nu\left(x,\overline{\theta}\right)\cdot\partial_\mu\mathscr{B}^{(d)}\left(x,\overline{\theta}\right) - i\partial_\mu\overline{F}^{(d)}\left(x,\overline{\theta}\right) \\ &\quad\cdot\partial^\mu F^{(d)}\left(x,\overline{\theta}\right) = \varepsilon^{\mu\nu}A_\nu(x)\cdot\partial_\mu\mathscr{B}(x) - i\partial_\mu\overline{C}(x) \\ &\quad\cdot\partial^\mu C(x). \end{aligned} \tag{B.4}$$

The substitution of the expansion of $F(x,\overline{\theta})$ from (A.1) into the top relationship, in the above, leads to the condition $B_1\times\mathscr{B} = 0$. One of the nontrivial solutions is $B_1 = -\mathscr{B}$ so that we obtain the following useful expansion:

$$F^{(d)}\left(x,\overline{\theta}\right) = C(x) + \overline{\theta}(-i\mathscr{B}) \equiv C(x) + \overline{\theta}\left(s_d C\right). \tag{B.5}$$

Finally, when we substitute the expansions for $B_\mu(x,\overline{\theta})$ from (A.1), $\mathscr{B}^{(d)}(x,\overline{\theta})$ and $\overline{F}^{(d)}(x,\overline{\theta})$ from (B.2), and $F^{(d)}(x,\overline{\theta})$ from (B.5), we obtain $R_\mu(x) = -\varepsilon_{\mu\nu}\partial^\nu\overline{C}$, which *also* satisfies both the additional conditions in (B.3) and leads to

$$\begin{aligned} B_\mu^{(d)}\left(x,\overline{\theta}\right) &= A_\mu(x) + \overline{\theta}\left(-\varepsilon_{\mu\nu}\partial^\nu\overline{C}\right) \\ &\equiv A_\mu(x) + \overline{\theta}\left(s_d A_\mu\right). \end{aligned} \tag{B.6}$$

The super expansions in (B.2), (B.5), and (B.6) demonstrate that we have derived $s_d\overline{B} = s_d\mathscr{B} = s_d\overline{C} = 0$, $s_d C = -i\mathscr{B}$, and $s_d A_\mu = -\varepsilon_{\mu\nu}\partial^\nu\overline{C}$. We mention, in passing, that $s_d B = 0$ implies that we have *no* antichiral expansion for $B(x)$ as it is a co-BRST invariant quantity.

To derive the anti-co-BRST symmetry transformations, we invoke the chiral expansions for the superfields as given in (A.14) and (A.20). In this context, first of all, we look for the useful anti-co-BRST invariant quantities and generalize them onto $(2,1)$-dimensional *chiral* super submanifold (of the general $(2,2)$-dimensional supermanifold on which our present theory is generalized). After this, we demand that such invariant quantities should be independent of the "soul" coordinate θ. In this context, we note the following:

$$\begin{aligned} s_{ad} C &= 0, \\ s_{ad} \left(\partial_\mu A^\mu\right) &= 0, \\ s_{ad} \left(D_\mu \partial^\mu C\right) &= 0, \\ s_{ad} \bar{B} &= 0, \\ s_{ad} \left(\bar{C} \times \mathscr{B}\right) &= 0, \\ s_{ad} B &= 0, \\ s_{ad} \mathscr{B} &= 0, \\ s_{ad} \left[\varepsilon^{\mu\nu} A_\nu \cdot \partial_\mu \mathscr{B} - i \partial_\mu \bar{C} \cdot \partial^\mu C\right] &= 0. \end{aligned} \tag{B.7}$$

The *trivial* chiral expansions are $B(x) \to \tilde{B}^{(ad)}(x,\theta) = B(x)$, $\bar{B}(x) \to \tilde{\bar{B}}^{(ad)}(x,\theta) = \bar{B}(x)$, $\mathscr{B}(x) \to \tilde{\mathscr{B}}^{(ad)}(x,\theta) = \mathscr{B}(x)$, and $C(x) \to F^{(ad)}(x,\theta) = C(x)$, which imply that $s_{ad} C = s_{ad} B = s_{ad} \bar{B} = s_{ad} \mathscr{B} = 0$. The nontrivial conditions are

$$\begin{aligned} \partial_\mu B^\mu (x,\theta) &= \partial_\mu A^\mu (x) \Longrightarrow \\ \partial_\mu \bar{R}^\mu &= 0, \\ \bar{F}(x,\theta) \times \mathscr{B}^{(ad)}(x,\theta) &= \bar{C}(x) \times \mathscr{B}(x) \Longrightarrow \\ \left(\bar{B}_2 \times \mathscr{B}\right) &= 0, \end{aligned} \tag{B.8}$$

which imply that if we choose $\bar{B}_2 = \mathscr{B}$, the condition $\bar{B}_2 \times \mathscr{B} = 0$ is satisfied and it leads to

$$\bar{F}^{(ad)}(x,\theta) = \bar{C}(x) + \theta (i\mathscr{B}) \equiv \bar{C}(x) + \theta \left(s_{ad} \bar{C}\right), \tag{B.9}$$

where the superscript (ad) denotes that the above superfield has been obtained after the application of (B.7). Thus, we observe that we have already derived the nontrivial anti-co-BRST symmetry transformation: $s_{ad} \bar{C} = i\mathscr{B}$. We now focus on the latter conditions:

$$\begin{aligned} &\partial_\mu \partial^\mu F^{(ad)}(x,\theta) + i B_\mu (x,\theta) \\ &\quad \times \partial^\mu F^{(ad)}(x,\theta) = \partial_\mu \partial^\mu C(x) + i A_\mu (x) \\ &\quad \times \partial^\mu C(x), \\ &\varepsilon^{\mu\nu} B_\nu (x,\theta) \cdot \partial_\mu \mathscr{B}(x) - i \partial_\mu \bar{F}^{(ad)}(x,\theta) \\ &\quad \cdot \partial^\mu F^{(ad)}(x,\theta) = \varepsilon^{\mu\nu} A_\nu (x) \cdot \partial_\mu \mathscr{B}(x) - i \partial_\mu \bar{C}(x) \\ &\quad \cdot \partial^\mu C(x), \end{aligned} \tag{B.10}$$

where we have to use $F^{(ad)}(x,\theta) = C(x)$ and (B.9) to obtain the following conditions:

$$\begin{aligned} \bar{R}_\mu \times \partial^\mu C &= 0, \\ \bar{R}_\mu + \varepsilon_{\mu\nu} \partial^\nu C &= 0 \Longrightarrow \\ \bar{R}_\mu &= -\varepsilon_{\mu\nu} \partial^\nu C. \end{aligned} \tag{B.11}$$

Thus, ultimately, we obtain the chiral expansion:

$$\begin{aligned} B_\mu^{(ad)}(x,\theta) &= A_\mu (x) + \theta \left(-\varepsilon_{\mu\nu} \partial^\nu C\right) \\ &\equiv A_\mu (x) + \theta \left(s_{ad} A_\mu\right), \end{aligned} \tag{B.12}$$

where the superscript (ad) denotes that the above superfield has been obtained after the application of (B.7). It is evident, by now, that we have obtained *all* the anti-co-BRST symmetry transformations s_{ad} (cf. Section 2) of our theory by exploiting the symmetry invariant restrictions on the *chiral* superfields. We point out that the choice $R_\mu = -\varepsilon_{\mu\nu} \partial^\nu C$ satisfies both additional conditions $\partial_\mu \bar{R}^\mu = 0$ and $\bar{R}_\mu \times \partial^\mu C = 0$ that are present in (B.9) and (B.11). We comment that we have chosen $B_1 = -\mathscr{B}$ and $\bar{B}_2 = +\mathscr{B}$ (which imply that $s_d C = -i\mathscr{B}$ and $s_{ad} \bar{C} = i\mathscr{B}$) because these choices satisfy the absolute anticommutativity property ($s_d s_{ad} + s_{ad} s_d = 0$) of the (anti-)co-BRST symmetry transformations.

C. On the Symmetry Invariance in the Theory

We have concentrated on the (anti-)BRST as well as (anti-)co-BRST invariance(s) of our present 2D non-Abelian theory within the framework of AVSA to BRST formalism. In this appendix, we capture the (anti-)BRST and (anti-)co-BRST invariance of the Lagrangian densities (1) and (39) within the framework of AVSA to BRST formalism (which are explicitly quoted in (3), (7), and (40)). Towards this goal in mind, first of all, we generalize the Lagrangian densities (1) onto $(2,2)$-dimensional supermanifold as follows:

$$\begin{aligned} \mathscr{L}_B \longrightarrow & \\ &\tilde{\mathscr{L}}_B \\ &= \mathscr{B}^{(g)}(x,\theta,\bar{\theta}) \cdot \tilde{E}^{(h)}(x,\theta,\bar{\theta}) - \frac{1}{2} \mathscr{B}^{(g)}(x,\theta,\bar{\theta}) \\ &\quad \cdot \mathscr{B}^{(g)}(x,\theta,\bar{\theta}) + B(x) \cdot \partial_\mu B^{\mu(h)}(x,\theta,\bar{\theta}) \\ &\quad + \frac{1}{2} \left(B(x) \cdot B(x) + \tilde{\bar{B}}^{(g)}(x,\theta,\bar{\theta}) \cdot \tilde{\bar{B}}^{(g)}(x,\theta,\bar{\theta}) \right) \\ &\quad - i \partial_\mu \bar{F}^{(h)}(x,\theta,\bar{\theta}) \cdot D^\mu F^{(h)}(x,\theta,\bar{\theta}), \\ \mathscr{L}_{\bar{B}} \longrightarrow & \\ &\tilde{\mathscr{L}}_{\bar{B}} \\ &= \mathscr{B}^{(g)}(x,\theta,\bar{\theta}) \cdot \tilde{E}^{(h)}(x,\theta,\bar{\theta}) - \frac{1}{2} \mathscr{B}^{(g)}(x,\theta,\bar{\theta}) \\ &\quad \cdot \mathscr{B}^{(g)}(x,\theta,\bar{\theta}) - \bar{B}(x) \cdot \partial_\mu B^{\mu(h)}(x,\theta,\bar{\theta}) \end{aligned}$$

$$+\frac{1}{2}\left(B^{(g)}\left(x,\theta,\bar{\theta}\right)\cdot B^{(g)}\left(x,\theta,\bar{\theta}\right)\right)+\frac{1}{2}\bar{B}(x)\cdot\bar{B}(x)$$
$$-iD_\mu\bar{F}^{(h)}\left(x,\theta,\bar{\theta}\right)\cdot\partial^\mu F^{(h)}\left(x,\theta,\bar{\theta}\right), \quad \text{(C.1)}$$

where the superfields with superscript (h) are the ones that have been derived in the main body of the text. It is to be noted that we have defined the covariant derivatives as $D_\mu F^{(h)}(x,\theta,\bar{\theta}) = \partial_\mu F^{(h)}(x,\theta,\bar{\theta}) + i(B_\mu^{(h)}(x,\theta,\bar{\theta})\times F^{(h)}(x,\theta,\bar{\theta}))$ and $D_\mu\bar{F}^{(h)}(x,\theta,\bar{\theta}) = \partial_\mu\bar{F}^{(h)}(x,\theta,\bar{\theta}) + i(B_\mu^{(h)}(x,\theta,\bar{\theta})\times\bar{F}^{(h)}(x,\theta,\bar{\theta}))$. The superfields with superscript (g) denote the *ones* that have been obtained after GIR. We elaborate here a few of them. For instance, let us focus on the explicit expression of $\mathscr{B}^{(g)}(x,\theta,\bar{\theta})$. In this context, we note that

$$s_b(E\cdot\mathscr{B}) = 0,$$
$$s_{ab}(E\cdot\mathscr{B}) = 0. \quad \text{(C.2)}$$

At this stage, we exploit the basic tenets of AVSA to BRST formalism, which state that any arbitrary (anti-)BRST invariant quantity must remain independent of the "soul" coordinates $(\theta,\bar{\theta})$ when it is generalized onto an appropriately chosen supermanifold on which our basic gauge theory is generalized. Thus, we have the following equality:

$$\tilde{E}^{(h)}\left(x,\theta,\bar{\theta}\right)\cdot\mathscr{B}\left(x,\theta,\bar{\theta}\right) = E(x)\cdot\mathscr{B}(x). \quad \text{(C.3)}$$

In the above, the full expansions for $E^{(h)}(x,\theta,\bar{\theta})$ and $\mathscr{B}(x,\theta,\bar{\theta})$ are

$$E^{(h)}\left(x,\theta,\bar{\theta}\right) = E(x)+\theta\left(iE\times\bar{C}\right)+\bar{\theta}(iE\times C)$$
$$+\theta\bar{\theta}\left[-E\times B-(E\times C)\times\bar{C}\right], \quad \text{(C.4)}$$
$$\mathscr{B}\left(x,\theta,\bar{\theta}\right) = \mathscr{B}(x)+\theta\bar{S}(x)+\bar{\theta}S(x)+i\theta\bar{\theta}P(x),$$

where $E^{(h)}(x,\theta,\bar{\theta})$ has been derived from (18) and the general super expansion for the superfield $\mathscr{B}(x,\theta,\bar{\theta})$ has been quoted in (C.4), where the secondary fields $(S(x),\bar{S}(x))$ are fermionic and $P(x)$ is bosonic in nature. The substitution of (C.4) into (C.3) produces the following expressions for the secondary fields in terms of the basic and auxiliary fields:

$$S(x) = i(\mathscr{B}\times C),$$
$$\bar{S}(x) = i\left(\mathscr{B}\times\bar{C}\right), \quad \text{(C.5)}$$
$$P(x) = i\left[(\mathscr{B}\times B)+(\mathscr{B}\times C)\times\bar{C}\right].$$

Thus, we have the final expansion for the superfield $\mathscr{B}^{(g)}(x,\theta,\bar{\theta})$ as

$$\mathscr{B}^{(g)}\left(x,\theta,\bar{\theta}\right) = \mathscr{B}(x)+\theta\left(i\mathscr{B}\times\bar{C}\right)+\bar{\theta}(i\mathscr{B}\times C)$$
$$+\theta\bar{\theta}\left[-\mathscr{B}\times B-(\mathscr{B}\times C)\times\bar{C}\right]$$
$$\equiv \mathscr{B}(x)+\theta\left(s_{ab}\mathscr{B}\right)+\bar{\theta}\left(s_b\mathscr{B}\right) \quad \text{(C.6)}$$
$$+\theta\bar{\theta}\left(s_b s_{ab}\mathscr{B}\right).$$

In other words, we have derived the (anti-)BRST symmetry transformations for the auxiliary field $\mathscr{B}(x)$ and, in the process, we have obtained the explicit form of $\mathscr{B}^{(g)}(x,\theta,\bar{\theta})$ which has been used in the explicit expression for the super Lagrangian densities (C.1). We discuss here about the derivations of $\tilde{\bar{B}}^{(g)}(x,\theta,\bar{\theta})$ and $\tilde{B}^{(g)}(x,\theta,\bar{\theta})$ that are present in the expressions for the super Lagrangian densities $\tilde{\mathscr{L}}_B$ and $\tilde{\mathscr{L}}_{\bar{B}}$ (cf. (C.1)). Using the (anti-)BRST symmetry transformations from (2), we note that

$$s_b\left(E\cdot\bar{B}\right) = 0,$$
$$s_{ab}(E\cdot B) = 0,$$
$$s_{ab}\bar{B} = 0, \quad \text{(C.7)}$$
$$s_b B = 0$$

are the BRST and anti-BRST invariant quantities. According to the basic tenets of AVSA to BRST formalism, the BRST invariance of B (i.e., $s_b B = 0$) and anti-BRST invariance of $\bar{B}$ (i.e., $s_{ab}\bar{B} = 0$) imply that the general super expansions

$$B(x) \longrightarrow$$
$$\tilde{B}\left(x,\theta,\bar{\theta}\right) = B(x)+\theta\bar{M}(x)+\bar{\theta}M(x)+i\theta\bar{\theta}N(x),$$
$$\bar{B}(x) \longrightarrow \quad \text{(C.8)}$$
$$\tilde{\bar{B}}\left(x,\theta,\bar{\theta}\right) = \bar{B}(x)+\theta\bar{L}(x)+\bar{\theta}L(x)+i\theta\bar{\theta}K(x),$$

would remain independent of $\bar{\theta}$ and θ, respectively, in view of the mapping $s_b \leftrightarrow \partial_{\bar{\theta}}$ and $s_{ab} \leftrightarrow \partial_\theta$. Thus, the reduced forms of the superfields in (C.8) are

$$\tilde{B}^{(r)}\left(x,\theta,\bar{\theta}\right) = B(x)+\theta\bar{M}(x),$$
$$\tilde{\bar{B}}^{(r)}\left(x,\theta,\bar{\theta}\right) = \bar{B}(x)+\bar{\theta}\bar{L}(x). \quad \text{(C.9)}$$

In the above expansions (C.8) and (C.9), the secondary fields $(M(x),\bar{M}(x),L(x),\bar{L}(x))$ are fermionic and $(N(x),K(x))$ are bosonic in nature due to the fermionic nature (i.e., $\theta^2 = \bar{\theta}^2 = 0$, $\theta\bar{\theta}+\bar{\theta}\theta = 0$) of the Grassmannian variables $(\theta,\bar{\theta})$ and bosonic nature of the superfields $\tilde{B}(x,\theta,\bar{\theta})$ and $\tilde{\bar{B}}(x,\theta,\bar{\theta})$. The superscript (r) on the superfields in (C.9) corresponds to the reduced form of the general super expansion in (C.8) when $\bar{\theta} = 0$ and $\theta = 0$, respectively. Basically, these reduced forms become chiral and antichiral superfields.

We exploit now the (anti-)BRST invariance that has been expressed in (C.7). In fact, we have the following restrictions:

$$\tilde{E}^{(h)}\left(x,\theta,\bar{\theta}\right)\cdot\bar{B}^{(r)}\left(x,\theta,\bar{\theta}\right) = E(x)\cdot\bar{B}(x),$$
$$\tilde{E}^{(h)}\left(x,\theta,\bar{\theta}\right)\cdot B^{(r)}\left(x,\theta,\bar{\theta}\right) = E(x)\cdot B(x), \quad \text{(C.10)}$$

where the expansion for $E^{(h)}(x,\theta,\bar{\theta})$ is given in (C.4) and the reduced forms of $\bar{B}^{(r)}(x,\theta,\bar{\theta})$ and $B^{(r)}(x,\theta,\bar{\theta})$ are quoted in

(C.9). Ultimately, with the substitution of these into (C.10), we obtain the following results:

$$M(x) = i\left(B \times \overline{C}\right), \qquad \overline{L}(x) = i\left(\overline{B} \times C\right). \tag{C.11}$$

Thus, we have the following explicit super expansions:

$$\begin{aligned} \widetilde{B}^{(g)}\left(x,\theta,\overline{\theta}\right) &= B(x) + \theta\left(iB \times \overline{C}\right) \\ &\equiv B(x) + \theta\left(s_{ab}B\right), \\ \widetilde{\overline{B}}^{(g)}\left(x,\theta,\overline{\theta}\right) &= \overline{B}(x) + \overline{\theta}\left(i\overline{B} \times C\right) \\ &\equiv \overline{B}(x) + \overline{\theta}\left(s_b\overline{B}(x)\right). \end{aligned} \tag{C.12}$$

The above expressions for $\widetilde{B}^{(g)}(x,\theta,\overline{\theta})$ and $\widetilde{\overline{B}}^{(g)}(x,\theta,\overline{\theta})$ have been used in the super Lagrangian densities (C.1). The rest of the other terms in (C.1) are straightforward and clear.

We are now in the position to express the (anti-)BRST invariance of the Lagrangian densities (1) which change to the total spacetime derivatives under the above symmetry transformations (cf. (3) and (4)). It is straightforward to check that

$$\begin{aligned} \frac{\partial}{\partial\theta}\widetilde{\mathscr{L}}_{\overline{B}}\Big|_{\overline{\theta}=0} &= -\partial_\mu\left(\overline{B}\cdot D^\mu\overline{C}\right), \\ \frac{\partial}{\partial\overline{\theta}}\widetilde{\mathscr{L}}_{B}\Big|_{\theta=0} &= \partial_\mu\left(B\cdot D^\mu C\right), \end{aligned} \tag{C.13}$$

which are nothing but our earlier results (cf. (3)), where we have shown that $s_{ab}\mathscr{L}_{\overline{B}} = -(\overline{B}\cdot D^\mu\overline{C})$ and $s_b\mathscr{L}_B = \partial_\mu(B\cdot D^\mu\overline{C})$. Geometrically, the above observations show that super Lagrangian densities (C.1) are the sum of composite (super)fields, obtained after (anti-)BRST invariant restrictions and HC, such that their translation along the $(\theta,\overline{\theta})$ directions of the $(2,2)$-dimensional supermanifold produces the total spacetime derivatives.

In exactly similar fashion, we can discuss the (anti-)co-BRST invariance of the Lagrangian densities (1), where these are generalized onto the $(2,2)$-dimensional supermanifold as

$$\begin{aligned} \mathscr{L}_B \longrightarrow{}& \\ \widetilde{\mathscr{L}}_B &= \mathscr{B}(x)\cdot\widetilde{E}^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) - \frac{1}{2}\mathscr{B}(x) \\ &\cdot\mathscr{B}(x) + B(x)\cdot\partial_\mu B^{\mu(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) \\ &+ \frac{1}{2}\left(B(x)\cdot B(x) + \overline{B}(x)\cdot\overline{B}(x)\right) \\ &- i\partial_\mu\overline{F}^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right)\cdot\partial^\mu F^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right) \\ &+ \partial_\mu\overline{F}^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right) \\ &\cdot\left(B^{\mu(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right)\times F^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right)\right), \\ \mathscr{L}_{\overline{B}} \longrightarrow{}& \\ \widetilde{\mathscr{L}}_{\overline{B}} &= \mathscr{B}(x)\cdot\widetilde{E}^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) - \frac{1}{2}\mathscr{B}(x) \\ &\cdot\mathscr{B}(x) - \overline{B}(x)\cdot\partial_\mu B^{\mu(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) \\ &+ \frac{1}{2}\left(B(x)\cdot B(x) + \overline{B}(x)\cdot\overline{B}(x)\right) \\ &- i\partial_\mu\overline{F}^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right)\cdot\partial^\mu F^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right) \\ &+ \left(B_\mu^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right)\times\overline{F}^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right)\right) \\ &\cdot\partial^\mu F^{(\mathrm{dh})}\left(x,\theta,\overline{\theta}\right), \end{aligned} \tag{C.14}$$

where the superscripts (dh) and (dg) on the superfields have already been explained in the main body of the text. We would like to comment here that the expression for $\widetilde{E}^{(\mathrm{dg})}(x,\theta,\overline{\theta})$ has been derived (i.e., $F_{01}^{(\mathrm{dg})}(x,\theta,\overline{\theta}) = \widetilde{E}^{(\mathrm{dg})}(x,\theta,\overline{\theta})$) from the superfield corresponding to the field strength tensor; namely,

$$\begin{aligned} \widetilde{F}_{\mu\nu}^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) &= \partial_\mu B_\nu^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) + \partial_\nu B_\mu^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) \\ &+ i\left(B_\mu^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right)\times B_\nu^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right)\right), \end{aligned} \tag{C.15}$$

where the expansion of $B_\mu^{(\mathrm{dg})}(x,\theta,\overline{\theta})$ has been illustrated in (32). In fact, the explicit substitution of this superfield into the above equation leads to the following:

$$\begin{aligned} \widetilde{E}^{(\mathrm{dg})}\left(x,\theta,\overline{\theta}\right) &= E(x) + \theta\left(D_\mu\partial^\mu C\right) + \overline{\theta}\left(D_\mu\partial^\mu\overline{C}\right) \\ &+ \theta\overline{\theta}\left(-iD_\mu\partial^\mu\mathscr{B} - i\varepsilon_{\mu\nu}\left(\partial^\nu\overline{C}\times\partial^\mu C\right)\right) \\ &\equiv E(x) + \theta\left(s_{ad}E(x)\right) + \overline{\theta}\left(s_dE(x)\right) \\ &+ \theta\overline{\theta}\left(s_ds_{ad}E(x)\right). \end{aligned} \tag{C.16}$$

We note that the substitution of the super expansions from (26) and (32) into the super Lagrangian densities (C.14) would express them in terms of the coefficients of $(1,\theta,\overline{\theta},\theta\overline{\theta})$. It can be now checked that the following are true:

$$\begin{aligned} \frac{\partial}{\partial\theta}\widetilde{\mathscr{L}}_{\overline{B}}\Big|_{\overline{\theta}=0} &= \partial_\mu\left[\mathscr{B}\cdot\partial^\mu C\right] \Longleftrightarrow \\ s_{ad}\mathscr{L}_{\overline{B}} &= \partial_\mu\left[\mathscr{B}\cdot\partial^\mu C\right], \\ \frac{\partial}{\partial\overline{\theta}}\widetilde{\mathscr{L}}_{B}\Big|_{\theta=0} &= \partial_\mu\left[\mathscr{B}\cdot\partial^\mu\overline{C}\right] \Longleftrightarrow \\ s_d\mathscr{L}_B &= \partial_\mu\left[\mathscr{B}\cdot\partial^\mu\overline{C}\right]. \end{aligned} \tag{C.17}$$

Hence, we have provided the equivalence of the (anti-)co-BRST invariance of the Lagrangian densities (1) in the language of AVSA to BRST formalism. Consequently, the (anti-)co-BRST invariance can be explained within the framework of AVSA to BRST formalism as follows. The translation of the super Lagrangian densities (C.14) along $(\theta, \bar{\theta})$-directions of the $(2, 2)$-dimensional supermanifold is such that it results in the total spacetime derivatives, thereby rendering the action integrals (corresponding to the appropriate Lagrangian densities) invariant under the (anti-)co-BRST symmetry transformations. We end this *appendix* with a concise remark that we can *also* capture the (anti-)co-BRST invariance of the coupled Lagrangian densities (39) exactly in the same manner as we have done for our starting Lagrangian densities (1) for the present 2D non-Abelian 1-form gauge theory.

Conflicts of Interest

The authors declare that there are no conflicts of interest of any kind.

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References

[1] J. Thierry-Mieg, "Geometrical reinterpretation of Faddeev-Popov ghost particles and BRS transformations," *Journal of Mathematical Physics*, vol. 21, article 2834, 1980.

[2] M. Quirós, F. J. De Urries, J. Hoyos, M. L. Mazón, and E. Rodriguez, "Geometrical structure of Faddeev–Popov fields and invariance properties of gauge theories," *Journal of Mathematical Physics*, vol. 22, no. 8, pp. 1767–1774, 1981.

[3] R. Delbourgo and P. D. Jarvis, "Extended BRS invariance and OSp (4/2) supersymmetry supersymmetry," *Journal of Physics A: Mathematical and Theoretical*, vol. 15, no. 2, p. 611, 1982.

[4] L. Bonora and M. Tonin, "Superfield formulation of extended BRS symmetry," *Physics Letters B*, vol. 98, no. 1-2, pp. 48–50, 1981.

[5] L. Bonora, P. Pasti, and M. Tonin, "Extended BRS symmetry in non-Abelian gauge theories," *Il Nuovo Cimento A*, vol. 64, no. 3, pp. 307–331, 1981.

[6] L. Bonora, P. Pasti, and M. Tonin, "Superspace approach to quantum gauge theories," *Annals of Physics*, vol. 144, no. 1, pp. 15–33, 1982.

[7] L. Baulieu and J. Thierry-Mieg, "The principle of BRS symmetry: an alternative approach to Yang-Mills theories," *Nuclear Physics B*, vol. 197, no. 3, pp. 477–508, 1982.

[8] L. Alvarez-Gaumé and L. Baulieu, "The two quantum symmetries associated with a classical symmetry," *Nuclear Physics B*, vol. 212, no. 2, pp. 255–267, 1983.

[9] R. P. Malik, "Abelian 2-form gauge theory: superfield formalism," *The European Physical Journal C*, vol. 60, no. 3, pp. 457–470, 2009.

[10] R. P. Malik, "Superfield approach to nilpotent symmetries for QED from a single restriction: an alternative to the horizontality condition," *Journal of Physics A: Mathematical and General*, vol. 39, no. 33, p. 10575, 2006.

[11] R. P. Malik, "An alternative to the horizontality condition in the superfield approach to BRST symmetries," *The European Physical Journal C*, vol. 51, pp. 169–177, 2007.

[12] R. P. Malik, "Superfield approach to symmetries for matter fields in Abelian gauge theories," *Journal of Physics A: Mathematical and General*, vol. 37, no. 19, pp. 5261–5274, 2004.

[13] R. P. Malik, "New topological field theories in two dimensions," *Journal of Physics A: Mathematical and General*, vol. 34, no. 19, pp. 4167–4181, 2001.

[14] N. Srinivas and R. P. Malik, "Nilpotent symmetries and Curci-Ferrari-type restrictions in 2D non-Abelian gauge theory: superfield approach," *International Journal of Modern Physics A*, vol. 32, no. 33, Article ID 1750193, 21 pages, 2017.

[15] N. Srinivas and R. P. Malik, "Nilpotent symmetries and Curci-Ferrari type restrictions in 2D non-Abelian gauge theory: superfield approach," *International Journal of Modern Physics A*, vol. 32, no. 33, pp. 1–21, 2017.

[16] S. Kumar, B. K. Kureel, and R. P. Malik, "(Anti-)BRST and (anti-)co-BRST symmetries in 2D non-Abelian gauge theory: some novel observations," https://arxiv.org/abs/1703.09099.

[17] S. Gupta and R. P. Malik, "Rigid rotor as a toy model for Hodge theory," *The European Physical Journal C*, vol. 68, no. 1, pp. 325–335, 2010.

[18] R. Kumar, S. Krishna, A. Shukla, and R. P. Malik, "Abelian p-form (p = 1, 2, 3) gauge theories as the field theoretic models for the Hodge theory," *International Journal of Modern Physics*, vol. 29, Article ID 1450135, 2014.

[19] E. Witten, "Topological quantum field theory," *Communications in Mathematical Physics*, vol. 117, no. 3, pp. 353–386, 1988.

[20] A. S. Schwarz, "The partition function of degenerate quadratic functional and Ray-Singer invariants," *Letters in Mathematical Physics*, vol. 2, no. 3, pp. 247–252, 1978.

[21] T. Eguchi, P. B. Gilkey, and A. J. Hanson, "Gravitation, gauge theories and differential geometry," *Physics Reports*, vol. 66, no. 6, pp. 213–393, 1980.

[22] S. Mukhi and N. Mukunda, Introduction to topology, differential geometry and group theory for physicists, Wiley Eastern Private Limited, New Delhi, 1990.

[23] K. Nishijima, "Observable states in the representations of BRS algebra," *Progress of Theoretical and Experimental Physics*, vol. 80, no. 5, pp. 905–912, 1988.

[24] J. W. van Holten, "Becchi-Rouet-Stora-Tyutin cohomology of compact gauge algebras," *Physical Review Letters*, vol. 64, no. 24, pp. 2863–2865, 1990.

[25] S. Gupta and R. P. Malik, "A field-theoretic model for Hodge theory," *The European Physical Journal Particles and Fields*, vol. 58, no. 3, pp. 517–529, 2008.

[26] S. Krishna, A. Shukla, and R. P. Malik, "Topologically massive non-Abelian theory: superfield approach," *International Journal of Modern Physics A*, vol. 26, no. 25, pp. 4419–4450, 2011.

[27] D. Dudal, V. E. R. Lemes, M. S. Sarandy, S. P. Sorella, and M. Picariello, "On the SL(2, $\mathbb{R}$) symmetry in Yang-Mills theories in the Landau, Curci-Ferrari and maximal Abelian gauge," *Journal of High Energy Physics*, vol. 212, no. 8, 2002.

[28] D. Dudal, H. Verschelde, V. E. Lemes et al., "The anomalous dimension of the gluon-ghost mass operator in Yang-Mills theory," *High Energy Physics*, vol. 569, no. 1-2, pp. 57–66, 2003.

[29] R. P. Malik, "Hodge duality operation and its physical applications on supermanifolds," *International Journal of Modern Physics A*, vol. 21, no. 16, pp. 3307–3336, 2006.

[30] N. Srinivas, T. Bhanja, and R. P. Malik, "(Anti)chiral superfield approach to nilpotent symmetries: self-dual chiral bosonic theory," *Advances in High Energy Physics*, vol. 2017, Article ID 6138263, 2017.

[31] D. Shukla, T. Bhanja, and R. P. Malik, "Supervariable approach to the nilpotent symmetries for a toy model of the Hodge theory," *Advances in High Energy Physics*, vol. 2016, Article ID 2618150, 13 pages, 2016.

[32] N. Srinivas, A. Shukla, and R. P. Malik, "Nilpotent symmetries of a 4D model of the hodge theory: augmented (anti-)chiral superfield formalism," https://arxiv.org/abs/1501.06770.