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Binary Neutron-Star Mergers with a Crossover Transition to Quark Matter

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Abstract: This paper summarizes recent work on the possible gravitational-wave signal from binary neutron-star mergers in which there is a crossover transition to quark matter. Although this is a small piece of a much more complicated problem, we discuss how the power spectral density function may reveal the presence of a crossover transition to quark matter.

Keywords: neutron stars; equation of state; quark matter; gravitational waves

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1. Introduction

I am honored to have a chance to contribute to this Festschrift in honor of Remo Ruffini's 80th birthday. I have enjoyed collaboration with Remo on papers [1,2] exploring the physics of the X-ray afterglow associated gamma-ray bursts. Indeed, this collaboration inspired me to return to simulations of the relativistic hydrodynamics associated with binary neutron stars and their merger. As Remo has correctly pointed out during this Festschrift and elsewhere, the electromagnetic evolution will dominate the dynamics of binary neutron-star mergers, and moreover, there are enormous uncertainties associated with detecting and calculating the gravitational radiation emanating from binary neutron-star mergers. Nevertheless, in this presentation, I describe recent work [3] with my collaborators in which we have simulated the relativistic merger of neutron stars and explored effects on the emergent gravitational waves of a crossover transition to quark matter.

It has been discussed for some time that neutron stars (NSs) within binary systems could be used to probe the equation of state (EoS) at high densities (e.g., Refs. [4,5]). Gravitational waves (GWs) from the GW170817 event by the LIGO-Virgo Collaboration [6,7] may have provided new insight into neutron-star matter [8]. Also, NS masses and radii determined by the NICER mission constrain the EoS of nuclear matter [9–11].

Moreover, differences in the EoS can lead to a variety of observable effects (cf. [12]). Such changes in the EoS may lead to a change in the maximum peak frequency f_{peak} (sometimes denoted as f_2) in the inferred power spectral density (PSD) [13–15]. A shift may violate the proposed universality relations between f_{peak} and tidal deformability for pure hadronic EoSs [16–21]. In [3], we analyzed how such an observed shift might also probe the quark matter phase. This is, however, model dependent (e.g., [22,23]) and depends somewhat on the time duration of the merged system [12,24,25].

There have been many recent works considering EoS effects on the detected GWs. Some have considered the formation of quark matter [12–14,23–33]. Often these studies, however, were limited to a first-order phase transition. In a first order transition, a mixed phase of quarks and hadrons develops. This mixed phase diminishes the pressure support of the remnant, resulting in a prompt collapse. However, a crossover or a weak first-order transition remains a possibility [34–38]. The matter pressure during the crossover could be enormous. This could extend the duration of the postmerger phase. We proposed that observing such a long-duration post-merger system might signal both the phase-transition order and the strength of the coupling of quark matter [3].

In Ref. [3], we calculated the GW signal from the postmerger phase and showed that it is sensitive to the presence of quark matter in the equation of state. We demonstrated that the properties of quark matter in the crossover phase increase the duration of the postmerger GW emission. Hence, this probes the properties of quark matter. Various parameterizations of the quark-hadron crossover (QHC19) EoS of [39] were investigated in Ref. [3] . A similar study was made [40] based on the more recent version of the (QHC21) EoS with similar conclusions. The crossover is treated as continuous in the QHC19 EoS. In Ref. [3], the maximum chirp frequency f_{max}, the tidal deformability, and the peak in the power spectral density f_{peak} were used to identify observational characteristics of a crossover to quark matter during mergers of equal-mass binary neutron stars. The crucial high frequency range (1–4 kHz) is associated with the postmerger gravitational waves. Although this frequency is not within the sensitivity limits of the LIGO/aVirgo/KAGRA observatories, the next generation of gravitational-wave detectors, e.g., the Einstein Telescope [41] and Cosmic Explorer [42], should be sensitive to such high frequency emissions. We argue that this observation in the next generation detectors might indicate both the order of the transition and the parameters characterizing the crossover to quark matter.

2. Equations of State

At high baryon density a non-perturbative approach to QCD is required. This approach must include chiral symmetry breaking [43], the generation of constituent quark masses, quark pairing, the possibility of color superconductivity [44], etc. In the hadronic regime, we considered both the SLy [45] and the GNH3 [46] EoSs. These bracket the properties of an extremely soft or a rather stiff equation of state.

The QHC19 EoS is based upon the the NJL Lagrangian [47–49]. The four coupling constants are: (1) (G) the scalar coupling; (2) (K) the coefficient of the Kobayashi-Maskawa-'t Hooft vertex; (3) (g_v) the vector coupling for quark repulsion; and (4) (H) the di-quark strength. Only two coupling constants (g_v/G and H/G), were used to construct various versions of the EoS. The three parameter sets used in Ref. [3] are labeled as [39]: QHC19B [$(g_V, H) = (0.8, 1.49)$], QHC19C [$(g_V, H) = (1.0, 1.55)$], and QHC19D [$(g_V, H) = (1.2, 1.61)$]. At the crossover densities ($2\,n_0 < n < 5\,n_0$), the pressure is given by fifth-order polynomials in terms of the baryonic chemical potential.

3. Simulation Details

Binary merger simulations were run in [3] using the `Einstein Toolkit` [50] numerical relativity software. This includes full general relativity in three spatial dimensions with differential equations based upon the BSSN-NOK framework [51–55]. The hydrodynamics was evolved with the use of the `GRHydro` code [56–58] based on the Valencia formulation [59,60]. The initial conditions were generated using `LORENE` [61,62]. The thorn `Carpet` [63,64] was used for adaptive mesh refinement based upon six mesh refinement levels and a minimum grid of 0.3125 in Cactus units (≈461 m). A constant adiabatic index $\Gamma_{\rm th} = 1.8$ was used to account for the thermal pressure in `GRHydro` as described in Ref. [65].

The Newman–Penrose formalism was employed to extract the gravitational waves emitted during the binary merger. This minimizes numerical noise by fitting a multipole expansion in spherical harmonics of the Weyl scalar $\Psi_4^{(l,m)}(\theta, \phi, t) = \ddot{h}_+^{(l,m)}(\theta, \phi, t) + i\ddot{h}_\times^{(l,m)}(\theta, \phi, t)$. The two polarizations of the strain $h_+(\theta, \phi, t)$ and $h_\times(\theta, \phi, t)$ result from a sum over the (l, m) modes followed by integrating twice. The neutron star models were based upon baryonic masses of $M_B = 1.45$, 1.50, 1.55 $M_\odot$. These were chosen because gravitational masses associated with these baryonic masses for various equations of state are similar ∼1.35–1.4 $M_\odot$. Simulations began at an initial coordinate separation of 45 km between centers.

In Ref. [3] it was shown that even during inspiral, the central densities in the neutron stars achieved densities in the crossover range (2–5 n_0). During the merger, the maximum density increases until it exceeds $\sim$5–6 n_0. The central region of the system then collapses.

It was also shown in Ref. [3] that the postmerger GW emission continues for a much longer time for the simulations with a QHC EoS. When going from QHCB to QHCC the postmerger GW emission becomes longer, corresponding to increasing the quark coupling. This led to the suggestion that the strength of the quark–matter couplings might be deduced from the duration of the post merger phase. Indeed, the lifetime of the postmerger intermediate hyper-massive neutron star (HMNS) depends rather significantly on the stiffness of the equation of state at the crossover densities. One interesting finding is that the postmerger remnants from mergers including the QHC19D EoS had so much pressure that no black hole formed during the simulations. As the EoS stiffness within the QHC models increased lifetimes of their HMNS remnants were apparent. Even the QHC19B EoS produces a much longer postmerger duration than that of a pure hadronic EoSs.

A waveform analysis of the strain can be performed in the frequency domain. This highlights the dominant frequencies of the waveform. Specifically, the effective Fourier amplitude is obtained from

$$\tilde{h}_{+,\times}(f) = \int h_{+,\times}(t) e^{-i2\pi f t} dt \ . \quad (1)$$

This is presented in Figure 1, which shows an example of the normalized power spectral density $2\tilde{h}(f) f^{1/2}$ [66] based upon the simulations of Ref. [3]. The lower blue and orange curves show the anticipated sensitivity of the Einstein Telescope and Cosmic Explorer, respectively, while the upper green curve shows the current LIGO sensitivity. The initial inspiral up to contact between the merging neutron stars ends with the first peak at around 1 kHz. For probing the crossover to quark matter, however, the peaks, f_{peak}, at around 2.5–3.5 kHz are most useful. These arise from the extended postmerger phase. The amplitude of f_{peak} correlates with the time duration of the postmerger remnant. Therefore, it correlates with the strength of the coupling constants in the QHC19 equations of state. As discussed in [3], one can also infer the maximum chirp strain amplitude, $f_{max} = \frac{1}{2\pi} \frac{d\phi}{dt}|_{max}$, where ϕ is the phase of the strain (see [66]). This is not apparent in the PSD, but is deduced from the phase of the strain during the merger. Although this is referred to at the maximum chirp strain, this is not to be confused with the instantaneous gravitational-wave frequency at the time of merger.

4. Discussion and Conclusions

Although the amplitude of the f_{peak} PSD becomes larger for crossover equations of state with increasing coupling strengths, this might also be realized in other equations of state as demonstrated in [66]. Therefore, one desires another signature to uniquely show the formation of quark matter. In [3], it was pointed out that the QHC19 equations of state show behavior consistent with a soft EoS at low density, $\sim 3n_0$. This affects the merger regime of f_{max}. On the other hand, the postmerger phase represented in the f_{peak} frequency exhibits the behavior of a stiff EoS.

This dual nature of the QHC19 EoSs might be revealed by correlating f_{max} and f_{peak} in a GW event [3]. This is illustrated in Figure 2. For pure hadronic EoSs, there appears to be a linear correlation between f_{max} and f_{peak}. However, a crossover EoS deviates from this correlation as indicated by the circled points on this figure. Thus, an observation of events in the circled region might indicate the crossover to quark matter. We note, however, that this deviation is not entirely robust as an indicator. For example, the hadronic Sly EoS also deviates from the linear relation. What is needed is a more exhaustive set of calculations to better clarify this trend. That, however, is left to a future work.

Additionally, in Ref. [3], the relation between f_{peak} and the pseudo-averaged rest-mass density [17,66] was considered. For this case, the f_{peak} frequencies tend to cluster in a region in between a soft and stiff EoS [3]. Hence, although there are enormous uncertainties in this suggestion, observing a transition from soft to stiffness in the correlations of f_{max} and f_{peak} could indicate that quark matter had formed during the merger. Moreover, the amplitude of the PSD at the frequency of f_{peak} may suggest the quark–matter coupling strengths.

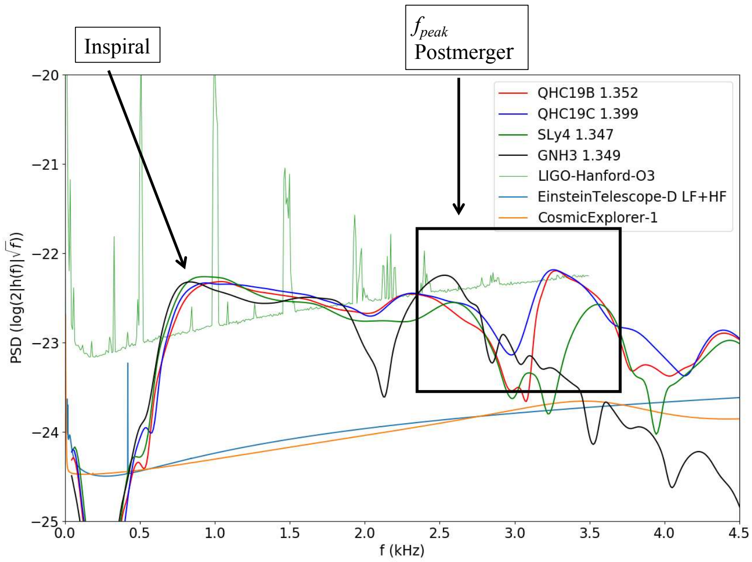

Figure 1. Power spectral density $(2\tilde{h}(f)f^{1/2})$ vs. frequency f for various simulations. The the lower blue and orange curves show anticipated sensitivity of the Einstein Telescope and Cosmic Explorer, respectively, while the upper green curve shows the LIGO sensitivity. The first peak at around 1 kHz is the initial contact of the merging binaries. The second peaks near 2.5–3.5 kHz correspond to the long postmerger phase, f_{peak}.

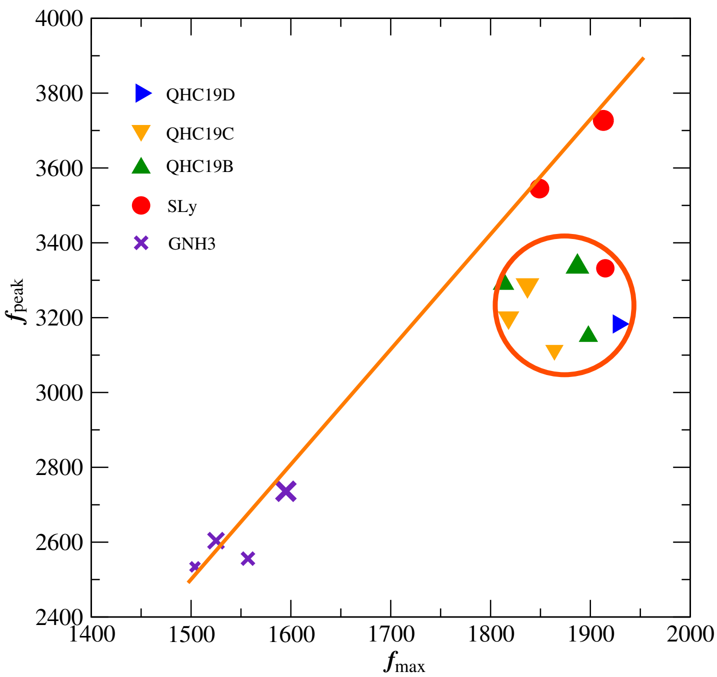

Figure 2. Correlation between f_{max} and f_{peak}. There appears to be a linear correlation for normal hadronic EoSs as indicated by the straight line. However, the existence of a crossover regime to quark matter leads to outliers from this correlation as indicated by the circled points.

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