

# A QCD analysis of high statistics $F_2$ data on $H_2$ and $D_2$ targets, with determination of higher twists

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**Abstract** We present the preliminary results of a QCD analysis of the high statistics BCDMS and SLAC  $F_2$  data on  $H_2$  and  $D_2$ . At high  $Q^2$ , the data are in good agreement with the predictions of perturbative QCD and lead to an improved measurement of  $\Lambda_{QCD}$  and  $\alpha_S$ . At lower  $Q^2$ , a precise measurement of non-perturbative effects ("higher-twists") is obtained : they are very small below  $x = 0.40$  and small, positive and increasing with  $x$  at higher  $x$ . Altogether, the data give a clear indication for the running of  $\alpha_S$ .

In the past months, the final results of the two highest statistics measurements of  $F_2$  on deuterium and hydrogen targets have been presented. This set of results covers a wide kinematic range : 0.07 to 0.85 in  $x$  and 0.5 to 260  $\text{GeV}^2$  in  $Q^2$ . These data are thus well suited for a test of perturbative QCD as well as for a measurement of possible "higher-twist" (non-perturbative) effects in the  $Q^2$ -evolution of  $F_2$ . We present here the preliminary results of such a study.

The high  $Q^2$  data (7 to 260  $\text{GeV}^2$ ) are those obtained by the BCDMS Collaboration [1] with their muon scattering experiment using a toroidal iron spectrometer; these data have already been used for QCD analyses of  $F_2$  [2] at high  $Q^2$ , where non-perturbative effects are expected to be small. The low  $Q^2$  data (0.5 to 30  $\text{GeV}^2$ ) come from a coherent global reanalysis of electron scattering data from a number of experiments at SLAC spanning the time period 1970 to 1985 [3]. The main improvements compared to previous publications are a better determination of  $R(x, Q^2)$  and a correct treatment of radiative corrections. This allows to increase the useable kinematical range. In the present analysis, we have used all the published data, apart from the last  $x$ -bin (0.85), where only SLAC data exist.

The data are shown in Figure 1, interpolated where necessary to the  $x$ -bins used here. The errors shown on Figure 1 are "total" errors, i.e. statistical and systematic combined in quadrature. In addition to these point-to-point errors, there are global normalisation errors of 3% and 2% respectively for the BCDMS and SLAC data. We do not discuss here the comparison and compatibility of these data sets, which

can be found in [4]. We nevertheless emphasize a specific point from ref. [4] that is important for the treatment of systematic errors in our fits : the kinematical region where the systematic errors are largest in the muon scattering data corresponds to high  $x$  ( $x > 0.50$ ) and low  $Q^2$ . In this region, the systematic error originates predominantly from uncertainties on the calibrations of the measurement of the incident and scattered muon energy and on the resolution of the spectrometer. These three sources of errors have a similar  $x$  and  $Q^2$  dependence and can thus be combined quadratically into a one-standard-deviation 100% correlated error which we call here the "main systematic error" of the BCDMS data (see [5] for full tables of errors and [4] for a more detailed discussion). Unfortunately, this dominant systematic error is largest precisely where the low  $Q^2$  SLAC and high  $Q^2$  BCDMS data overlap to some extent.

We have employed for these fits a computer program developed by members of the BCDMS Collaboration [6] that has already been used to fit the predictions of perturbative QCD to the BCDMS data (see e.g. [2]). This program performs a fully numerical integration of the Altarelli-Parisi equations (both singlet and non-singlet in next-to-leading order).

The free parameters in these fits correspond to

- a description of the  $x$ -dependence of the non-singlet and singlet part of  $F_2$  ( $F_2^{NS}(x, Q_0^2)$  and  $F_2^{SI}(x, Q_0^2)$ ), and of the gluon distribution  $xG(x, Q_0^2)$ , where  $Q_0^2$  is taken to be 20  $\text{GeV}^2$  and

$$xG(x, Q_0^2) = 0.48(1 + \eta)(1 - x)^\eta$$

with  $\eta$  fixed to 7 (when free, the typical error on this exponent is  $\pm 2$ , and the fits are not sensitive to the gluon distribution above  $x = 0.30$ ).

- the value of  $\Lambda_{\overline{MS}}^{(4)}$  for four active quark flavours,
- coefficients  $C_i$  (one per  $x$ -bin and by target material) describing the twist-four effects (HT) in the  $Q^2$ -evolution of  $F_2$ , such that

$$F_2^{HT}(x_i, Q^2) = F_2^{LT}(x_i, Q^2) \left(1 + C_i/Q^2\right),$$

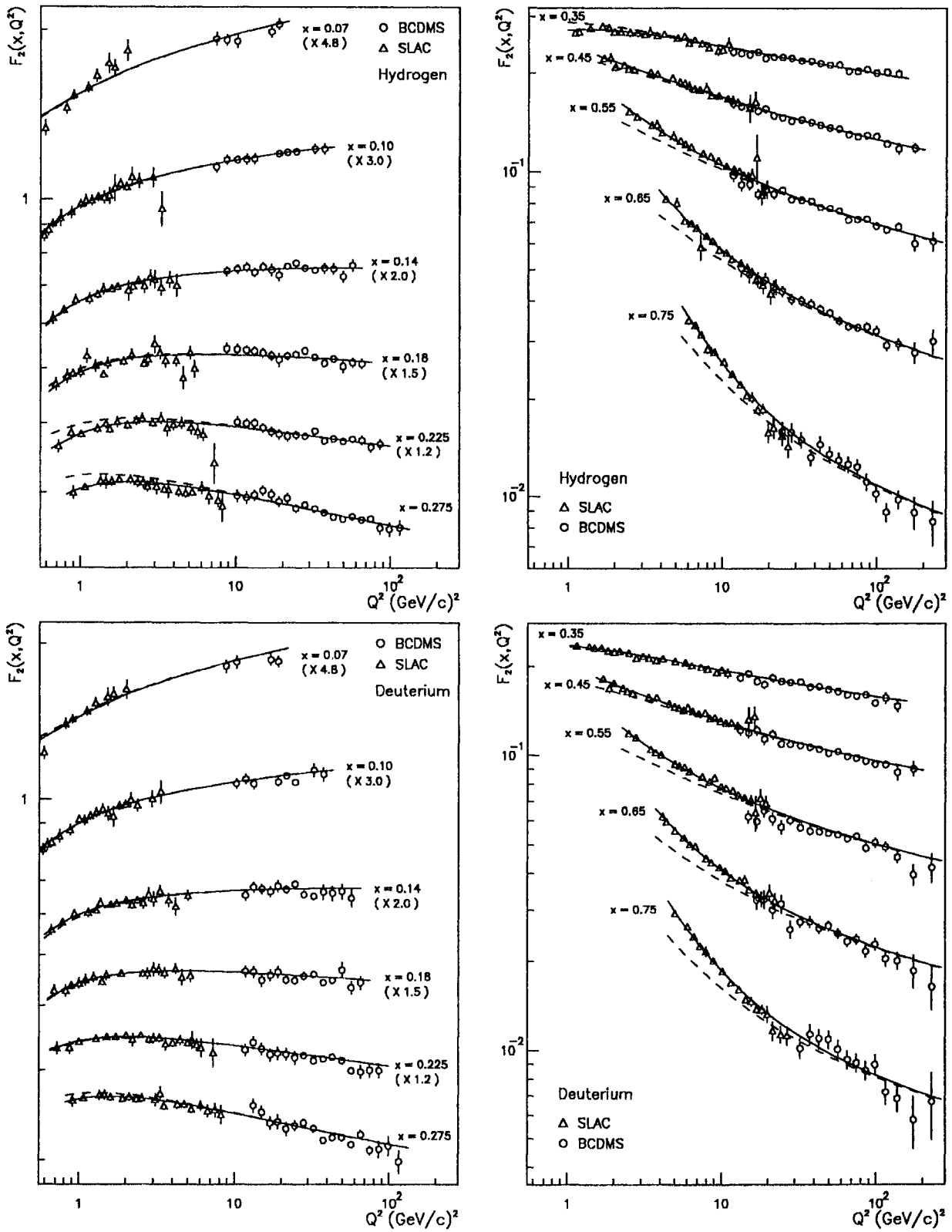


Figure 1: High statistics measurements of  $F_2$  on hydrogen (up) and deuterium (bottom) targets. The fits shown are described in the text.

where  $F_2^{HT}$  is the function that is fitted to the data and  $F_2^{LT}$  obeys the perturbative QCD  $Q^2$ -evolution according to Altarelli-Parisi equations.

In addition to these phenomenological and theoretical parameters that enter in the fit, we include four (two for each target material) “experimental” parameters describing the systematic errors

- one for the relative normalisation of the SLAC and BCDMS data sets,
- one for the dominant systematic error of the BCDMS data discussed above. In that case, we have taken into account all correlation effects; more explicitly, if  $F_2(x_i, Q_j^2)$  and  $\Delta F_2^{sys}(x_i, Q_j^2)$  are respectively the values of  $F_2$  and of the (one standard deviation) dominant systematic error in each bin  $(x_i, Q_j^2)$ , then the fitted quantity is  $F_2(x_i, Q_j^2) + \lambda \Delta F_2^{sys}(x_i, Q_j^2)$ , where  $\lambda$  is the free parameter describing the “amount of BCDMS dominant systematic error”.

All the other sources of systematic error in the BCDMS and SLAC data are notably smaller (comparable to or smaller than the statistical error) and we have chosen to combine them quadratically with the statistical errors, and to use the resulting errors in the fits as if they were purely statistical. We thus ignore their possible correlations but this is of minor importance given their sizes.

The QCD fits described above have been performed simultaneously on the  $H_2$  and  $D_2$  data and both with and without the inclusion of target mass corrections (TMC, from reference [7]). These corrections are computed numerically from the measured  $F_2$ ’s themselves and do not involve any additional free parameter. The results of the fits with TMC are summarized in Table 1.

	Hydrogen	Deuterium
$\Lambda$	$250 \pm 40 \text{ MeV}$	
$\chi^2 / \text{dof}$	$(325+270) / (378+360-49)$	
BCDMS/SLAC rel. norm.	-1.0%	0.2%
$\lambda$ (BCDMS main syst.)	1.5	1.3

Table 1: Results of combined QCD (NLO) fits to BCDMS and SLAC data

We now comment on the general features of these (preliminary) fit results. The  $\chi^2$ ’s are good - smaller than one per degree of freedom, partly because we

have included some of the systematic errors in the total errors. They are slightly better with TMC included, but this is not very significant. As an example of the fit quality, we show in Figure 1 the fit including TMC. The solid lines represent the full fit, while the dashed lines represent the leading twist contribution ( $F_2^{LT}$ ). The overall description of the data by the fit is good. It is clear, from the difference between the solid and dashed curves that the influence of twist-four terms in the  $Q^2$ -evolution of  $F_2$  are negligible above  $\sim 2 \text{ GeV}^2$  at low  $x$  ( $x < 0.30$ ) and  $\sim 10 \text{ GeV}^2$  at higher  $x$ .

The value of  $\Lambda_{\overline{MS}}^{(4)}$  is almost the same in the two fits (it is here rounded to the nearest 10 MeV), and the total error on  $\Lambda$  is rather small (40 MeV) : in terms of  $\alpha_S$ , we get :

$$\alpha_S(50 \text{ GeV}^2) = 0.177 \pm 0.008.$$

This corresponds to :

$$\alpha_S(M_Z^2) = 0.112 \pm 0.003 \text{ (total error)}.$$

The recent results from the LEP experiments (typically  $0.118 \pm 0.012$  [8]) are in agreement with this value. Our error on  $\Lambda$  is dominated by systematic errors. The central value of 250 MeV is in agreement with recent measurements of  $\Lambda$  at high  $Q^2$  [2]. The relative normalisation of the two data sets is everywhere smaller than 1.0% perfectly compatible with the absolute normalisation uncertainties of 3% and 2% on the BCDMS and SLAC data. The amount of BCDMS main systematic error ( $\lambda$  parameter) that corresponds to the best  $\chi^2$  is of order 1.4 times the published errors.

We illustrate the good agreement between the measured  $Q^2$ -evolution of  $F_2$  and the one predicted by perturbative QCD on Figure 2. In this Figure, the points represent the values of the logarithmic derivatives  $d \ln F_2 / d \ln Q^2$  for the hydrogen data at high  $Q^2$  (larger than 8 to 20  $\text{GeV}^2$ , depending on  $x$ ), and the solid line is the prediction obtained from the fit (with  $\Lambda_{\overline{MS}}^{(4)} = 250 \text{ MeV}$ ). The dashed line corresponds to the fit result where the higher-twist coefficients  $C_i$  are arbitrarily put to zero : this fit is also in good agreement with the data (at high  $Q^2$ ). The dotted line corresponds to the fit with no higher-twists and no target mass corrections : the difference is visible for  $x > 0.55$ . Our conclusions on the Deuterium data are similar.

In Figure 3, we show the values of the coefficients  $C_i$ , both for  $H_2$  and  $D_2$ , with and without inclusion

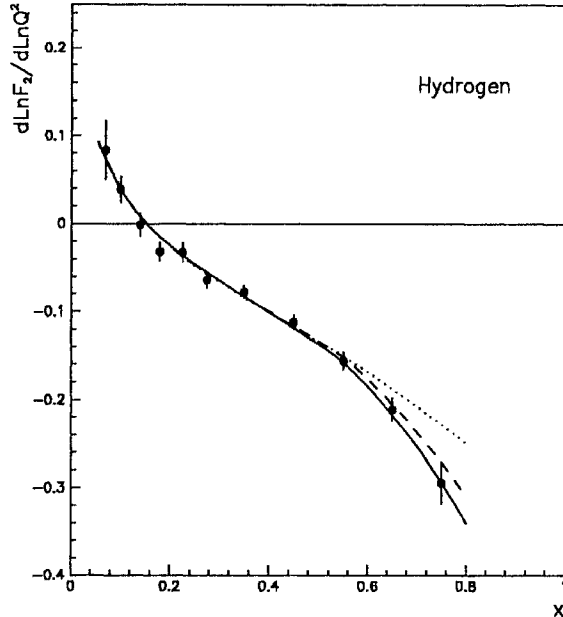


Figure 2: The logarithmic derivatives  $d \ln F_2 / d \ln Q^2$  at high  $Q^2$  for the  $H_2$  data. The solid line is the QCD prediction from the fit with HT and TMC (dashed line : TMC and no HT; dotted line : no TMC and no HT) for  $\Lambda = 250$  MeV.

of TMC's. The  $x$ -dependences of these higher-twist terms are very similar in  $H_2$  and  $D_2$  data. They are small for  $x < 0.40$ , and even almost compatible with zero for fits with TMC's. As the  $C_i$  parameters are nearly mutually uncorrelated in the fits, this fact is of clear physical significance. For  $x > 0.40$ , the higher-twist terms increase with  $x$ , as expected; they are clearly smaller in the case of fits including TMC's. Because of the high statistical power of these data, this determination of higher-twist terms in deep inelastic scattering is presently the most precise. We consider *remarkable* the fact that the inclusion of TMC's in the fits reduces everywhere the magnitude of the higher-twist terms needed to describe the data, especially so at low  $x$  where this reduction is almost a cancellation.

The behaviour of these higher-twist terms has two interesting consequences : first, concerning the large  $x$  domain ( $x > 0.25$ ), the values of  $\Lambda_{\overline{MS}}$  resulting from QCD fits on high  $Q^2$  data ( $Q^2 > 20$  GeV<sup>2</sup>) are not significantly affected by these higher-twist terms; second, concerning the lower  $x$  domain, the higher-twist influence on the  $Q^2$ -evolution of  $F_2$  is so small that even data at rather low  $Q^2$  (down to 1 GeV<sup>2</sup>) can be used in the estimation of the gluon distribution.

Finally, we have performed pseudo-QCD fits to the same data, identical to the previous ones apart from

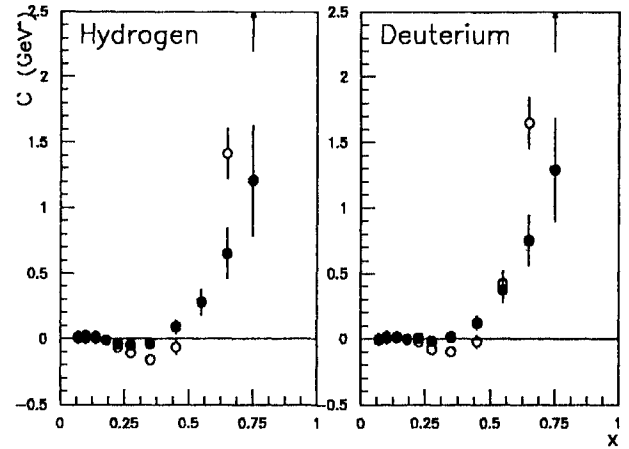


Figure 3: The higher-twist coefficients  $C_i$  as a function of  $x$  for  $H_2$  and  $D_2$  data. Full (open) circles are for fits with (without) TMC.

the fact that we have imposed that  $\alpha_s$  show no  $Q^2$ -variation. In perturbative QCD,  $\alpha_s(Q^2)$  is expected to decrease significantly from 1 to 250 GeV<sup>2</sup> (by a factor 2.5), and we want to see if the data are statistically powerful enough to favor the running of  $\alpha_s$ . These “ $\alpha_s = \text{constant}$ ” fits have slightly worse  $\chi^2$ 's, the difference with QCD fits being a bit over 10 units. The resulting higher-twist coefficients  $C_i$ , however, are significantly larger than in the QCD fits. This is illustrated in Figure 4 (analogous to Figure 1 bottom-left), where the amount of higher-twist terms needed to describe the  $Q^2$ -evolution of  $F_2$  is indicated in each  $x$  bin by the difference between the solid and dashed lines. Obviously, it is much more natural to have a running  $\alpha_s$  with very small higher-twist terms than a constant  $\alpha_s$  and large higher-twist contributions. We consider that this comparison gives a *strong physical indication* for the running of  $\alpha_s$ .

We have presented combined QCD fits to the two highest statistics  $F_2$  data on hydrogen and deuterium targets. These data are in good agreement and are complementary : the high  $Q^2$  data of BCDMS allow to test the perturbative QCD predictions and the low  $Q^2$  data of SLAC lead to a precise determination of the magnitude of non-perturbative effects in the  $Q^2$ -evolution of  $F_2$ . The data are well described over the whole  $Q^2$ -range (0.5 to 250 GeV<sup>2</sup>) by perturbative QCD fits including target mass corrections and higher-twist terms; these terms are very small or negligible at low  $x$  ( $x < 0.40$ ) and they are small, positive and rise with  $x$  at higher  $x$ . The value of  $\alpha_s$  obtained from these fits constitutes the most precise measurement of this fundamental quantity. Moreover, the data give an indication for the running of  $\alpha_s$ .

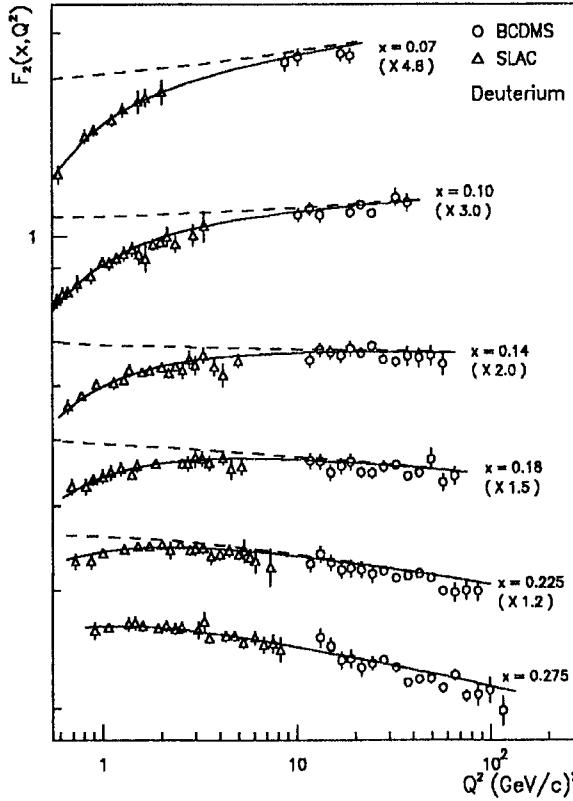


Figure 4:  $\alpha_S = cst$  fit to the deuterium data for  $x < 0.30$ . The solid lines are the result of the fit and the dashed lines visualize the  $Q^2$  evolution with no HT. This can be compared directly with Figure 1 (bottom-left).

The work presented here has been done in collaboration with A. Milsztajn, A. Staude, K.M. Teichert, M. Virchaux and R. Voss.

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