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Matrix of response functions for deconvolution of gamma-ray spectra

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Abstract

An approach for creation the response functions` matrix for the xenon gamma-ray detector is discussed. A set of gamma-ray spectra was obtained by Geant4 simulation to generate the matrix. Iterative algorithms used allow to deconvolve and restore initial gamma-ray spectra. Processed spectrum contains peaks that help to identify and estimate a activity of a radioactive source. Results and analysis of experimental spectra deconvolution are shown.

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Keywords: xenon gamma detector; radioactive waste; gamma ray

1. Introduction

Registered by a gamma-ray detector gamma-ray spectrum from a radioactive source can be described [1-3] using of the 1st kind integral Fredholm equation [4]:

$$\int K(x,y)F(x)dx = Z(y) \tag{1}$$

$$\vec{y} = K_{ij}\vec{x} \tag{2}$$

K(x,y) is the kernel of integral equation, *F(x)* is the spectrum corresponding to original gamma-ray source, *Z(y)* is the spectrum registered by a xenon gamma-rays detector. Experimental spectra *Z(y)* were obtained by standard radioactive calibration point sources located 20 cm from detector.

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Equation (1) can be transformed into a system of linear equations (2). $\vec{x}$ is energy distribution of the incident gamma-rays. $\vec{y}$ is energy distribution of the measured gamma-rays. And K_{ij} is matrix of response functions and corresponding to kernel of integral equation in discrete form.

The solution of (1), (2) is an ill-posed problem because it has many solutions and depends on fluctuation of *Z(y)*. Such problem can be solved using numeric calculus. To solve these equations, one must use matrix of response functions. Matrix contains a set of spectra corresponding to the detector's response to gamma-rays of specified energy range. In our case a matrix of the size 2000 by 2000 was used for xenon gamma-rays detector [5,6].

2. Spectra Simulation for the xenon gamma-ray detector

Spectra for the matrix of the xenon gamma-ray detector were obtained by simulation using framework for high energy particle physics Geant4 [7]. The modeled spectra $\vec{M}$ have the energy bin width of 1 keV, that matches the bin width of experimental spectra. Before the matrix creation, comparison of experimental gamma-ray spectra with simulation was made. Total area spectrum deviations of simulated data from experiment are 3.2% for ^{137}Cs and 2.3 % for ^{60}Co. Created matrix is shown in Fig. 1.

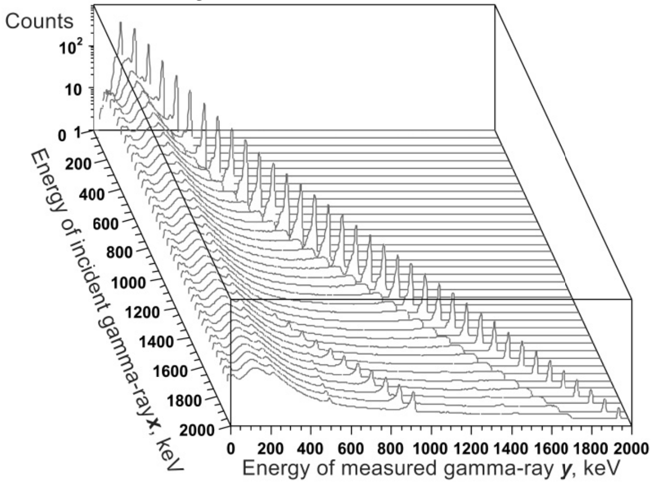

Fig. 1. The Matrix of response functions for the xenon gamma-ray detector.

Virtual radioactive sources have the delta function energy distribution $\vec{x}$ .We have simulated the set of spectrum $\vec{M}$ that response to delta functions energy distribution of incident gamma-rays for each energy of 0 to 2000 keV range. Each value M_j was divided by amount of emitted gamma-ray photons x_i and was stored in the element of the matrix K_{ij}. The simulated radioactive source was located at 20 cm from the detector at the same distance for experimental spectra. The intensity of the source at another distance was corrected on the solid angle between the source and the detector.

3. Deconvolution methods

There are different methods of the ill-posed equation (2) solution. One of these methods is the algorithm based on Gauss elimination. One of conditions for a correct solution is the fact that all components x_i of $\vec{x}$ should be of positive values. Due to triangular shape of the matrix of response functions, Gauss elimination can be used for the solution of (1) equation as fast method by means of reducing matrix in echelon form.

Other methods of solution are iterative. Richardson-Lucy (RL), Maximum A Posteriory (MAP) (Richardson [8]) methods are based on Bayes theorem. Using the matrix of response functions as the conditional probability *p(y|x)*, one may find *p(x)* and original *F(x)* spectrum of gamma-rays irradiating the detector. *p(y|x)* is probability that gamma-rays of *x* energy will be registered in bin corresponding to energy *y*. Gold method (Gold [9]) is based on the relaxation iterative algorithm:

$$\vec{x}^{n+1} = \vec{x}^{n} + \mu(K^T \vec{y} - K^T K \ \vec{x}^{n}) \quad (3)$$

where μ is the relaxation factor (Gold [9]), n is number of a iteration. In this paper we consider Gold, MAP and RL methods for deconvolution of experimental spectra registered by the xenon gamma-ray detector.

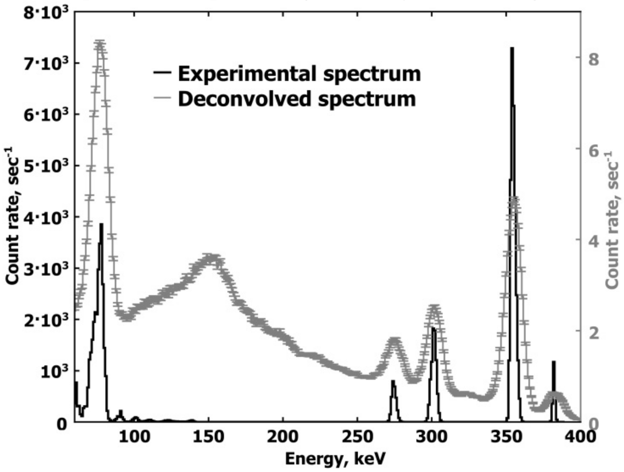

Fig. 2. Deconvolved spectrum solved by Gold method from experimental spectrum obtained from ^{133}Ba source

4. Deconvolution of experimental spectrum

Using deconvolution methods one may find the original spectrum of the radioactive source of incident gamma-rays irradiating the detector. To check and compare methods, experimental spectra of each standard calibration point-like source were obtain. Sources have been placed at 20 cm from the xenon gamma-ray detector. Multichannel analyzer with the energy bin width of 1 keV was used. Experimental spectra were processed by the iterative deconvolution methods (see Fig. 2). The fact that deconvolved spectrum contains narrow peaks helps and facilitates us to identification of radioactive nuclei.

Also, these methods allow to separate closely spaced peaks (see Fig. 3). To study this process and find a limitation, a set of spectra of gamma-ray sources with close energy values (660 and 661 keV, 660 and 662 keV and so on) was simulated. Each virtual source located at 20 cm from the detector and was irradiated by 10^8 gamma-ray photons. Gold method gives better results than MAP (see Fig. 4) and allows to separate even 660 and 662 keV gamma-ray lines (see Fig. 5).

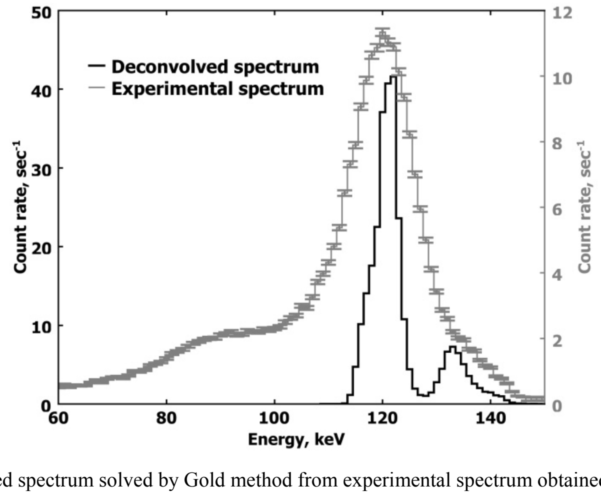

Fig. 3.Deconvolved spectrum solved by Gold method from experimental spectrum obtained from ^{60}Co source

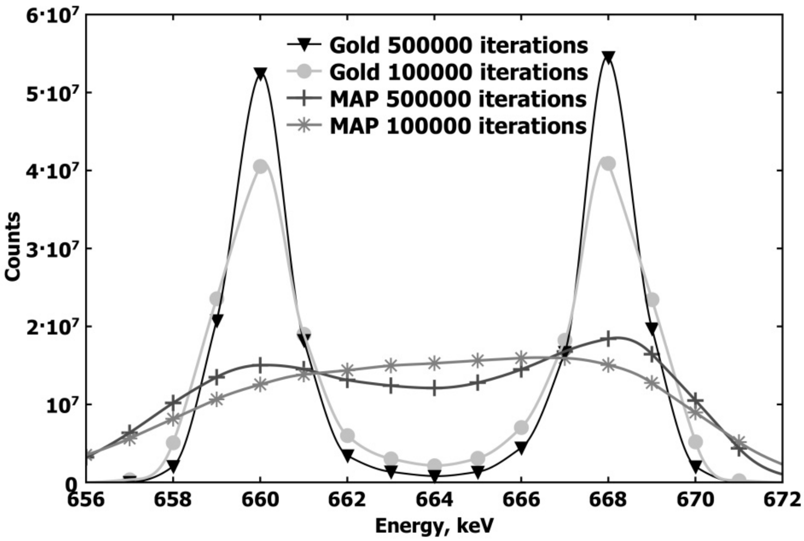

Fig. 4. Comparison of deconvolved spectra by Gold and MAP methods

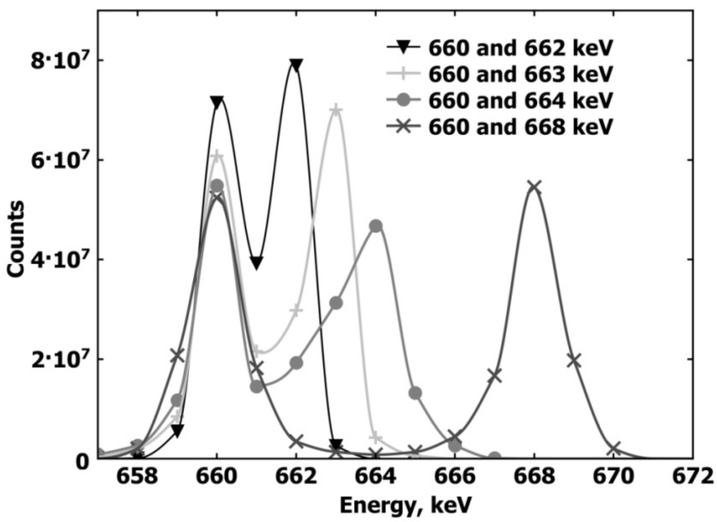

Fig. 5. Deconvolved after 500000 iterations by Gold method spectra of two closely spaced gamma-ray lines of different doublets

The closer gamma-ray lines, the more iterations are required to separate them. Results also depend on energy bin width of spectra. In our case we have bin width of 1 keV and it's impossible to separate 660 and 661 keV gamma-ray lines.

5. Analysis of energy resolution and estimation of source activity

After deconvolution, spectrum contains peaks corresponding to lines of the original radioactive source. The energy resolution of deconvolved peaks is higher, than of ones measured by the detector (see Fig. 6). For the example, after 10000 iterations of Gold method processing the full width at half maximum (FWHM) of 662 keV energy peak reduces to 2.9 times (from 14.45 to 4.87 keV). The result depends on a method and number of iterations. Gold method gives better results comparing to MAP one (see Fig. 6).

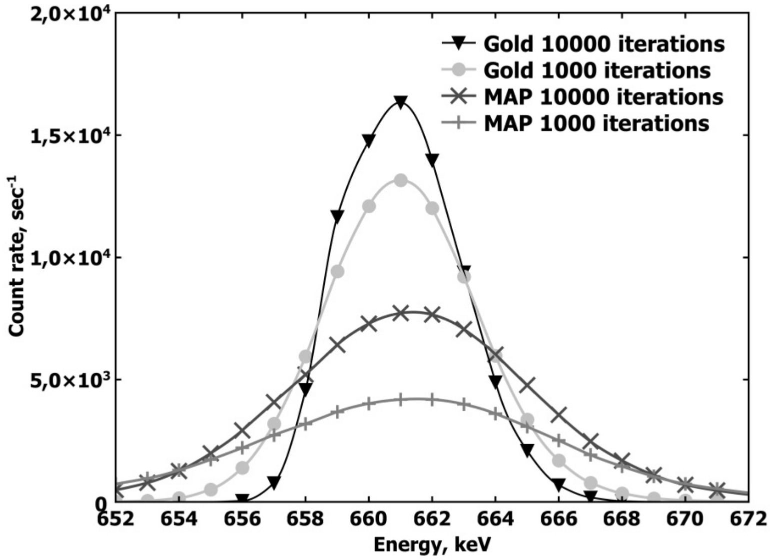

Fig. 6. Comparison of peak FWHM of ^{137}Cs after deconvolution by Gold and MAP methods

Number of iterations is an important question. Iterative methods require a lot of time for solution and we should choose a stop condition. After 5000 iterations the area under peak area tends to the constant. And we chose 10000 iterations as optimal iteration number from the point of view of a ratio result/time.

The area under deconvolved peaks let to estimate a activity of a radioactive source. For example, ^{133}Ba has few gamma-ray lines and most intensive ones are 81.0, 276.4, 302.9, 356.0 and 383.8 keV. Using area of count rates S_i under each i peak and information about relative intensities I_i (number of photons per decays ratio), one may estimate the activity A of a radioactive source (see formula (4)).

$$A = \frac{\sum_{i=1}^{m} S_i}{\sum_{i=1}^{m} I_i} \tag{4}$$

Where m is number of peaks used for calculation (m = 5 for ^{133}Ba)MAP and Gold methods can be applied for activity estimation. Richardson-Lucy method just reallocates events from measured spectrum to deconvolved peaks, but area under these peaks doesn't correspond to the real activity of a source. The estimated activity deviates from the real value up to 17 % ^{133}Ba.Best results are given by Gold method (1.77 % deviation for ^{137}Cs).

Table 1. Estimation of the activity, Bq.

Isopote	Real Value, Bq	Gold Method, Bq	MAP, Bq
^{133}Ba	46532±2327	52041±595	54312±609
^{57}Co	16357±818	16399±189	16250±188
^{137}Cs	95082±4754	93396±994	83079±968
^{60}Co	76622±3831	70504±562	62503±528

Real activities of radioactive sources and deconvolved results are shown in Table 1. The uncertainty of the activity for standard calibration gamma-ray sources is 5% according to the datasheet.

6. Conclusion

The matrix of response functions was obtained for the xenon gamma-ray detector by simulation using Geant4 framework. The methods of spectra processing used in this research can be applied to different types of gamma-ray detectors. Deconvolution methods allow to obtain original spectra of incident gamma-rays irradiating a detector. These methods may be applied in the spectrometry. Deconvolved spectra contain narrow peaks, which helps to identify the radioactive materials and estimate their activity. FWHM for 662 keV peak of ^{137}Cs decreases from 14.45 to 4.88 keV. The difference between the estimation of activity obtained by Gold method for ^{137}Cs and the real one is 1.77 %, that less than 5 % uncertainty of the source activity.

Acknowledgements

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