Fermi National Accelerator Laboratory

FERMILAB-PUB-86/59-T
May, 1986

The Adler-Weisberger Sum Rule and g_A in the Large-N Expansion

M. SOLDATE

Fermi National Accelerator Laboratory

Batavia, Illinois 60510

ABSTRACT

The consistency of the Adler-Weisberger sum rule in the large-N expansion is examined. It is shown that the Δ saturates the sum rule in the nonrelativistic quark model to all orders in $\frac{1}{N}$, and in the Skyrme model to leading order in $\frac{1}{N}$. Phenomenologically, it is evident that either g_A or the integral over the difference of πp total cross sections appearing in the sum rule is poorly behaved as an expansion in $\frac{1}{N}$. In the Skyrme model, based on the calculation of the Δ contribution to the sum rule it appears that it is g_A itself which has large higher-order corrections. It seems likely that in a direct calculation of g_A it is necessary to include the first two subleading orders before an accurate result for g_A can be obtained.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

One of the most serious phenomenological problems with the Skyrme model is its prediction for g_A, $\simeq .61$,[1] about a factor of 2 less than the measured value. On the other hand, one notable success of the theory of chiral symmetry breaking is the Adler-Weisberger (A-W) sum rule[2] which accurately relates g_A to total π-proton scattering cross sections.

$$g_A^2 = 1 + \frac{2f_\pi^2}{\pi} \int_{\nu_\pi}^{\infty} d\nu \frac{\sqrt{\nu^2 - m_\pi^2}}{\nu^2} \left[\sigma_{\pi^+ p}(\nu) - \sigma_{\pi^- p}(\nu)\right] \tag{1}$$

Here $\nu = \frac{p \cdot q}{m_p}$ where p (q) is the proton (pion) momentum, and $\nu_\pi = m_\pi + \frac{m_\pi^2}{2m_p}$. As one expects on the basis of the quark model, the dominant contribution experimentally to the integral is from the Δ. Simple isospin relations dictate that the Δ^{++} contribution is three times that of the Δ^0. Therefore, it is surprising at first glance to see a calculation of g_A that is less than 1 in a model which both incorporates the spontaneous breaking of chiral symmetry and describes to some degree the Δ.

A partial resolution to the problem is that the Skyrme model predictions are to be interpreted in large N. By explicit evaluation in the Skyrme model, g_A is $O(N)$. The term 1 on the right-hand side of eqn.(1) is not present then in a leading order evaluation of the sum rule. However, f_π^2 is $O(N)$ and generically $\sigma_{\pi N}$ is $O(1)$[3] . Thus, one is led to the more basic question of how the sum rule is consistent in large N. The Δ will play a central part in the following discussion. In order to make clear the special role it has in the large-N limit, it is convenient to review briefly a derivation of the sum rule.

The derivation[4] sketched here begins with the identity

$$\int d^4x d^4y\, e^{ik\cdot x} e^{-iq\cdot y} \{ k^\alpha q^\beta \langle p' | T[A_\alpha^+(x) A_\beta^-(y)] |p\rangle$$
$$- \langle p' | T[\partial^\alpha A_\alpha^+(x) \partial^\beta A_\beta^-(y)] |p\rangle - \delta(x^0 - y^0) \langle p' | [A_0^-(y), \partial^\alpha A_\alpha^+(x)] |p\rangle \qquad (2)$$
$$-iq^\beta \delta(x^0 - y^0) \langle p' | [A_0^+(x), A_\beta^-(y)] |p\rangle \} = 0.$$

It is to be analyzed in the soft pion limit, q^2 and $k^2 \simeq 0$. We will remove single pion poles from A_α and $\partial^\alpha A_\alpha$, denoting them by $\bar{A}_\alpha$ and $\bar{\partial}$ afterwards. Eqn.(2) gives several equations obtained by setting the collection of terms with the same pole structure in $\frac{1}{k^2 - m_\pi^2}$ and $\frac{1}{q^2 - m_\pi^2}$ to zero. The third term in eqn.(2), a σ -term, is small and can be dropped, as can matrix elements of $\bar{\partial}$. The terms which are proportional to $\frac{1}{(k^2 - m_\pi^2)(q^2 - m_\pi^2)}$ cancel. The information contained in the terms with single poles can be used to simplify the equation for the non-pole terms. Afterwards, it is convenient to cancel an overall 4-momentum delta function, and then set $p' = p$. The contribution of the fourth term in eqn.(2) is normalized to $\mathcal{O}(\nu^2)$ by the isospin of the proton. The non-pole terms assemble to give

$$q^\alpha q^\beta \int d^4x\, e^{iq\cdot x} \langle p | T[\bar{A}_\alpha^+(x) \bar{A}_\beta^-(0)] |p\rangle + i f_\pi^2 \mathcal{T}_-(\nu, q^2 \simeq 0)$$
$$= constant + i m_p \nu + \mathcal{O}(\nu^2) \qquad (3)$$

where $\mathcal{T}_-(\nu, q^2) = \langle p\pi^-(q) | T | p\pi^-(q) \rangle$ is the $\pi^- p$ T matrix element. The A-W sum rule follows from the terms of eqn.(3) linear in ν, so that the constant in eqn.(3) is irrelevant.

The usual derivation of eqn.(1) from eqn.(3) proceeds in two steps. To begin with, the neutron pole is extracted from the first term of the left-hand side of eqn. (3) with the result

$$q^\alpha q^\beta \int d^4x\, e^{iq\cdot x} \langle p| T[\bar{A}^+_\alpha(x)\bar{A}^-_\beta(0)] |p\rangle \Big|_{neutron\ pole} = ig_A^2 \frac{m_p \nu^2}{\nu - \nu_n}, \tag{4}$$

where $\nu_n = \frac{m_n^2 - m_p^2}{2m_p}$. Note that there is an apparent sensitivity to the neutron-proton mass difference; if $\nu_n = 0$ there is a term in eqn.(4) linear in ν, otherwise there is not. The result of the full calculation should not have this feature. Secondly, a dispersive treatment is given for $\mathcal{T}_-(\nu, q^2 \simeq 0)$ at small ν. In the real world $m_\pi \neq 0$ and the Δ is a resonance in the continuum. Taking into account the neutron pole at ν_n,

$$\begin{aligned} \mathcal{T}_-(\nu, q^2) = & -g^2_{\pi NN} \frac{4m_p}{(m_n + m_p)^2} \frac{\nu_n^2}{\nu - \nu_n} \\ & + \frac{2m_p}{\pi} \int_{\nu_\pi}^{\infty} d\nu' \sqrt{\nu'^2 - m_\pi^2} \left[\frac{\sigma_{\pi^- p}(\nu')}{\nu' - \nu} + \frac{\sigma_{\pi^+ p}(\nu')}{\nu' + \nu} \right]. \end{aligned} \tag{5}$$

Eqn.'s(4) and (5) are to be substituted into eqn.(3). As expected, the neutron pole terms assemble to a quantity which is insensitive to ν_n.

$$ig_A^2 \frac{m_p \nu^2}{\nu - \nu_n} - if_\pi^2 g^2_{\pi NN} \frac{4m_p}{(m_n + m_p)^2} \frac{\nu_n^2}{\nu - \nu_n} \simeq constant + ig_A^2 m_p \nu, \tag{6}$$

where the Goldberger-Treiman relation has been used. Eqn.(1) follows from eqn.(3) directly upon combining eqn.'s (4), (5), and (6).

The discussion of the sum rule at large N is strongly influenced by the properties of the Δ as $N \to \infty$. The first consideration is that $m_\Delta - m_N \sim \mathcal{O}(\frac{1}{N})$ as can be seen either by calculating the color magnetic spin splitting[5] in large N, or in the Skyrme model calculation. Using the results of ref.[1] it is straightforward to show that $\Gamma(\Delta \to \pi N) \sim O(\frac{1}{N^2})$, if this decay mode of the Δ is kinematically allowed. (Throughout the paper electroweak effects will be ignored. For example, we will not include the partial width $\Gamma(\Delta \to \gamma N)$ in the full width. Therefore, if the decay $\Delta \to \pi N$ is kinematically disallowed, the Δ is stable.) At this point, there are two simple alternatives to consider. The first is the case $m_\pi \neq 0$, $m_u = m_d$. The nucleon multiplet is degenerate ($\nu_n = 0$), as is the Δ multiplet. For sufficiently large N, $m_\Delta - m_N < m_\pi$ because $m_\pi \sim O(1)$. Then the Δ is no longer a resonance in the continuum of πp scattering, but is instead a pole below threshold, like the neutron. The second case is that when $m_u = m_d = m_\pi = 0$. Again $\nu_n = 0$, and the Δ multiplet is degenerate. Here, though, the neutron is a pole at threshold in $\pi^{\mp} - p$ scattering , and the Δ is a narrow resonance, to leading order a pole, in the continuum. The case with $m_\pi \neq 0$ is closer in spirit to the original case considered by Adler and Weisberger, and will be investigated first. The second case will be discussed briefly afterwards.

In the first situation the treatment of the neutron in the sum rule is unchanged; however, the Δ should now be treated on an equal footing with the neutron, not as a continuum resonance. In the two terms of the left-hand side of eqn.(3), the Δ^0 (Δ^{++}) contributes as an s (u) - channel pole. It should be remarked that in the nonrelativistic quark model (NRQM) in large N and in the Skyrme model there are $I = J = \frac{5}{2}, \frac{7}{2}, \ldots$ states nearly degenerate with the nucleon and Δ; they have the wrong isospin, though, to give pole contributions.

It is convenient to evaluate the various terms involving the Δ when describing the Δ by a massive Rarita - Schwinger spinor $U^{\mu;a}_{\alpha;\alpha_I}$, where μ (α) is a vector (Dirac) Lorentz index, and a (α_I) is an isospin vector (doublet) index. The

Dirac and isodoublet indices will be suppressed frequently. To project out the spin- and isospin-$\frac{3}{2}$ states, the Δ spinor satisfies

$$(\gamma_\mu)_{\alpha\beta} U^{\mu;a}_{\beta;\alpha_I} = 0, \ (\tau^a)_{\alpha_I\beta_I} U^{\mu;a}_{\alpha;\beta_I} = 0. \tag{7}$$

One quantity which will appear often is the nucleon -Δ matrix element of the axial vector current:

$$\begin{aligned} {}_N\langle p', s', I'| A^\mu_a(0) |p, s, I\rangle_\Delta = \bar{u}(p', s', I')[F(q^2)g^\mu{}_\nu + G(q^2)\gamma^\mu q_\nu \\ + H(q^2)q^\mu q_\nu + iI(q^2)\sigma^{\mu\rho} q_\rho q_\nu] U^{\nu;a}(p, s, I), \end{aligned} \tag{8}$$

where $p, s,$ and I ($p', s',$ and I') label the momentum, spin, and isospin of the Δ (nucleon), and $q = p - p'$. Using the Dirac equations for u and U, eqn.(7), and the P invariance of strong interactions, the four form factors above are a maximal set, and by time reversal invariance they are real. The $\pi - N - \Delta$ coupling will be taken to be

$$\frac{g_{\pi N\Delta}}{m_\Delta} \pi^a [(\partial_\mu \bar{\psi}) \Psi^{\mu;a} + \bar{\Psi}^{\mu;a} (\partial_\mu \psi)], \tag{9}$$

where $\Psi^{\mu;a}$ (ψ) is the field of the Δ (nucleon). There is a Goldberger-Treiman relation. One finds in the usual way,

$$\frac{f_\pi g_{\pi N\Delta}}{m_\Delta} = F + G\ (m_\Delta - m_N). \tag{10}$$

By naive counting F and G can grow at most like N as $N \to \infty$, because the coupling of the current to any one quark is $\mathcal{O}(1)$. Explicitly, we will see in the NRQM and Skyrme model that $F \sim \mathcal{O}(N)$. Because $m_\Delta - m_N \sim \mathcal{O}(\frac{1}{N})$, to leading order in N the term $G\ (m_\Delta - m_N)$ may be dropped in eqn.(10). A final useful identity is that for the Δ propagator,

$$\Delta^{\mu\nu;ab} = \Delta^{\mu\nu}(p) T^{ab} \tag{11}$$

with

$$\begin{aligned} \Delta^{\mu\nu}(p) = \frac{(\not p + m_\Delta)}{p^2 - m_\Delta^2}\left[-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3m_\Delta^2}p^\mu p^\nu + \frac{1}{3m_\Delta}(\gamma^\mu p^\nu - p^\mu\gamma^\nu)\right], \\ T^{ab} = (\delta^{ab} - \frac{1}{3}\tau^a\tau^b). \end{aligned} \tag{12}$$

Return to the analysis of eqn.(3), and evaluate the Δ-pole contributions. Both the Δ^{++} and the Δ^0 give a one pole term to the first term in eqn.(3).

$$\begin{aligned} & q^\alpha q^\beta \int d^4x\, e^{iq\cdot x} \langle p|\, T[\bar{A}^+_\alpha(x)\bar{A}^-_\beta(0)]\, |p\rangle \,\big|_{\Delta\ pole} \\ &= \frac{i}{2}\bar{u}(p)(F + G\not q)q^\mu \Delta_{\mu\nu}(p+q) q^\nu T_{11}^{1+i2,1-i2}(F - G\not q)u(p) + \begin{pmatrix} q \to -q \\ 1 + i2 \leftrightarrow 1 - i2 \end{pmatrix} \\ &= \frac{2i}{9}[F^2\ (m_\Delta + m_N)\nu^2 + O(\nu^3)](\frac{1}{\nu - \nu_\Delta} - \frac{3}{\nu + \nu_\Delta}) \end{aligned} \tag{13}$$

where $\nu_\Delta = \frac{m_\Delta^2 - m_N^2}{2m_N}$. There appears to be a noncommutivity in limits. For finite N as $\nu \to 0$ there is no term linear in ν, while when $\nu_\Delta = 0$ at infinite N there is. Again, one expects that the inclusion of the second term on the left-hand side of eqn.(3) will eliminate the sensitivity to the small mass difference. Eqn.(5) must be changed to include the Δ-pole contribution below threshold

$$f_\pi^2\ T_-(\nu, q^2 \simeq 0)\,\big|_{\Delta\ pole} = \frac{4}{9}\frac{f_\pi^2 g_{\pi N\Delta}^2}{m_\Delta^2}\frac{m_N(m_\Delta + m_N)^2}{4m_\Delta^2}(\frac{\nu_\Delta^2}{\nu_\Delta - \nu} + \frac{3\nu_\Delta^2}{\nu_\Delta + \nu}) \tag{14}$$

Only for finite N does eqn.(14) have a term linear in ν. With the appropriate

Goldberger-Treiman relation, eqn.(10), one has

$$\begin{aligned}\{q^\alpha q^\beta \int d^4x\, e^{iq\cdot x}\, \langle p|\, T[\bar{A}^+_\alpha(x)\bar{A}^-_\beta(0)]\,|p\rangle + if_\pi^2\ T_-(\nu, q^2 \simeq 0)\}_{\Delta\ pole} \\ = constant - i\frac{8}{9}m_N[F + G\ (m_\Delta - m_N)]^2\frac{(m_\Delta + m_N)^2}{4m_\Delta^2}\nu + O(\nu^2)\end{aligned} \tag{15}$$

In the limit $N = \infty$ when $\nu_\Delta = 0$, eqn.(14) does not contain a term linear in ν. However, the term linear in ν in eqn.(15) matches that of eqn.(13) in this same limit by the discussion following eqn.(10).

The modified Adler-Weisberger sum rule appropriate for $N \to \infty, m_\pi \neq 0$, and $m_u = m_d$ is obtained upon combining eqn.'s (1),(2), and (15),

$$\begin{aligned}g_A^2 =& 1 + \frac{8}{9}[F + G\ (m_\Delta - m_N)]^2\frac{(m_\Delta + m_N)^2}{4m_\Delta^2} \\ &+ \frac{2f_\pi^2}{\pi}\int_{\nu_\pi}^{\infty} d\nu \frac{\sqrt{\nu^2 - m_\pi^2}}{\nu^2}\left[\sigma_{\pi^+p}(\nu) - \sigma_{\pi^-p}(\nu)\right]\end{aligned} \tag{16}$$

Essentially the same result can be obtained in the limit $m_\pi = 0$ by slighty different manipulations. Treating the Δ as a zero-width resonance, as is valid to leading order in $\frac{1}{N}$, from eqn.(9)

$$\sigma_{\pi^-p\to\Delta^0} = \frac{\pi}{4m_N^2\nu}\delta(\nu - \nu_\Delta)\left[\frac{g_{\pi N\Delta}^2}{18}\frac{(m_\Delta + m_N)^2}{m_\Delta^4}(m_\Delta^2 - m_N^2)^2\right], \tag{17}$$

where the term in brackets is the matrix-element squared evaluated at $\nu = \nu_\Delta$. Using eqn.(10) and isospin relations gives

$$\frac{2f_\pi^2}{\pi}\int_{\nu_\Delta-\epsilon}^{\nu_\Delta+\epsilon}\frac{d\nu}{\nu}[\sigma_{\pi^+p\to\Delta^{++}}(\nu) - \sigma_{\pi^-p\to\Delta^0}(\nu)] = \frac{8}{9}[F + G\ (m_\Delta - m_N)]^2\frac{(m_\Delta + m_N)^2}{4m_\Delta^2} + \ldots \tag{18}$$

where there are additional corrections due to the finite width of the Δ.

Assuming for the moment that $F \sim O(N)$, it is rather clear at this point how one of the paradoxes presented in the introduction may be resolved. Although $\sigma_{\pi p}(\nu)$ - in particular $\sigma_{\pi p \to \Delta}(\nu)$ - is $O(1)$, the Δ component is concentrated at $\nu \simeq \nu_\Delta \sim O(\frac{1}{N})$. The Δ contribution to the sum rule is of order $\frac{f_\pi^2}{\nu_\Delta}$ which is proportional to N^2, as is g_A^2. The fact that resonance contributions to the integral over the cross sections can be order $O(N^2)$ was pointed out by Kakuto and Toyoda,[6] although apparently it was not appreciated that the Δ-nucleon mass difference is $O(\frac{1}{N})$, (or, as will become clear, that the Δ alone appears to saturate the sum rule in leading order).

There are two immediate issues to address in regards to this explanation of the consistency of the sum rule in large N. The first is to establish indeed that $F \sim O(N)$, (and review that $g_A \sim O(N)$). The second is whether or not the Δ saturates the sum rule in leading order; if it does not , then there should probably be other states with isospin $\frac{1}{2}$ or $\frac{3}{2}$ whose masses differ from m_N by order $\frac{1}{N}$ as well. Although definitive conclusions about these issues can not be reached, it is possible to examine them in models. Here, g_A and the Δ contribution to the sum rule will be evaluated in the NRQM, and the Skyrme model to leading order in $\frac{1}{N}$.

To obtain the Δ contribution in these two models is not difficult. Label the nucleon or Δ state by $\{J = I;\ J_3\ I_3\}$. In the (most naive) NRQM $m_N = m_\Delta$, and it is only necessary to calculate F. With static states

$$\frac{1}{2} g_A = \left\langle \frac{1}{2};\ \frac{1}{2}\ \frac{1}{2} \right| A_3^3(0) \left| \frac{1}{2};\ \frac{1}{2}\ \frac{1}{2} \right\rangle , \tag{19}$$

and similarly,

$$\frac{2}{3} F = \left\langle \frac{1}{2};\ \frac{1}{2}\ \frac{1}{2} \right| A_3^3(0) \left| \frac{3}{2};\ \frac{1}{2}\ \frac{1}{2} \right\rangle . \tag{20}$$

When working to leading order in $\frac{1}{N}$, $m_N = m_\Delta$ and the same formulas hold for our analysis of the Skyrme model.

The NRQM calculations are straightforward. In constructing the wave- functions color indices can be dropped because the axial current is diagonal in color. Consider, for example, the u quarks in a proton. They are in a state anti-symmetric in color, but symmetric in flavor and space, and therefore spin as well. Their state can be labeled simply as $\left|\frac{N_u}{2},\ m_s\right\rangle_u$ where N_u is the number of up quarks. Assume N is odd. The spin up proton NRQM wavefunction is

$$\left|\frac{1}{2};\ \frac{1}{2}\ \frac{1}{2}\right\rangle_{NRQM} = \sum_{m=-(\frac{N+1}{4})}^{\frac{N+1}{4}} \left\langle \frac{N+1}{4}, m; \frac{N-1}{4}, \frac{1}{2} - m \,\middle|\, \frac{1}{2}\frac{1}{2} \right\rangle \left|\frac{N+1}{4}, m\right\rangle_u \left|\frac{N-1}{4}, \frac{1}{2} - m\right\rangle_d . \tag{21}$$

The required Δ wavefunction is

$$\left|\frac{3}{2};\ \frac{1}{2}\ \frac{1}{2}\right\rangle_{NRQM} = \sum_{m=-(\frac{N+1}{4})}^{\frac{N+1}{4}} \left\langle \frac{N+1}{4}, m; \frac{N-1}{4}, \frac{1}{2} - m \,\middle|\, \frac{3}{2}\frac{1}{2} \right\rangle \left|\frac{N+1}{4}, m\right\rangle_u \left|\frac{N-1}{4}, \frac{1}{2} - m\right\rangle_d . \tag{22}$$

As usual, $A_3^3(0)\Big|_{NRQM} = (\frac{1}{2}\sigma^3\tau^3)_{quarks}$. Then in the NRQM[6] ,

$$\frac{1}{2}g_A = \sum_{m=-(\frac{N+1}{4})}^{\frac{N+1}{4}} \left\langle \frac{N+1}{4}, m; \frac{N-1}{4}, \frac{1}{2} - m \,\middle|\, \frac{1}{2}\frac{1}{2} \right\rangle^2 (2m - \frac{1}{2}) = \frac{N+2}{6}. \tag{23}$$

Similarly,

$$\frac{2}{3}F = \sum_{m=-(\frac{N+1}{4})}^{\frac{N+1}{4}} \left\langle \frac{N+1}{4}, m; \frac{N-1}{4}, \frac{1}{2} - m \,\middle|\, \frac{1}{2}\frac{1}{2} \right\rangle \left\langle \frac{N+1}{4}, m; \frac{N-1}{4}, \frac{1}{2} - m \,\middle|\, \frac{3}{2}\frac{1}{2} \right\rangle (2m - \frac{1}{2}) = \frac{1}{3\sqrt{2}}\sqrt{(N+5)(N-1)} . \tag{24}$$

Finally, in this model one finds

$$g_A^2 = 1 + \frac{8}{9}F^2, \tag{25}$$

or that the Δ completely saturates the sum rule. This was known long ago for $N = 3$ [7] and one expects the result to hold for arbitrary N as well. However, it does provide an example where g_A and F are $O(N)$, and the Δ saturates the sum rule in leading order, (to all orders in fact).

The Skyrme model as developed in ref.[1] can provide information about the leading N dependence of g_A and F. In leading order the relevant quantity for the sum rule is $\frac{F}{g_A}$. By eqn.'s (20) and (19),

$$\frac{F}{g_A} = \frac{3}{4}\frac{\left\langle \frac{1}{2};\ \frac{1}{2}\ \frac{1}{2}\right| A_3^3(0) \left|\frac{3}{2};\ \frac{1}{2}\ \frac{1}{2}\right\rangle}{\left\langle \frac{1}{2};\ \frac{1}{2}\ \frac{1}{2}\right| A_3^3(0) \left|\frac{1}{2};\ \frac{1}{2}\ \frac{1}{2}\right\rangle}. \tag{26}$$

The ratio is only sensitive to the collective coordinate wavefunction; it is in a sense a group theoretic quantity. Applying a general result of Manohar [8] ,

$$\frac{F}{g_A}\Big|_{Skyrme} = \lim_{N\to\infty} \frac{F}{g_A}\Big|_{NRQM} = \frac{3}{2\sqrt{2}}. \tag{27}$$

This may be seen explicitly as follows. The collective coordinate wavefunctions are just those of the spherical top, $\sqrt{\frac{2j+1}{8\pi^2}}D^{(j)}_{m_J m_I}(A)$, where A is an SU(2) matrix collective coordinate. From ref.[1], the collective coordinate component of the operator $A_3^3(0)$ is proportional to $Tr(\tau^3 \hat{A}\tau^3\hat{A}^\dagger) \propto D^{(1)}_{0\ 0}(\hat{A})$, where $\hat{A}$ is the collective coordinate operator diagonal on $|A\rangle$. The matrix elements in the collective coordinates are given in terms of 3-j symbols.

$$\begin{aligned}\left\langle j_1;\ m_1'\ m_1\right| D^{(1)}_{0\ 0}(\hat{A}) \left| j_2;\ m_2'\ m_2\right\rangle_{col\ coord} \\ =(-)^{m_1'-m_1}\sqrt{2j_1+1}\sqrt{2j_2+1}\begin{pmatrix} j_2 & j_1 & 1 \\ m_2' & -m_1' & 0\end{pmatrix}\begin{pmatrix} j_2 & j_1 & 1 \\ m_2 & -m_1 & 0\end{pmatrix}\end{aligned} \tag{28}$$

Evaluation of the relevant 3-j symbols gives the desired result, eqn.(27). By eqn.(27), the Δ saturates the sum rule in leading order in N. It seems likely that this holds in large-N QCD as well, although we have no proof of this.

Phenomenologically, one may ask how well the Skyrme model reproduces the experimental result for the sum of the Δ and continuum contributions in eqn.(16), (equivalently, the continuum in eqn.(1)). Apparently, it does passably giving to leading order in N (from the Δ) the result $\simeq (.61)^2$, while experimentally the result is $\simeq .5$. If one then computes g_A^2 indirectly from eqn.(16) and this result, one gets a rather good value for $g_A^2 \simeq 1.36$. Higher order corrections have not been systematically included here. The result $(.61)^2$ is not a purely leading order result but includes some higher order effects because the experimental values for m_N and m_Δ have been used in calculating it. Also, explicitly higher order effects in the Δ plus continuum contribution have not been evaluated. If indeed the Δ plus continuum series behaves well here (the series sums to a value close to .36), it would be necessary apparently in a direct calculation of g_A to include the first two subleading terms before an accurate value could be obtained.

The particular situation in the Skyrme model illustrates a rather general point. The issue is whether both of the two physical quantities g_A^2 and the Δ plus continuum contribution can be well-behaved as series in $\frac{1}{N}$ in the most naive sense, that is that the first term is reasonably close to the final result, the magnitude of each successive term is progressively smaller, and each successive partial sum is closer to the final result. Suppose that the sum of each series accurately reproduces the measured values. The two series are identical except for the $\mathcal{O}(1)$ terms which differ by 1. In absolute magnitude the larger of the $\mathcal{O}(1)$ terms is minimized if the $\mathcal{O}(1)$ term of g_A^2 (Δ plus continuum) is +.5 (-.5). On the other hand, a reasonable first approximation to each series is about 1.

Then, the $\mathcal{O}(N)$ term of each series must be rather small and one of the second partial sums must move away from the final result. Therefore, it is not possible for both series to be well-behaved in the most naive sense, and either g_A^2 or the Δ plus continuum contribution to the Adler-Weisberger sum rule will be somewhat problematic in any large-N expansion.

ACKNOWLEDGEMENTS

I thank M. Karliner, M. Mattis, M. Peskin, and M. Weinstein for first stimulating my interest in this problem, and for many helpful discussions. I have also benefitted from conversations with E. Braaten, M. Cvetic, F. Gilman, and R. Pisarski.

REFERENCES

1. G.S.Adkins, C.R.Nappi, and E.Witten, *Nucl.Phys.* **B228**(1983)552.
2. S.Adler, *Phys.Rev.Lett.***14**(1965)1051, *Phys.Rev.***140**(1965)B736; W.Weisberger, *Phys.Rev.Lett.***14**(1965)1047, *Phys.Rev.***143**(1966)1302.
3. E.Witten, *Nucl.Phys.***B160**(1979) 57.
4. W.Weisberger, ref.2; M.Weinstein, Lectures on Chiral Symmetry, Springer Tracts in Modern Physics, Vol.60, Ed. G.Hohler.
5. T.A.DeGrand, R.L.Jaffe, K.Johnson, and J.Kiskis *Phys.Rev.***D12**(1975) 2060; A.De Rujula, S.L.Glashow, and H.Georgi, *Phys.Rev.***D12**(1975)147.
6. A.Kakuto and F.Toyoda, *Prog.Theor.Phys.***66**(1981)2307; G.Karl and J.E. Paton, *Phys.Rev.***D30**(1984)238.
7. B.Lee, *Phys.Rev.Lett.***14**(1965)676.
8. A.Manohar, *Nucl.Phys.***B248**(1984)19.