

Research on Quantum State Implementation Based on Topological Insulators

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Abstract. This paper first introduces the mainstream technology route of quantum computing. The analysis provides the role and advantages of topological insulators in quantum computing, such as providing stable quantum bits, enhancing the coherence and non-locality of quantum bits, and enabling the transmission of quantum information and entanglement allocation. This paper then proposes a quantum state implementation scheme based on Bi_2Se_3 , based on the characteristics and advantages of the quantum Hall effect of topological insulators. It includes calculating the eigenstates of the Hamiltonian of topological insulators, obtaining the spatial distribution of wave functions, and generating a quantum bit platform based on topological insulators. A quantum state implementation system based on topological insulator (Bi_2Se_3) was constructed, and the system framework and reference configuration were provided. The solution presented in this article is an important research achievement in topological quantum computing, providing more possibilities for the implementation of feasible fault-tolerant quantum computer prototypes.

Keywords. Quantum; quantum state; quantum computing; topological insulator; topological quantum computing; qubit.

1. Introduction

Quantum information technology is a technology based on quantum mechanics, which involves observing and manipulating microscopic particles and their systems such as photons and electrons to obtain quantum states, thereby achieving information acquisition, transmission, and processing that cannot be achieved by classical theory.

One of the key technological links in the development of quantum computing is the physical implementation of quantum chips. To achieve quantum computing, like classical computing, information needs to be encoded on energy levels or states that can be distinguished under certain conditions. The simplest is a two-level system, which is a typical research object of atomic and molecular physics and solid-state physics. Correspondingly, the encoded information is called a quantum bit, which is a fundamental unit in quantum information technology. The decoherence time of a quantum bit is particularly important for its manipulation. Decoherence time refers to the length of time after entanglement of quantum bits in quantum computing that generates correlation. To complete measurement and manipulation of quantum bits, the decoherence time must be much longer than the measurement and manipulation time. Under certain conditions, the larger the decoherence time parameter, the shorter the gate operation time, which is more advantageous for the implementation of quantum computing.

At present, the main quantum computing technology routes include optical quantum,



superconducting quantum circuits, semiconductor quantum dots, ion traps, etc. Although various implementation platforms have made significant progress, it is still unknown which system can achieve universal quantum computing service capabilities earlier in the future. Superconducting quantum circuits. Superconducting quantum computing utilizes the motion of ultra-low temperature frozen electrons (holes) to achieve control of particle states. Superconducting qubits come in three forms: superconducting phase, superconducting magnetic flux, and superconducting charge. The core unit of superconducting quantum circuits is the Josephson junction, which is a sandwich structure composed of superconductors insulators superconductors. Control of superconducting qubits can be achieved by adding electromagnetic signals and increasing the waveguide frequency. Superconducting quantum circuits are currently the fastest and best solid-state physics implementation platform. However, due to the openness of quantum systems, environmental noise, magnetic flux noise, and other difficult to manipulate degrees of freedom, energy dissipation and decoherence occur. In addition, the implementation of extremely low temperatures also poses certain problems in the implementation of superconducting quantum circuits. A single electron transistor prepared on a planar semiconductor electronic device follows the motion laws of quantum mechanics, and the system composed of electron spin up and down can serve as a quantum bit. A typical example is the diamond NV color center, which can be controlled by ordinary electronic switches through the correlation between electrons and spin, completing the manipulation of a single quantum bit. However, semiconductor quantum dot systems are severely affected by nuclear spin in the surrounding environment, facing issues of decoherence and low fidelity. Ion trap quantum computing utilizes the interaction between charge and magnetic field to restrict the motion of charged particles and utilizes the ground and excited states of confined particles as two-level systems, thus serving as quantum bits. Ion traps face significant problems, among which are related to this article: poor scalability, large volume, difficulty in miniaturization, and decoherence caused by ion sensitivity to electric field noise.

The quantum state implementation method based on topological insulator material (Bi_2Se_3) proposed in this article defines a quantum bit that encodes information on the surface of the topological insulator and controls the quantum bit by applying a finite electric field. The topological insulator becomes one of the candidate materials for integration into transistors due to its topologically protected, non-dissipative surface state, which is a normal insulation state. This scheme has the advantages of high integration, easy fabrication of thin films, and saving on raw materials and integration costs when applied to quantum bits.

2. Related Work

In the 1980s, the quantum Hall effect was observed in two-dimensional electron gases under low-temperature and strong magnetic field conditions, opening the door for condensed matter physicists to study topological quantum physics [1-3]. In recent years, there have been some research achievements on the quantum effects achieved by topological insulation materials. [4] conducted analysis and outlook the future research directions of magnetic topological insulators and the quantum transport phenomena in this system that have not yet been fully understood. Research on the existence of topologically non mediocre Majorana zero energy modes based on non-local quantum transport measurements [5]. Some experiments have studied the quantum Hall effect of topological surface states [6] and provided prospects for the study of quantum Hall states in topological insulators. By adding non topological defects (lattice vacancies) to a topological insulator with particle hole symmetry, the energy level distribution and particle density of states distribution of the system were obtained through strict numerical diagonalization. The global phase diagram of the system was calculated using Chen numbers, and the novel quantum states protected by particle hole symmetry in topological insulators were studied [7]. In addition, the combination of topological quantum bits and superconducting quantum computing technology can achieve rapid manipulation and reading of quantum states within a limited quasi particle poisoning time [8].

3. The Role of Topological Insulators in Quantum Computing

Topological insulators are a special type of condensed matter physical material with band isolation and boundary state characteristics. Topological insulators have a unique electronic structure, and compared to traditional insulators, they can exist in a special boundary state on the material surface or boundary that does not require an external magnetic field or bias. The existence of this boundary state is closely related to topological invariants, and electrons exhibit very special behavior in topological insulators, which is called topological protection. The main advantages of topological insulators in quantum computing are:

(1) Based on the characteristics of boundary state "topological protection", topological insulators can provide stable quantum bits with strong stability and anti-interference ability, which makes the storage and operation of quantum information in topological insulators more reliable, thereby improving the stability of quantum computing.

(2) Enhancing the coherence and non-locality of quantum bits: Topological insulators have strong coherence and non-locality, which makes the interaction between quantum bits easier to achieve, providing more efficient resources and faster computing speed for quantum computing.

(3) Implementing the transmission and entanglement allocation of quantum information: By utilizing the characteristics of topological insulators, the transmission and entanglement allocation of quantum information can be achieved, further expanding the application scope of quantum computing. Efficient quantum gate operation: Topological insulators have a very low energy threshold, which can effectively reduce the error rate of quantum gates. Meanwhile, it has a very strong quantum bit energy level structure, which can effectively ensure the stability and controllability of quantum bits. Overall, topological insulators provide an ideal physical platform for quantum computing due to their unique electronic structure and stable quantum states.

4. A Quantum State Implementation Method Based on Topological Insulators

The flowchart of quantum state implementation based on topological insulators is shown in figure 1, with the specific steps as follows:

- (1) Step S1: Calculate the eigenstates of the Hamiltonian of the topological insulator.
- (2) Step S2: Based on the calculated eigenstates of the Hamiltonian, obtain the wave function spatial distribution of a topological insulator with a thickness of five atomic layers at high symmetry points.
- (3) Step S3: Select two surface states in the spatial distribution of the wave function, namely hole type and electron type, with spin up energy bands, to generate a quantum bit platform based on topological insulators.

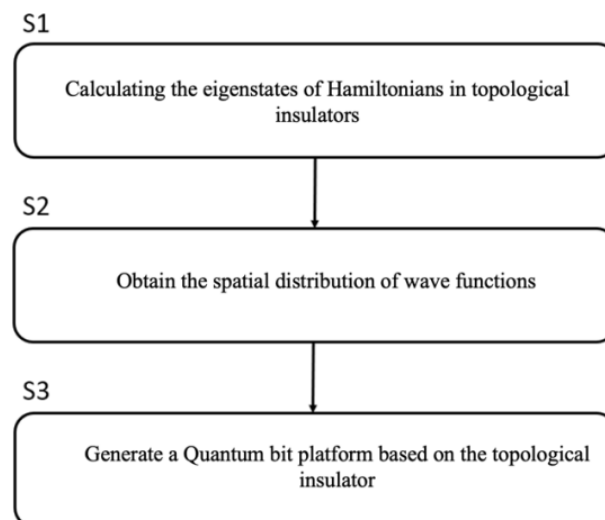


Figure 1. Flow chart of quantum state implementation based on topological insulators.

4.1. Calculating the Eigenstates of Hamiltonians in Topological Insulators

Select the topological insulator Bi_2Se_3 and calculate its Hamiltonian eigenstates, as shown in figure 2. The specific description is as follows:

(1) Step S11: Obtain the effective low-energy Hamiltonian model of the topological insulator (Bi_2Se_3);

(2) Step S12: Based on the effective low-energy Hamiltonian model, determine the state closest to the Fermi level caused by spin orbit interaction as the basis vector of the Hamiltonian

(3) Step S13: Based on the determined basis vector of the Hamiltonian, use the difference finite method to solve the eigenstates of the Hamiltonian.

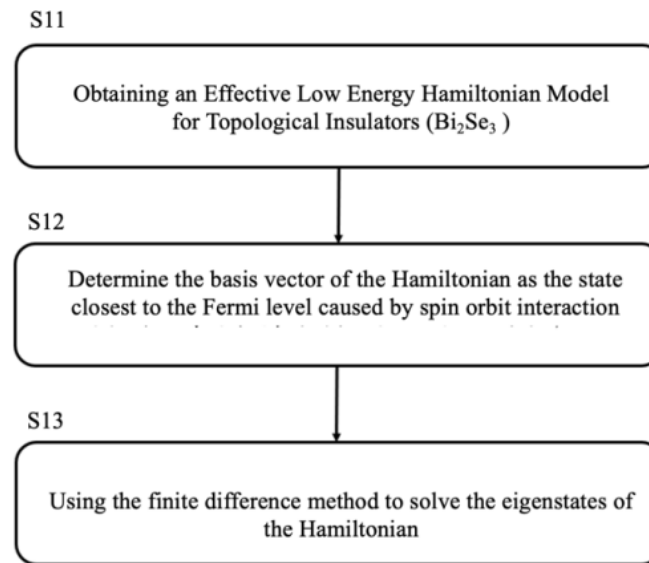


Figure 2. Calculation flowchart of Hamiltonian eigenstates for topological insulator (Bi_2Se_3).

1) Step S11 Effective Low Energy Hamiltonian Model

The effective low-energy Hamiltonian model in step S11 is shown in formula (1).

$$H_0 = C(k)I_4 + M(k)\Gamma_4 + B_0\Gamma_3k_z + A_0(\Gamma_1k_y - \Gamma_2k_x) \quad (1)$$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \Gamma_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \Gamma_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (2)$$

Among them, I_4 is a 4 * 4 identity matrix, as shown in (3).

$$C(k) = C_0 + C_1k_z^2 + C_2|k_{||}|^2, \quad (3)$$

$$M(k) = M_0 + M_1k_z^2 + M_2|k_{||}|^2,$$

$$k_{||} = k_x - ik_y$$

The first class matrix is obtained from the principle of symmetry, and the parameters $C_0, C_1, C_2, M_0, M_1, M_2, A_0$ and B_0 are the relevant parameters obtained from dispersion experiments. k represents the

wave vector of the electron, and k_x, k_y, k_z represent the x, y, z axial component of the electron wave vector; $C(\mathbf{k})$ represents the kinetic energy of electrons in this type of material, $M(\mathbf{k})$ represents the coupling term between the spin orbit of electrons and their kinetic energy in topological insulators, M_0, M_1 are the decisive parameters of the topological phase, A_0 is the Rashba type spin orbit interaction, and B_0 represents the coupling strength between Bi and Se atoms with the same spin. Changing the size of C_0 can regulate the energy of topological insulators, but it does not change the energy level difference. A_0 is the main reason for forming topological protected surface states.

2) Step S12 Determine the base vector

In step S12, select the state closest to the Fermi level caused by spin orbit interaction in the above model as the basis vector of the Hamiltonian, as shown in (4).

$$\begin{aligned}
 |\Psi_1\rangle &= \left| P1\pm, m_j = \frac{1}{2} \right\rangle \approx \left| P1\pm, p_z, \uparrow \right\rangle, \\
 |\Psi_2\rangle &= \left| P2\mp, m_j = \frac{1}{2} \right\rangle \approx \left| P2\mp, p_z, \uparrow \right\rangle, \\
 |\Psi_3\rangle &= \left| P1\pm, m_j = -\frac{1}{2} \right\rangle \approx \left| P1\pm, p_z, \downarrow \right\rangle, \\
 |\Psi_4\rangle &= \left| P2\mp, m_j = -\frac{1}{2} \right\rangle \approx \left| P2\mp, p_z, \downarrow \right\rangle,
 \end{aligned} \tag{4}$$

Among them, $P1$ represents the atomic state of Bi element, $P2$ represents the atomic state of Se element, \pm represents positive parity, \mp represents negative parity, m_j represents spin quantum number, characterizes the spin state of electrons, and $1/2$ represents the spin up of electrons; P_z represents the z -axis component of the p-level orbital of an electron, \uparrow represents spin up, and \downarrow represents spin down.

3) Step S13 Solving Hamiltonian eigenstates

Using Finite Difference Algorithm to Solve the Eigenstates of Hamiltonians.

4.2. Obtain the Spatial Distribution of Wave Functions

Based on the eigenstates of the Hamiltonian, obtain the wave function spatial distribution of a topological insulator (Bi_2Se_3) with a thickness of five atomic layers at the high symmetry point Γ . The high symmetry points include: $|S1\rangle$ is a surface state with a hole type energy band spin down; $|S2\rangle$ is the surface state of the hole type energy band with spin up; $|S3\rangle$ is a surface state with an electron type band spin up; And $|S4\rangle$ is a surface state with electron type band spin down.

Each surface state contains four wave components (composed of intrinsic states).

4.3. Generate a Quantum bit Platform Based on the Topological Insulator

Select the two surface states of hole type and electron type energy bands with spin up in the spatial distribution of the wave function mentioned above to generate a quantum bit platform based on topological insulator (Bi_2Se_3). Combine the two surface states so that the quantum states are located on the upper and lower surfaces of the topological insulator for information encoding.

To achieve quantum computing, it is first necessary to define a suitable quantum two-level system. Naturally, by combining the above surface states, the state where the majority of charges are located on the lower surface of the sample (when the sample thickness is 0) can be defined as a quantum state; The quantum state where the majority of charges are located on the surface of the sample (when the sample thickness is 5 atomic layers) is defined as the quantum state $|\Phi_1\rangle = |S2\rangle + |S3\rangle$. By combining the two surface states of hole type and electron type with spin up energy bands, the obtained quantum state is located on the upper and lower surfaces of the topological insulator, and the information can be encoded

on the upper and lower surfaces, forming a quantum bit platform based on the topological insulator, as shown in figures 3a and 3b, which are the spatial distribution of the wave function of the quantum state generated based on the topological insulator (Bi_2Se_3) on the lower surface of the sample.

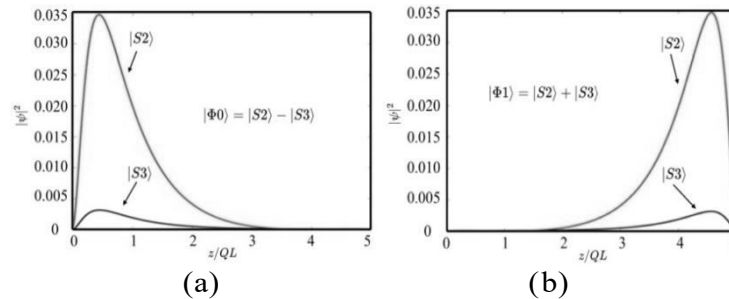


Figure 3. Spatial distribution map of quantum state wave functions generated based on topological insulator (Bi_2Se_3).

5. Quantum Bit Gate Operation Based on Topological Insulators

For quantum computing, it is necessary to manipulate quantum bits. In the actual calculation process, it is to rotate quantum bits on Bloch balls (in any direction), which is also called logic gate operation.

Figure 4 shows a schematic diagram of quantum bit gate operation based on topological insulators on Bloch balls. Taking an applied electric field as an example, rotation operation is performed, where E represents the applied electric field. In this embodiment, a rotation operation is performed on the quantum state $|\Phi_0\rangle$ to obtain the operation time at different sample layer thicknesses.

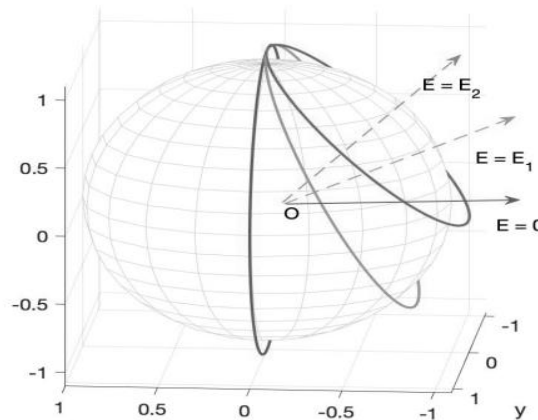


Figure 4. Schematic diagram of quantum bit gate operation based on topological insulators implemented on Bloch balls.

Figure 5 is a schematic diagram of the time scale for the rotation operation of quantum bits generated based on topological insulators. As shown in figure 5, the time scale for the rotation operation of quantum bits in topological insulator quantum dots is approximately 10^{-3} - 10^{-1} ps, and for topological insulator quantum bits with five atomic layers thick, the rotation operation time is 0.36ps. In quantum logic gate operations, there is a type of gate operation called Pauli Z gate operation, which is called Pauli-Z gate (σ_z) operation, For example, this gate operation keeps the basic state $|0\rangle$ unchanged and changes $|1\rangle$ to $-|1\rangle$. Perform the quantum states at any position in the topological insulator quantum bit defined in the above embodiment $|\Phi_0\rangle$ σ_z -gate operation, resulting in effective results σ . The fidelity of σ_z -gate operation is very high, above 99.9%.

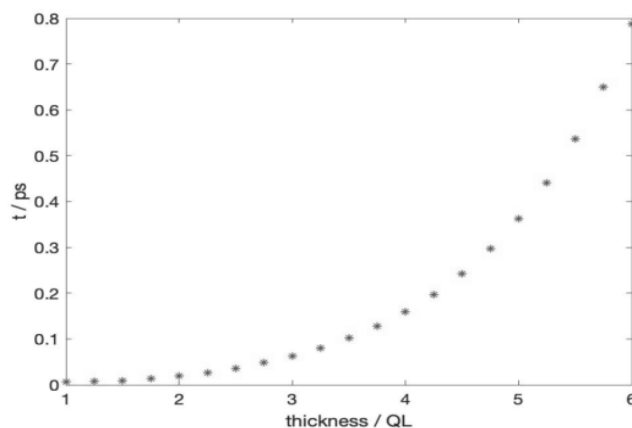


Figure 5. Schematic diagram of the time scale for the rotation operation of quantum bits generated based on topological insulators.

6. Conclusion

This article combines the characteristics and advantages of the quantum Hall effect of topological insulators and proposes a quantum state implementation method based on topological insulators (Bi_2Se_3), which encodes information into quantum bits on the surface of topological insulators. Quantum states implemented by topological insulators with "topological protection" have the advantages of long decoherence time and high fidelity, providing a new choice for the physical implementation platform of quantum computing.

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References

- [1] Klitzing K V, Dorda G and Pepper M 1980 *Phys. Rev. Lett.* **45** 494.
- [2] Klitzing K V 1986 *Rev. Mod. Phys.* **58** 519.
- [3] Tsui D C, Stormer H L and Gossard A C 1982 *Phys. Rev. Lett.* **48** 1559.
- [4] Liu C and W Y Y 2023 Quantum transport phenomena in magnetic topological insulators *Acta Physica Sinica* **72**(17): 225-242.
- [5] Hou Y L 2022 *Research on Nonlocal Quantum Transport Based on Topological Insulator Multi Terminal Devices* (Institute of Physics, Chinese Academy of Sciences).
- [6] Zhang S and Fenglin S 2023 Research on nonlocal quantum transport based on topological insulator multi terminal devices [D]. University of Chinese Academy of Sciences (Institute of Physics, Chinese Academy of Sciences) **72**(17): 243-253.
- [7] Tu W and Wu Y J 2022 Novel quantum states protected by particle hole symmetry in topological insulators *Journal of Xi'an University of Technology* **42**(06): 547-551+558.
- [8] Sun P X 2023 Superconducting devices and hybrid superconducting qubits based on topological insulator nanowires [D]. University of Chinese Academy of Sciences (Institute of Physics, Chinese Academy of Sciences).