F_L proton structure function from the unified approach

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We compute the longitudinal proton structure function F_L from the k_T factorization scheme, using the unified DGLAP/BFKL resummation approach at small x for the unintegrated gluon density. The differences between the k_T factorization, collinear factorization and dipole approaches are analyzed and discussed. We present the comparisons with the experimental data from HERA collider.

1 Introduction

The longitudinal nucleon structure function F_L, measured in the deep inelastic lepton-nucleon scattering, is proportional to the cross section for the interaction of the longitudinally polarized virtual photon with a nucleon. This observable is of particular interest since it is directly sensitive to the nucleon gluon distribution. In the naive quark-parton model F_L vanishes (the Callan-Gross relation). This is due to the quark spin 1/2 and the fact that the struck quark has limited transverse momentum in the naive parton model. In the QCD improved parton model, however, the gluon interactions cause the average quark transverse momentum $\langle \kappa_T^2 \rangle$ to grow with increasing value of the (minus) photon virtuality Q^2. As a result, F_L acquires a nonzero leading twist contribution proportional to $\alpha_s(Q^2)$. At small values of the Bjorken variable x, F_L is driven mainly by gluons through the transition $g \to q\bar{q}$. Therefore, it can be used for the extraction of the gluon distribution in a nucleon providing a crucial test of the validity of perturbative QCD in this kinematical range.

At small x, the nucleon structure functions receive large logarithmic corrections coming from resummation of large powers of $\alpha_s \ln 1/x$. This procedure goes beyond the standard collinear factorization and is achieved by the use of the k_T factorization formalism [2] with the unintegrated gluon density found as a solution to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [3]. Since the small x expansion receives large corrections at higher orders, resummation at small x is in general necessary in order to obtain predictions which are in agreement with data.

In this talk we present the calculation of F_L within the k_T factorization formalism using the unintegrated gluon density obtained from the Kwieciński-Martin-Staśto (KMS) approach [4], which provides a convenient framework for the unification of the conventional Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) and small x BFKL evolution equations. From the point of view of the small x hierarchy, the KMS approach includes important effects of higher order resummation. We systematically analyze the relation between this approach and the collinear and dipole approaches, investigating the role of different contributions to F_L in various kinematical regions. We especially emphasize the role of the exact gluon kinematics in the k_T factorization formulae and demonstrate numerically that this kinematics have a sizable effects on the predictions for F_L, and thus, on the extracted gluon density. We compare our computations with the experimental data at small x from the H1 [5] and ZEUS [6] collaborations.

The results presented in this talk [1] have been obtained in collaboration with Krzysztof Golec-Biernat and published in [7].

2 Unified BFKL/DGLAP formalism

We argue that the unintegrated gluon distribution $f(x, k_T^2)$ and the k_T factorization theorem provides the natural framework for describing observables at small x. To determine f we arrange the BFKL equation so that we only need to solve it in the perturbative domain $k_T^2 > k_0^2$ [4]. We also include the residual DGLAP contributions. To be precise we have

$$\begin{aligned} f(x,k_T^2) &= \frac{\alpha_S}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z}\, g(\frac{x}{z}, k_0^2) \; + \\ &+ \frac{3\alpha_S}{\pi}\, k_T^2 \int_x^1 \frac{dz}{z} \int_{k_0^2} \frac{dk_T'^2}{k_T'^2} \left[\frac{f(\frac{x}{z}, k_T'^2)\, \theta\,(\frac{k_T^2}{z} - k_T'^2) - f(\frac{x}{z}, k_T^2)}{|k_T'^2 - k_T^2|} + \frac{f(\frac{x}{z}, k_T^2)}{(4k_T'^4 + k_T^4)^{\frac{1}{2}}} \right] \\ &+ \frac{3\alpha_S}{\pi} \int_x^1 dz \left(\frac{P_{gg}(z)}{6} - \frac{1}{z} \right) \int_{k_0^2}^{k_T^2} \frac{dk_T'^2}{k_T'^2}\, f\left(\frac{x}{z}, k_T'^2\right) + \frac{\alpha_S}{2\pi} \int_x^1 dz P_{gq}\, \Sigma\left(\frac{x}{z}, k_T^2\right) \end{aligned} \tag{1}$$

where $-1/z$ is taken from DGLAP because it is already included in BFKL. The input term comes from two sources: the $k_T^2 < k_0^2$ parts of BFKL and DGLAP terms. We specify the input in terms of a simple two parameter form $g(x, k_0^2) = N(1-x)^\beta$. In addition to restricting the solution of the BFKL equation to the perturbative region $k_T^2 > k_0^2$ and to including the DGLAP terms, we have also introduced a θ function which imposes the constraint $k_T'^2 < k_T^2/z$ on the real gluon emissions. We take a running coupling $\alpha_S(k_T^2)$. The final term in (1) depends on the quark singlet momentum distribution Σ. At small x the sea quark components S_q of Σ dominate. They are driven by the gluon via the $g \to q\overline{q}$ transitions, that is $S_q = B_q \otimes f$ where at lowest order B_q is the box (and crossed box) contribution of the boson-gluon fusion process. Besides the z and k_T^2 integrations symbolically denoted by $\otimes$ the box contribution implicitly includes an integration over the transverse momentum κ_T of the exchanged quark. The evolution equation for Σ may be written in the form

$$\Sigma = S^{(0)} + \sum_q B_q(k_T^2 = 0) \overline{\otimes}\, z g(z, k_0^2) + \sum_q B_q \otimes f + P_{qq} \otimes S_q + V \tag{2}$$

where the first three terms on the right hand side are the "$B_q \otimes f$" contributions coming from three different regions of the k_T^2 and κ_T^2 integrations. First, in the non-perturbative domain, $k^2, \kappa'^2 < k_0^2$, the u, d, s sea quark contribution is parametrized in the form $S^{(0)} = C_{I\!P}\, x^{-0.08}(1-x)^8$ consistent with soft pomeron and counting rule expectations, where $C_{I\!P}$ is independent of Q^2. The constant $C_{I\!P}$ is fixed in terms of the two parameters, N and β, by the momentum sum rule. In the second region, $k^2 < k_0^2 < \kappa'^2$, we apply the strong k_T ordering approximation with $B_q \approx B_q(k_T^2 = 0)$ so that the k_T^2 integration can be carried out to give a contribution proportional to $g(x/z, k_0^2)$.

Finally in third region, $k^2 > k_0^2$, we evaluate the full box contribution; this gives the main contribution and is responsible for the rise of F_2 with decreasing x. The last two terms in

(2) give the sea $\to$ sea evolution contribution, and the valence contribution $V(x, Q^2)$ which is taken directly from a recent parton set. The charm quark component of the sea is given totally by *perturbative* QCD, since for $k_T^2 < k_0^2$ the box $B(k_T^2 = 0)$ is finite as $\kappa_T^2 \to 0$ due to $m_c \neq 0$.

3 k_T factorization vs collinear and dipole picture

The formula used for the calculation of the longitudinal structure function F_L from the k_T factorization formalism with the KMS approach reads:

$$\begin{aligned} F_L(x, Q^2) &= \frac{Q^4}{\pi^2} \sum_q e_q^2 \int \frac{dk^2}{k^4}\, \theta(k^2 > k_0^2) B^L \otimes f(x_g, k^2) \\ &+ \frac{\alpha_s(Q^2)}{\pi} \left\{ \frac{4}{3} \int_x^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 F_2(y, Q^2) + \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) yg(y, k_0^2) \right\}. \end{aligned}$$

where the cutoff for the gluon momentum $k_0^2 = 1$ GeV^2. The function B^L denotes boson-gluon fusion in the k_t factorization approach. In the above formula the gluon kinematics is kept exact, and therefore x_g is a function of the internal momenta in the boson-gluon fusion box. As a result of this $x_g > x$.

The k_T factorization formula reduces to the collinear one, in the on-shell approximation. This is obtained in the limit of very small gluon transverse momentum $k_T^2 \ll Q^2$. In this case the expression given by the above formula gives the collinear limit

$$F_L^{(\text{on-shell})}(x, Q^2) = 2 \sum_q e_q^2 \left[J_q^{(1)} - 2 \frac{m_q^2}{Q^2} J_q^{(2)} \right] \tag{3}$$

where

$$J_q^{(1)} = \frac{\alpha_s}{\pi} \int_{\bar{x}_q}^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}\, yg(y, Q^2)\,,$$

and

$$J_q^{(2)} = \frac{\alpha_s}{\pi} \int_{\bar{x}_q}^1 \frac{dy}{y} \left(\frac{x}{y}\right)^3 \ln \left[\frac{1 + \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}}{1 - \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}} \right] yg(y, Q^2)$$

Let us emphasize that in order to obtain the limit of the k_T factorization formula consistent with the collinear factorization, it is crucial to take the exact kinematics for the argument of the gluon density and drop the terms proportional to k^2/Q^2.

On the other hand, by taking the small x approximation in the argument of the unintegrated gluon density one can show that the k_T factorization formula reduces to the dipole formula It is obtained after the Fourier transformation of k_T factorized expression from the space of quark transverse momenta $\boldsymbol{\kappa}$ into the the space of the transverse coordinates, $\mathbf{r}$. It is important to note that one also needs to perform the small x approximation in the argument of the gluon density in formula, $x_g \to x$. This is obviously justified only in the

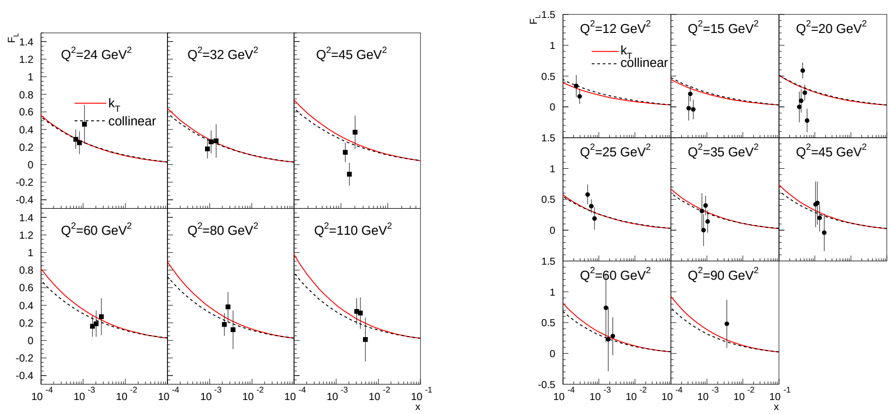

Figure 1: Comparison between the collinear and the k_T factorization calculations with exact gluon kinematics. The data are from the ZEUS and H1 experiment [6, 5]. The light quarks u, d, s are treated as massless, the charm quark mass is set to be $m_c = 1.5\,\mathrm{GeV}$. The solid (red) line denotes the calculation using the k factorization with exact kinematics, the black (dashed) line is calculation using the collinear factorization with the massive charm quark.

limit of very small x. In this way the Fourier integrals over the transverse momenta can be easily performed.

Thus the k_T factorization formula contains both collinear and dipole limits. The collinear formula is obtained upon expanding the k_T factorization formula in powers of k^2/Q^2 and retaining the lowest order in this expansion. The exact gluon kinematics has to be taken into account in that procedure. On the other hand, the dipole approach is obtained from the k_T factorization expression in the limit when x is very small, which amounts to approximating the gluon longitudinal momentum fraction x_g by the Bjorken x.

3.1 Comparison with the HERA data

In Figure 1 we show the calculations obtained using the collinear approach and the results from the k_T factorization formalism with the exact kinematics compared with the ZEUS and H1 data. The agreement between the experimental data and our calculations is good. In the case of the k_T factorization with the exact kinematics the results are rather close to the ones obtained from the collinear approach. The only regions where the results differ is the regime of very small x and high Q^2, where the k_T factorization with exact kinematics tends to give higher values, and the regime of small Q^2 (below 10 GeV^2) where the k_T factorization-based approach falls below collinear one.

We stress the importance of the exact kinematics in the evaluation of the gluon density. The collinear and k_T factorization approaches give very similar results only in the case when the gluon density is evaluated at x_g in the k_T factorization formula. In Figure 2 we show also the calculation where in the k_T factorization the argument of the gluon density equals

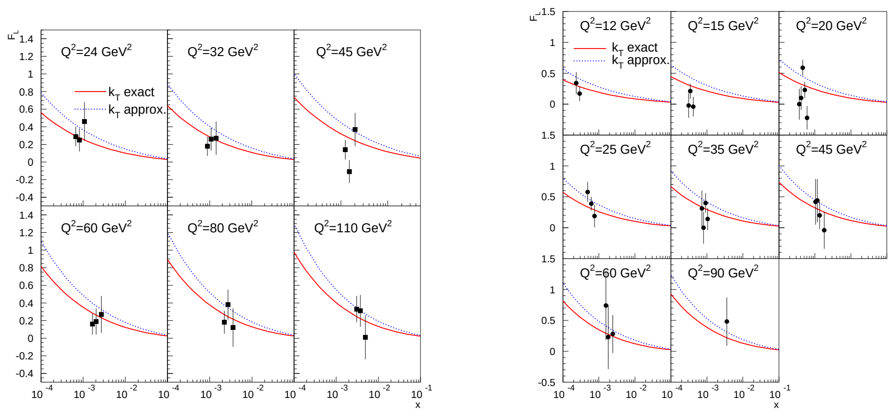

Figure 2: Comparison between the exact and the approximate (dipole like) kinematics in the k_T factorization formula. The data are from the ZEUS and H1 experiment. The light quarks u, d, s are treated as massless, the charm quark mass is set to be $m_c = 1.5\,\mathrm{GeV}$. The solid (red) line denotes the calculation using the k_T factorization with exact kinematics; the dotted (blue) line is calculation using the k_T factorization with the approximate kinematics.

the Bjorken x. Clearly, the results which do not include the exact kinematics are much higher than those with the exact kinematics. This is understandable as we are taking into account that finite energy has been used for the production of the $q\bar{q}$ pair, and as a result the argument of the gluon density $x_g > x$. We see that the differences are quite pronounced, they are typically larger than the differences between the collinear and the k_T factorization with the exact kinematics.

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