

QUARK MIXING AND CP VIOLATION

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Abstract

This article covers three topics

- i) the status of the flavor mixing matrix, where two changes of the past year are included,
- ii) a review of our understanding of CP violation in the standard model, and
- iii) a summary of the properties of a fourth generation (family) of quarks, including modifications of low energy predictions.

The Flavor Mixing Matrix

The basic constants which occur in the flavour mixing matrix together with the fermion masses comprise the larger set of unexplained parameters of the electroweak theory. Their values are known accurately and are used in order to test the completeness of the theory by probing the existence of additional generations of quarks and the origin of CP violating phenomena. The conclusions from such analyses depend crucially on the precise values of the parameters and especially their estimated errors. A recent development is the publication of more accurate data in several experiments which instead of bringing the data together indicate several inconsistencies. Before these inconsistencies are attributed to the theory, it is necessary to look again into the assumptions going into the analyses.

1. Beta Decay. The V_{ud} matrix element is most accurately known. It is determined by comparing superallowed nuclear beta decays to muon decay.^{1,2)} the experimental ft -values are corrected for radiative corrections (δ_R) and for isospin mixing in the nuclear levels. From the experimental ft -values one obtains corrected $\mathcal{F}t$ -values through the equation

$$\mathcal{F}t = ft(1 + \delta_R)(1 - \delta_C)$$

which should be the same for all transitions. Improved experimental data²⁾ indicate that the corrected $\mathcal{F}t$ -values instead of coming together, they cluster at two distinct values: for the lighter nuclei (^{14}O , ^{26}A , ^{34}Cl , ^{38}K) the values are consistent with $\mathcal{F}t = 3075.2 \pm 2.1$ sec, whereas for the four heavier nuclei (^{42}Sc , ^{46}V , ^{50}Mn , ^{54}Co) are consistent with $\mathcal{F}t = 3087.7 \pm 2.6$ sec; there is a five standard deviation inconsistency. Towner and Hardy³⁾ conjecture that the root of the discrepancy must be the calculation of δ_R and δ_C . A recent summary of deviations in theoretical estimates of δ_C shows¹⁾ that they are comparable to the discrepancy in the $\mathcal{F}t$ -values discussed above and could be the origin of the discrepancy.

Until the discrepancy of the $\mathcal{F}t$ -values is resolved we must allow for a range which spans both values. From table 2 we obtain for the weighted average of the four lighter nuclei

$$v_{ud}^{(\text{light})} = 0.9750 \pm 0.0002$$

and for the four heavier one

$$v_{ud}^{(\text{heavy})} = 0.9729 \pm 0.0001$$

where the errors are statistical. The weighted average for all eight decays is $v_{ud} = 0.9742 \pm 0.0004 \pm 0.0009 \pm 0.0012$. Here, the first error is statistical, the second is the uncertainty from the inner radiative corrections⁴⁾ and the third one is our estimate¹⁾ of the uncertainty in the isospin corrections. Combining the theoretical errors in quadrature we recommend a range

$$v_{ud} = 0.9742 \pm 0.0004 \pm 0.0015 \quad (1)$$

which covers the values for $v_{ud}^{(\text{light})}$ and $v_{ud}^{(\text{heavy})}$. To this the reader may include a reduction of the central value by 1 o/oo due to part of the higher order radiative corrections.⁴⁾

2. Determination of V_{us} . Of the various ways for determining V_{us} the K_{e3} decays are the cleanest. We adopt the value⁵⁾

$$V_{us} = 0.221 \pm 0.002 \quad (2)$$

Values for V_{us} from hyperon decays are traditionally larger but there are many more theoretical loose points. The larger values from hyperon decays must carry a larger error. This is also supported by the fact that a χ^2 -fit of the hyperon decays with $V_{us} = 0.221 \pm 0.002$ as an input parameter gave⁶⁾ an acceptable χ^2 .

3. B-Meson Decays. B-meson lifetime measurements gave

$$V_{cb} = 0.053 \pm 0.004$$

which still remains unchanged. There are two methods for extracting V_{ub} from the lepton momentum spectrum. In the first, one fits the measured spectrum over the entire momentum range to a mix of $b \rightarrow u\ell\nu$, $b \rightarrow c\ell\nu$ and $b \rightarrow c \rightarrow s\ell\nu$. In the second method, one counts the leptons beyond the end-point for $b \rightarrow c\ell\nu$ (the 2.4 to 2.6 GeV/c region). The published results from CLEO⁷⁾ and CUSB⁸⁾ have used the first method and obtained

$$\frac{\text{Br}(b \rightarrow ue\nu)}{\text{Br}(b \rightarrow ce\nu)} \leq 0.04 \text{ (CLEO)} \text{ and } \leq 0.045 \text{ (CUSB)}.$$

In the meanwhile CLEO acquired more data and made a combined fit⁹⁾ of old and recent data. For the best value of the Fermi-momentum they found a negative value for the ratio

$$R\left(\frac{b \rightarrow u}{b \rightarrow c}\right) = \frac{|v_{bu}|^2}{|v_{bc}|^2 f_{ph.}}$$

Under the circumstances it is prudent to add to the minimum χ^2 five units and obtain the upper bound

$$\frac{|v_{bu}|^2}{|v_{bc}|^2 f_{ps}} \leq 0.08$$

which gives⁹⁾

$$|v_{bu}| \leq 0.012 \quad . \quad (3)$$

4. Implications for the Six-Quark Model. Unitarity. The above results for the three matrix elements v_{ud} , v_{us} and v_{ub} can be used to test the unitarity of the KM matrix

$$|v_{ud}|^2 + |v_{us}|^2 + |v_{ub}|^2 = 0.997 \pm 0.005 \quad (4)$$

Thus, the KM matrix is consistent with unitarity; however, the existence of a fourth generation of quarks (T,B) with a coupling

$$|v_{ub}| \lesssim 0.08$$

can not be excluded. Combining the above bounds with the constraint from the measurement of the B-lifetime, we can use the unitarity of the 3×3 KM-matrix to derive bounds on the other matrix elements. Thus, we arrive at a mixing matrix whose elements satisfy

$$v = \begin{bmatrix} 0.972 \dots 0.976 & 0.218 \dots 0.222 & 0.000 \dots 0.012 \\ 0.217 \dots 0.222 & 0.973 \dots 0.975 & 0.048 \dots 0.057 \\ 0.000 \dots 0.024 & 0.044 \dots 0.058 & 0.998 \dots 0.999 \end{bmatrix} \quad (5)$$

In summary, we note that more precise data for several elements did not restrict their values but point to difficulties which demand enlarged ranges (weaker bounds).

CP Violation

The observation of CP violation can originate either in the mass matrix or from the presence of phases in the decay amplitudes. So far all observations of CP-violation can be attributed to the formation of K mesons, which are not pure CP-eigenstates but instead the following admixtures

$$K_S = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}} \{ (K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0) \}$$

$$K_L = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}} \{ \epsilon(K_0 - \bar{K}_0) + (K_0 + \bar{K}_0) \}$$

The ϵ parameter is routinely computed in terms of the box-diagrams and the observed $K_L - K_S$ mass differences. It gives correlations among the unknown parameters: the mass of the top quark, the CP-violating phase δ' and the reduced matrix element

$$B_K = \langle K^0 | [\bar{d}\gamma_\mu (1-\gamma_5)s]^2 | \bar{K}^0 \rangle / (\frac{4}{3} f_K^2 M_K)$$

For B_K theoretical calculations give a wide range of values. Recently, Pich and de Rafael¹⁰⁾ obtained $|B_K| = 0.33$ from QCD sum rules, but other articles using the same sum rules give different values.¹¹⁾ The value $B_K = 1/3$ is also in good agreement with an older determination using PCAC¹²⁾ and SU(3) symmetry. Taking this value seriously a lower bound for the top quark is derived

$$B_K = 1/3 \rightarrow m_t \gtrsim 50 \text{ GeV} .$$

On the other hand, the vacuum saturation value $B_K = 1$ does not improve the existing experimental bound for $m_t \geq 22 \text{ GeV}$. Alternatively, since $|\epsilon|$ is proportional to the product $|V_{ub} V_{cb} V_{us}| \sin\delta'$, the small experimental values for V_{ub} and V_{cb} force $\sin\delta'$ to be large. In figure 1, we plot lower and upper bounds on δ' as a function of the top-mass for $B_K = 1/3$ and $B_K = 1$. We shall use these correlations for the remaining part of this article.

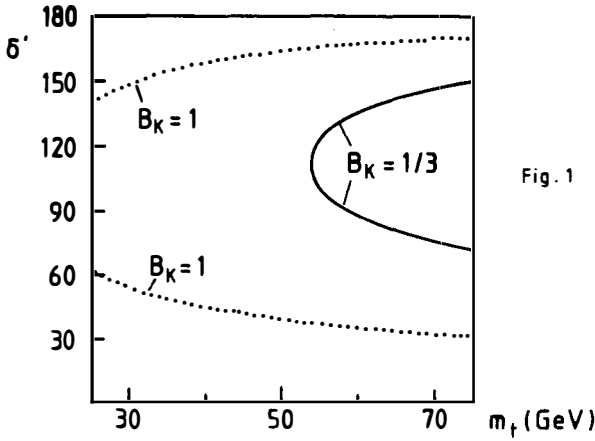


Fig. 1

The existence of an ϵ -parameter can be attributed to Wolfenstein's superweak theory,¹³⁾ which introduces a superweak force that violates CP and forms the K meson eigenstates. CP-violation can also occur in the decay amplitudes. In this case the decay amplitudes of a state into two distinct final states have different phases (the reference here is to phases over and beyond final state interactions). In the decays of $K_0 \rightarrow 2\pi$ we can distinguish two isospin states of the pions and define two amplitudes

$$\tilde{A}_0 = A_0(K_0 \rightarrow 2\pi, I=0) e^{-i\delta_0}$$

$$\tilde{A}_2 = A_2(K_0 \rightarrow 2\pi, I=2) e^{-i\delta_2}$$

The phases δ_0 and δ_2 come from final state interactions and are removed. If the amplitudes \tilde{A}_0 and \tilde{A}_2 still have different phases, then the origin of the effect can not be the mass matrix and could be attributed to the phase occurring in the Kobayashi-Maskawa matrix. In the quark phase convention the first row of the matrix is real and a phase can be produced only through the Penguin diagrams. They produce $\Delta I = 1/2$ transitions and thus contribute to the A_0 amplitude, which is written as

$$\tilde{A}_0 = |A_0| e^{i\xi}$$

In general we define

$$\eta_{+-} = \frac{\text{Amp}(K_L^0 \rightarrow \pi^+ \pi^-)}{\text{Amp}(K_S^0 \rightarrow \pi^+ \pi^-)} = \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\text{Amp}(K_L^0 \rightarrow 2\pi^0)}{\text{Amp}(K_S^0 \rightarrow 2\pi^0)} = \varepsilon - 2\varepsilon'$$

In the quark phase convention

$$\sqrt{2} \varepsilon' = - \frac{R_e A_2}{R_e A_0} \frac{\text{Im} A_0}{A_0} e^{i(\pi/2 + \delta_2 - \delta_0)}$$

and $\varepsilon'/\varepsilon = -15.5 \xi$ (6)

Our next problem is to determine ξ .

K-meson Issues: In addition to the B_K parameter there are two other effects of the K mesons, which are still very debatable: the ratio ε'/ε and the enhancement of the $\Delta I = 1/2$ amplitude. In the standard model the ε' parameter is a short distance effect since it involves the imaginary part of the diagram in fig. 2 with only charm and top quarks in the intermediate states. The $\Delta I = 1/2$ amplitude, on the other hand, contains short and long range contributions and it is harder to estimate. The two effects are closely related to each other and I try to give an up to date summary of the situation.

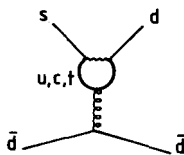


Fig. 2

Penguin Diagrams

Most, but not all, investigations begin with an effective Hamiltonian generated from the operator product expansion (OPE)

$$\mathcal{H}_W^{\Delta S=1} = \sqrt{2} G_F \sin\theta \cos\theta \sum_{n=1}^6 C_n Q_n$$

where $C_n(g, \frac{m_i}{\mu})$ are the Wilson coefficients obtained from renormalization group equations in QCD and Q_n are the following four-fermion operators

$$\begin{aligned} Q_1 &= \bar{s}_\alpha d_\alpha(V-A) \bar{u}_\beta u_\beta(V-A), \\ Q_2 &= \bar{s}_\alpha d_\beta(V-A) \bar{u}_\beta u_\alpha(V-A), \\ Q_3 &= \bar{s}_\alpha d_\alpha(V-A) [\bar{u}_\beta u_\beta(V-A) + \bar{d}_\beta d_\beta(V-A) + \bar{s}_\beta s_\beta(V-A)], \\ Q_5 &= \bar{s}_\alpha d_\alpha(V-A) [\bar{u}_\beta u_\beta(V+A) + \bar{d}_\beta d_\beta(V+A) + \bar{s}_\beta s_\beta(V+A)], \\ Q_6 &= \bar{s}_\alpha d_\beta(V-A) [\bar{u}_\beta u_\alpha(V+A) + \bar{d}_\beta d_\alpha(V+A) + \bar{s}_\beta s_\alpha(V+A)]. \end{aligned} \quad (7)$$

Finally, Q_4 is the linear combination

$$Q_4 = -Q_1 + Q_2 + Q_3.$$

This is a short distance expansion and it is justified for the imaginary part, mentioned above, but could be problematic for the real part which is important for the decay amplitudes. The operator product expansion is general and holds for hyperon, as well as K-meson decays.

The first papers¹⁴⁾ noticed an enhancement of the $\Delta I = 1/2$ amplitudes originating in the Wilson coefficients. Subsequently Shifman, Vainstein and Zakharov¹⁵⁾ observed that an additional enhancement comes from the Penguin type diagrams and were able to account for many amplitudes in hyperon- and K-meson decays. In their work it was necessary to select the coefficient $c_5 + 3/16 c_6 = -0.25$ which is a factor of 2 to 4 larger than their estimate from the renormalization group analysis (see their Table I). This point has been criticized, as it seems that the renormalization calculation does not give a sufficient $\Delta I = 1/2$ enhancement. Here one can either abandon the OPE or take the attitude, as I do, that the operators represent a basis for a "useful" expansion with the coefficient to be taken from the experiments.

To be specific the renormalization analysis gives

$$\text{Re } H_{\text{eff}} = \frac{G}{\sqrt{2}} \{-1.17(Q_1 - Q_2) + 0.62 Q^{3/2} - 0.03 Q_3 + 0.02 Q_5 - 0.10 Q_6\} \theta \quad (8)$$

$$\text{Im } H_{\text{eff}} = \frac{G}{\sqrt{2}} \{-0.03(Q_1 - Q_2) + 0.01 Q_3 + 0.08 Q_6\} \text{Im } \xi_c^* \quad (9)$$

The coefficients were computed for $\Lambda = 0.1 \text{ GeV}$, $\alpha_s(\mu^2) = 1.0$, $m_c = 1.5 \text{ GeV}$ and for a range of the top-quark mass: $30 \text{ GeV} \leq m_t \leq 60 \text{ GeV}$. The dominant terms in the real part are two $\Delta I = 1/2$ terms: $\{-1.17(Q_1 - Q_2); -0.10 Q_6\}$ and the $\Delta I = 3/2$ term $0.62 Q^{3/2}$. The $\Delta I = 3/2$ operator has a large Wilson coefficient which accounts for 50% of the $K^+ \rightarrow \pi^+ \pi^0$ amplitude.¹⁷⁾ It is also important to remark that only up and charm quarks contribute to the real part while in the imaginary part unitarity requires $\text{Im} \xi_c = -\text{Im} \xi_t^*$ and there is a cancellation between the charm- and top-quark Wilson coefficients. For this reason I believe that estimates of the imaginary part must be quite reliable.

We now proceed and estimate ξ

$$\xi = \frac{\text{Im} A_0}{\text{Re} A_0} = \frac{-\beta \gamma \sin \delta}{\sin \theta} R$$

with

$$R = \frac{0.08 + 0.01 \cdot \langle Q_3 \rangle / \langle Q_6 \rangle}{-0.10 + (-0.03 \langle Q_3 \rangle - 1.17 \langle Q_1 - Q_2 \rangle + 0.02 \langle Q_5 \rangle + L) / \langle Q_6 \rangle}$$

We can obtain a reasonable estimate of R with the following assumptions

- (i) $\langle Q_6 \rangle$ must interfere constructively with the other terms of the amplitude, because if cancellations occur then the other terms must be immense in order for the left-over term to give a large $\Delta I = 1/2$ amplitude.
- (ii) (a) For dominant $\langle Q_6 \rangle$ we obtain $R = -0.80$.
- (b) In order to allow for long distance effects, I introduced in the denominator the term $L / \langle Q_6 \rangle$. If we assume an extreme case, where $\langle Q_6 \rangle$ is only 25% of the total amplitude then $R = -0.02$. For this range of R we find the bounds

$$10^{-4} \leq |\xi| \leq 5.2 \times 10^{-4} \tag{10}$$

for $\beta = 0.012$, $\gamma = 0.06$ and $\delta' = \pi/2$,

and
$$10^{-3} \leq \epsilon'/\epsilon \leq 8.4 \times 10^{-3} \tag{11}$$

Alternatively, we can repeat the standard analysis for ϵ'/ϵ , where in the numerator of equ. (6) we take a value three times smaller than that used earlier¹⁷⁾

$$\langle 2\pi, I=0 | Q_6 | K^0 \rangle = 0.47 \text{ GeV}^3$$

and for the denominator the experimental value for $|A_0|$. Fig. 3 depicts the allowed region for ϵ'/ϵ for a range of the top quark mass below the W-mass and for $B_K = 1/3$ and $B = 1$. The positive sign of ϵ'/ϵ depends on

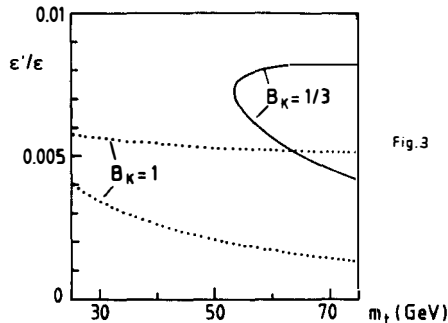


Fig.3

the hypothesis that the penguin contribution interferes constructively with the rest of the $\Delta I = 1/2$ amplitudes. The results of equ. (11) and Fig. 3 are consistent with the experimental¹⁸⁾ value $\epsilon'/\epsilon = -0.003 \pm 0.005$.

The above discussion represents one of the numerous approaches to this problem. Other proposals fall into two general categories.

- (i) To adopt the short distance expansion and compute the matrix elements of the Q_n operators in one of the following schemes: Factorization and chiral symmetry,¹⁹⁾ $1/N$ -expansion,²⁰⁾ These groups obtain ϵ'/ϵ in the range discussed in this article, but the enhancement of the A_0 amplitude is not large enough as required by the observations. The difficulty is the small Wilson coefficients discussed earlier.
- (ii) The second group uses dispersion relations²¹⁾ or saturation with

intermediate states²²⁾ and obtains a consistent $\Delta I = 1/2$ amplitude, but a prediction for ϵ'/ϵ larger than the experimental upper bound. My concern here is that such calculations can miss cancellations and in particular the cancellation in $\text{Im } H_{\text{eff}}$ which makes it a short distance operator and this may be the origin of their large value for ϵ'/ϵ .

iii) For other approaches, I refer to the articles by Eeg, Pich, Soni, and Pham in these proceedings.

This brief summary shows that the experimental observation of ϵ'/ϵ at the level of $(\text{few}) \times 10^{-3}$ will be an important achievement and will lead to several wonderful conclusions:

- i) It will show that CP-violation can be attributed to the phase occurring in the Kobayashi-Maskawa matrix.
- ii) It will give a lower bound for V_{ub} , in the neighborhood of the upper bound given in inequality (3).
- iii) It will provide new insight in the structure of QCD with the $\langle \pi\pi, I=0 | Q_6 | K \rangle$ amplitude determined by the experiment and will indicate its enhancement.

The Fourth Generation

A modest extension of the standard model is the introduction of a fourth generation (family) of quarks and leptons.²³⁻²⁵⁾ Direct evidence for their existence can come from the production of a charged heavy lepton and indirect evidence from low energy effects induced by the heavy intermediate states.

Renormalization studies of the mass matrices for heavy quarks show that they are attracted at low energies to fixed points values. The attraction occurs for masses heavier than 70 GeV. Thus a very heavy pair of quarks is expected to have masses

$$m_T \approx m_B \approx 200 \text{ GeV}$$

$$m_T - m_B \approx 5 \text{ GeV} \quad \text{from } U(1) \text{ breaking}$$

and small mixing angles of the heavy generation to the lighter ones. When

the flavor mixing matrix is extended to a 4x4 matrix there are six mixing angles and three phases. The constraints discussed in the first section allow two solutions, obtained some time ago²³⁾

i) $V_{KM} = 1 + V_{anti}$, with V_{anti} an antisymmetric matrix with very small elements, or

ii) V_{KM} breaks into block diagonal form

$$V_{KM} = \begin{bmatrix} 1 & \theta & O(\epsilon) \\ -\theta & 1 & \\ O(\epsilon) & 1 & \sigma \\ & -\sigma & 1 \end{bmatrix}$$

with θ and σ large and the remaining elements much smaller.

The fourth generation (family) makes several predictions which are different than those in the standard model.

- 1) The CP-violating parameter ϵ_K has a finite value²⁴ even when the element v_{ub} vanishes.
- 2) The extension modifies the ratio ϵ'/ϵ by changing the dependence on the mixing angles²⁵⁾

$$\epsilon'/\epsilon = \{40.7\beta\gamma\sin\delta' - 52.0\sigma\tau\sin\delta_2\} \frac{\langle 2\pi, I=0 | Q_6 | K^0 \rangle}{1.4 \text{ GeV}^3}$$

where σ and τ are two new mixing angles, δ_2 one of the new phases and dominance of the Q_6 operator was assumed.

- 3) Finally, the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has additional intermediate states and the upper bound for the branching ratio is relaxed²⁶

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 8.10^{-9}$$

This bound is accessible to the experiments now running at Brookhaven.²⁷⁾

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